Resonant conversions of extremely high energy neutrinos in dark matter halos

R. Horvat
''Rudjer Bošković'' Institute, P.O. Box 1016, 10001 Zagreb, Croatia
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We study the effect of adiabatically resonant conversion in galactic halos of neutrinos at the highest energies ($\sim 10^{20}$–10^{22} eV), when the ν source is in the center of a galaxy. Using the standard neutrino properties and the standard cosmological scenario for hot dark matter, we find that interesting conversions may take place just for neutrino parameters relevant to the solar and atmospheric neutrino problem. The effect is due to the large enhancement in the ν density in galactic halos and to the form of the effective matter potential both below and above the pole of the Z resonance. [S0556-2821(99)03210-5]

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It has recently been suggested [1–3] that annihilation of extremely high energy (EHE) ($\gtrsim 10^{21}$ eV) primary cosmic neutrinos on a background of clustered relic neutrinos may be able to account for the detected flux of cosmic rays of the highest energy ($\gtrsim 10^{20}$ eV) [4,5]. The Fly's Eye [4] and Akemo Giant Air Shower Array (AGASA) [5] experiments confirmed the existence of protonlike cosmic rays with energies $\gtrsim 10^{20}$ eV which, being well above the Greisen-Zatsepin-Kuzmin [6] cutoff, may create a rather serious problem in terms of a possible astrophysical explanation. In the above scenario [1–3] this problem is elegantly circumvented by the fact that the highest energy cosmic rays may actually be generated very close to Earth, possibly in our hot-dark-matter galactic halo [1,2]. In order that this new mechanism for generating the highest energy cosmic rays have a chance to work, a relic electronvolt mass neutrino would be needed as the gravitational clustering of relic cosmic neutrinos is more efficient for heavier and slower species. If the Hubble and age parameter allow for a critical-density universe [7] as suggested by inflation, then the mixed cold + hot dark matter model with $m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2.5$ eV (giving $\Omega_\nu \simeq 0.2$) was found to be consistent with all available observations. Indeed this model was shown to agree well with observations of cosmic background temperature anisotropy on large scales performed by the Cosmic Background Explorer (COBE) satellite [8] as well as with those on the number density of clusters [9]. Although in accordance with the theory of the formation of cosmic structure, the scenario is beset by the problem [3] that one requires a very large energy generation rate of high energy neutrinos (which poses a difficult astrophysical problem) and the fact that $Z,W^{\pm}$ decay produces a large population of π^0 secondaries and hence we have more photons (from π^0 decay) than protons.

The interaction of the EHE neutrino with the relic one had been studied first in [10] and reconsidered afterwards in [11]. It was shown [11] that neutrinos with the standard electroweak interaction can travel through cosmic distances with a little hindrance, and only those neutrinos with energies close to the Z resonance value may be absorbed while crossing galactic halos, leading to a dip in the spectrum around $E_{res} \simeq 10^{21}$ eV. This idea was then taken over in [1,2] to show that secondaries (such as protons and photons) of the EHE-relic-neutrino annihilation in our galactic halo may be responsible for the observed high energy cosmic ray energy spectrum.

Here we examine whether the condition for adiabatically resonant conversion of these EHE neutrinos while traversing the halo around the source is met, if the oscillation parameters are taken to be just those responsible for the solution to the solar and atmospheric neutrino anomaly. In the following we will be considering ν_e-$\nu_{\tau(\mu)}$ and ν_e-ν_s (ν_s being a sterile neutrino) as well as ν_μ-ν_s oscillations.

Assuming that the halo is nearly round we take for the halo density a density profile such as

$$\rho(r) = \frac{\rho(r=0)}{1+(r/a)^2}, \tag{1}$$

where a is the core radius of the halo. By considering the values for our local galactic halo it was found [1] that, notwithstanding the fact that light neutrinos constitute a smaller fraction of the density of the galactic halo [12] than they do of the cosmological density in a typical mixed-dark-matter scenario [13], the neutrino number density inside the core might be a factor $\sim 10^5$–10^7 larger than the average cosmic ν density. Hence, $N_\nu \simeq 10^7$–10^9 cm^{-3} was assumed in [1], together with a relic neutrino mass $m_\nu = 10$ eV. In the present work, however, we treat N_ν as a free parameter, presenting first the conditions under which conversion may occur, and then adding a discussion of whether or not these conditions may be met in the ''realistic neutrino halo'' allowed by the Tremaine-Gunn phase-space constraint [14]. In addition, we assume that the halo consists predominantly of heavier (ν_μ and ν_τ) neutrinos with $m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2.5$ eV. It is just the fact that the background is *not* flavor symmetric that allows us to study not only active-sterile but active-active neutrino oscillations as well.

Effects of a medium on neutrino propagation are determined by the difference of potentials, whose standard-model contribution in the context of thermal field theory may readily be obtained from the relevant thermal self-energies of a neutrino: charged current, neutral current, and tadpole. Of these, only the neutral-current and tadpole contributions are of relevance here as the halo background consists of pure neutrinos (up to some of whatever dark-matter particles which appear as a major constituent of the halo and which are irrelevant for the present discussion). The contribution from the tadpole diagram, in addition to being a constant independent of the external neutrino momentum and hence irrelevant (see below), is always proportional to the asymmetry for certain neutrino species. Since the cosmic neutrino background is likely to be CP symmetric, its contribution is completely negligible, leaving us with the neutral-current contribution only.

Since the ''target'' neutrinos are massive ($m_\nu \simeq 2.5$ eV), the Z boson may be considered ''massless'' always when $s \simeq 2E_\nu m_\nu \gg M_Z^2$. This corresponds to energies in the laboratory frame $E_\nu \gtrsim 10^{21}$ eV. In the opposite limit, $s \ll M_Z^2$, the Z boson should be considered ''massive'' and the usual contact approximation for the Z propagator is adequate. Using the real-time version of thermal field theory, we find by explicit calculation a contribution to the effective matter potentials in the ''massless'' limit as

$$V_e \simeq 0.02(T_\nu^2/E_\nu)\left(\sum_{i=2,3} |U_{ei}|^2\right), \tag{2}$$

$$V_\mu \simeq V_\tau \simeq 0.02(T_\nu^2/E_\nu). \tag{3}$$

Although the heaviest neutrinos are so massive that they are firmly nonrelativistic today, they still should be described by the relativistic distribution function as their total number density is fixed at about 100 cm^{-3} in the uniform, nonclustered background and at about a factor of few larger in the galactic halo. Hence the temperature T_ν from Eqs. (2) and (3) is always related to the neutrino number density via the relation $T_\nu \simeq (N_\nu/0.18)^{1/3}$. Notice, in addition, that the appearance of a unitary mixing matrix in the vacuum U in the expression for V_e in Eq. (2) is due to the fact that internal lines of the thermal self-energy graph should necessarily be written in the (vacuum) mass eigenstate basis. If in a world with just three neutrinos all of them have nearly the same mass, of about ~ 1.5 eV, then the suppression due to $(\Sigma_i |U_{ei}|^2)$ will be absent.

Let us first consider the values of the parameters of the small mixing angle ν_e-ν_s solution to the solar neutrino problem which lie in the region [15]

$$2.9\times 10^{-6} \lesssim \Delta m_{es}^2/eV^2 \lesssim 7.7\times 10^{-6},$$
$$3.5\times 10^{-3} \lesssim \sin^2 2\theta \lesssim 1.4\times 10^{-2}, \tag{4}$$

where $\Delta m_{es}^2 = m_s^2 - m_e^2$ and θ is the vacuum mixing angle of the ν_e-ν_s mixing system. The Mikheyev-Smirnov-Wolfenstein (MSW) resonant condition for all neutrinos with $E_\nu > 10^{21}$ eV now reads

$$0.04T_\nu^2\left(\sum_i |U_{ei}|^2\right) \simeq \Delta m_{es}^2. \tag{5}$$

Note that Eq. (5) is independent of neutrino energy. For the central value (5×10^{-6}) in Eq. (4) we find that the MSW resonance for ν_e-ν_s oscillations occurs within the core radius of the halo if the neutrino number density inside the core of a galaxy is $3\times 10^7(\Sigma_i |U_{ei}|^2)^{-3/2}$ cm^{-3}. The same occurs also for $\overline{\nu_e} \to \overline{\nu_s}$ oscillations as $V_{\overline{e}} > 0$.

The condition for unsuppressed oscillations is that the oscillation frequency must be real; otherwise, the system is critically overdamped, the oscillations would be fully incoherent, and hence in fact there will be no oscillations. This translates into the requirement that the oscillation frequency should be much greater than the total neutrino interaction rate Γ:

$$2\pi\omega_{osc} > \frac{1}{2}\Gamma. \tag{6}$$

At the resonance this gives

$$\frac{\Delta m^2 \sin 2\theta}{2E_\nu} > \frac{1}{2}\Gamma. \tag{7}$$

For $E_\nu = 10^{22}$ eV we have $(\Delta m^2 \sin 2\theta)/2E_\nu \simeq 2\times 10^{-29}$ eV, whereas $(1/2)\Gamma = (1/2)N_\nu\sigma \simeq 5\times(10^{-32}\text{--}10^{-30})$ eV for $N_\nu \simeq 10^7\text{--}10^9$ cm^{-3}. The latter estimate follows from [11], where the total neutrino absorption cross section was calculated to be $\sigma(E_\nu = 10^{22}\text{ eV}) \simeq 5\times 10^{-34}$ cm^2. Thus ν_e-ν_s oscillations in the halo of a galaxy can be considered as unsuppressed for $E_\nu \sim 10^{22}$ eV. However, level crossing alone is not sufficient for a complete conversion. The extent of conversion is determined by a degree of adiabacity; if there is only level crossing but no sufficient adiabacity at the resonance, then there will be no complete conversion of electron neutrinos into sterile neutrinos. The condition for adiabatic transition at the resonance may be written in our case as

$$\frac{3\Delta m^2 \sin^2 2\theta}{4E_\nu \cos 2\theta |dN_\nu/N_\nu dr|_0} \gg 1, \tag{8}$$

where the subscript 0 denotes that this quantity should be evaluated at the resonance. Using the distribution (1) one can easily show that for $E_\nu \sim 10^{22}$ eV adiabatically resonant conversion may only take place in the regions very close to the center of a galaxy, namely, for

$$r_0 \lesssim 2\times 10^{-3} a \simeq \mathcal{O}\quad(10\text{ pc}). \tag{9}$$

For the last estimate in Eq. (9) we have taken $a \simeq 10$ kpc [16]. Equation (9) is due to the fact that the adiabacity parameter, as defined in Eq. (8), can be larger than unity for such EHEs only where the flatness of the distribution (1) is appreciable. There is also another solution for r_0 from Eq. (8) but it is located well outside the core where the number density is smaller; besides, Eq. (1) is not a good fit to $\rho(r)$ at $r \gg a$ [17]. However, even the restriction (9) is consistent with the models [18,19] in which neutrino production takes place in active galactic nuclei (AGN) jets. In these models protons are accelerated in the jets that stream highly collimated out of their cores to distances of several parsec. In the jets the neutrinos are Lorentz boosted to energies much higher than in AGN cores, which has important implications for their detection. Notice, for comparison, that V_e obtained here is always larger [for $(\Sigma_i |U_{ei}|^2)^{3/2} \gtrsim 10^{-2}$ and N_ν

$\simeq 10^7$ cm^{-3}] than the effective matter potential in AGN cores whose magnitude was estimated to be 10^{-29}–10^{-32} eV [20]. One should stress, however, here that as $\rho(r)$ has a flat region for $r < r_0$, the neutrinos are always produced at densities very close to the resonant one. Thus, the mixing angle at the production point $\theta_{mp} \simeq \pi/4$ rather than $\theta_{mp} \to \pi/2$, and consequently the oscillation probability $P(\nu_e \to \nu_e)$ is centered around 1/2 rather than around $\sin^2\theta$. One can therefore speak of partial ($\gtrsim 50\%$) conversion.

Let us now turn to active-active $\nu_e \to \nu_{\tau(\mu)}$ oscillations. Notice that there is no resonant transition in the ''massless'' limit as $\Delta m^2_{e\tau(\mu)} > 0$, but $V_e - V_{\tau(\mu)} < 0$ [see Eqs. (2) and (3)]. The same holds for $\bar{\nu}_e \to \bar{\nu}_{\tau(\mu)}$ oscillations as $V_{\bar{e}} - V_{\overline{\tau(\mu)}} < 0$. The situation turns out to be completely different if we concentrate on the ''massive'' limit for the Z-boson propagator. In this case the effective matter potential is given as a sum of the finite-density and finite-temperature contribution

$$V(E_\nu) = V_\rho + V_T(E_\nu), \tag{10}$$

where [for $\nu_\tau(\nu_\mu)$]

$$V_\rho \simeq \sqrt{2} G_F N_\gamma L_\nu \tag{11}$$

and

$$V_T(E_\nu) \simeq -24 G_F^2 T_\nu^4 E_\nu . \tag{12}$$

In Eq. (11), N_γ represents the number density of photons and L_ν the tau- (muon-) like asymmetry. We are now in position to examine a possibility for resonant conversion as we have $V_e - V_{\tau(\mu)} \simeq -V_\tau > 0$. It is to be noted that for $L_\nu = 10^{-10}$ and $E_\nu \sim 10^{20}$ eV, $|V_T|$, which represents a contribution coming from the low energy tail of the Z resonance, is about seven orders of magnitude larger than V_ρ for $N_\nu \simeq 10^7$ cm^{-3}, and therefore from Eq. (10) $V \simeq V_T < 0$.

Let us now concentrate to the hypothesis of two-neutrino vacuum oscillations which was found to provide a good quality description of the solar neutrino data for values of the two oscillation parameters belonging approximately to the region [21]

$$5.0 \times 10^{-11} \lesssim \Delta m^2_{e\tau(\mu)}/eV^2 \lesssim 10^{-10}, \quad 0.65 \lesssim \sin^2 2\theta \lesssim 1.0. \tag{13}$$

The resonant energy for the central values in Eq. (13) (7.5×10^{-11}, $\sin^2 2\theta \simeq 0.8$) turns out to lie in the range 10^{19}–10^{21} eV for the input neutrino number density 10^9–10^7 cm^{-3}. The same occurs also for $\bar{\nu}_e \to \bar{\nu}_{\tau(\mu)}$ oscillations as $V_{\bar{e}} - V_{\overline{\tau(\mu)}} > 0$. Here we would like to stress the importance of ν_e-ν_τ oscillations as the initial fluxes of the ultrahigh energy neutrinos originating from AGNs are estimated to have a ratio [22] $\nu_\tau/\nu_\mu \lesssim 10^{-3}$ (also $\nu_e/\nu_\mu \simeq 1/2$ is expected from AGNs). Hence if an enhanced ν_τ/ν_e ratio as compared to the nonoscillation case is observed correlated to the direction of the source for ultrahigh energy neutrinos, then this may represent a piece of evidence for neutrino oscillations taking place in the halo around the source. The model of Ref. [18] with maximum energy of 10^{18}–10^{19} eV can therefore be very relevant to our scenario where level crossing occurs at a bit higher energies. We find that resonant conversion with sufficient adiabacity does occur in the part of a dark halo with

$$r_0 \lesssim \mathcal{O}\ (\text{few pc}). \tag{14}$$

This is very similar to the estimate (9) for ν_e-ν_s oscillations in the ''massless'' limit and hence still compatible to the proposed sites for neutrino production in the vicinity of AGNs.

Let us finally consider the ν_μ-ν_s [23] oscillation hypothesis that provides an appealing explanation for the large up-down asymmetry observed by SuperKamiokande for atmospheric neutrino induced μ events [24]. The oscillations ν_μ-ν_s can solve the atmospheric problem if oscillation parameters lie in the approximate range

$$3 \times 10^{-4} \lesssim \Delta m^2_{\mu s}/eV^2 \lesssim 7 \times 10^{-3}, \quad 0.82 \lesssim \sin^2 2\theta \lesssim 1.0. \tag{15}$$

We note that it has been claimed [25] recently that at present there is no constraint from cosmology to the ν_μ-ν_s solution with the aforementioned parameters.

Since $|2E_\nu V_\mu| \ll |\Delta m^2_{\mu s}|$, level crossing never occurs in a halo except when $\sin^2 2\theta$ is extremely close to unity. Rather than considering such a coincidence we note out the adiabatic condition

$$(\sin^2 2\theta)^{-1} \frac{(\Delta m^2_{\mu s})^2}{2E_\nu^2 |V_\mu|} \gg a^{-1}. \tag{16}$$

While in the ''massless'' limit with $E_\nu \sim 10^{22}$ eV the condition (16) is well satisfied for the whole range $0 \lesssim r \lesssim a$, it is always extremely well satisfied in the ''massive'' limit. It is well known [26] that actually it does not matter whether a resonance has been crossed or not while a neutrino propagates adiabatically from one region to another as a similar formula for the propagation probability applies:

$$P(\nu_\mu \to \nu_\mu) = \frac{1}{2}(1 + \cos\theta_{im} \cos\theta_{fm}). \tag{17}$$

Equation (17) describes a situation where ν_μ produced at some density near the galactic center, with mixing angle θ_{im}, propagates adiabatically through the halo to the region of diffuse neutrino background, with mixing angle θ_{fm}. Since the mass-squared difference $\Delta m^2_{\mu s}$ is always larger than the induced mass squared of the muon neutrino, the neutrino does not go through the resonance but is produced below it, and therefore we have

$$\theta_{im} \simeq \theta_{fm} \simeq \theta. \tag{18}$$

In this case the propagation probability (17) reduces just to the classical probability in the vacuum. With the parameters from Eqs. (15) we get a constraint

$$0.5 \lesssim P(\nu_\mu \to \nu_\mu) \lesssim 0.59. \tag{19}$$

We have seen that all of the above effects except that based on the ν_μ-ν_s oscillation hypothesis require at least $N_\nu \simeq 10^7$ cm^{-3}. There is, however, a strong bound based on Pauli's exclusion principle on the number density N_ν^{max} of neutrinos clustered in the halo around a galaxy. The neutrino density in the halo is approximately constrained because of this requirement (Tremaine-Gunn constraint [14]) by $N_\nu^{max} \leqslant (4\pi/3)(p_{max}/h)^3$, where $p_{max}=m_\nu v_{esc}$ is the maximum momentum for a neutrino of mass m_ν and v_{esc} is the escape velocity of bound neutrinos. If, in addition, we assume that massive neutrinos that fill up dark matter halos today result from gravitational instability of three cosmic seas of relic neutrinos, then the bound is improved by a factor of 2, giving

$$N_\nu^{max} < 1 \times 10^5 \left(\frac{m_\nu}{2.5 \text{ eV}} \right)^3 \left(\frac{v_{esc}}{500 \text{ km/s}} \right)^3 \text{ cm}^{-3}. \qquad (20)$$

It is clear from Eq. (20) that to have sufficiently large N_ν in halos where the neutrinos have typical escape velocities, larger values of m_ν would be preferable. Conversely, if we stick to the cold + hot dark matter scenario of structure formation ($m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 2.5$ eV), the increased neutrino density can be achievable only in halos where the neutrinos have large escape velocities. Notice, however, that larger values of m_ν are still tolerable by the cosmological mass bound on light neutrinos [27] ($m_{\nu_e}+m_{\nu_\mu}+m_{\nu_\tau} \lesssim 37$ eV), and thus for example $m_{\nu_\mu} \simeq m_{\nu_\tau} \simeq 10$ eV may result in $N_\nu \simeq 10^7$ cm^{-3}. The effect of the higher neutrino masses would be to lower the resonant energy E_ν^{res} (the energy which discriminates between the ''massive'' and ''massless'' limits) by a factor of few, thereby increasing the ν_e-ν_s conversion and depressing ν_e-$\nu_{\tau(\mu)}$ conversion.

In conclusion, we have considered a possibility for adiabatically resonant conversion of EHE neutrinos originating from the center of a galaxy, while traversing its dark matter halo. Owing to the large enhancement of the effective neutrino potentials at energies immediately above the pole of the Z resonance, we have found that ν_e's can resonantly be converted into unobservable ν_s's, if their parameters are consistent with the MSW interpretation of the solar neutrino data. On the other hand, because the hot dark matter around galaxies is expected not to be flavor symmetric, ν_e's can resonantly be converted into ν_τ's, if their parameters are consistent with the vacuum mixing solution of the solar neutrino anomaly. This kind of transition is especially important because there is a negligible number of ν_τ's in the initial fluxes of high energy neutrinos of astrophysical origin. Since the resonance density approaches the central neutrino production region, the above conversions turn out to be only partial. It is striking that in some proton blazar models a maximum neutrino energy is very close to the resonant energy. With regard to the Tremaine-Gunn constraint, these effects require either neutrino masses close to the cosmological mass bound or such halos where the neutrinos have large escape velocities, or both. Finally, if the parameters of active-sterile mixing system are consistent with the ν_μ-ν_s solution to the atmospheric neutrino anomaly, then neutrino propagation inside a dark matter halo is always adiabatic for $E_\nu \lesssim 10^{24}$ eV and the propagation probability simply reduces to the classical probability in the vacuum.

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