

Lorentzian condition in quantum gravity

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The wave function of the Universe is usually taken to be a functional of the three-metric on a spacelike section, Σ , which is measured. It is sometimes better, however, to work in the conjugate representation, where the wave function depends on a quantity related to the second fundamental form of Σ . This makes it possible to ensure that Σ is part of a Lorentzian universe by requiring that the argument of the wave function be purely imaginary. We demonstrate the advantages of this formalism first in the well-known examples of the nucleation of a de Sitter or a Nariai universe. We then use it to calculate the pair creation rate for sub-maximal black holes in de Sitter space, which had been thought to vanish semi-classically. [S0556-2821(99)00810-3]

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I. INTRODUCTION

The no boundary proposal [1] is formulated in terms of Euclidean path integrals. But the world we live in is Lorentzian, or at least we interpret our observations in terms of Lorentzian spacetime. One therefore has to continue the results from the Euclidean path integrals analytically to the Lorentzian regime.

The approach to quantum cosmology that has been followed in the past is to examine the behavior of the wave function, as a function of the overall scale, a , of the metric, h_{ij} , on the spacelike surface, Σ . If the dependence on a was exponential, this was interpreted as corresponding to a Euclidean spacetime, while an oscillatory dependence on a was interpreted as corresponding to a Lorentzian spacetime.

For example, in the case of Einstein gravity with a cosmological constant Λ , the path integral for the wave function of a three-sphere of radius a will be dominated by an instanton which is part of a four-sphere of radius $R_0 = \sqrt{3/\Lambda}$. In this saddlepoint approximation, the wave function will be given by e^{-I} , where I is the Euclidean action of the saddlepoint geometry; we are neglecting a prefactor. For $a < R_0$, there will be a real Euclidean geometry, bounded by the three-sphere, Σ , of radius a . The wave function, Ψ , will be 1 for $a = 0$, and will increase rapidly with a , up to $a = R_0$. For $a > R_0$, there are no Euclidean solutions with the given boundary conditions.

There are, however, two complex solutions, each of which can be thought of as half the Euclidean four-sphere, joined to part of the Lorentzian de Sitter solution. The real part of the action of these complex solutions is equal to the action of the Euclidean half-four-sphere, and is the same for all values of a . On the other hand, the imaginary part of the action comes from the Lorentzian de Sitter part of the solution, and depends on a . Thus the wave function for large a oscillates rapidly with constant amplitude.

This shows the association between an oscillatory wave function and a Lorentzian spacetime, but the distinction be-

tween exponential and oscillatory is not precise, and does not identify which part of the wave function describes which physical situation. In more complicated situations, the saddlepoint complex solutions will not separate neatly into Euclidean and Lorentzian parts. So it is not clear how to calculate the probability of Lorentzian geometries.

One might apply certain operators to the wave function to recover information about whether a given spacelike surface is part of a Lorentzian or a Euclidean spacetime. But the use of operators is cumbersome and requires the evaluation of Ψ for a range of arguments. It would be preferable if the observable geometric properties, such as the Lorentzian character of the Universe, were manifest in the argument of the wave function. The square of its amplitude would then yield a probability measure for any given set of such quantities.

We therefore want to put forward an approach which focuses on the defining characteristic of a Lorentzian geometry in the neighborhood of Σ . This is that the induced metric, h_{ij} , on Σ should be real, but the second fundamental form,

$$K_{ij} = \nabla_i n_j, \quad (1.1)$$

defined for Euclidean signature, should be purely imaginary. Here n^j is the unit normal to the surface Σ . The second fundamental form is also called the extrinsic curvature of the surface Σ in the manifold M . It can be regarded as the derivative of the metric, h_{ij} , on Σ , as Σ is moved in its normal direction in M . Thus requiring the second fundamental form to be purely imaginary means that h_{ij} has a real derivative with respect to the Lorentzian time coordinate, $t = \text{Im}(\tau)$, where τ is Euclidean time. This is the condition for a Lorentzian geometry in a neighborhood of Σ .

The second fundamental form, K_{ij} , is trivially related to π_{ij} , the momentum conjugate to h_{ij} :

$$\pi_{ij} = -h^{1/2}(K_{ij} - h_{ij}K_{kl}h^{kl}), \quad (1.2)$$

where h is the determinant of the metric h_{ij} . Clearly, for real metrics h_{ij} , taking K_{ij} to be purely imaginary is equivalent

to taking π_{ij} to be purely imaginary. It is easy to transform from the usual representation of the wave function, $\Psi[h_{ij}]$, to the momentum representation, in which the wave function is a functional of π_{ij} . The two representations are related by a Laplace transform:

$$\Psi[\pi^{ij}] = \int d[h_{ij}] \Psi[h_{ij}] \exp\left(-\int_{\Sigma} d^3x \pi^{ij} h_{ij}\right), \quad (1.3)$$

where the integral over the metric components at each point of Σ is taken to be over all h_{ij} with positive determinant h [1]. This Laplace transform can be analytically continued to complex values of π^{ij} . The wave function for a universe that is Lorentzian in a neighborhood of Σ is then obtained by taking π^{ij} to be purely imaginary.

Thus the requirement that we live in a Lorentzian universe can be made manifest in the argument of the wave function. Further support for choosing the momentum representation comes from the fact that we cannot measure the metric globally on a spacelike section, but that the expansion rate of the universe, which is related to the second fundamental form, is easily observable.

Our arguments are independent of the semiclassical approximation, but they can be illustrated by it. The saddlepoint approximation to the wave function will be

$$\Psi[\pi^{ij}] = e^{-I}, \quad (1.4)$$

where we neglect a prefactor;

$$I = -\frac{1}{16\pi} \int d^4x g^{1/2} (R - 2\Lambda) \quad (1.5)$$

is the Euclidean action¹ of a complex solution of the field equations with the imaginary given values of π^{ij} on Σ . This complex saddlepoint solution will be Lorentzian near Σ by construction. Further away it may be complex or Euclidean but this does not matter because one is making measurements only on Σ . One therefore has to perform a path integral over the metric everywhere except on Σ . The use of a complex saddlepoint solution does not mean that spacetime is complex. It can just be regarded as a mathematical trick to evaluate the path integral.

II. HOMOGENEOUS ISOTROPIC UNIVERSE WITHOUT BLACK HOLES

We can illustrate the above discussion by a consideration of general relativity without matter fields but with a cosmological constant Λ . Because we are not interested in gravitational waves, we shall restrict ourselves to spherically symmetric solutions. This means that the second fundamental form K_{ij} has two independent components, K_s and K_l . By a gauge choice, we can consider only cases with K_l constant on Σ :

¹Note that this action does not contain the usual surface term, which is cancelled exactly in the Laplace transform.

$$K_{ij} = \text{diag}[K_l(\tau), K_s(\tau, x), K_s(\tau, x) \sin^2 \theta]. \quad (2.1)$$

A homogeneous isotropic universe without black holes is the background with respect to which we have to compare the probability of a universe containing a pair of black holes. This is the familiar de Sitter model, with the Euclidean saddlepoint metric

$$ds^2 = V(r) d\tau^2 + V(r)^{-1} dr^2 + r^2 d\Omega^2, \quad (2.2)$$

where

$$V(r) = 1 - \frac{\Lambda}{3} r^2. \quad (2.3)$$

We can make a choice of coordinates in which the spacelike surfaces Σ will be round three-spheres. Then the metric takes the form

$$ds^2 = d\hat{\tau}^2 + a(\hat{\tau})^2 d\Omega_3^2, \quad (2.4)$$

where $d\Omega_3^2 = dx^2 + \sin^2 x d\Omega_2^2$ is the metric on the unit three-sphere, and

$$a(\hat{\tau}) = R_0 \sin(R_0^{-1} \hat{\tau}). \quad (2.5)$$

The second fundamental form² K_l^j contains only one independent component, $K = K_l$, since

$$K_l = K_s = \frac{\dot{a}}{a}; \quad (2.6)$$

an overdot denotes differentiation with respect to Euclidean time $\hat{\tau}$. For K real (i.e. Euclidean), there will always be a real Euclidean solution. For positive K , this will be less than half the Euclidean four-sphere of radius R_0 and for K negative, it will be more than half. The action will be

$$I_{\text{ds}}(K) = -\frac{3\pi}{2\Lambda} \left[1 - \frac{(3 + 2K^2)K}{2(1 + K^2)^{3/2}} \right]. \quad (2.7)$$

The saddlepoint approximation to the wave function, neglecting the prefactor A , will be

$$\Psi(K) = \exp[-I_{\text{ds}}(K)]. \quad (2.8)$$

For $K=0$, the saddlepoint solution will be half the Euclidean four-sphere and the wave function will be

²As we pointed out in the previous section, we should strictly be working with the canonical momentum, π_{ij} . The Lorentzian condition that the argument of the wave function be purely imaginary, however, can equally well be implemented for various combinations of π_{ij} and h_{ij} , such as K_{ij} or K_l^j . Here we are choosing the latter quantity for the sake of clarity, since it leads to rather simple equations. It is straightforward to repeat the treatment using components of π_{ij} .

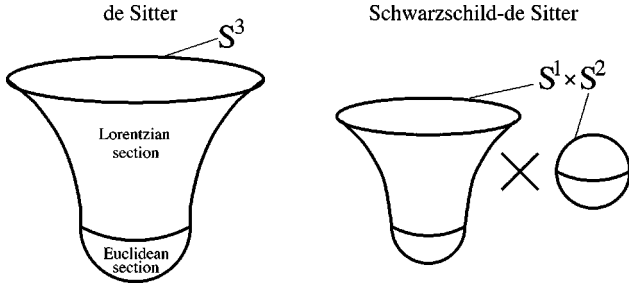


FIG. 1. The creation of a de Sitter universe (left) can be visualized as half of a Euclidean four-sphere joined to a Lorentzian four-hyperboloid. The picture on the right shows the corresponding nucleation process for a de Sitter universe containing a pair of black holes. In this case the spacelike slices have non-trivial topology.

$$\Psi = \exp\left(\frac{3\pi}{2\Lambda}\right). \quad (2.9)$$

Having calculated the wave function for real K , one can now analytically continue to complex values. Up the imaginary K axis, only the imaginary part of the action will change, as can be seen from Eq. (2.7). Thus, the amplitude of the wave function will remain at the value for $K=0$ given in Eq. (2.9). But the phase of the wave function will vary rapidly with the imaginary part of K . The wave function for positive imaginary K will be given by just one of the two complex solutions we had before. It is the one that consists of the half Euclidean four-sphere, joined to an expanding de Sitter solution across a minimal three-sphere spatial section (see Fig. 1).

Thus this approach separates the expanding and contracting phases of the de Sitter universe, which occur when one looks at the wave function in the h_{ij} representation. This makes contact with the tunneling proposal for the wave function [2] (see also [3] for earlier work). In this one selects the solution of the Wheeler-DeWitt equation that is *outgoing* at large values of the scale factor a . One can regard the Lorentzian condition as a precise definition of outgoing. However, the probability according to the tunneling proposal is e^{+I} rather than e^{-I} as with the no boundary proposal.

III. UNIVERSE WITH MAXIMAL BLACK HOLES

To get a universe containing black holes, one would like to calculate the probability for a Lorentzian geometry on a spacelike surface Σ with n handles. This would represent an expanding universe, with n pairs of black holes, that inflated from spacetime foam. It seems reasonable to suppose that the probability of n handles is roughly the n th power of the probability of a single handle, with appropriate phase space factors. Thus it is sufficient to consider the relative probabilities for zero and one handles. We shall restrict ourselves to spherical symmetry, to make the problem tractable, but it is reasonable to assume that spherical configurations dominate the path integral.

The zero handle surfaces (topology S^3) correspond to the Lorentzian de Sitter solution, while the one handle surfaces

(topology $S^1 \times S^2$) correspond to Schwarzschild-de Sitter, with the Lorentzian metric

$$ds^2 = -V(r)dt^2 + V(r)^{-1}dr^2 + r^2 d\Omega^2, \quad (3.1)$$

where

$$V(r) = 1 - \frac{2\mu}{r} - \frac{\Lambda}{3}r^2. \quad (3.2)$$

This represents a pair of black holes in a de Sitter background. The mass parameter, μ , of the black holes can be in the range from zero up to a maximum value of $1/(3\sqrt{\Lambda})$. For mass less than the maximum value, the surface gravity of the black hole horizon is greater than that of the cosmological horizon. This means that if one tries to turn the Schwarzschild-de Sitter solution into a compact Euclidean instanton ($d\tau = idt$), one gets a conical singularity either on the black hole horizon or on the cosmological horizon. For this reason, it has been thought that black holes could spontaneously nucleate in a de Sitter background only if they had the maximum mass [4–6]. We shall show in the next section that these conditions can in fact be relaxed.

For now, we shall focus on the maximal case. In this limit, the Schwarzschild-de Sitter solution degenerates into the Nariai solution, in which the two horizons have the same area and surface gravity, and a compact Euclidean instanton is possible without conical singularities:

$$ds^2 = d\hat{\tau}^2 + a(\hat{\tau})^2 dx^2 + R_1^2 d\Omega_2^2, \quad (3.3)$$

where $a(\hat{\tau}) = R_1 \sin(R_1^{-1}\hat{\tau})$. The two-spheres on Σ all have the same radius, $R_1 = 1/\sqrt{\Lambda}$, so $K_s = 0$ and there will be only one independent component of the second fundamental form, $K = K_l$. The Euclidean saddlepoint is a direct product of two round two-spheres of radius R_1 . The Lorentzian Nariai solution is the direct product of $(1+1)$ -dimensional de Sitter space with a round two-sphere.

The value of K will govern the size of the first Euclidean two-sphere in the same way it did for the de Sitter four-sphere in the previous section. For real K , the geometry is entirely Euclidean, while for imaginary K , it will consist of half of $S^2 \times S^2$, joined to the expanding half of the Lorentzian Nariai solution (see Fig. 1). The action will be given by

$$I_N(K) = -\frac{\pi}{\Lambda} \left(1 - \frac{K}{\sqrt{1+K^2}} \right), \quad (3.4)$$

yielding the wave function

$$\Psi_N(K) = \exp[I_N(K)]. \quad (3.5)$$

To obtain a Lorentzian universe, we must choose K to be purely imaginary. Then the real part of the Euclidean action, which gives the amplitude of the wave function, will be $-2\pi/\Lambda$. As in the de Sitter case, this is independent of K as long as $\text{Re}(K)=0$. The imaginary part of the action, which gives the phase of the wave function, depends on K .

To calculate the pair creation rate of Nariai black holes on a de Sitter background, we note that $\Psi^* \Psi$ is a probability measure. It is important to stress that the probability measure depends only on the real part of the saddlepoint action,

which stems from the Euclidean sector. In accordance with other instanton methods, the pair creation rate Γ_N can thus be obtained by normalizing this probability with respect to de Sitter space:

$$\Gamma_N = \frac{\Psi_N^* \Psi_N}{\Psi_{\text{dS}}^* \Psi_{\text{dS}}} = \exp\{-2[\text{Re}(I_N) - \text{Re}(I_{\text{dS}})]\} = \exp\left(\frac{-\pi}{\Lambda}\right). \quad (3.6)$$

Therefore the pair creation of black holes is highly suppressed except when the (effective) cosmological constant is close to the Planck value, as it may have been in the earliest stages of inflation.

IV. UNIVERSE WITH SUB-MAXIMAL BLACK HOLES

In the previous section, we chose to consider only black holes of maximal size in order to avoid a conical singularity in the Euclidean saddlepoint solution. For a metric to dominate the path integral, it has to be a solution of the Einstein equations at every point of the manifold; but on a conical singularity clearly it is not. Thus the action will not be stationary with respect to general variations of a metric containing a conical singularity.

However, conical singularities are expected in general on the measurement surface Σ if one is working in the metric representation. The solution of the field equations for given h_{ij} on Σ will in general have a non-zero second fundamental form K_{ij} on Σ . When this solution is joined to its reflection across Σ to calculate $\Psi^* \Psi$, one gets a conical singularity in general.

The rule is that conical singularities are expected on Σ if they correspond to components of the metric that are measured. For example, if one wants the probability of an $S^1 \times S^2$ handle with a two-sphere cross section, σ , of area A , one can impose the Lorentzian condition that the real part of the second fundamental form vanish everywhere on Σ except for σ . One cannot specify the second fundamental form on σ , because one is prescribing the metric there. On the other hand, one can impose the Lorentzian condition, that the real part of the second fundamental form is zero, everywhere else on Σ . This allows one to find a saddlepoint solution, bounded by a surface Σ with a handle of area A , for any area up to the maximum, $4\pi/\Lambda$. Therefore the nucleation of Schwarzschild–de Sitter black hole pairs of any size can be analyzed in the instanton formalism. We choose the cosmological horizon to be regular in the Euclidean sector, which will lead to a conical singularity on the black hole horizon. This is allowed as long as the surface of measurement, Σ , contains the conical singularity (since this means that the metric is not varied there).

The cross section σ corresponds to the black hole horizon; it will be the smallest S^2 in the spacelike surface Σ . Assume it is not. Then the σ will not correspond to the conical singularity, whose metric will then not be fixed on the boundary. But such configurations will not dominate in the path integral and can be neglected.) One can now choose some slicing of Schwarzschild–de Sitter which must have the property that the proper time between points on different

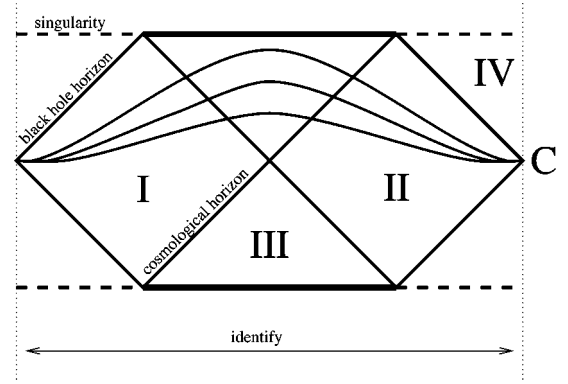


FIG. 2. Carter-Penrose diagram of the Schwarzschild–de Sitter spacetime. The point C is the location of the conical singularity in the Euclidean sector. The curved lines indicate a family of spacelike slices which all pass through the conical singularity. This is necessary since one must specify the metric there in order to ensure that the Euclidean solution is a saddlepoint. Regions I and II lie between the black hole and the cosmological horizon. Region III corresponds to an asymptotic de Sitter region, and region IV to the black hole interior.

slices goes to zero at least quadratically as a function of proper distance from the black hole horizon. This type of slicing is shown schematically in a Carter-Penrose diagram in Fig. 2.

It ensures that all Lorentzian spacelike slices will be regular on the black hole horizon. We shall not give any such slicing explicitly. Once a particular slicing is chosen, there will again be only one degree of freedom in the second fundamental form, say $K = \int d^3x h^{1/2} K_{ij} h^{ij}$.

Thus, in the Schwarzschild–de Sitter case, the wave function has two arguments, A and K . The first determines the size of the black hole, while the second selects a spacelike slice in the saddlepoint metric. The de Sitter and Nariai cases are included for $A=0$ and $A=4\pi/\Lambda$, respectively.

The Euclidean part of the saddlepoint metric has a boundary with zero second fundamental forms everywhere except on σ , where it is a delta function. This boundary will split the full Euclidean solution in half in the same way as in the de Sitter and Nariai solutions. This half of the Euclidean geometry will give the real part of the action. Choosing K to be purely imaginary leads to a Lorentzian universe, which once again can be obtained by analytically continuing the Euclidean solution. Similar to the de Sitter and Nariai solutions, the Lorentzian section will contribute only to the imaginary part of the action. Therefore the real part of the action will be independent of K for imaginary K :

$$I_{\text{dS}}(A, K) = I_{\text{dS}}^{\text{Re}}(A) + i I_{\text{dS}}^{\text{Im}}(A, K). \quad (4.1)$$

To calculate the probability measure, and thus the nucleation rate for a Schwarzschild–de Sitter black hole pair, we need only calculate the real part of the action, since

$$\Psi_{\text{dS}}^* \Psi_{\text{dS}} = \exp[-2 \text{Re}(I_{\text{dS}})]. \quad (4.2)$$

But $2 \operatorname{Re}(I_{\text{SdS}}) = 2I_{\text{SdS}}^{\text{Re}}(A)$, which is twice the action of the Schwarzschild–de Sitter instanton, which in turn is equal to the action of the full Euclidean Schwarzschild–de Sitter solution, $I_{\text{SdS}}^{\text{full}}$.

Using Eq. (1.5) and $R=4\Lambda$, one can show that

$$I_{\text{SdS}}^{\text{full}} = -\frac{\Lambda \mathcal{V}}{8\pi} - \frac{A\delta}{8\pi}, \quad (4.3)$$

where $\mathcal{V}$ is the four-volume of the Euclidean solution. The extra term gives the contribution from a conical deficit angle δ at a two-surface of area A [4].

In order to facilitate the calculation of this action, it is useful to parametrize the Schwarzschild–de Sitter solutions by the radii b and c of the black hole and the cosmological horizon. The parameters Λ and μ can be expressed in terms of the new parameters b and c :

$$\Lambda = \frac{3}{b^2 + c^2 + bc}, \quad (4.4)$$

$$\mu = \frac{bc(b+c)}{2(b^2 + c^2 + bc)}. \quad (4.5)$$

The Euclidean Schwarzschild–de Sitter metric is

$$ds^2 = V(r)d\tau^2 + V(r)^{-1}dr^2 + r^2d\Omega^2, \quad (4.6)$$

where $V(r)$ is given by Eq. (3.2); in terms of b and c it takes the form

$$V(r) = \frac{(r-b)(c-r)(r+b+c)}{r(b^2 + c^2 + bc)}. \quad (4.7)$$

To avoid a conical singularity at the cosmological (black hole) horizon, the Euclidean time τ must be identified with the period $\tau_c^{\text{id}}(\tau_b^{\text{id}})$, where

$$\tau_{c,b}^{\text{id}} = 2\pi \sqrt{g_{rr}} \Big|_{r=c,b} \left| \frac{\partial}{\partial r} \sqrt{g_{\tau\tau}} \right|_{r=c,b}^{-1}, \quad (4.8)$$

where $g_{\tau\tau} = 1/g_{rr} = V(r)$. This gives

$$\tau_{c,b}^{\text{id}} = 4\pi \left| \frac{\partial V}{\partial r} \right|_{r=c,b}. \quad (4.9)$$

We choose to get rid of the conical singularity at $r=c$, so the volume will be

$$\mathcal{V} = \frac{4\pi}{3}(c^3 - b^3)\tau_c^{\text{id}}. \quad (4.10)$$

The conical deficit angle at the black hole horizon is by definition

$$\delta = 2\pi \left(1 - \frac{\tau_c}{\tau_b} \right); \quad (4.11)$$

the two-sphere area A is obviously $4\pi b^2$.

With $\Lambda, \mathcal{V}, A$, and δ expressed in terms of b and c , Eq. (4.3) evaluates to

$$I_{\text{SdS}}^{\text{full}} = -\pi(b^2 + c^2). \quad (4.12)$$

Note that this action is related to the geometric entropy, S , and the total horizon area in the usual way [7–11]:

$$-I = S = \frac{A + A_c}{4}, \quad (4.13)$$

where $A_c = 4\pi c^2$ is the area of the cosmological horizon. Thus we obtain for the pair creation rate of arbitrary-size Schwarzschild–de Sitter black holes in de Sitter space:

$$\Gamma_{\text{SdS}} = \exp[-(I_{\text{SdS}}^{\text{full}} - I_{\text{ds}}^{\text{full}})] = \exp(-\pi bc). \quad (4.14)$$

Using Eq. (4.4) and $A = 4\pi b^2$, this result can easily be rewritten in terms of Λ and A , the argument we specified in the wave function. However, the physical implications are quite clear from Eq. (4.14): a decreasing cosmological constant corresponds to increasing cosmological horizon size c and thus, as in the maximal case, to increasing suppression. At a fixed value of the cosmological constant, the suppression increases with the black hole radius, b , which is physically sensible. Considering the Planck length to be the lower bound on the black hole size ($b \geq 1$), we find that even the smallest black holes are highly suppressed unless the cosmological constant is also near the Planck value.

Wu has recently proposed [12] that one should calculate the saddlepoint approximation to the wave function using “constrained instantons,” which include spacetimes with a conical singularity. He conjectures the conical singularities should be allowed on the “equator,” i.e. the $K_{ij} = 0$ surface on which the real Euclidean geometry is matched to a real Lorentzian one. This is essentially equivalent to what we have done but the motivation for his calculation is maybe not so clear. He obtains the same result for the pair creation probability of sub-maximal black holes.

V. SUMMARY AND CONCLUSIONS

We have argued that the momentum representation of the wave function of the Universe has several advantages over the metric representation. Most importantly, the requirement that we live in a Lorentzian universe can be implemented straightforwardly in this formulation: one must take the argument of the wave function to be purely imaginary. Moreover, unlike the three-metric, the canonical momentum is closely related to observable quantities like the expansion rate of the universe, and it distinguishes between expanding and contracting branches. While the momentum and metric representations are related by a Laplace transform and thus contain the same information, we conclude that many of the most relevant physical properties of a spacetime are manifest only in the momentum representation.

We have clarified how, and under which conditions, Euclidean solutions with a conical singularity may be used as saddlepoints. We showed that this is possible in the case of sub-maximal Schwarzschild–de Sitter universes if the space-like boundary, Σ , is chosen to contain the conical singularity

and the metric is specified there. On the rest of Σ , a purely imaginary second fundamental form is specified to ensure that the observed universe is Lorentzian. This enabled us to describe the quantum nucleation of such spacetimes and calculate their creation rate on a de Sitter background.

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