

Quark and lepton mass matrices in the SO(10) grand unified theory with generation flipping

Yasunori Nomura and Takashi Sugimoto

Department of Physics, University of Tokyo, Tokyo 113-0033, Japan

(Received 15 March 1999; published 3 April 2000)

We investigate the SO(10) grand unified model with generation flipping. The model contains one extra matter multiplet $\psi(\mathbf{10})$ and it mixes with the usual matter multiplets $\psi_i(\mathbf{16})$ when SO(10) is broken down to SU(5). We find the parameter region of the model in which the observed quark masses and mixings are well reproduced. The resulting parameter region is consistent with the observation that only $\psi_i(\mathbf{16})$ have a source of hierarchies and indicates that the mixing between second and third generations tends to be large in the lepton sector, which is consistent with the observed maximal mixing of the atmospheric neutrino oscillation. We also show that the model can accommodate MSW and vacuum oscillation solutions to the solar neutrino deficit depending on the form of the Majorana mass matrix for the right-handed neutrinos.

PACS number(s): 12.15.Ff, 12.10.Dm, 12.60.Jv, 14.60.Pq

I. INTRODUCTION

The grand unified theory (GUT) [1] is a very attractive idea which unifies all gauge interactions of the standard model (SM) and explains otherwise peculiar U(1) hypercharge assignments. In particular, its supersymmetric (SUSY) version [2] has achieved the great success of gauge coupling [3] and $b-\tau$ Yukawa coupling [4] unifications, so that various SUSY GUT models have been proposed so far.

Recently, the Super-Kamiokande Collaboration has reported convincing evidence for atmospheric neutrino oscillations with a mass squared difference $\delta m_{\text{atm}}^2 \approx 5 \times 10^{-3} \text{ eV}^2$ and nearly maximal mixing angle $\sin^2 2\theta_{\text{atm}} \gtrsim 0.8$ [5]. In the SUSY SM, neutrino masses come from the effective superpotential

$$W_{\text{eff}} = \sum_{i,j=1}^3 \frac{\kappa_{ij}}{M_R} l_i l_j H_u H_u, \quad (1)$$

where l_i, H_u are SU(2)_L-doublet lepton chiral multiplets and the Higgs doublet chiral multiplet which gives masses for the up-type quarks, respectively. Here, $i, j = 1, \dots, 3$ are generation indices, κ_{ij} denote dimensionless coupling constants, and M_R is the scale at which this operator is generated. The observed mass squared difference δm_{atm}^2 implies that the scale M_R is substantially lower than the gravitational scale $M_G \approx 2.4 \times 10^{18} \text{ GeV}$. Thus, it is natural to think that these effective operators arise from the virtual exchange of some heavy fields of masses M_R through the seesaw mechanism [6]. This motivates SO(10) GUT's [7], which attain complete unification of all quarks and leptons in a single generation together with the right-handed neutrinos.

The minimal version of SUSY SO(10) GUT contains three generations of quarks and leptons $\psi_i(\mathbf{16})$ belonging to $\mathbf{16}$ of the SO(10)_{GUT} and one Higgs field $H(\mathbf{10})$. This minimal model is known to yield a mass degeneracy of up-type and down-type quarks and vanishing quark flavor mixing [Cabibbo-Kobayashi-Maskawa (CKM) [8] mixing] [9], so that it should obviously be extended in order to be reconciled with the observed pattern of quark masses and mixings.

However, these predictions are not altogether ridiculous as a zeroth order approximation (though not fully realistic) in the quark sector, since both up-type and down-type quarks have hierarchical mass patterns and the CKM mixings are all small in nature. What requires large deviation from these predictions seems to be only the large mixing observed in the atmospheric neutrino oscillation. Thus, we would like to extend the minimal SO(10) model to be able to accommodate the large mixing in the lepton sector, keeping their successful approximate relations in the quark sector. One way to achieve this extension is to slightly modify SU(5)-5* components of $\psi_i(\mathbf{16})$ (nonparallel family structure [10]).¹ Since 5* of the standard SU(5)_{GUT} contains right-handed (down-type) quarks and left-handed leptons, this modification is directly transmitted only to the lepton flavor mixing matrix [Maki-Nakagawa-Sakata (MNS) [12] matrix]. As for the quark sector, the effect of the modification is transmitted to the CKM matrix through a diagonalization of the down-type quark mass matrix, inducing small CKM mixings.

In a previous paper, Yanagida and one of the authors (Y.N.) have proposed a SO(10) GUT model which can accommodate realistic quark and lepton mass matrices along the above line of reasoning [13]. The model contains one extra matter multiplet $\psi(\mathbf{10})$ belonging to $\mathbf{10}$ of the SO(10)_{GUT} and it mixes with three $\psi_i(\mathbf{16})$ when the SO(10)_{GUT} is broken down to SU(5) [14]. As a result, the low-energy quarks and leptons are three linear combinations of $\psi_i(\mathbf{16})$ and $\psi(\mathbf{10})$, and realistic masses and mixings are obtained with the right-handed neutrino Majorana mass matrix M_N proportional to the unit one.

In this paper, we extend the previous model [13] and investigate whether it can really accommodate the observed values of quark masses and mixings quantitatively. We find that the model reproduces well the observed quark masses and the CKM matrix elements. The resulting parameter region is consistent with the observation that only $\psi_i(\mathbf{16})$ have

¹For other attempts to realize realistic quark and lepton mass matrices in SO(10) GUT, see Ref. [11].

a source of hierarchies such as $U(1)$ flavor symmetry charges [15]. Furthermore, we discuss flavor mixings in the lepton sector under certain simplifying assumptions. We find that the model can naturally explain the observed atmospheric neutrino deficit [16,5] and solar neutrino deficit [17,18] by the neutrino oscillation [19,12]. The resulting mass squared differences and mixing angles depend on the form of the Majorana mass matrix M_N . We consider two cases that M_N is proportional to the unit matrix and that it has a hierarchy consistent with the above observation that only $\psi_i(\mathbf{16})$ have a source of hierarchies.

This paper is organized as follows. In Sec. II, we briefly review the model in Ref. [13] and slightly extend it removing unwanted mass degeneracy between down-type quarks and charged leptons. In Sec. III, we investigate the model and find a parameter region where it is consistent with all the observed quark masses and mixings including a CP -violating phase. We discuss masses and mixings in the lepton sector, particularly solutions to the solar neutrino deficit, in Sec. IV. Section V is devoted to our conclusions. We also add an Appendix, in which we give a detailed derivation of quark and lepton mass matrices in the model.

II. THE MODEL

In this section, we explain the model proposed in Ref. [13] and set up the framework for the phenomenological analyses given in the next section.

The model has three generations of $\psi_i(\mathbf{16})$ and one $\psi(\mathbf{10})$ as matter multiplets. We take a basis where the original Yukawa coupling matrix of the Higgs field $H(\mathbf{10})$ to the matter fields $\psi_i(\mathbf{16})$ is real and diagonal:

$$W = \frac{1}{2} \sum_{i=1}^3 h_i \psi_i(\mathbf{16}) \psi_i(\mathbf{16}) H(\mathbf{10}). \quad (2)$$

This leads to a diagonal mass matrix for the up-type quarks as

$$M_u = m_t \begin{pmatrix} \hat{m}_u & 0 & 0 \\ 0 & \hat{m}_c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (3)$$

where $\hat{m}_u$ and $\hat{m}_c$ are defined as

$$\hat{m}_u = m_u / m_t, \quad \hat{m}_c = m_c / m_t, \quad (4)$$

and m_u, m_c and m_t are given by

$$m_u = h_1 \langle \mathbf{5}_H \rangle, \quad m_c = h_2 \langle \mathbf{5}_H \rangle, \quad m_t = h_3 \langle \mathbf{5}_H \rangle. \quad (5)$$

Here, $\mathbf{5}_H$ is a $SU(5)$ -5 component of $H(\mathbf{10})$. At this stage, the down-type quark mass matrix $M_{d/l}$ and neutrino Dirac mass matrix $M_{\nu D}$ are completely the same with M_u except for the difference between the vacuum expectation values of the Higgs fields.

We now assume that the $SO(10)_{\text{GUT}}$ is broken down to $SU(5)$ by condensation of Higgs fields $\langle \chi(\mathbf{16}) \rangle = \langle \bar{\chi}(\mathbf{16}^*) \rangle$

TABLE I. $U(1)_R$ charges.

	$H(\mathbf{10})$	$H(\mathbf{16})$	$\bar{H}(\mathbf{16}^*)$	$\chi(\mathbf{16})$	$\bar{\chi}(\mathbf{16}^*)$	$\psi_i(\mathbf{16})$	$\psi(\mathbf{10})$
R	0	0	2	0	0	1	1

$= V$ with V being $\sim 10^{16}$ GeV. This GUT breaking also induces a mass term for the matter multiplets through the following superpotential:

$$W = \sum_{i=1}^3 f_i e^{i\eta_i} \psi_i(\mathbf{16}) \psi(\mathbf{10}) \langle \chi(\mathbf{16}) \rangle. \quad (6)$$

Namely, a linear combination $\tilde{\mathbf{5}}^* \propto \sum_{i=1}^3 f_i e^{i\eta_i} \mathbf{5}_i^*$ in $\psi_i(\mathbf{16})$ receives a GUT scale mass together with $\mathbf{5}_\psi$ in $\psi(\mathbf{10})$, and the other two linear combinations, $\mathbf{5}_1^*$ and $\mathbf{5}_3^*$, in $\psi_i(\mathbf{16})$ remain as massless fields. The relation between these fields can be parametrized as

$$\begin{pmatrix} \mathbf{5}_1^{*'} \\ \mathbf{5}_2^{*'} \\ \mathbf{5}_3^{*'} \end{pmatrix} = \begin{pmatrix} e^{-i\delta_1 - i\delta_2} & 0 & 0 \\ 0 & e^{-i\delta_1 + i\delta_2} & 0 \\ 0 & 0 & e^{2i\delta_1} \end{pmatrix} \times \begin{pmatrix} \cos \alpha_1 & -\sin \alpha_1 & 0 \\ \sin \alpha_1 & \cos \alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \cos \alpha_2 & 0 & e^{-i\delta_3} \sin \alpha_2 \\ 0 & 1 & 0 \\ -e^{i\delta_3} \sin \alpha_2 & 0 & \cos \alpha_2 \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{5}}^* \\ \mathbf{5}_1^* \\ \mathbf{5}_3^* \end{pmatrix}, \quad (7)$$

without a loss of generality (see Appendix). Here, α_1, α_2 and δ_i are functions of f_i and η_i . Thus, the low-energy quark and lepton fields belonging to $\mathbf{5}^*$ of $SU(5)$ are $\mathbf{5}_1^*, \mathbf{5}_3^*$ and $SU(5)$ - $\mathbf{5}^*$ of $\psi(\mathbf{10})$ (i.e. $\mathbf{5}_\psi^*$), so that the relations $M_u = M_{\nu D} \propto M_{d/l}$ are removed.

The down-type quark mass matrix is, however, incomplete, since $\mathbf{5}_\psi^*$ does not have any Yukawa coupling to $H(\mathbf{10})$. To solve this problem we introduce a pair of Higgs fields $H(\mathbf{16})$ and $\bar{H}(\mathbf{16}^*)$ and consider a superpotential²

$$W = \lambda H(\mathbf{10}) \bar{H}(\mathbf{16}^*) \bar{\chi}(\mathbf{16}^*) + M H(\mathbf{16}) \bar{H}(\mathbf{16}^*). \quad (8)$$

$U(1)$ R -symmetry may be useful to have this form of superpotential. The $U(1)_R$ charges are given in Table I. The GUT

²We can take λ and M to be real using phase degrees of freedom in $H(\mathbf{16})$ and $\bar{H}(\mathbf{16}^*)$. Strictly speaking, ϕ_i in Eq. (11) are defined in this basis.

condensation $\langle \bar{\chi}(\mathbf{16}^*) \rangle \neq 0$ induces a mass mixing between $\mathbf{5}^*$'s of $H(\mathbf{10})$ and $H(\mathbf{16})$ (i.e., $\mathbf{5}_{H(\mathbf{10})}^*$ and $\mathbf{5}_{H(\mathbf{16})}^*$). Thus, a linear combination

$$\mathbf{5}_H^* = \cos \theta \mathbf{5}_{H(\mathbf{10})}^* + \sin \theta \mathbf{5}_{H(\mathbf{16})}^*, \quad (9)$$

$$\tan \theta = -\frac{\lambda \langle \bar{\chi}(\mathbf{16}^*) \rangle}{M}, \quad (10)$$

remains as a massless Higgs field in the standard $SU(5)_{\text{GUT}}$ and contributes to the quark and lepton mass matrices. Then, $\mathbf{5}_H^*$ can couple to $\mathbf{5}_{\psi}^*$ through the superpotential

$$W = \sum_{i=1}^3 g_i e^{i\phi_i} \psi_i(\mathbf{16}) \psi(\mathbf{10}) H(\mathbf{16}), \quad (11)$$

and the down-type quark and charged-lepton mass matrix is given by

$$M_{d/l} = m_t \frac{\cos \theta}{\tan \beta} \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & x e^{i\phi_x} & \hat{m}_u \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c \cos \alpha_1 & y e^{i\phi_y} & \hat{m}_c \sin \alpha_1 \sin \alpha_2 \\ 0 & z & \cos \alpha_2 \end{pmatrix}. \quad (12)$$

Here, $\tan \beta \equiv \langle \mathbf{5}_H \rangle / \langle \mathbf{5}_H^* \rangle$ and

$$x = \frac{g_1}{h_3} \tan \theta, \quad y = \frac{g_2}{h_3} \tan \theta, \quad z = \frac{g_3}{h_3} \tan \theta, \quad (13)$$

$$\begin{aligned} \phi_x &= \phi_1 - \phi_3 + 3\delta_1 + \delta_2 + \delta_3, \\ \phi_y &= \phi_2 - \phi_3 + 3\delta_1 - \delta_2 + \delta_3 \end{aligned} \quad (14)$$

(for a detailed derivation, see Appendix). Note that in the present model $\tan \beta$ can take a value in a wide range due to the presence of $\cos \theta$ in Eq. (12).

The Dirac mass matrix $M_{\nu D}$ for neutrinos is also incomplete, since $\mathbf{5}_{\psi}^*$ never couples to $SU(5)$ singlets of $\psi_i(\mathbf{16})$ (i.e., $\mathbf{1}_i$). However, the following nonrenormalizable interactions give desired couplings:

$$W = \sum_{i=1}^3 k_i e^{i\varphi_i} \psi_i(\mathbf{16}) \psi(\mathbf{10}) H(\mathbf{10}) \frac{\bar{\chi}(\mathbf{16}^*)}{M_G}. \quad (15)$$

Together with the original couplings in Eq. (2), the nonrenormalizable interactions Eq. (15) yield

$$M_{\nu D} = m_t \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & \delta_x e^{i\varphi_x} & \hat{m}_u e^{i\varphi_u} \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c e^{i\varphi_c} \cos \alpha_1 & \delta_y e^{i\varphi_y} & \hat{m}_c e^{i\varphi_v} \sin \alpha_1 \sin \alpha_2 \\ 0 & \delta_z e^{i\varphi_z} & e^{i\varphi_w} \cos \alpha_2 \end{pmatrix}. \quad (16)$$

Here,

TABLE II. The running quark and lepton masses evaluated at the energy of Z-boson mass m_Z [22–24] and the GUT scale $M_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV [23].

	$\mu = m_Z$	$\mu = M_{\text{GUT}}$
$m_u(\mu)$	2.2 ± 0.7 MeV	0.98 ± 0.31 MeV
$m_c(\mu)$	626 ± 106 MeV	279 ± 47 MeV
$m_t(\mu)$	175 ± 6 GeV	110 ± 19 GeV
$m_d(\mu)$	4.1 ± 1.1 MeV	1.2 ± 0.3 MeV
$m_s(\mu)$	85 ± 19 MeV	24 ± 5 MeV
$m_b(\mu)$	3.02 ± 0.19 GeV	1.01 ± 0.06 GeV
$m_e(\mu)$	0.487 MeV	0.325 MeV
$m_\mu(\mu)$	102.7 MeV	68.6 MeV
$m_\tau(\mu)$	1.747 GeV	1.171 GeV

$$\delta_x = \frac{k_1}{h_3} \frac{\langle \bar{\chi}(\mathbf{16}^*) \rangle}{M_G}, \quad \delta_y = \frac{k_2}{h_3} \frac{\langle \bar{\chi}(\mathbf{16}^*) \rangle}{M_G}, \quad (17)$$

$$\delta_z = \frac{k_3}{h_3} \frac{\langle \bar{\chi}(\mathbf{16}^*) \rangle}{M_G},$$

$$\begin{aligned} \varphi_x &= \varphi_1 + \delta_1 - \delta_2, & \varphi_y &= \varphi_2 + \delta_1 - \delta_2, & \varphi_z &= \varphi_3 + \delta_1 - \delta_2, \\ \varphi_t &= 2\delta_2, & \varphi_u &= -3\delta_1 - \delta_2 - 2\delta_3, \end{aligned} \quad (18)$$

$$\varphi_v = -3\delta_1 + \delta_2 - 2\delta_3, \quad \varphi_w = -\delta_3$$

(see Appendix). Notice that $\delta_x, \delta_y, \delta_z \simeq O(10^{-2})$ as long as $k_i/h_3 = O(1)$. The Majorana masses for the right-handed neutrinos $\mathbf{1}_i$ are given by the following nonrenormalizable superpotential:

$$W = \frac{1}{2} \sum_{i,j=1}^3 j_{ij} \psi_i(\mathbf{16}) \psi_j(\mathbf{16}) \frac{\bar{\chi}(\mathbf{16}^*) \bar{\chi}(\mathbf{16}^*)}{M_G}. \quad (19)$$

After the $SO(10)_{\text{GUT}}$ breaking we obtain the Majorana mass matrix for the right-handed neutrinos,

$$(M_N)_{ij} = \frac{j_{ij}}{|j_{33}|} M_R, \quad (20)$$

where $M_R = |j_{33}| V^2 / M_G$. Then, the mass matrix M_ν for the light neutrinos is given by $M_\nu = M_{\nu D}^T M_N^{-1} M_{\nu D}$ through the seesaw mechanism [6].

Now, we have $SU(5)$ -invariant mass matrices $M_u, M_{d/l}$ and M_ν . However, they yield wrong $SU(5)_{\text{GUT}}$ mass relations, $m_\mu = m_s$ and $m_e = m_d$, so that we have to remove these unwanted mass relations in order to obtain realistic quark and lepton masses. It can be done by introducing $SU(5)$ breaking effects into $M_{d/l}$ [20]. We assume that the $SU(5)$ is broken down to the SM gauge group by a Higgs multiplet $\Sigma(\mathbf{45})$ belonging to $\mathbf{45}$ of the $SO(10)_{\text{GUT}}$. Then, if $\Sigma(\mathbf{45})$ has nonrenormalizable interactions to the matter and Higgs fields, the $SU(5)$ breaking effect of order $\langle \Sigma(\mathbf{45}) \rangle / M_G$ can be transmitted to the quark and lepton mass matrices [21]. For this purpose, we introduce the following nonrenormalizable superpotential:

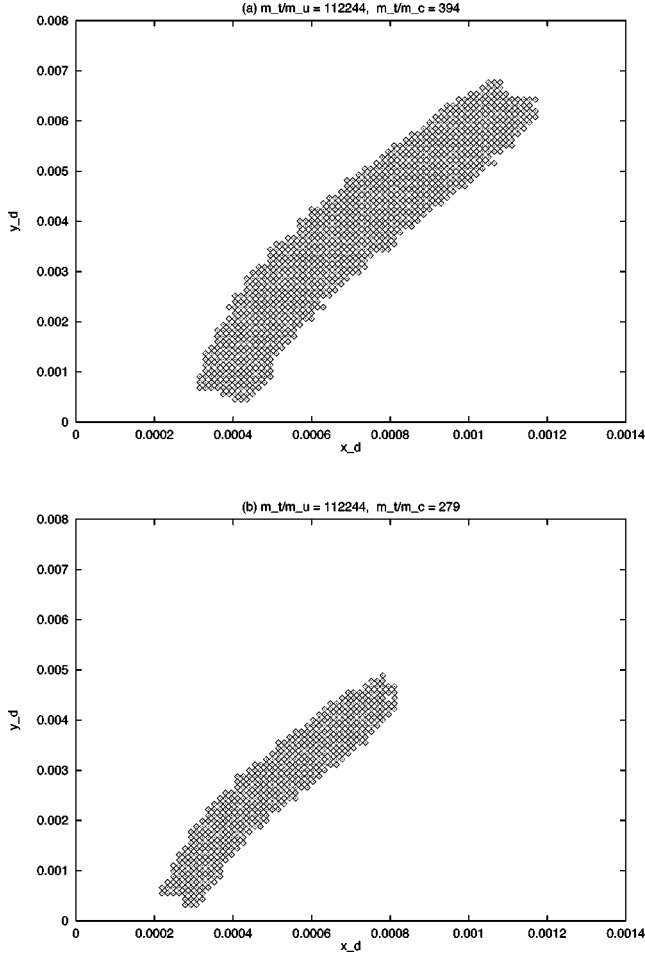


FIG. 1. The parameter region of x_d and y_d where all quark masses and the CKM parameters are reproduced. We have set $\hat{m}_u^{-1}$ to be 112244 and $\hat{m}_c^{-1}$ to be (a) 394 (central value) and (b) 279 (lowest value).

$$W = \sum_{i=1}^3 g'_i e^{i\phi'_i} \psi_i(\mathbf{16}) \psi(\mathbf{10}) H(\mathbf{16}) \frac{\langle \Sigma(45) \rangle}{M_G}. \quad (21)$$

We consider that only (1,2) and (2,2) components of the down-type quark mass matrix M_d and the charged-lepton mass matrix M_l are modified from those of $M_{d/l}$ for simplicity, since the value of z which gives realistic masses and mixings is relatively larger than those of x and y as we shall see later. We represent these modified components with the subscript d and l (i.e., $x_d, y_d, \phi_{xd}, \phi_{yd}, x_l, y_l, \phi_{xl}$ and ϕ_{yl}).

With the mass matrices M_u, M_d, M_l and M_ν , we can reproduce well the observed quark and lepton flavor structure. In the next section, we search a parameter region of M_u and M_d which gives the observed quark masses and mixings including a CP -violating phase.

III. THE MASSES AND MIXINGS IN THE QUARK SECTOR

The quark masses are estimated using various methods such as QCD sum rules. However, the estimated quark masses still have some uncertainties. In our analysis, we

TABLE III. The observed CKM parameters (at $\mu = m_Z$) [22,25] and their RG-evolved values at the GUT scale $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV [23].

	$\mu = m_Z$	$\mu = M_{\text{GUT}}$
$(V_{CKM})_{us}(\mu)$	0.215~0.224	0.215~0.224
$(V_{CKM})_{cb}(\mu)$	0.036~0.043	0.031~0.037
$(V_{CKM})_{ub}/(V_{CKM})_{cb}(\mu)$	0.060~0.12	0.061~0.12
$J_{CKM}(\mu)$	$(1.5 \sim 4.4) \times 10^{-5}$	$(1.1 \sim 3.3) \times 10^{-5}$

adopt the running quark and lepton masses evaluated at the energy of Z-boson mass given in Table II [22–24] to constrain the mass matrices derived in Sec. II.

The CKM mixing angles are specified by three real parameters $(V_{CKM})_{us}, (V_{CKM})_{cb}$ and $(V_{CKM})_{ub}/(V_{CKM})_{cb}$ whose observed values are given in Table III [22,25]. CP violation effects are parametrized by the quantity called Jarlskog parameter J_{CKM} [26] defined by $J_{CKM} \equiv \text{Im}\{(V_{CKM})_{ud}(V_{CKM})_{td}^*(V_{CKM})_{tb}(V_{CKM})_{ub}^*\}$, which is independent of phase conventions of the SM fields. The value of J_{CKM} obtained from neutral meson experiments is given in Table III [25]. In the following, we search a parameter region of the model consistent with these observed masses and mixings.

To compare with the mass matrices obtained in Sec. II, we have to use renormalization group (RG)-evolved values of the masses and mixings at the GUT scale. We assume particle content of the minimal SUSY SM between the weak scale and the GUT scale. Then, the GUT scale values are not sensitive to the precise mass spectrum of the SUSY particles and also to $\tan\beta$ as long as it has a moderate value $5 \lesssim \tan\beta \lesssim 30$ [27]. Thus, we fix a value $\tan\beta = 10$ for simplicity. The obtained masses and mixings at the GUT scale using two-loop RG equations for the Yukawa couplings are given in Tables II and III [23]. We will use these values to constrain the parameter space of the model.³

Now, we search values of $x_d, y_d, z, \alpha_1, \alpha_2, \phi_{xd}, \phi_{yd}$ which reproduce down-type quark mass ratios in Table II, $32.8 < m_b/m_s < 56.3$ and $12.7 < m_s/m_d < 32.2$, and all the CKM parameters given in Table III. We take $\hat{m}_c$ to be $\hat{m}_c^{-1}(M_{\text{GUT}}) = 394$ (central value) and 279 (lowest value) in order to show the dependence of the results on up-type quark masses at the GUT scale. The results hardly depend on $\hat{m}_u$ due to its smallness, so that we fix $\hat{m}_u^{-1}(M_{\text{GUT}})$ to be its central value 112244.

We find a parameter space which reproduces all quark masses and mixings including a CP -violating phase. The parameter region of x_d and y_d is shown in Fig. 1. As is readily seen, the slope of this x_d - y_d graph roughly gives the reciprocal of the Cabibbo angle $\sim (0.22)^{-1}$. Since ratios of x_d, y_d and z are related to the CKM mixing angles, we find

³The GUT scale value of m_t is highly dependent on the input value of $m_t(m_Z)$, especially in the case that $m_t(m_Z) \gtrsim 180$ GeV. As we shall see, however, our qualitative conclusions hardly depend on the value of $m_t(M_{\text{GUT}})$.

that x_d, y_d and z also have hierarchical structure. Indeed, the region of z is around $0.02 < z < 0.12$. The required hierarchy is milder than that of h_1, h_2 and h_3 , and can be roughly parametrized as

$$h_1 : h_2 : h_3 \approx \epsilon^4 : \epsilon^2 : 1,$$

$$x_d : y_d : z \approx g_1 : g_2 : g_3 \approx \epsilon^2 : \epsilon : 1, \quad (22)$$

with $\epsilon = O(0.1)$. This may indicate that only $\psi_i(\mathbf{16})$ have a source of hierarchies such as U(1) flavor symmetry charges [see Eqs. (2) and (11)]. To reproduce the observed CP violation (the value of J_{CKM} given in Table III) ϕ_{yd} has to be around $3\pi/2$ ($10\pi/8 \lesssim \phi_{yd} \lesssim 15\pi/8$ in both cases of $\hat{m}_c^{-1} = 394$ and $\hat{m}_c^{-1} = 279$). The results hardly depend on ϕ_{xd} , since it can be transferred into the phase of $\hat{m}_u$ by an appropriate phase rotation of the quark doublet (u, d) and $\hat{m}_u$ is very small.

The parameter region of α_1 and α_2 ($0 \leq \alpha_1, \alpha_2 \leq \pi/2$) is shown in Fig. 2. We find that both α_1 and α_2 are close to $\pi/2$ [$\cos \alpha_1 \sim \cos \alpha_2 = O(0.1)$]. This implies that the extra matter multiplet $\psi(\mathbf{10})$ is dominantly coupled with $\psi_3(\mathbf{16})$ in the superpotential Eq. (6).⁴ The hierarchy of the couplings f_i is roughly

$$\cos \alpha_1 \cos \alpha_2 : \sin \alpha_1 \cos \alpha_2 : \sin \alpha_2 = f_1 : f_2 : f_3 \approx \epsilon^2 : \epsilon : 1, \quad (23)$$

which also is consistent with the above observation that only $\psi_i(\mathbf{16})$ have a source of hierarchies [see Eq. (6)].

Finally, we plot the down-type quark mass ratios m_b/m_s and m_s/m_d as functions of $z/\cos \alpha_2$ in Figs. 3 and 4, respectively. We have ascertained that m_b/m_s and m_s/m_d take any value in a range $32.8 < m_b/m_s < 56.3$ and $12.7 < m_s/m_d < 32.2$, so that there is no relation between them. Note that the allowed region of $z/\cos \alpha_2$ is hardly dependent on $\hat{m}_c^{-1}$, since z and $\cos \alpha_2$ respond to the change of $\hat{m}_c^{-1}$ in the same way. Thus, our qualitative discussion below is almost independent of $\hat{m}_c^{-1}(M_{GUT})$. The quantity $z/\cos \alpha_2$ almost corresponds to the tangent of the MNS mixing angle, $\tan \theta_{\mu 3}$, between second and third generations, since it dictates the mixing between left-handed charged leptons of second and third generations. Actual mixing angle is the sum of it and an additional contribution from neutrino mass matrix M_ν . Considering that δ_z is given by Eq. (17) and $h_3 = O(1)$, the additional contribution can be of order the Cabibbo angle. Thus, if two contributions are added up constructively, the region $z/\cos \alpha_2 \gtrsim 0.4$ is consistent with the observed near maximal mixing of the atmospheric neutrino oscillation

⁴If some components of the down-type quark mass matrix other than (1,2) and (2,2) ones are modified by nonrenormalizable interactions, the parameter region of (α_1, α_2) is not necessarily very close to $(\pi/2, \pi/2)$. This possibility will be considered elsewhere.

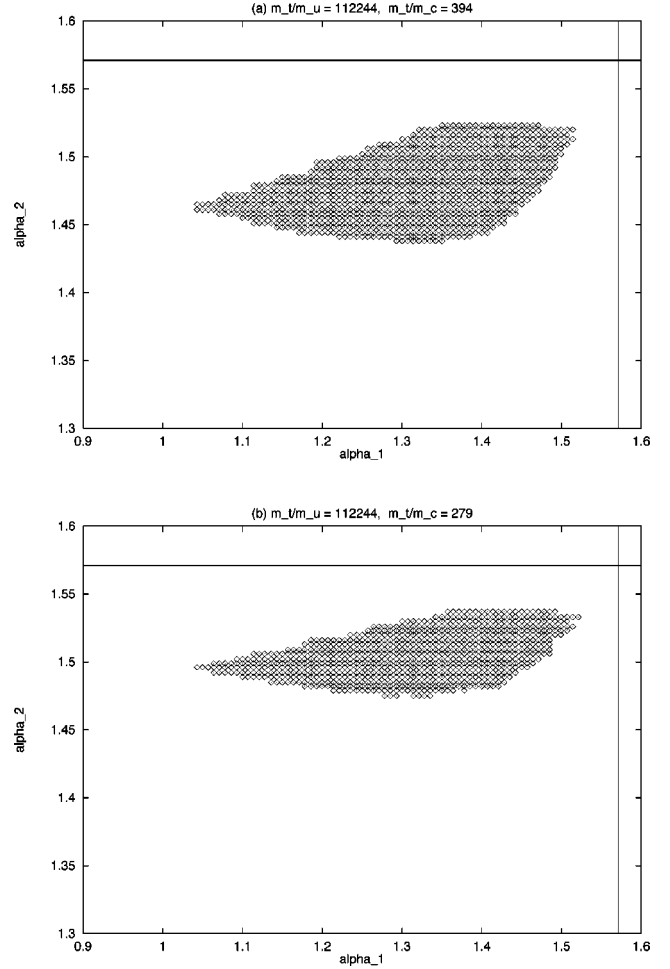


FIG. 2. The parameter region of α_1 and α_2 where all quark masses and the CKM parameters are reproduced. α_1 and α_2 are represented with radians and their domains are $0 \leq \alpha_1 \leq \pi/2$ and $0 \leq \alpha_2 \leq \pi/2$. Solid lines denote $\alpha_1 = \pi/2$ and $\alpha_2 = \pi/2$. We have set $\hat{m}_u^{-1}$ to be 112244 and $\hat{m}_c^{-1}$ to be (a) 394 (central value) and (b) 279 (lowest value).

($\nu_\mu \leftrightarrow \nu_\tau$) [16,5].⁵ We find that significant region is consistent with the large angle between ν_μ and ν_τ .

IV. THE MASSES AND MIXINGS IN THE LEPTON SECTOR

In this section, we discuss the lepton sector of the model taking $\hat{m}_u^{-1}$ and $\hat{m}_c^{-1}$ to be their central values for simplicity. First, we begin with the charged-lepton masses whose precise values are known experimentally as in Table II. The mass matrix for the charged leptons is different from that for the down-type quarks due to the presence of nonrenormalizable interactions Eq. (21). We assume that the nonrenormal-

⁵Even if we have no contribution from the neutrino sector, the region $z/\cos \alpha_2 \gtrsim 0.6$ is consistent with the observed near maximal mixing of the atmospheric neutrino oscillation, $\sin^2 2\theta_{\mu 3} \gtrsim 0.8$. In this case, a relatively large value of m_b/m_s is favorable.

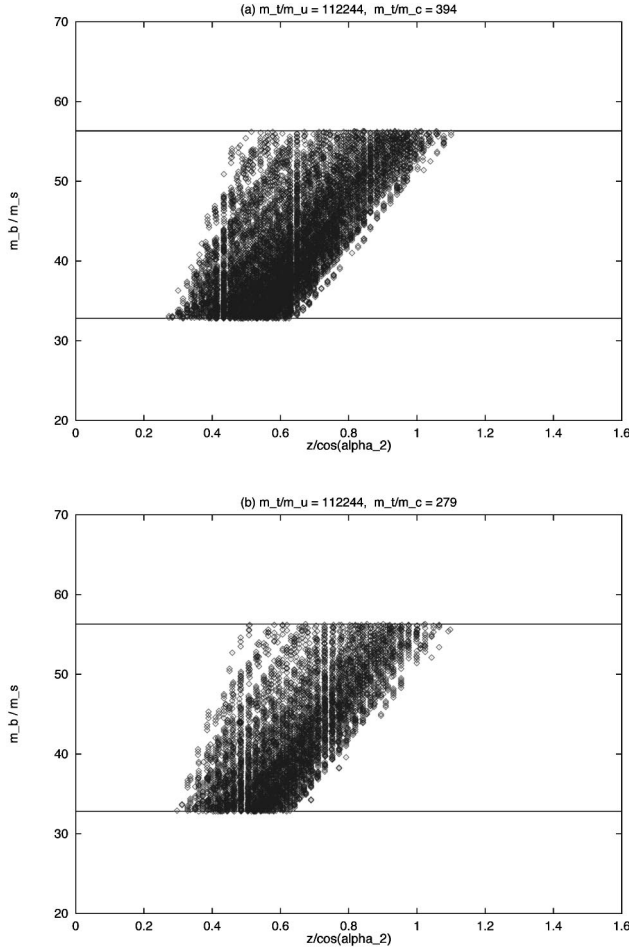


FIG. 3. The mass ratio m_b/m_s as a function of $z/\cos \alpha_2$. The allowed region is between two horizontal lines ($32.8 < m_b/m_s < 56.3$). We have set $\hat{m}_u^{-1}$ to be 112244 and $\hat{m}_c^{-1}$ to be (a) 394 (central value) and (b) 279 (lowest value).

izable interactions Eq. (21) modify only (1,2) and (2,2) components of the mass matrices. Thus, we search a parameter region of x_l and y_l which can well reproduce the known lepton mass ratios at the GUT scale, $m_\tau/m_\mu = 17.07$ and $m_\mu/m_e = 211.1$, using the values of z , α_1 and α_2 obtained in Sec. III which reproduce the quark mass ratios and the CKM parameters.

The resulting region of x_l and y_l is given in Fig. 5 together with that of x_d and y_d obtained in Fig. 1. We find that there is significant region consistent with the observed charged-lepton masses. The preferred region is $x_l \sim x_d$ and $y_l \sim 3y_d$, which agrees with the earlier observation [20]. Thus, we conclude that the presence of nonrenormalizable interactions Eq. (21) with $g_i' = O(1)$ is sufficient to remove unwanted $SU(5)_{\text{GUT}}$ mass relations, $m_\mu = m_s$ and $m_e = m_d$, also in the present model.

Next, we discuss the neutrino masses and mixings qualitatively. We call the mass eigenstates for three neutrinos ν_1 , ν_2 and ν_3 such that $m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$. In the present model, we have a mass hierarchy $m_{\nu_2} \ll m_{\nu_3}$ due to the fact that $\delta_x, \delta_y, \delta_z \lesssim 10^{-2}$. Then, the data of atmospheric neutrino oscillation ($\nu_\mu \leftrightarrow \nu_\tau$) from Super-Kamiokande implies that

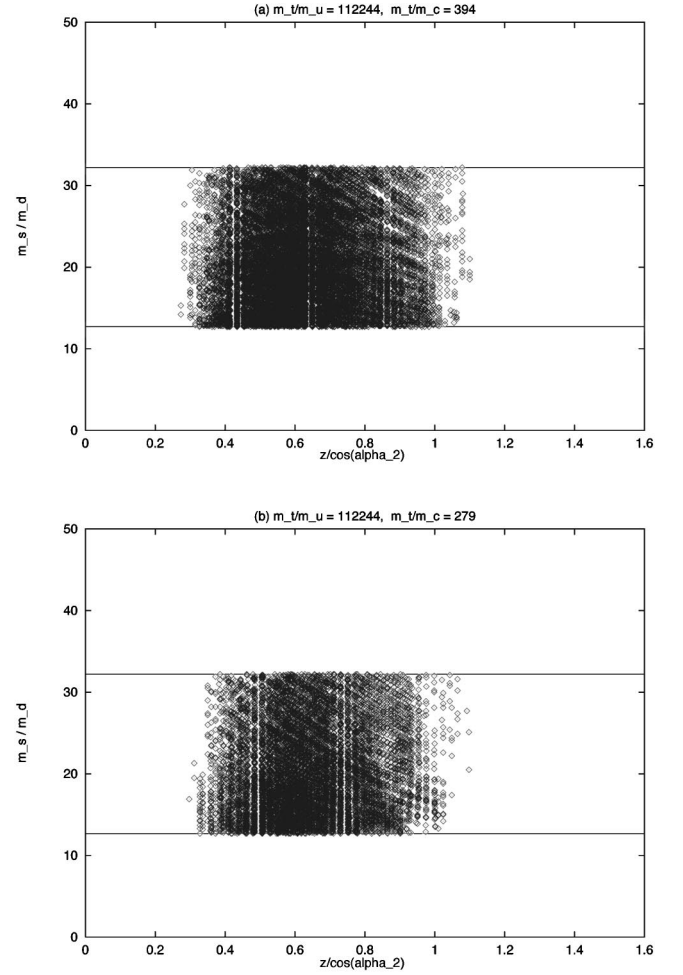


FIG. 4. The mass ratio m_s/m_d as a function of $z/\cos \alpha_2$. The allowed region is between two horizontal lines ($12.7 < m_b/m_s < 32.2$). We have set $\hat{m}_u^{-1}$ to be 112244 and $\hat{m}_c^{-1}$ to be (a) 394 (central value) and (b) 279 (lowest value).

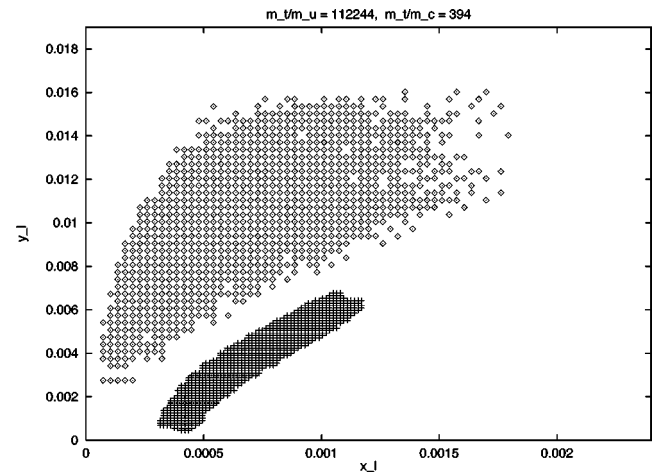


FIG. 5. The parameter region of x_l and y_l where all lepton mass ratios are reproduced. The hatched region represents that of x_d and y_d given in Fig. 1(a). We have set $\hat{m}_u^{-1} = 112244$ and $\hat{m}_c^{-1} = 394$.

TABLE IV. The allowed regions of mass squared differences and mixing angles which reproduce the observed solar neutrino deficit in terms of the neutrino oscillation [30,31]. We have also shown mass ratios m_{ν_2}/m_{ν_3} under the mass hierarchy $m_{\nu_2} \ll m_{\nu_3}$.

Solutions	δm_{sol}^2	$\sin^2 2\theta_{\text{sol}}$	m_{ν_2}/m_{ν_3}
Small angle MSW	$(4-12) \times 10^{-6} \text{ eV}^2$	$(2-12) \times 10^{-3}$	~ 0.03
Large angle MSW	$(8-25) \times 10^{-6} \text{ eV}^2$	$0.5-0.8$	~ 0.05
	$(6-7) \times 10^{-10} \text{ eV}^2$	$0.8-1.0$	$\sim 10^{-4}$
Vacuum oscillation	$(4-5) \times 10^{-10} \text{ eV}^2$	$0.7-1.0$	$\sim 10^{-4}$
	$(5-9) \times 10^{-11} \text{ eV}^2$	$0.6-0.9$	$\sim 10^{-4}$

$m_{\nu_3} \approx 7 \times 10^{-2} \text{ eV}$. The neutrino mass matrix M_ν is given by $M_\nu = M_{\nu D}^T M_N^{-1} M_{\nu D}$. Thus, the scale M_R of the Majorana masses is determined as $M_R \approx 10^{12} - 10^{13} \text{ GeV}$, which is close to the natural scale derived from Eq. (19) with $j_{33} = O(1)$. The observed maximal mixing is also naturally obtained as a result of fitting quark masses and mixings. In the following, we discuss the implications of the model on the solar neutrino deficit. For simplicity, we ignore CP -violating effects in the lepton sector and take all values appear in the neutrino mass matrix to be real, $\varphi_x = \varphi_y = \varphi_z = \varphi_t = \varphi_u = \varphi_v = \varphi_w = 0$ or π .

The observed solar neutrino deficit is explained by either matter-enhanced neutrino oscillation [Mikheyev-Smirnov-Wolfenstein (MSW [28])] solution or the vacuum oscillation solution [29] ($\nu_e \leftrightarrow \nu_\mu, \nu_\tau$). Then, the allowed regions of mass squared differences and mixing angles are as shown in Table IV [30,31]. The MSW solution has two distinct regions, that is, the small and the large angle ones. Below, we investigate which solutions the model can realize in two cases that the neutrino Majorana mass matrix M_N is propor-

tional to the unit matrix or that it has a hierarchy consistent with the observation that $\psi_i(\mathbf{16})$ have a source of hierarchies. The RG effects between the GUT scale and the weak scale are negligible for our qualitative argument, so that we evaluate the masses and mixing angles at the GUT scale and compare them with the observed values given in Table IV.

A. The case of unit Majorana mass matrix

We first consider the unit Majorana mass matrix case:

$$M_N = M_R \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (24)$$

Here, we simply assume that M_N is proportional to the unit matrix for some reasons in the basis that the original Yukawa couplings are diagonal, and regard δ_x , δ_y and δ_z as free parameters. Then, the neutrino mass matrix, $M_\nu = M_{\nu D}^T M_N^{-1} M_{\nu D}$, is given as

$$M_\nu = \frac{m_t^2}{M_R} \begin{pmatrix} -\hat{m}_u s_1 & \hat{m}_c c_1 & 0 \\ \delta_x & \delta_y & \delta_z \\ \hat{m}_u c_1 s_2 & \hat{m}_c s_1 s_2 & c_2 \end{pmatrix} \begin{pmatrix} -\hat{m}_u s_1 & \delta_x & \hat{m}_u c_1 s_2 \\ \hat{m}_c c_1 & \delta_y & \hat{m}_c s_1 s_2 \\ 0 & \delta_z & c_2 \end{pmatrix} \\ \simeq \frac{m_t^2}{M_R} \begin{pmatrix} \hat{m}_u^2 c_1^2 & -\hat{m}_u \delta_x + \hat{m}_c \delta_y c_1 & \hat{m}_c^2 c_1 \\ -\hat{m}_u \delta_x + \hat{m}_c \delta_y c_1 & \delta_x^2 + \delta_y^2 + \delta_z^2 & \hat{m}_u \delta_x c_1 + \hat{m}_c \delta_y + \delta_z c_2 \\ \hat{m}_c^2 c_1 & \hat{m}_u \delta_x c_1 + \hat{m}_c \delta_y + \delta_z c_2 & c_2^2 \end{pmatrix}, \quad (25)$$

where $c_i \equiv \cos \alpha_i$ and $s_i \equiv \sin \alpha_i$ ($i=1,2$).

The contribution to the mixing angle between second and third generations from the neutrino mass matrix Eq. (25) is given by $\theta_{\mu 3}^\nu \sim \delta_z / c_2$. In view of $c_2 = O(0.1)$, we choose $\delta_z \sim \epsilon^2$ to give naturally the near maximal mixing in the atmospheric neutrino oscillation (see discussion at the end of Sec. III). Then, the relevant mass ratio and mixing to the solar neutrino oscillation are given as

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{\delta_y^2}{c_2^2} \sim \frac{\delta_y^2}{\epsilon^2}, \quad (26)$$

$$\theta_{e2}^\nu \sim \frac{\hat{m}_c c_1}{\delta_y} \sim \frac{\epsilon^3}{\delta_y}, \quad (27)$$

TABLE V. $U(1)_X$ charges. The other fields have vanishing charges.

	$\psi_1(\mathbf{16})$	$\psi_2(\mathbf{16})$	$\psi_3(\mathbf{16})$	$\psi(\mathbf{10})$	$H(\mathbf{10})$	$\chi(\mathbf{16})$	$\bar{\chi}(\mathbf{16}^*)$	ϕ
$U(1)_X$	2	1	0	0	0	0	0	-1

as long as $\delta_x \lesssim \delta_y$ and δ_y is not very small compared with δ_z .

From Eq. (26), we find that if we choose $\delta_y \sim \epsilon^2$ we obtain the mass ratio consistent with the MSW solutions. Then, the mixing angle θ_{e2}^ν is of order of ϵ from Eq. (27). The actual mixing angle is the sum of θ_{e2}^ν and the corresponding contribution from charged-lepton mass matrix θ_{e2}^l . Since analyses of the previous section indicate $\theta_{e2}^l \simeq 0.04 - 0.19$, the resulting mixing angle is consistent with the small angle MSW solution if two contributions are added up destructively, while it cannot reach that of the large angle MSW solution even if two contributions are added up constructively. The required cancellation is rather mild such that the reduction of factor two or three is sufficient. We also mention that the mixing angle θ_{e3} is small, so that it is not conflict with the result of long baseline reactor neutrino oscillation experiment (CHOOZ), $\theta_{e3} \lesssim 0.22$ [32].

If we choose $\delta_y \sim \epsilon^3$, on the other hand, both mass ratio and mixing angle are consistent with vacuum oscillation solution. The reason for this coincidence can be traced back to the fact that the required neutrino mass ratio $m_{\nu_2}/m_{\nu_3} \sim 10^{-4}$ is approximately equal to $(m_c/m_t)^2$ as predicted by a simple (unit M_N) seesaw model in the $SO(10)$ GUT. That is, if we intend to obtain large solar-neutrino mixing angle we have to choose $\delta_y \simeq \hat{m}_c c_1$, and then we necessarily have the mass ratio $m_{\nu_2}/m_{\nu_3} \sim \delta_y^2/c^2 \sim \hat{m}_c^2$. The resulting MNS matrix has the so-called bimaximal form [13,33] and evades the constraint from CHOOZ experiment. We note that this vacuum oscillation solution is realized with $k_1:k_2:k_3 \simeq \epsilon^2:\epsilon:1$ which is implied by the hierarchy Eqs. (22), (23) [see Eq. (15)].

B. The case of hierarchical Majorana mass matrix

If $\psi_i(\mathbf{16})$ have a source of hierarchies indicated by Eqs. (22), (23), it is natural to think that the Majorana mass matrix M_N also has hierarchy consistent with it. Thus, we consider the case where the Majorana mass matrix has the hierarchical form,

$$M_N \simeq M_R \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon \\ \epsilon^2 & \epsilon & 1 \end{pmatrix}, \quad (28)$$

up to order-one coefficients.

In this subsection, we stick to the possibility that $\psi_i(\mathbf{16})$ have a source of hierarchies indicated by Eqs. (22), (23), so that we assume that $k_1:k_2:k_3 \simeq \epsilon^2:\epsilon:1$. Considering that $\delta_z \sim \epsilon^2$ in order to give naturally the near maximal mixing between ν_μ and ν_τ , elements of the $M_{\nu D}$ have the following order of magnitude:

$$\delta_x \simeq \epsilon^4, \quad \delta_y \simeq \epsilon^3, \quad \delta_z \simeq \epsilon^2. \quad (29)$$

Then, the neutrino mass matrix, $M_\nu = M_{\nu D}^T M_N^{-1} M_{\nu D}$, is given as

$$M_\nu \simeq \frac{m_t^2}{M_R} \begin{pmatrix} \epsilon^{-4} \tilde{m}_u^2 & \tilde{m}_u & \epsilon^{-3} \tilde{m}_u \tilde{m}_c \\ \tilde{m}_u & \epsilon^4 & \epsilon \tilde{m}_c \\ \epsilon^{-3} \tilde{m}_u \tilde{m}_c & \epsilon \tilde{m}_c & \epsilon^{-2} \tilde{m}_c^2 \end{pmatrix}, \quad (30)$$

where $\tilde{m}_u \equiv \hat{m}_u + \epsilon \hat{m}_c c_1$ and $\tilde{m}_c \equiv \hat{m}_c + \epsilon c_2$. This gives the relevant mass ratio and mixing to the solar neutrino oscillation as

$$\frac{m_{\nu_2}}{m_{\nu_3}} \sim \frac{\epsilon^6}{\tilde{m}_c^2} = \frac{\epsilon^6}{(\hat{m}_c + \epsilon c_2)^2}, \quad (31)$$

$$\theta_{e2}^\nu \sim \frac{\tilde{m}_u}{\epsilon^4} = \frac{\hat{m}_u + \epsilon \hat{m}_c c_1}{\epsilon^4}. \quad (32)$$

From Eq. (31), we obtain $m_{\nu_2}/m_{\nu_3} \simeq 10^{-2} - 10^{-1}$, which is consistent with both small and large angle MSW solutions. Also, the angle is given as $\theta_{e2}^\nu \simeq 0.1 - 1$ from Eq. (32), so that we can reproduce the large angle MSW solution naturally. To reproduce the small angle solution, θ_{e2}^ν and θ_{e2}^l have to be added up destructively, but the required cancellation is mild. (The reduction of factor three to five is sufficient.) Thus, we conclude that the the model can accommodate both large and small angle MSW solutions in the case of the hierarchical Majorana mass matrix.

V. CONCLUSIONS

In this paper, we have investigated the $SO(10)$ GUT model with generation flipping. The model contains one extra matter multiplet $\psi(\mathbf{10})$ in addition to the usual matter multiplets $\psi_i(\mathbf{16})$, and it mixes with $\psi_i(\mathbf{16})$ when the $SO(10)_{\text{GUT}}$ is broken down to $SU(5)$ by the vacuum expectation values of $\chi(\mathbf{16})$ and $\bar{\chi}(\mathbf{16}^*)$ fields.⁶ The low-energy quarks and leptons are three linear combinations of $\psi_i(\mathbf{16})$ and $\psi(\mathbf{10})$.

We have found the parameter region of the model in which the observed quark masses and mixings are well reproduced. The required hierarchies of the coupling constants are consistent with the observation that only $\psi_i(\mathbf{16})$ have a source of hierarchies such as $U(1)$ flavor symmetry charges. As for the lepton sector, the model can reproduce the

⁶This structure may be realized in a framework of an E_6 GUT [34].

charged-lepton masses if there are suitable nonrenormalizable interactions which introduce SU(5) breaking effects into $M_{d/l}$. The obtained charged-lepton mass matrix indicates that the mixing between second and third generations tends to be large in the lepton sector, which is consistent with the observed maximal mixing of the atmospheric neutrino oscillation.

We have also discussed neutrino masses and mixings qualitatively. We have considered two cases that the Majorana mass matrix M_N for the right-handed neutrinos is proportional to the unit matrix and it has the hierarchical form. In the former case, the model can accommodate small angle MSW solution and vacuum oscillation solution to the solar neutrino deficit, depending on the values of the coupling constants k_i . The vacuum oscillation solution is realized when k_i have the hierarchy indicated in the quark sector, and the resulting MNS mixing matrix has the form of so-called bimaximal mixing.

In the latter case, the model can accommodate both small and large angle MSW solutions and the parameter region is consistent with the hierarchy required in the quark sector. It is remarkable that the single anomalous $U(1)_X$ flavor symmetry with the ϕ field which has the vacuum expectation value of order 10^{17} GeV realizes this possibility.⁷ Then, the operators in the superpotential have suppression factors of appropriate powers of $\langle\phi\rangle/M_G \simeq \epsilon$ depending on their $U(1)_X$ charges [35]. An example of the $U(1)_X$ charges for various fields is given in Table V.⁸ Note, however, that this $U(1)_X$ alone is incomplete to obtain the superpotential of the present model, since we have to suppress unwanted terms such as $M_G \psi(\mathbf{10}) \psi(\mathbf{10})$ by hand. Thus, additional symmetries are required to make the model natural.

To summarize, we have found that in the present model the observed solar neutrino deficit is explained by different neutrino-oscillation solutions depending on the form of the Majorana mass matrix M_N . We hope that future experiments will distinguish among different solutions and we will be

able to approach the structure of the Majorana mass matrix for the right-handed neutrinos.

ACKNOWLEDGMENTS

We would like to thank T. Yanagida for valuable discussions and comments on the present work. We also thank J. Hashiba, K. Kurosawa, N. Okamura, and J. Sato for discussions. Y.N. is supported by the Japan Society for the Promotion of Science.

APPENDIX: THE MASS MATRICES IN THE MODEL

In this appendix, we give a detailed derivation of the mass matrices in the model, clarifying a phase convention we have used.

The model contains matter fields $\psi_i(\mathbf{16})$, $\psi(\mathbf{10})$ and Higgs fields $H(\mathbf{10})$, $H(\mathbf{16})$. After the spontaneous breakdown of $SO(10)_{\text{GUT}}$ to $SU(5)$, these fields decompose as

$$\begin{aligned} \mathbf{10}_1 + \mathbf{5}_1^{*'} + \mathbf{1}_1 &= \psi_1(\mathbf{16}), \\ \mathbf{10}_2 + \mathbf{5}_2^{*'} + \mathbf{1}_2 &= \psi_2(\mathbf{16}), \\ \mathbf{10}_3 + \mathbf{5}_3^{*'} + \mathbf{1}_3 &= \psi_3(\mathbf{16}), \\ \mathbf{5}_\psi + \mathbf{5}_\psi^* &= \psi(\mathbf{10}), \end{aligned} \tag{A1}$$

and

$$\begin{aligned} \mathbf{5}_H + \mathbf{5}_{H(\mathbf{10})}^* &= H(\mathbf{10}), \\ \mathbf{10}_{H(\mathbf{16})} + \mathbf{5}_{H(\mathbf{16})}^* + \mathbf{1}_{H(\mathbf{16})} &= H(\mathbf{16}). \end{aligned} \tag{A2}$$

The original Yukawa couplings are given by the superpotential Eq. (2) and the Majorana masses for the right-handed neutrinos by Eq. (19). At this stage, the quark and lepton masses are written as

$$\begin{aligned} W = \frac{1}{2} (\mathbf{10}_1 \mathbf{10}_2 \mathbf{10}_3) & \begin{pmatrix} h_1 \langle \mathbf{5}_H \rangle & 0 & 0 \\ 0 & h_2 \langle \mathbf{5}_H \rangle & 0 \\ 0 & 0 & h_3 \langle \mathbf{5}_H \rangle \end{pmatrix} \begin{pmatrix} \mathbf{10}_1 \\ \mathbf{10}_2 \\ \mathbf{10}_3 \end{pmatrix} + (\mathbf{10}_1 \mathbf{10}_2 \mathbf{10}_3) \begin{pmatrix} h_1 \langle \mathbf{5}_{H(\mathbf{10})}^* \rangle & 0 & 0 \\ 0 & h_2 \langle \mathbf{5}_{H(\mathbf{10})}^* \rangle & 0 \\ 0 & 0 & h_3 \langle \mathbf{5}_{H(\mathbf{10})}^* \rangle \end{pmatrix} \begin{pmatrix} \mathbf{5}_1^{*'} \\ \mathbf{5}_2^{*'} \\ \mathbf{5}_3^{*'} \end{pmatrix} \\ & + (\mathbf{1}_1 \mathbf{1}_2 \mathbf{1}_3) \begin{pmatrix} h_1 \langle \mathbf{5}_H \rangle & 0 & 0 \\ 0 & h_2 \langle \mathbf{5}_H \rangle & 0 \\ 0 & 0 & h_3 \langle \mathbf{5}_H \rangle \end{pmatrix} \begin{pmatrix} \mathbf{5}_1^{*'} \\ \mathbf{5}_2^{*'} \\ \mathbf{5}_3^{*'} \end{pmatrix} + \frac{1}{2} (\mathbf{1}_1 \mathbf{1}_2 \mathbf{1}_3) \begin{pmatrix} M_R \tilde{J}_{11} & M_R \tilde{J}_{12} & M_R \tilde{J}_{13} \\ M_R \tilde{J}_{21} & M_R \tilde{J}_{22} & M_R \tilde{J}_{23} \\ M_R \tilde{J}_{31} & M_R \tilde{J}_{32} & M_R \tilde{J}_{33} \end{pmatrix} \begin{pmatrix} \mathbf{1}_1 \\ \mathbf{1}_2 \\ \mathbf{1}_3 \end{pmatrix}, \end{aligned} \tag{A3}$$

in terms of SU(5) language. Here, $\tilde{J}_{ij} = j_{ij}/|j_{33}|$ and M_R is given as $M_R = |j_{33}|V^2/M_G$. If we assume that the Majorana mass

⁷The nonrenormalizable operators in Eq. (21) might be generated via exchanges of some heavy fields, so that they are suppressed by the mass scale of the heavy fields instead of M_G . We think that the masses for these heavy fields, the GUT scale V and the mass M are not related to the anomalous $U(1)_X$ breaking scale, $\langle\phi\rangle$.

⁸Under the charge assignment of Table V, the mass M in Eq. (8) should be 10^{17} GeV. On the other hand, if $\psi(\mathbf{10})$ has the $U(1)_X$ charge 1, M should be the GUT scale, 10^{16} GeV.

matrix is proportional to the unit matrix, $j_{ij} = j \delta_{ij}$, in the basis that the original Yukawa couplings are diagonal, $\tilde{J}_{ij}$ and M_R are reduced to $\tilde{J}_{ij} = \delta_{ij}$ and $M_R = jV^2/M_G$.

The superpotential Eq. (6) mixes $\psi(\mathbf{10})$ with $\psi_i(\mathbf{16})$. This term can be written in terms of SU(5) language as

$$W = \sqrt{|f_1|^2 + |f_2|^2 + |f_3|^2} \langle \chi(\mathbf{16}) \rangle \tilde{\mathbf{5}}^* \mathbf{5}_\psi, \quad (\text{A4})$$

where $\tilde{\mathbf{5}}^*$ is a linear combination of $\mathbf{5}_i^{*'}$, $\tilde{\mathbf{5}}^* = \sum_{i=1}^3 f_i e^{i\eta_i} \mathbf{5}_i^{*'} / \sqrt{|f_1|^2 + |f_2|^2 + |f_3|^2}$. We call the remaining two linear combinations $\mathbf{5}_1^*$ and $\mathbf{5}_3^*$. The fields $\tilde{\mathbf{5}}^*, \mathbf{5}_1^*$ and $\mathbf{5}_3^*$ are related to $\mathbf{5}_i^{*'}$ by a unitary transformation. Thus, using arbitrariness of defining $\mathbf{5}_1^*$ and $\mathbf{5}_3^*$ and phases of the fields $\tilde{\mathbf{5}}^*, \mathbf{5}_1^*$ and $\mathbf{5}_3^*$, we can write this relation as

$$\begin{pmatrix} \mathbf{5}_1^{*'} \\ \mathbf{5}_2^{*'} \\ \mathbf{5}_3^{*'} \end{pmatrix} = \begin{pmatrix} e^{-i\delta_1 - i\delta_2} & 0 & 0 \\ 0 & e^{-i\delta_1 + i\delta_2} & 0 \\ 0 & 0 & e^{2i\delta_1} \end{pmatrix} \begin{pmatrix} \cos \alpha_1 & -\sin \alpha_1 & 0 \\ \sin \alpha_1 & \cos \alpha_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos \alpha_2 & 0 & e^{-i\delta_3 \sin \alpha_2} \\ 0 & 1 & 0 \\ -e^{i\delta_3 \sin \alpha_2} & 0 & \cos \alpha_2 \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{5}}^* \\ \mathbf{5}_1^* \\ \mathbf{5}_3^* \end{pmatrix}, \quad (\text{A5})$$

without a loss of generality. Here, α_1, α_2 and δ_i are functions of f_i and η_i . Note that $\tilde{\mathbf{5}}^*$ receives a GUT scale mass together with $\mathbf{5}_\psi$ by Eq. (A4), so that the low-energy quark and lepton fields belonging to $\mathbf{5}^*$ of SU(5) are $\mathbf{5}_1^*, \mathbf{5}_3^*$ and $\mathbf{5}_\psi^*$.

The $\psi(\mathbf{10})$ is coupled with Higgs fields and $\psi_i(\mathbf{16})$ by superpotentials Eqs. (11), (15). Then, from Eqs. (A3), (A5) we obtain down-type quark (charged-lepton) mass matrix M_{10-5^*} and neutrino Dirac mass matrix M_{1-5^*} defined by

$$W = (\mathbf{10}_1 \mathbf{10}_2 \mathbf{10}_3) M_{10-5^*} \begin{pmatrix} \mathbf{5}_1^* \\ \mathbf{5}_\psi^* \\ \mathbf{5}_3^* \end{pmatrix} + (\mathbf{1}_1 \mathbf{1}_2 \mathbf{1}_3) M_{1-5^*} \begin{pmatrix} \mathbf{5}_1^* \\ \mathbf{5}_\psi^* \\ \mathbf{5}_3^* \end{pmatrix}, \quad (\text{A6})$$

as

$$M_{10-5^*} = \begin{pmatrix} -h_1 \langle \mathbf{5}_{H(10)}^* \rangle e^{-i\delta_1 - i\delta_2} \sin \alpha_1 & g_1 \langle \mathbf{5}_{H(16)}^* \rangle e^{i\phi_1} & h_1 \langle \mathbf{5}_{H(10)}^* \rangle e^{-i\delta_1 - i\delta_2 - i\delta_3} \cos \alpha_1 \sin \alpha_2 \\ h_2 \langle \mathbf{5}_{H(10)}^* \rangle e^{-i\delta_1 + i\delta_2} \cos \alpha_1 & g_2 \langle \mathbf{5}_{H(16)}^* \rangle e^{i\phi_2} & h_2 \langle \mathbf{5}_{H(10)}^* \rangle e^{-i\delta_1 + i\delta_2 - i\delta_3} \sin \alpha_1 \sin \alpha_2 \\ 0 & g_3 \langle \mathbf{5}_{H(16)}^* \rangle e^{i\phi_3} & h_3 \langle \mathbf{5}_{H(10)}^* \rangle e^{2i\delta_1} \cos \alpha_2 \end{pmatrix},$$

$$M_{1-5^*} = \begin{pmatrix} -h_1 \langle \mathbf{5}_H \rangle e^{-i\delta_1 - i\delta_2} \sin \alpha_1 & k_1 \langle \mathbf{5}_H \rangle \frac{V}{M_G} e^{i\varphi_1} & h_1 \langle \mathbf{5}_H \rangle e^{-i\delta_1 - i\delta_2 - i\delta_3} \cos \alpha_1 \sin \alpha_2 \\ h_2 \langle \mathbf{5}_H \rangle e^{-i\delta_1 + i\delta_2} \cos \alpha_1 & k_2 \langle \mathbf{5}_H \rangle \frac{V}{M_G} e^{i\varphi_2} & h_2 \langle \mathbf{5}_H \rangle e^{-i\delta_1 + i\delta_2 - i\delta_3} \sin \alpha_1 \sin \alpha_2 \\ 0 & k_3 \langle \mathbf{5}_H \rangle \frac{V}{M_G} e^{i\varphi_3} & h_3 \langle \mathbf{5}_H \rangle e^{2i\delta_1} \cos \alpha_2 \end{pmatrix}. \quad (\text{A7})$$

SU(5)- $\mathbf{5}^*$ of the Higgs fields $H(\mathbf{10})$ and $H(\mathbf{16})$ are also mixed by the superpotential Eq. (8). One linear combination $\tilde{\mathbf{5}}_H^*$ gets a GUT scale mass together with SU(5)- $\mathbf{5}$ of $\overline{H}(\mathbf{16}^*)$ and the other combination $\mathbf{5}_H^*$ remains as a massless Higgs field in the standard SU(5)_{GUT}. We can parametrize these fields as

$$\begin{pmatrix} \mathbf{5}_{H(10)}^* \\ \mathbf{5}_{H(16)}^* \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} \mathbf{5}_H^* \\ \tilde{\mathbf{5}}_H^* \end{pmatrix}, \quad (\text{A8})$$

where $\tan \theta = -\lambda V/M$. That is,

$$\begin{aligned} \langle \mathbf{5}_{H(10)}^* \rangle &= \cos \theta \langle \mathbf{5}_H^* \rangle, \\ \langle \mathbf{5}_{H(16)}^* \rangle &= \sin \theta \langle \mathbf{5}_H^* \rangle. \end{aligned} \quad (\text{A9})$$

Substituting Eq. (A9) for Eq. (A7) and combining with Eq. (A3), we obtain complete mass matrices. Defining

$$m_u \equiv h_1 \langle 5_H \rangle, \quad m_c \equiv h_2 \langle 5_H \rangle, \quad m_t \equiv h_3 \langle 5_H \rangle, \quad \tan \beta \equiv \langle 5_H \rangle / \langle 5_H^* \rangle, \quad (\text{A10})$$

the quark and lepton masses are written as

$$\begin{aligned} W = & (uct) \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \begin{pmatrix} \bar{u} \\ \bar{c} \\ \bar{t} \end{pmatrix} + \frac{\cos \theta}{\tan \beta} (dsb) \begin{pmatrix} -m_u e^{-i\delta_1 - i\delta_2} \sin \alpha_1 & m_t x_d e^{i\phi_{1d}} & m_u e^{-i\delta_1 - i\delta_2 - i\delta_3} \cos \alpha_1 \sin \alpha_2 \\ m_c e^{-i\delta_1 + i\delta_2} \cos \alpha_1 & m_t y_d e^{i\phi_{2d}} & m_c e^{-i\delta_1 + i\delta_2 - i\delta_3} \sin \alpha_1 \sin \alpha_2 \\ 0 & m_t z e^{i\phi_3} & m_t e^{2i\delta_1} \cos \alpha_2 \end{pmatrix} \\ & \times \begin{pmatrix} \bar{d} \\ \bar{s} \\ \bar{b} \end{pmatrix} + \frac{\cos \theta}{\tan \beta} (e\mu\tau) \begin{pmatrix} -m_u e^{-i\delta_1 - i\delta_2} \sin \alpha_1 & m_t x_l e^{i\phi_{1l}} & m_u e^{-i\delta_1 - i\delta_2 - i\delta_3} \cos \alpha_1 \sin \alpha_2 \\ m_c e^{-i\delta_1 + i\delta_2} \cos \alpha_1 & m_t y_l e^{i\phi_{2l}} & m_c e^{-i\delta_1 + i\delta_2 - i\delta_3} \sin \alpha_1 \sin \alpha_2 \\ 0 & m_t z e^{i\phi_3} & m_t e^{2i\delta_1} \cos \alpha_2 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} + (\bar{\nu}_1 \quad \bar{\nu}_2 \quad \bar{\nu}_3) \\ & \times \begin{pmatrix} -m_u e^{-i\delta_1 - i\delta_2} \sin \alpha_1 & m_t \delta_x e^{i\varphi_1} & m_u e^{-i\delta_1 - i\delta_2 - i\delta_3} \cos \alpha_1 \sin \alpha_2 \\ m_c e^{-i\delta_1 + i\delta_2} \cos \alpha_1 & m_t \delta_y e^{i\varphi_2} & m_c e^{-i\delta_1 + i\delta_2 - i\delta_3} \sin \alpha_1 \sin \alpha_2 \\ 0 & m_t \delta_z e^{i\varphi_3} & m_t e^{2i\delta_1} \cos \alpha_2 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} + \frac{1}{2} (\bar{\nu}_1 \quad \bar{\nu}_2 \quad \bar{\nu}_3) \\ & \times \begin{pmatrix} M_R \tilde{J}_{11} & M_R \tilde{J}_{12} & M_R \tilde{J}_{13} \\ M_R \tilde{J}_{21} & M_R \tilde{J}_{22} & M_R \tilde{J}_{23} \\ M_R \tilde{J}_{31} & M_R \tilde{J}_{32} & M_R \tilde{J}_{33} \end{pmatrix} \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \\ \bar{\nu}_3 \end{pmatrix}, \quad (\text{A11}) \end{aligned}$$

in terms of the SM fields. Here, we have defined various parameters as

$$x \equiv \frac{g_1}{h_3} \tan \theta, \quad y \equiv \frac{g_2}{h_3} \tan \theta, \quad z \equiv \frac{g_3}{h_3} \tan \theta, \quad (\text{A12})$$

$$\delta_x \equiv \frac{k_1}{h_3} \frac{V}{M_G}, \quad \delta_y \equiv \frac{k_2}{h_3} \frac{V}{M_G}, \quad \delta_z \equiv \frac{k_3}{h_3} \frac{V}{M_G}, \quad (\text{A13})$$

and taken the effects of nonrenormalizable superpotential Eq. (21) into account by changing (1,2) and (2,2) components of the down-type quark and charged-lepton mass matrices from their original values $x e^{i\phi_1}$ and $y e^{i\phi_2}$.

Now, we redefine various SM fields as

$$\begin{aligned} (u, d) & \rightarrow (u, d) e^{i\delta_1 + i\delta_2}, \quad \bar{u} \rightarrow \bar{u} e^{-i\delta_1 - i\delta_2}, \quad \bar{d} \rightarrow \bar{d}, \\ (c, s) & \rightarrow (c, s) e^{i\delta_1 - i\delta_2}, \quad \bar{c} \rightarrow \bar{c} e^{-i\delta_1 + i\delta_2}, \quad \bar{s} \rightarrow \bar{s} e^{-i\phi_3 + 2i\delta_1 + i\delta_3}, \\ (t, b) & \rightarrow (t, b) e^{-2i\delta_1 - i\delta_3}, \quad \bar{t} \rightarrow \bar{t} e^{2i\delta_1 + i\delta_3}, \quad \bar{b} \rightarrow \bar{b} e^{i\delta_3}, \\ (\nu_1, e) & \rightarrow (\nu_1, e) e^{i\delta_1 + i\delta_2}, \quad \bar{\nu}_1 \rightarrow \bar{\nu}_1, \quad \bar{e} \rightarrow \bar{e}, \\ (\nu_2, \mu) & \rightarrow (\nu_2, \mu) e^{i\delta_1 - i\delta_2}, \quad \bar{\nu}_2 \rightarrow \bar{\nu}_2, \quad \bar{\mu} \rightarrow \bar{\mu} e^{-i\phi_3 + 2i\delta_1 + i\delta_3}, \\ (\nu_3, \tau) & \rightarrow (\nu_3, \tau) e^{-2i\delta_1 - i\delta_3}, \quad \bar{\nu}_3 \rightarrow \bar{\nu}_3, \quad \bar{\tau} \rightarrow \bar{\tau} e^{i\delta_3}, \end{aligned} \quad (\text{A14})$$

in order to simplify the form of the quark and lepton mass matrices. Then, from Eq. (A11) we obtain the mass matrices defined by

$$W = (uct)M_u \begin{pmatrix} \overline{u} \\ \overline{c} \\ \overline{t} \end{pmatrix} + (dsb)M_d \begin{pmatrix} \overline{d} \\ \overline{s} \\ \overline{b} \end{pmatrix} + (\overline{e\mu\tau})M_l \begin{pmatrix} \overline{e} \\ \overline{\mu} \\ \overline{\tau} \end{pmatrix} + (\overline{\nu_1\nu_2\nu_3})M_{\nu D} \begin{pmatrix} \overline{\nu_1} \\ \overline{\nu_2} \\ \overline{\nu_3} \end{pmatrix} + \frac{1}{2}(\overline{\nu_1\nu_2\nu_3})M_N \begin{pmatrix} \overline{\nu_1} \\ \overline{\nu_2} \\ \overline{\nu_3} \end{pmatrix}, \quad (\text{A16})$$

as

$$\begin{aligned} M_u &= m_t \begin{pmatrix} \hat{m}_u & 0 & 0 \\ 0 & \hat{m}_c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \\ M_d &= m_t \frac{\cos \theta}{\tan \beta} \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & x_d e^{i\phi_{xd}} & \hat{m}_u \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c \cos \alpha_1 & y_d e^{i\phi_{yd}} & \hat{m}_c \sin \alpha_1 \sin \alpha_2 \\ 0 & z & \cos \alpha_2 \end{pmatrix}, \\ M_l &= m_t \frac{\cos \theta}{\tan \beta} \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & x_l e^{i\phi_{xl}} & \hat{m}_u \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c \cos \alpha_1 & y_l e^{i\phi_{yl}} & \hat{m}_c \sin \alpha_1 \sin \alpha_2 \\ 0 & z & \cos \alpha_2 \end{pmatrix}, \\ M_{\nu D} &= m_t \begin{pmatrix} -\hat{m}_u \sin \alpha_1 & \delta_x e^{i\varphi_x} & \hat{m}_u e^{i\varphi_u} \cos \alpha_1 \sin \alpha_2 \\ \hat{m}_c e^{i\varphi_c} \cos \alpha_1 & \delta_y e^{i\varphi_y} & \hat{m}_c e^{i\varphi_v} \sin \alpha_1 \sin \alpha_2 \\ 0 & \delta_z e^{i\varphi_z} & e^{i\varphi_w} \cos \alpha_2 \end{pmatrix}, \\ M_N &= M_R \begin{pmatrix} \tilde{J}_{11} & \tilde{J}_{12} & \tilde{J}_{13} \\ \tilde{J}_{21} & \tilde{J}_{22} & \tilde{J}_{23} \\ \tilde{J}_{31} & \tilde{J}_{32} & \tilde{J}_{33} \end{pmatrix}, \end{aligned} \quad (\text{A17})$$

where $\hat{m}_u \equiv m_u/m_t$, $\hat{m}_c \equiv m_c/m_t$. Here, we have defined various phases as

$$\begin{aligned} \phi_{xd} &\equiv \phi_{1d} - \phi_3 + 3\delta_1 + \delta_2 + \delta_3, & \phi_{yd} &\equiv \phi_{2d} - \phi_3 + 3\delta_1 - \delta_2 + \delta_3, \\ \phi_{xl} &\equiv \phi_{1l} - \phi_3 + 3\delta_1 + \delta_2 + \delta_3, & \phi_{yl} &\equiv \phi_{2l} - \phi_3 + 3\delta_1 - \delta_2 + \delta_3, \end{aligned} \quad (\text{A18})$$

$$\varphi_x \equiv \varphi_1 + \delta_1 - \delta_2, \quad \varphi_y \equiv \varphi_2 + \delta_1 - \delta_2, \quad \varphi_z \equiv \varphi_3 + \delta_1 - \delta_2,$$

$$\varphi_t \equiv 2\delta_2, \quad \varphi_u \equiv -3\delta_1 - \delta_2 - 2\delta_3, \quad (\text{A19})$$

$$\varphi_v \equiv -3\delta_1 + \delta_2 - 2\delta_3, \quad \varphi_w \equiv -\delta_3.$$

Note that we have retained the form of M_N . As a result, not all phases are physically independent. The domains of α_1 and α_2 can be restricted to

$$0 \leq \alpha_1 \leq \pi/2, \quad 0 \leq \alpha_2 \leq \pi/2, \quad (\text{A20})$$

without a loss of generality by redefining the phases of the SM fields appropriately.

Finally, the mass matrix for the light neutrinos defined by

$$W = \frac{1}{2}(\nu_1 \nu_2 \nu_3) M_\nu \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (\text{A21})$$

is given by

$$M_\nu = M_{\nu D}^T M_N^{-1} M_{\nu D}, \quad (\text{A22})$$

through the seesaw mechanism [6].

-
- [1] J.C. Pati and A. Salam, Phys. Rev. D **10**, 275 (1974); H. Georgi and S.L. Glashow, Phys. Rev. Lett. **32**, 438 (1974).
- [2] S. Dimopoulos and H. Georgi, Nucl. Phys. **B193**, 150 (1981); N. Sakai, Z. Phys. C **11**, 153 (1981).
- [3] J. Ellis, S. Kelley, and D.V. Nanopoulos, Phys. Lett. B **260**, 131 (1991); U. Amaldi, W. de Boer, and H. Fürstenau, *ibid.* **260**, 447 (1991); P. Langacker and M.-X. Luo, Phys. Rev. D **44**, 817 (1991); C. Giunti, C.W. Kim, and U.W. Lee, Mod. Phys. Lett. A **6**, 1745 (1991).
- [4] E.B. Einhorn and D.R.T. Jones, Nucl. Phys. **B196**, 175 (1982); J. Ellis, D.V. Nanopoulos, and S. Rudaz, *ibid.* **B202**, 43 (1982).
- [5] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1562 (1998).
- [6] T. Yanagida, in Proceedings of the Workshop on the Unified Theory and Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D.Z. Freedman (North-Holland, Amsterdam, 1979), p. 315.
- [7] H. Georgi, in *Particles and Fields*, edited by C.E. Carlson (AIP, New York, 1975), p. 575; H. Fritzsch and P. Minkowski, Ann. Phys. (N.Y.) **93**, 193 (1975).
- [8] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1964); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).
- [9] P. Langacker, Phys. Rep. **72**, 185 (1981).
- [10] T. Yanagida and J. Sato, Nucl. Phys. B (Proc. Suppl.) **77**, 293 (1999).
- [11] J.K. Elwood, N. Irges, and P. Ramond, Phys. Rev. Lett. **81**, 5064 (1998); C.H. Albright, K.S. Babu, and S.M. Barr, *ibid.* **81**, 1167 (1998); C.H. Albright and S.M. Barr, Phys. Lett. B **452**, 287 (1999); K.S. Babu, J.C. Pati, and F. Wilczek, hep-ph/9812538; K. Oda, E. Takasugi, M. Tanaka, and M. Yoshimura, Phys. Rev. D **59**, 055001 (1999).
- [12] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).
- [13] Y. Nomura and T. Yanagida, Phys. Rev. D **59**, 017303 (1999).
- [14] See also, J. Hisano, H. Murayama, and T. Yanagida, Phys. Rev. D **49**, 4966 (1994); K.S. Babu and S.M. Barr, Phys. Rev. Lett. **75**, 2088 (1995); Phys. Rev. D **56**, 2614 (1997).
- [15] C.D. Froggatt and H.B. Nielsen, Nucl. Phys. **B147**, 277 (1979).
- [16] Kamiokande Collaboration, K.S. Hirata *et al.*, Phys. Lett. B **280**, 146 (1992); IMB Collaboration, R. Becker-Szendy *et al.*, Phys. Rev. D **46**, 3720 (1992); SOUDAN-2 Collaboration, W.W.M. Allison *et al.*, Phys. Lett. B **391**, 491 (1997).
- [17] Super-Kamiokande Collaboration, Y. Fukuda *et al.*, Phys. Rev. Lett. **81**, 1158 (1998); **81**, 4279(E) (1998); Kamiokande Collaboration, Y. Fukuda *et al.*, *ibid.* **77**, 1683 (1996).
- [18] GALLEX Collaboration, W. Hampel *et al.*, Phys. Lett. B **388**, 384 (1996); SAGE Collaboration, D.N. Abdurashitov *et al.*, Phys. Rev. Lett. **77**, 4708 (1996).
- [19] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957) [Sov. Phys. JETP **6**, 429 (1958)]; *ibid.* **53**, 1717 (1967) [*ibid.* **26**, 984 (1968)]; V. Gribov and B. Pontecorvo, Phys. Lett. **28B**, 495 (1969).
- [20] H. Georgi and C. Jarlskog, Phys. Lett. **86B**, 297 (1979).
- [21] J. Ellis and M.K. Gaillard, Phys. Lett. **88B**, 315 (1979).
- [22] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
- [23] H. Fusaoka and Y. Koide, Phys. Rev. D **57**, 3986 (1998).
- [24] K. Hagiwara and N. Okamura, Nucl. Phys. **B548**, 60 (1999).
- [25] S. Mele, hep-ph/9808411; Phys. Rev. D **59**, 113011 (1999).
- [26] C. Jarlskog, Phys. Rev. Lett. **55**, 1039 (1985); Z. Phys. C **29**, 491 (1985).
- [27] J.A. Aguilar-Saavedra and M. Masip, Phys. Rev. D **54**, 6903 (1996).
- [28] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978); S.P. Mikheyev and A. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento C **9**, 17 (1986).
- [29] V. Barger, R.J.N. Phillips, and K. Whisnant, Phys. Rev. D **24**, 538 (1981); S.L. Glashow and L.M. Krauss, Phys. Lett. B **190**, 199 (1987).
- [30] N. Hata and P. Langacker, Phys. Rev. D **56**, 6107 (1997); J.N. Bahcall, P.I. Krastev, and A. Yu. Smirnov, *ibid.* **58**, 096016 (1998); G.L. Fogli, E. Lisi, A. Marrone, and G. Scioscia, *ibid.* **59**, 033001 (1999).
- [31] V. Barger and K. Whisnant, Phys. Lett. B **456**, 54 (1999).
- [32] CHOOZ Collaboration, M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998).
- [33] V. Barger, S. Pakvasa, T.J. Weiler, and K. Whisnant, Phys. Lett. B **437**, 107 (1998); S.M. Bilenky and C. Giunti, *ibid.* **444**, 379 (1998); A.J. Baltz, A.S. Goldhaber, and M. Goldhaber, Phys. Rev. Lett. **81**, 5730 (1998).
- [34] M. Bando and T. Kugo, Prog. Theor. Phys. **101**, 1313 (1999).
- [35] L. Ibanez and G.G. Ross, Phys. Lett. B **332**, 100 (1994); P. Binetruy and P. Ramond, *ibid.* **350**, 49 (1995); V. Jain and R. Shrock, *ibid.* **352**, 83 (1995).