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Investigation of the Paradox of Einstein, Podolsky, and Rosen for Ultrasmall Space-Time Intervals

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It is demonstrated that the Einstein-Podolsky-Rosen paradox (EPRP) can be investigated for ultrasmall space-time intervals by a reaction called "proximity correlation." One of the examples of proximity correlation analyzed in detail is the emission from a state $J=0$ of two spin-1 particles, A and B , in a relative orbital angular momentum state $L=1$, A and B decaying subsequently into spin-0 particles. It is shown that the quantity of interest in EPRP, $C(\hat{a}, \hat{b})$, the probability of particle A having polarization $M_{SA}=0$ with respect to $\hat{a}$, and particle B having polarization $M_{SB}=0$ with respect to $\hat{b}$, is predicted according to quantum mechanics as being $C(\hat{a}, \hat{b}) \equiv C(\theta) \propto \sin^2 \theta$. It is further shown that this relation can be tested experimentally by the decay of a $0^-(J^P)$ meson into two vector mesons. Experimental details are discussed.

INTRODUCTION

Einstein, Podolsky, and Rosen¹ gave an example of a hypothetical experiment capable of testing certain apparently paradoxical predictions of the current quantum theory. A simplified form of the experiment is the following.^{2,3} Two spin- $\frac{1}{2}$ particles, A and B , are emitted from a $J_{AB}=0$ state, and have a relative orbital angular momentum zero. The state vector for the system with respect to the z axis is

$$|\psi\rangle = (2)^{-1/2} [|M_{SA} = +\frac{1}{2}\rangle_z |M_{SB} = -\frac{1}{2}\rangle_z - |M_{SA} = -\frac{1}{2}\rangle_z |M_{SB} = +\frac{1}{2}\rangle_z]. \quad (1)$$

Measuring the z component of the spin of particle A , when A and B are sufficiently separated so that they are noninteracting, we can conclude that the spin component of particle B is exactly opposite to that of particle A , since $J_{AB}=0$. Assume that particle A is found to have spin up along the z axis. The state vector (1) then becomes

$$|\psi\rangle \rightarrow |\psi\rangle = |M_{SA} = +\frac{1}{2}\rangle_z |M_{SB} = -\frac{1}{2}\rangle_z. \quad (2)$$

The state vector (2) describes a particle B with a definite spin component in the z direction, but with spin components, in the x and y directions

randomly fluctuating. The J_{AB} of (2), in particular, can be zero or one. For example, the state vector for $J_{AB}=1$, $M_{JAB}=0$ is

$$|\psi\rangle = (2)^{-1/2} [|M_{SA} = +\frac{1}{2}\rangle_z |M_{SB} = -\frac{1}{2}\rangle_z + |M_{SA} = -\frac{1}{2}\rangle_z |M_{SB} = +\frac{1}{2}\rangle_z]. \quad (2')$$

On the other hand, (1) describes a particle B with all its spin components correlated to those of particle A such that $J_{AB}=0$. The randomization of the x and y spin components of B in (2) was accomplished with no direct interaction with B .

The indeterminacy principle has been regarded as representing the disturbance of an observed system by a measuring apparatus. This interpretation leads to no difficulty for a single particle. For example, there is no problem according to this interpretation in explaining the random fluctuation of the x and y spin components of A in (2). In general this view implies that the definiteness of any desired component of spin, as well as the indefiniteness of the other two components, is a potentiality which is realized with the aid of a spin measuring apparatus. This interpretation does not explain how particle B realizes the potentiality of the indefiniteness of the x and y components of its spin without any apparatus having acted upon it.

Furry⁴ suggested a possible resolution of the above Einstein-Podolsky-Rosen paradox (EPRP). Essentially the suggestion was that before the measurement of particle A the state vector has the form (2) and not (1), with the direction z in (2) in an arbitrary direction. For many events, $\vec{a}$ is distributed uniformly over 4π sr. Bohm and Aharonov,³ however, showed that the Wu-Shaknov experiment⁵ of positron annihilation into two photons negated this possibility, at least for the space-time intervals covered by the experiment.

The polarization correlation $C(\vec{a}, \vec{b})$ predicted by Eq. (1) is

$$C(\vec{a}, \vec{b}) = \sin^2(\frac{1}{2}\theta), \quad (3)$$

where $C(\vec{a}, \vec{b})$ is the probability of particle A having polarization $M_{s_A} = +\frac{1}{2}$ with respect to $\vec{a}$, and particle B having polarization $M_{s_B} = +\frac{1}{2}$ with respect to $\vec{b}$, θ being the angle between $\vec{a}$ and $\vec{b}$. Until now, relation (3) has not been verified for even large space-time intervals.

It is of great interest to investigate EPRP and $C(\vec{a}, \vec{b})$, with as wide a range of parameters as possible. Experiments using massless spin-1 particles (photons)^{6,7} as well as spin- $\frac{1}{2}$ particles (protons)⁸⁻¹⁰ have been proposed. A reaction called "proximity correlation" (PC) is studied here and is shown to provide examples of reactions suitable for investigating EPRP, in particular for ultrasmall space-time intervals. In Sec. II polarization measurements in ultrasmall space-time intervals are discussed. In Sec. III proximity correlation is defined, and examples are analyzed. Finally, in Sec. IV the decay of a $0^- (J^P)$ meson into two vector mesons is shown to be an example of proximity correlation capable of investigating EPRP for ultrasmall space-time intervals.

II. MEASUREMENT OF POLARIZATION IN ULTRASMALL SPACE-TIME INTERVALS

Conceptually, a polarization analyzer is most conveniently thought of as a Stern-Gerlach apparatus. For spin- $\frac{1}{2}$ particles the inhomogeneous field of the magnets splits the beam into two lines, $M = +\frac{1}{2}, -\frac{1}{2}$, with the quantization axis being parallel to the direction of the magnetic field. A detector is placed on one of the lines, for example, the one corresponding to $M = +\frac{1}{2}$. For spin-1 particles the inhomogeneous field splits the beam into three lines, $M = +1, 0, -1$. Again a detector is placed on one of the lines, for example, the central one ($M = 0$).

The Stern-Gerlach apparatus is obviously inconvenient if one wishes to make polarization measurements for ultrasmall space-time intervals. When the particle is unstable we will investigate another

method that can be used not only for large, but also for ultrasmall space-time intervals.

Let us take the example of an unstable spin-1 particle decaying into two spinless particles. (Though we will confine our discussion to this example, it is in fact fairly general.) The matrix element $T(M)$ for a spin-1 particle having a magnetic quantum number M with respect to a quantization axis $\vec{a}$ is¹¹

$$T(M) = G\vec{p}_1 \cdot \vec{\epsilon}(M), \quad (4)$$

where $G = G(s)$ is a function of the effective mass $s = (p_1 + p_2)^2$, p_1 and p_2 being four-vectors of the spinless decay particles, and $\vec{p}_1, \vec{p}_2$, their respective three-vectors. A term $G'\vec{p}_2 \cdot \vec{\epsilon}(M)$ does not appear explicitly, since $\vec{p}_2 = -\vec{p}_1$.

Taking the three Cartesian vectors as basic spin states and the quantization axis $\vec{a}$ in the z direction, we have

$$\vec{\epsilon} = \begin{pmatrix} \epsilon_x(M) \\ \epsilon_y(M) \\ \epsilon_z(M) \end{pmatrix}, \quad (5)$$

where

$$\vec{\epsilon}(\pm 1) = (2)^{-1/2} \begin{pmatrix} \mp 1 \\ -i \\ 0 \end{pmatrix} \quad (6)$$

and

$$\vec{\epsilon}(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \quad (7)$$

When the quantization axis $\vec{a}$ is in the xz plane ($\phi = 0$) at an angle θ with the z axis we have

$$\vec{\epsilon}(\pm 1) = (2)^{-1/2} \begin{pmatrix} \mp \cos \theta \\ -i \\ \pm \sin \theta \end{pmatrix}, \quad (8)$$

$$\vec{\epsilon}(0) = \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix}. \quad (9)$$

For $\vec{p}_1$ also in the xz plane ($\phi = 0$) and at an angle θ with the z axis

$$\vec{p}_1 = |\vec{p}_1| \begin{pmatrix} \sin \theta \\ 0 \\ \cos \theta \end{pmatrix}. \quad (10)$$

Substituting (8), (9), and (10) into (4) we obtain

$$\begin{aligned} T(0) &= G|\vec{p}_1|, \\ T(\pm 1) &= 0. \end{aligned} \quad (11)$$

Thus only spin-1 particles having $M=0$ with respect to the quantization axis $\vec{a}$ can emit a decay particle in the direction $\vec{a}$.

Relations (11) can now be used as a basis for a polarization analyzer which can be compared with a Stern-Gerlach apparatus. For this comparison let us assume for the moment that $v\tau$ is approximately several centimeters, where v is the velocity of the spin-1 particles and τ is their lifetime. We make this assumption so as to confine the analyzer, which we call a PC analyzer, to reasonable dimensions.

In a PC analyzer, similar to the Stern-Gerlach apparatus, we can use slits to define the entering beam, which we take to have intensity I_0 . The particles will decay in our analyzer at a distance $v\tau$ from the slits. At a distance $\gg v\tau$ in a direction $\vec{a}$ in the c.m. system of the spin-1 particles, we place a detector of solid angle $d\Omega_{PC}$ for a spinless decay particle. Again taking $\vec{a}$ as our polarization axis we have from (11) that only $M=0$ spin-1 particles are detected.

The transmission of the analyzer, T_{PC} , is then

$$\begin{aligned} T_{PC} &\equiv \frac{R_a(M=0) + [R_a(M=+1) + R_a(M=-1)]}{I_0} \\ &= \frac{R_a(M=0)}{I_0} \ll 1, \end{aligned} \quad (12)$$

where $R_a(M=0)$, $R_a(M=+1)$, $R_a(M=-1)$ are the count rates for spin-1 particles with $M=0, +1, -1$ with respect to the quantization axis $\vec{a}$.

The angular response of the analyzer $R_{PC}(\theta)$ to a polarized beam, all particles of which are in the state $M=0$ with respect to the z axis, is from (4), (7), and (10)

$$R_{PC}(\theta) \propto T^2 = G^2 |\vec{p}_1|^2 \cos^2 \theta \quad (13)$$

$$\propto \frac{d\sigma(\theta)}{d\Omega}, \quad (14)$$

where θ is the angle that the polarization axis $\vec{a}$ makes with the z axis, and $d\sigma(\theta)/d\Omega$ is the differential cross section for the emission of the spinless decay particle in the direction $\vec{a}$. The maximum value of T_{PC} for a polarized beam, $\bar{T}_{PC}$, is, from (13) and (14), and with θ equal to zero,

$$\begin{aligned} \bar{T}_{PC} &= \frac{\cos^2(\theta=0)d\Omega_{PC}}{\cos^2 \theta d\Omega} \\ &= \frac{3}{4\pi} d\Omega_{PC}. \end{aligned} \quad (15)$$

The efficiency of the analyzer $\bar{E}_{PC}$ to a polarized beam, again all particles of which are in the state

$M=0$ with respect to the z axis, is

$$\bar{E}_{PC} = \frac{R_a(M=0) - \frac{1}{2}[R_a(M=+1) + R_a(M=-1)]}{R_a(M=0) + \frac{1}{2}[R_a(M=+1) + R_a(M=-1)]} \quad (16)$$

or

$$\bar{E}_{PC} = 1. \quad (17)$$

In a Stern-Gerlach apparatus the $M=0$ component of a spin-1 beam can be cleanly separated from the $M=+1$ components. The apparatus can also be designed to have a transmission approaching unity. We thus have

$$\bar{E}_{SG} = 1, \quad (18)$$

$$\bar{T}_{SG} \cong 1. \quad (19)$$

The angular response of a Stern-Gerlach apparatus to a polarized beam of $M=0$ particles with respect to the z axis is similar to (13),

$$R_{SG}(\theta) \propto \cos^2 \theta. \quad (20)$$

It is thus seen that a PC analyzer is very similar to a Stern-Gerlach apparatus. A PC analyzer could in fact be used to analyze the polarization of a spin-1 beam, if $v\tau$ is of the order of a few centimeters.

The PC analyzer described above can be considerably improved, however, in the following way: The slits that were used to define the direction of the incoming beam and its intensity I_0 , and the detector of solid angle $d\Omega_{PC}$, can both be replaced by a single apparatus, a bubble chamber. Assuming that the spinless decay particle is charged and leaves a track in the bubble chamber, the number of spinless decay particles emitted in a direction $\vec{a}$, of solid angle $d\Omega_{PC}$, and differential cross section $d\sigma(\vec{a})/d\Omega$, can be measured. The direction and velocity of the parent spin-1 particle can be determined by measuring the direction and energy of the second spinless decay particle emitted in coincidence. The intensity I_0 is determined by counting the total number of events.^{11a}

If two spin-1 particles, A and B , are emitted in the same reaction, the polarization correlation $C(\vec{a}, \vec{b})$ can be determined, similar to (14), as

$$C(\vec{a}, \vec{b}) \propto \frac{d^2\sigma(\vec{a}, \vec{b})}{d\Omega_{A_1} d\Omega_{B_1}}, \quad (21)$$

where $C(\vec{a}, \vec{b})$ is the probability for particle A having polarization $M_{SA}=0$ with respect to $\vec{a}$ and particle B having polarization $M_{SB}=0$ with respect to $\vec{b}$. The spinless decay particles of A and B are A_1, A_2 and B_1, B_2 , respectively.

Of primary importance is the fact that a bubble chamber as a PC analyzer enables study of the polarization of spin-1 particles having very short lifetimes. In particular, we can make polarization

correlation measurements for spin-1 particles, such as vector mesons, with lifetimes $\sim 10^{-22}$ sec.

III. PROXIMITY CORRELATION

Proximity correlation is defined as the reaction in which an incident particle (I) strikes a target nucleus (T), forming a composite state (AB) of angular momentum J_{AB} , which breaks up into unstable particles (A) and (B) having spins S_A and S_B , respectively. Particle A decays into particles A_1 and A_2 , and particle B decays into particles B_1 and B_2 . The spins of particles A_1 , A_2 , B_1 , and B_2 are S_{A_1} , S_{A_2} , S_{B_1} , and S_{B_2} , respectively. Particles A and B have the relative orbital angular momentum L . The range of interactions is sufficiently short such that the particle description is valid. What is important is the measurement of $d^2(\tilde{a}, \tilde{b})/d\Omega_{A_1}d\Omega_{B_1}$, the differential cross section for particle A_1 being emitted in direction $\tilde{a}$ and solid angle $d\Omega_{A_1}$, and particle B_1 being emitted in the direction $\tilde{b}$ and solid angle $d\Omega_{B_1}$. The directions $\tilde{a}, \tilde{b}$ are evaluated in the c.m. systems of A, B , respectively. (We saw in Sec. II that this differential cross section is related to the polarizations of particles A and B .) Proximity correlation is similar to a reaction previously analyzed for the measurement of ultrashort lifetimes and cross sections of unstable particles.¹²

As an example of proximity correlation assume $J_{AB} = L = S_{A_1} = S_{A_2} = S_{B_1} = S_{B_2} = 0$ and $S_A = S_B = 1$. The state vector of the coupled system, in the c.m. system of $A-B$ with respect to a quantization axis $\tilde{a} = \tilde{z}$, is

$$|\psi\rangle = (3)^{-1/2} [|M_{S_A} = +1\rangle_z |M_{S_B} = -1\rangle_z + |M_{S_A} = -1\rangle_z |M_{S_B} = +1\rangle_z - |M_{S_A} = 0\rangle_z |M_{S_B} = 0\rangle_z]. \quad (22)$$

Let $\tilde{a} (= \tilde{z})$ and $\tilde{b}$ be the polarization axes of PC analyzers described in Sec. II for particles A_1, B_1 , respectively. Assume particle A decays, particle A_1 being emitted in the direction $\tilde{z}$ and solid angle $d\Omega_{A_1}$, and detected by the PC analyzer. Assume particle B has not yet had its polarization determined. From Sec. II we know for certain that system A (A_1, A_2) has $M_{S_A} = 0$. The state vector is then, instead of (22),

$$|\psi\rangle = |M_{S_A} = 0\rangle_z |M_{S_B} = 0\rangle_z. \quad (23)$$

The state vector (23) describes a particle B with a definite spin component in the z direction, but with spin components in the x and y directions randomly fluctuating. The J_{AB} of (23) is no longer zero, but has the probability of being also $J_{AB} = 2$. For example, the state vector for $J_{AB} = 2$, $M_{J_{AB}} = 0$ is

$$|\psi\rangle = (6)^{-1/2} [|M_{S_A} = +1\rangle_z |M_{S_B} = -1\rangle_z + |M_{S_A} = -1\rangle_z |M_{S_B} = +1\rangle_z + 2 |M_{S_A} = 0\rangle_z |M_{S_B} = 0\rangle_z]. \quad (24)$$

On the other hand, the state vector (22) describes a particle B with all its spin components correlated to those of particle A , such that $J_{AB} = 0$. The randomization of the x and y spin components of B in (23) has been accomplished with no direct interaction with particle B .

The above discussion is seen to be identical with that of EPRP in Sec. I. Proximity correlation thus provides examples of reactions capable of investigating EPRP.

In order to investigate EPRP experimentally, we wish to derive relations such as (3). For a spin-1 particle, the basis vectors $|M = +1\rangle_a$, $|M = 0\rangle_a$, and $|M = -1\rangle_a$ with respect to a quantization axis $\tilde{a}$ for a spin-1 particle, in terms of $|M = +1\rangle_b$, $|M = 0\rangle_b$, and $|M = -1\rangle_b$ with respect to a quantization axis $\tilde{b}$, are, using (5)–(9),

$$|M = +1\rangle_a = -(2)^{-1/2} \sin \theta |M = 0\rangle_b + \cos^2(\frac{1}{2}\theta) |M = +1\rangle_b + \sin^2(\frac{1}{2}\theta) |M = -1\rangle_b, \quad (25)$$

$$|M = 0\rangle_a = \cos \theta |M = 0\rangle_b + (2)^{-1/2} \sin \theta |M = +1\rangle_b - (2)^{-1/2} \sin \theta |M = -1\rangle_b, \quad (26)$$

$$|M = -1\rangle_a = (2)^{-1/2} \sin \theta |M = 0\rangle_b + \sin^2(\frac{1}{2}\theta) |M = +1\rangle_b + \cos^2(\frac{1}{2}\theta) |M = -1\rangle_b, \quad (27)$$

where θ is the angle between $\tilde{a}$ and $\tilde{b}$. Substituting (26) into (23), we obtain

$$|\psi\rangle = \cos \theta |M_{S_A} = 0\rangle_a |M_{S_B} = 0\rangle_b + (2)^{-1/2} \sin \theta |M_{S_A} = 0\rangle_a |M_{S_B} = +1\rangle_b - (2)^{-1/2} \sin \theta |M_{S_A} = 0\rangle_a |M_{S_B} = -1\rangle_b. \quad (28)$$

The polarization correlation $C(\tilde{a}, \tilde{b})$ for this case is then

$$C(\tilde{a}, \tilde{b}) \propto |{}_a\langle M_{S_A} = 0 | {}_b\langle M_{S_B} = 0 | \psi \rangle|^2 \propto \frac{d^2\sigma(\tilde{a}, \tilde{b})}{d\Omega_{A_1}d\Omega_{B_1}} \propto \cos^2 \theta. \quad (29)$$

As a second example, assume $L = 1$ instead of zero, and again $J_{AB} = S_{A_1} = S_{A_2} = S_{B_1} = S_{B_2} = 0$, $S_A = S_B = 1$. The state vector of the coupled system in the LS coupling scheme, in the c.m. system of $A-B$ with respect to a quantization axis $\tilde{a} = \tilde{z}$, is

$$|\psi\rangle = (6)^{-1/2} [|M_{S_A} = 0\rangle_z |M_L = +1\rangle_z |M_{S_B} = -1\rangle_z - |M_{S_A} = -1\rangle_z |M_L = +1\rangle_z |M_{S_B} = 0\rangle_z - |M_{S_A} = +1\rangle_z |M_L = 0\rangle_z |M_{S_B} = -1\rangle_z]$$

$$\begin{aligned}
& + |M_{S_A} = -1\rangle_z |M_L = 0\rangle_z |M_{S_B} = +1\rangle_z \\
& + |M_{S_A} = +1\rangle_z |M_L = -1\rangle_z |M_{S_B} = 0\rangle_z \\
& - |M_{S_A} = 0\rangle_z |M_L = -1\rangle_z |M_{S_B} = +1\rangle_z].
\end{aligned}
\tag{30}$$

If A decays and A_1 is emitted in the direction $\hat{z}$ and solid angle $d\Omega_{A_1}$, and detected by a PC analyzer, while B has not yet had its polarization determined, we have

$$\begin{aligned}
|\psi\rangle \rightarrow |\psi\rangle = (2)^{-1/2} [|M_{S_A} = 0\rangle_z [|M_L = +1\rangle_z |M_{S_B} = -1\rangle_z \\
- |M_L = -1\rangle_z |M_{S_B} = +1\rangle_z].
\end{aligned}
\tag{31}$$

Again J_{AB} of (31) is no longer zero and may also have, for example, $J_{AB} = 2$. The two state vectors corresponding to $J_{AB} = 2$, $M_{AB} = 0$ are

$$\begin{aligned}
|\psi\rangle = \frac{1}{2} [& |M_{S_A} = +1\rangle_z |M_L = 0\rangle_z |M_{S_B} = -1\rangle_z \\
& + |M_{S_A} = 0\rangle_z |M_L = +1\rangle_z |M_{S_B} = -1\rangle_z \\
& - |M_{S_A} = -1\rangle_z |M_L = 0\rangle_z |M_{S_B} = +1\rangle_z \\
& - |M_{S_A} = 0\rangle_z |M_L = -1\rangle_z |M_{S_B} = +1\rangle_z].
\end{aligned}
\tag{32}$$

or

$$\begin{aligned}
|\psi\rangle = (12)^{-1/2} [& +2 |M_{S_A} = +1\rangle_z |M_L = -1\rangle_z |M_{S_B} = 0\rangle_z \\
& -2 |M_{S_A} = -1\rangle_z |M_L = +1\rangle_z |M_{S_B} = 0\rangle_z \\
& + |M_{S_A} = +1\rangle_z |M_L = 0\rangle_z |M_{S_B} = -1\rangle_z \\
& - |M_{S_A} = 0\rangle_z |M_L = +1\rangle_z |M_{S_B} = -1\rangle_z \\
& - |M_{S_A} = -1\rangle_z |M_L = 0\rangle_z |M_{S_B} = +1\rangle_z \\
& + |M_{S_A} = 0\rangle_z |M_L = -1\rangle_z |M_{S_B} = +1\rangle_z].
\end{aligned}
\tag{33}$$

A discussion of the above equations leads us again to EPRP.

Substituting (25) and (27) into (31) we obtain for the polarization correlation, $C(\vec{a}, \vec{b})$,

$$\begin{aligned}
C(\vec{a}, \vec{b}) & \propto |_a \langle M_{S_A} = 0 | |_b \langle M_{S_B} = 0 | \psi \rangle|^2 \\
& \propto \frac{d^2 \sigma(\vec{a}, \vec{b})}{d\Omega_{A_1} d\Omega_{B_1}} \\
& \propto \sin^2 \theta.
\end{aligned}
\tag{34}$$

The above relation can be tested experimentally. An experiment capable of doing this is described in the next section.

IV. EXAMPLE OF PROXIMITY CORRELATION

Let us examine the decay of a $0^-(J^P)$ meson into two vector mesons, such as ρ 's, as an example of proximity correlation. Very few of the resonances above the two-vector-meson threshold have been identified and still fewer have had their J^P determined.¹³ A 0^- meson appears to exist slightly below threshold [the $E(1422)$ meson], and undoubtedly a 0^- meson will be found above threshold.

According to the terminology of Sec. III we have A and B as ρ mesons, A_1, A_2, B_1, B_2 as π mesons, $J_{AB} = S_{A_1} = S_{A_2} = S_{B_1} = S_{B_2} = 0$, and $S_A = S_B = L = 1$. The above reaction has the characteristics of the second example of proximity correlation studied in Sec. III.

The ϕ and ω can also be used as the vector mesons. A general discussion of three-pion decays was given by Zemach.¹⁴ If $\vec{p}_1, \vec{p}_2, \vec{p}_3$ are the momenta of the three pions, then the matrix element for the decay of a vector meson is proportional to $\vec{q} = \vec{p}_1 \times \vec{p}_2 = \vec{p}_2 \times \vec{p}_3 = \vec{p}_3 \times \vec{p}_1$. In the above, where we used the direction of a particular π meson, in the case of a three-pion decay we can substitute $\vec{q}$.

From the differential cross section, $d^2\sigma(\vec{a}, \vec{b})/d\Omega_{A_1} d\Omega_{B_1}$, we wish to examine, to as great an accuracy as possible, the predicted relation (34). In a breakdown of (34), $C(\vec{a}, \vec{b}) = C(\theta)$, where $F(\theta)$ could be quite a general function of θ . For example, in a particular hidden variable theory,¹⁵ $C(\theta) = 0$ for $\theta < 10^\circ$. Similarly, if a Furry-type solution^{4,3} of EPRP (see Sec. I) holds for ultrasmall space-time intervals, (34) is not true, in particular, $C(\theta = 0^\circ) \neq 0$. The number of events required to give a fair check of (34), for general possible deviations, is estimated to be approximately ten thousand.

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