

Probing matter effects with neutrino long baseline experiments

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(Received 12 April 1999; published 6 December 1999)

We consider $\nu_\mu \rightarrow \nu_e$ oscillations in long baseline experiments within a three flavor oscillation framework. A non-zero measurement of this oscillation probability implies that the (13) mixing angle ϕ is non-zero. We consider the effect of neutrino propagation through the matter of Earth's crust and show that, given the constraints from solar neutrino and CHOOZ data, matter effects enhance the mixing for neutrinos rather than for anti-neutrinos. We need data from two different experiments with different baseline lengths (such as K2K and MINOS) to discern the effect of the matter term.

PACS number(s): 14.60.Pq, 13.15.+g, 96.40.Tv, 96.60.Jw

Recent results from the Super-Kamiokande have sparked tremendous interest in neutrino physics [1,2]. The deficits seen in the solar and atmospheric neutrino fluxes can be very naturally explained in terms of neutrino oscillations. Since the energy and the distance scales of the solar neutrino problem are widely different from those of the atmospheric neutrino problem, the neutrino oscillation solution to these problems requires widely different values of mass-squared differences [3,4]. A minimum of three mass eigenstates, and hence three neutrino flavors, are needed for a simultaneous solution of solar and atmospheric neutrino problems. Since the CERN e^+e^- collider LEP has shown that three light, active neutrino species exist [5], it is natural to consider all neutrino data in the framework of oscillations between all the three active neutrino flavors.

The flavor eigenstates $\nu_\alpha (\alpha=e, \mu, \tau)$ are related to the mass eigenstates $\nu_i (i=1,2,3)$ by a unitary matrix U :

$$|\nu_\alpha\rangle = U |\nu_i\rangle. \quad (1)$$

As in the quark sector, U can be parametrized in terms of three mixing angles and one phase. A widely used parametrization, convenient for analyzing neutrino oscillations with matter effects, is [6]

$$U = U^{23}(\psi) \times U^{phase} \times U^{13}(\phi) \times U^{12}(\omega), \quad (2)$$

where $U^{ij}(\theta_{ij})$ is the two flavor mixing matrix between the i th and j th mass eigenstates with the mixing angle θ_{ij} . We assume that the vacuum mass eigenvalues have the pattern $\mu_3^2 \gg \mu_2^2 > \mu_1^2$. Hence $\delta_{31} \approx \delta_{32} \gg \delta_{21}$, where $\delta_{ij} = \mu_i^2 - \mu_j^2$. The larger δ sets the scale for the atmospheric neutrino oscillations and the smaller one for solar neutrino oscillations. The angles ω, ϕ and ψ vary in the range $[0, \pi/2]$. For this range of the mixing angles, there is no loss of generality due to the assumption that $\delta_{31}, \delta_{21} \geq 0$ [7].

It has been shown [6,8] that in the above approximation of one dominant mass, the oscillation probability for solar neutrinos is a function of only three parameters (δ_{21}, ω and ϕ) and that of the atmospheric neutrinos and long baseline neutrinos is also function of only three parameters (δ_{31}, ϕ and ψ). In each case, the three flavor nature of the problem is illustrated by the fact that the oscillation probability is a function of two mixing angles. The phase is unobservable in

the one dominant mass approximation because, in both solar and atmospheric neutrino oscillations, one of the three mixing angles can be set to zero [7]. The value of δ_{31} preferred by the Super-K atmospheric neutrino data is about $2 \times 10^{-3} \text{ eV}^2$ [1]. For this value of δ_{31} , CHOOZ data sets a very strong constraint [9,10]

$$\sin^2 2\phi \leq 0.2. \quad (3)$$

If $\phi=0$, then the solar and atmospheric neutrino oscillations get decoupled and become two flavor oscillations with relevant parameters being (δ_{21}, ω) and (δ_{31}, ψ) respectively.

It is interesting to look for the consequences of non-zero ϕ . A non-zero ϕ leads to $\nu_\mu \rightarrow \nu_e$ oscillations in atmospheric neutrinos and long baseline experiments. Due to theoretical uncertainty in the calculation of atmospheric neutrino fluxes, it will be very difficult to discern the effect of a small ϕ from the atmospheric neutrino data. Recently a proposal was made to look for matter enhanced $\nu_\mu \rightarrow \nu_e$ oscillations in atmospheric neutrino data, which can be significant even if ϕ is small [11]. Here we consider the effect of matter on $\nu_\mu \rightarrow \nu_e$ oscillations in a three flavor framework at long baseline experiments.

The relation between the flavor states and mass eigenstates can be written in the simple form

$$\begin{bmatrix} \nu_e \\ c_\psi \nu_\mu - s_\psi \nu_\tau \\ s_\psi \nu_\mu + c_\psi \nu_\tau \end{bmatrix} = \begin{pmatrix} c_\phi & 0 & s_\phi \\ 0 & 1 & 0 \\ -s_\phi & 0 & c_\phi \end{pmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}, \quad (4)$$

where c stands for cosine and s stands for sine. This equation is obtained by first setting $U^{phase}=I$ and $\omega=0$ in Eq. (2). Then substitute the resulting form U in Eq. (1) and multiply it from left by $U^{23}(-\psi)$. From Eq. (4) we see that ν_2 has no ν_e component and hence is decoupled from any oscillation involving ν_e . Thus the calculation of oscillation probability is essentially a two flavor problem. The three flavor nature is present in the fact that the oscillations occur between $[\nu_e \rightarrow (s_\psi \nu_\mu + c_\psi \nu_\tau)]$. The vacuum oscillation probability is calculated to be

$$P_{\mu e} = \sin^2 \psi \sin^2 2\phi \sin^2 \left(\frac{1.27 \delta_{31} L}{E} \right), \quad (5)$$

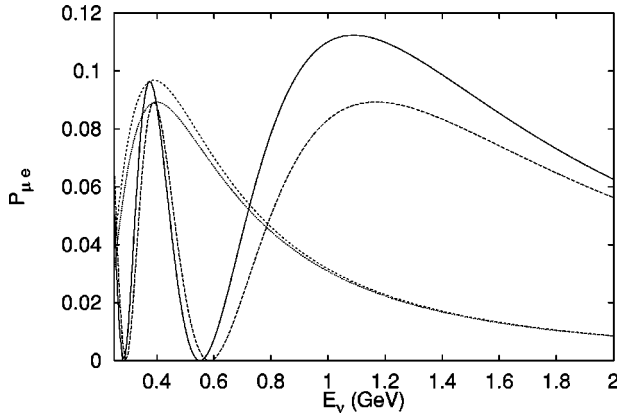


FIG. 1. Plots of $P_{\mu e}^m$ for MINOS (solid line), $P_{\mu e}$ for MINOS (long-dashed line), $P_{\mu e}^m$ for K2K (short-dashed line), $P_{\mu e}$ for K2K (dotted line) vs E for $\delta_{31}=2 \times 10^{-3} \text{ eV}^2$ and $\phi=12.5^\circ$.

where δ_{31} is in eV^2 , the baseline length L is in km and the neutrino energy E is in GeV. For low energies the phase $1.27\delta_{31}L/E$ oscillates rapidly and $P_{\mu e}$ becomes insensitive to it. However, for the range of energies for which the phase is close to $\pi/2$, $P_{\mu e}$ varies slowly as shown in Figs. 1–3. If the spectrum of the neutrinos in long baseline experiments peaks in this range, then δ_{31} and the mixing angles can be determined quite accurately.

Given the CHOOZ constraint on ϕ , Super-Kamiokande atmospheric neutrino data fix $\sin^2 2\psi \approx 1$ or $\sin^2 \psi \approx 0.5$. Substituting this value in Eq. (5), we find that $P_{\mu e} \leq 0.1$. The neutrino beams at K2K and MINOS have a 1% ν_e contamination. This background and the systematic uncertainties set a limit to the smallest value of $P_{\mu e}$ (and ϕ) that can be measured. With the current estimates of the systematic uncertainties, the sensitivity of K2K is similar to that of CHOOZ ($\sin^2 2\phi \sim 0.2$) but MINOS is capable of measuring values of ϕ as small as 3° ($\sin^2 2\phi \geq 0.01$) [12]. Once K2K starts running, its systematics will be better understood and its sensitivity is likely to improve.

In both K2K and MINOS experiments, the neutrino beam travels through Earth's crust, where it encounters matter of roughly constant density 3 g/cc. This traversal leads to the addition of the Wolfenstein term to the $e-e$ element of the

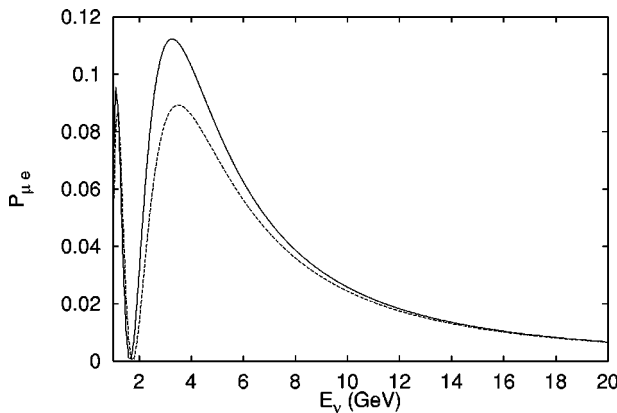


FIG. 2. Plots of $P_{\mu e}^m$ (solid line), $P_{\mu e}$ (long-dashed line) vs E for MINOS $\delta_{31}=6 \times 10^{-3} \text{ eV}^2$ and $\phi=12.5^\circ$.

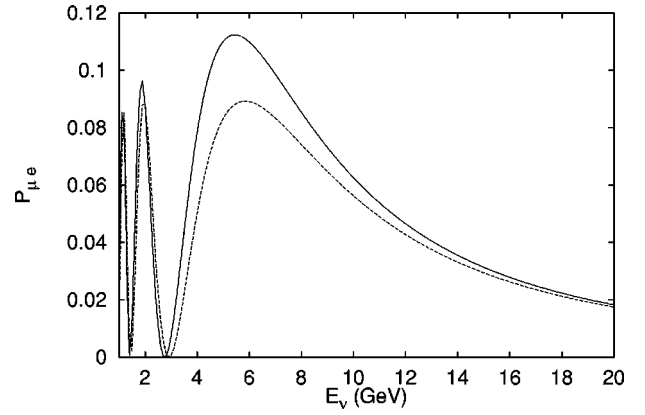


FIG. 3. Plots of $P_{\mu e}^m$ (solid line), $P_{\mu e}$ (long-dashed line) vs E for MINOS $\delta_{31}=10^{-2} \text{ eV}^2$ and $\phi=12.5^\circ$.

(mass)² matrix, when it is written in the flavor basis [13]. The Wolfenstein term is given by

$$A = 0.76 \times \rho(\text{g/cc}) \times E(\text{GeV}) \times 10^{-4} \text{ eV}^2. \quad (6)$$

For ρ of a few g/cc and E of a few GeV, A is comparable to the value of δ_{31} set by atmospheric neutrino data. This can lead to interesting and observable matter effects in long baseline experiments. The interactions of ν_μ and ν_τ with ordinary matter are identical. Hence Eq. (4) can be used to include matter effects in a simple manner because the problem, once again, is essentially a two flavor one. It is easy to see that ψ is unaffected by matter and the matter dependent mixing angle ϕ_m and the mass eigenvalues m_1^2 and m_3^2 are given by [8]

$$\tan 2\phi_m = \frac{\delta_{31} \sin 2\phi}{\delta_{31} \cos 2\phi - A}, \quad (7)$$

$$m_1^2 = \frac{1}{2} [(\delta_{31} + A) - \sqrt{(\delta_{31} \cos 2\phi - A)^2 + (\delta_{31} \sin 2\phi)^2}], \quad (8)$$

$$m_3^2 = \frac{1}{2} [(\delta_{31} + A) + \sqrt{(\delta_{31} \cos 2\phi - A)^2 + (\delta_{31} \sin 2\phi)^2}]. \quad (9)$$

The above equations hold for the propagation of neutrinos. For vacuum propagation, the mass eigenvalues and mixing angles for neutrinos and anti-neutrinos are the same. However, to include matter effects for anti-neutrinos, one should replace A by $-A$ in Eqs. (7), (8) and (9). Note that, since A is always positive, matter effects enhance the neutrino mixing angle and suppress the anti-neutrino mixing angle if $(\delta_{31} \cos 2\phi)$ is positive and vice versa if $(\delta_{31} \cos 2\phi)$ is negative. Since we have taken δ_{31} to be positive, matter effects enhance neutrino mixing if $\cos 2\phi$ is positive or $\phi < \pi/4$. The CHOOZ constraint from Eq. (3) on $\sin^2 2\phi$ sets the limit $\phi \leq 13^\circ$ or $\phi \geq 77^\circ$. For the latter possibility, $\cos 2\phi$ is negative. However, this large a value of ϕ is forbidden by the three flavor analysis of solar neutrino data, which yields the independent constraint $\phi \leq 50^\circ$ [3,4]. Hence the enhance-

ment of the mixing angle will be for neutrinos rather than for anti-neutrinos. This is good news because the beams in long baseline experiments consist of neutrinos overwhelmingly.

The matter dependent oscillation probability can be calculated in a straightforward manner from Eq. (4) in terms of ψ , ϕ_m and $\delta_{31}^m = m_3^2 - m_1^2$,

$$P_{\mu e}^m = \frac{1}{2} \sin^2 2\phi_m \sin^2 \left(\frac{1.27 \delta_{31}^m L}{E} \right), \quad (10)$$

where we have substituted the Super-Kamiokande best fit value $\sin^2 \psi = 1/2$. As discussed above, the matter effects enhance the mixing angle for neutrinos and the matter modified oscillation probability $P_{\mu e}^m$ will be greater than the vacuum oscillation probability $P_{\mu e}$, for a range of energies. For some neutrino energy, the Mikheyev-Smirnov resonance condition [14]

$$\delta_{31} \cos 2\phi = A \quad (11)$$

can be satisfied and near this energy $\phi_m \sim \pi/4$. Unfortunately, this does not lead to any dramatic increase in $P_{\mu e}^m$ because, near the resonance, when $\sin^2 2\phi_m$ is maximized, the matter dependent mass-squared difference δ_{31}^m is minimized [15]. In fact, near the resonance, $P_{\mu e}^m \geq P_{\mu e}$. By differentiating Eq. (10) with respect to E , we can calculate the energy at which $P_{\mu e}^m$ is maximized. The highest energy at which this occurs is just below the highest energy at which $P_{\mu e}$ is maximized, that is the energy for which the phase $(1.27 \delta_{31} L/E) = \pi/2$. At this energy, the ϕ_m is significantly higher than ϕ and hence $P_{\mu e}^m$ will be measurably higher than $P_{\mu e}$.

The long baseline experiments are primarily designed to be sensitive to the values of δ_{31} which provide a solution to the atmospheric neutrino problem. Since $\nu_\mu \leftrightarrow \nu_\tau$ oscillations are favored by the Super-Kamiokande data [1], long baseline experiments will find signatures of these oscillations first. In the following, we will assume that these measurements can determine δ_{31} with good precision and the energy of each long baseline experiment is tuned such that the phase $(1.27 \delta_{31} L/E) \sim \pi/2$. Since the baseline of K2K is 3 times smaller than that of MINOS, the energy where the phase becomes $\pi/2$ is smaller by a factor of 3 compared to similar energy for MINOS. Because the highest energy maximum occurs at different energies, the increase in $\nu_\mu \rightarrow \nu_e$ oscillation probability, due to the energy dependent enhancement of the mixing angle, is different for the two experiments. We find that, at their respective highest energy maxima, $P_{\mu e}^m \simeq 1.1 P_{\mu e}$ for K2K and $P_{\mu e}^m = 1.25 P_{\mu e}$ for MINOS. This is illustrated in Fig. 1 for $\phi \simeq 12.5^\circ$ (which is just below the CHOOZ limit) and $\delta_{31} = 2 \times 10^{-3} \text{ eV}^2$ (which is the best fit value for Super-Kamiokande atmospheric neutrino data). This conclusion is independent of the value of δ_{31} and is illustrated in Figs. 1–3, for different values of δ_{31} (and $\phi = 12.5^\circ$). A similar conclusion was obtained in a recent paper, where it was demonstrated that the relation between $P_{\mu e}^m$ and $P_{\mu e}$ is almost independent of ϕ [16].

To examine whether matter effects are playing a role in neutrino oscillations, we need data from at least two different

experiments with different baselines. Suppose we have data from only one experiment. If its energy is tuned such that its phase is $\pi/2$, then the measurement of the $\nu_\mu \rightarrow \nu_e$ oscillation probability gives us the $e - \mu$ mixing angle, as can be seen from Eqs. (5) and (10). The question then is: Is the measured quantity the vacuum mixing angle ϕ or the matter corrected mixing angle ϕ_m ? To answer this question, we need data from two different experiments with different baselines. For such experiments, matter effects lead to different enhancement of mixing angle ϕ , as illustrated in Fig. 1. This difference can be exploited to distinguish matter effects. Due to these, MINOS will find a higher value of the oscillation probability than K2K. The ratio of the oscillation probability of MINOS (P^M) to that of K2K (P^K) at their respective highest energy maxima is

$$\frac{P^M}{P^K} = \frac{\sin^2 2\phi_m^M}{\sin^2 2\phi_m^K}, \quad (12)$$

where ϕ_m^M and ϕ_m^K are the matter dependent mixing angles for MINOS and K2K respectively. ϕ_m is expected to be small because ϕ is small and the neutrino energy at the highest energy maximum is such that $A \ll \delta_{31}$. So we can use the approximation $\sin 2\phi_m \simeq \tan 2\phi_m$. From Eq. (7), we can easily show that

$$\frac{P^M}{P^K} \simeq 1 + 2 \frac{(A^M - A^K)}{\delta_{31}}, \quad (13)$$

where A^M and A^K are the respective Wolfenstein terms for MINOS and K2K, evaluated at their respective highest energy maxima. Note that this diagnostic for matter effects still stands even if the matter term, and hence the enhancement of ϕ , for K2K is small.

Finally we discuss the effect of the 1% ν_e background in the MINOS beam on our proposed diagnostic. Suppose MINOS observes N neutrino induced events. Of these, $0.01N$ events will be electron events because of the background. The uncertainty in the number of background events is $0.1\sqrt{N}$. The number of electron events which occur due to $\nu_\mu \leftarrow \nu_e$ oscillations is N times the oscillation probability. If the matter effects lead to an enhancement of the oscillation probability we expect an excess of electron events, above the number expected from vacuum oscillations,

$$\Delta N = (P_{\mu e}^m - P_{\mu e})N = \frac{1}{2} (\sin^2 2\phi_m^M - \sin^2 2\phi)N. \quad (14)$$

A estimate of $\sin^2 2\phi$ can be obtained from K2K from which $\sin^2 2\phi_m^K$, and hence ΔN can be calculated. For small ϕ and small ϕ_m^K , this becomes

$$\Delta N = \frac{A^M}{\delta_{31}} \sin^2 2\phi N. \quad (15)$$

To obtain a strong indication for matter effects, we need N large enough such that $\Delta N = 3 \times 0.1\sqrt{N}$.

In conclusion, we find that matter effects enhance $\nu_\mu \rightarrow \nu_e$ oscillations at long baseline experiments K2K and MINOS. This enhancement can be large enough to be observable, particularly for MINOS. However, analysis of the data

from both experiments is a prerequisite for elucidation of the effect of the matter term.

We thank Sameer Murthy for his help in the preparation of the figures.

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