

Relic neutrino asymmetries and big bang nucleosynthesis in a four neutrino model

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Oscillations between ordinary and sterile neutrinos can generate large neutrino asymmetries in the early universe. These asymmetries can significantly affect big bang nucleosynthesis (BBN) through modification of nuclear reaction rates. We study this phenomenon within a model consisting of the three ordinary neutrinos plus one sterile neutrino that can be motivated by the neutrino anomalies and the dark matter problem. We calculate how the lepton asymmetries produced, evolve at temperatures where they impact on BBN. The effect of the asymmetries on primordial helium production is determined, leading to an effective number of neutrino flavors during BBN of either about 2.7 or 3.1 depending on the sign of the lepton asymmetry. [S0556-2821(98)09720-3]

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I. INTRODUCTION

Models with sterile neutrinos are currently favored to accommodate all the neutrino oscillation measurements. It is difficult to simultaneously account for the solar, atmospheric and Liquid Scintillation Neutrino Detector (LSND) anomalies with just three ordinary neutrinos and hence only two mass-squared differences. Therefore we require at least one additional neutrino if we wish to explain all three of the observed neutrino anomalies. Constraints from the Z width suggest that any additional neutrinos must be sterile. Oscillations between ordinary and sterile neutrinos in the early universe have been studied in [1–5] and found to generate large neutrino asymmetries [4,5], which can significantly affect big bang nucleosynthesis (BBN) reaction rates, and can result in an effective neutrino number of less than three during BBN [6]. In this paper, the techniques developed in Ref. [6] will be applied to the four neutrino model of Ref. [7] which has almost degenerate ν_μ and ν_τ . This model is distinct from the four neutrino model of Ref. [6] where a mass hierarchy between ν_μ and ν_τ was assumed.

A motivation for studying how neutrino oscillations can affect N_ν^{eff} , the effective neutrino number during nucleosynthesis, arises from discrepancies between the observed deuterium abundance and standard BBN. The primordial D/H ratio can be used to give a sensitive determination of the baryon to photon ratio η which, given the inferred primordial ^4He mass fraction, can be used to predict the number of light neutrinos N_ν^{eff} .

Although there are numerous measurements of the deuterium abundance, obtaining an upper bound to the abundance is quite difficult, and there are conflicting observations. A high deuterium measurement of $D/H = (1.9 \pm 0.4) \times 10^{-4}$ was found in Ref. [8], leading to $\eta \sim 2 \times 10^{-10}$, while in Ref. [9] a low deuterium result of $D/H = (2.3 \pm 0.3 \pm 0.3) \times 10^{-5}$ was obtained, suggesting that $\eta \sim 7 \times 10^{-10}$. From these val-

ues of η , and the helium mass fraction Y_p , the effective number of neutrinos can be calculated. By way of example, the results

$$\begin{aligned} N_\nu^{eff} &= 2.9 \pm 0.3 \quad \text{high } D/H, \\ N_\nu^{eff} &= 1.9 \pm 0.3 \quad \text{low } D/H, \end{aligned} \quad (1)$$

were obtained in Ref. [10]. (For other calculations of N_ν^{eff} see Ref. [11].) Hence, if the low deuterium result is correct, standard BBN would require an effective neutrino number of less than three (provided that the value of Y_p used in Ref. [10] is correct). Clearly $N_\nu^{eff} < 3$ would imply new physics. Further observations are required to pin down the D/H value (and also Y_p) although it now seems as though the low D/H result is favored [9,12].

One possibility for the new physics is to look at the neutrino sector and study mechanisms by which the effective number of neutrinos can be made less than three. A possible way of doing this is to create an electron-neutrino asymmetry, as this would directly affect the reaction rates which determine the n/p ratio just before nucleosynthesis. The n/p ratio controls the helium mass fraction Y_p via the equation

$$\frac{dY_p}{dt} = -\lambda(n \rightarrow p)Y_p + \lambda(p \rightarrow n)(2 - Y_p), \quad (2)$$

where the reaction rates

$$\begin{aligned} \lambda(n \rightarrow p) &\simeq \lambda(n + \nu_e \rightarrow p + e^-) + \lambda(n + e^+ \rightarrow p + \bar{\nu}_e), \\ \lambda(p \rightarrow n) &\simeq \lambda(p + e^- \rightarrow n + \nu_e) + \lambda(p + \bar{\nu}_e \rightarrow n + e^+), \end{aligned} \quad (3)$$

depend on the momentum distributions of the species involved. The processes in Eq. (3) for determining $n \leftrightarrow p$ are valid for temperatures above about 0.4 MeV, below which the weak interaction rates become frozen out and neutron decay becomes the dominant factor affecting the n/p ratio. An excess of ν_e over $\bar{\nu}_e$ would reduce the n/p ratio, due to the modification of the neutrino momentum distributions through non-zero chemical potentials, thereby changing the rates for the processes in Eq. (3). Neutron decay is not sig-

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nificantly altered by lepton asymmetries. A change in the helium mass fraction Y_p can be related to a change in the effective number of neutrinos by [13]

$$\delta Y_p \approx 0.012 \times \delta N_\nu^{eff}. \quad (4)$$

A large neutrino asymmetry could simply be postulated, but it is more interesting to look at physics of the neutrino sector which will allow a large neutrino asymmetry to evolve as a dynamical variable.

A four-neutrino model was studied in Ref. [6], involving the three ordinary neutrinos and one sterile neutrino, where the mass hierarchy $m_{\nu_\tau} \gg m_{\nu_\mu}, m_{\nu_e}, m_{\nu_s}$ was considered by way of example. In this case oscillations between $\bar{\nu}_\tau$ and $\bar{\nu}_s$ resulted in an excess of ν_τ over $\bar{\nu}_\tau$ thereby generating a large tau-neutrino asymmetry and hence a large tau-lepton number. $\bar{\nu}_\tau - \bar{\nu}_e$ oscillations also occurred, and it was shown that this allowed some of the tau-neutrino asymmetry to be transferred to an electron-neutrino asymmetry. The effective number of neutrinos found in Ref. [6] was either 2.5 or 3.4. The two values arise because there is an ambiguity involving the sign of the asymmetry and hence the prediction for N_ν^{eff} (see Ref. [4] for a discussion of this issue). For a positive asymmetry, $\delta N_\nu^{eff} \approx -0.5$ was obtained over a range of mass differences $\delta m_{\tau s}^2 \sim 10 - 3000 \text{ eV}^2$, while for a negative asymmetry the result was $\delta N_\nu^{eff} \approx +0.4$. So we have the interesting situation that $N_\nu^{eff} < 3$ can be achieved although the precise value of N_ν^{eff} is model dependent as this paper will illustrate. If future measurements can pin down N_ν^{eff} precisely enough, then this quantity can be used to help discriminate various competing models explaining the observed neutrino physics anomalies.

II. MODEL OF NEUTRINO MASSES

The model we will consider in this paper is again a system of four neutrinos: ν_τ , ν_μ , ν_e and ν_s . In this model the ν_e and ν_s are assumed to be very light ($\ll 1 \text{ eV}$), and ν_τ and ν_μ are taken to have nearly degenerate masses. This model has been proposed in [7] as a pattern of masses that will allow the data from the solar, atmospheric, and LSND experiments to be satisfied as well as being consistent with dark matter models.

In this scenario, the solar neutrino problem is explained by the small angle Mikheyev-Smirnov-Wolfenstein (MSW) solution with oscillations between ν_e and ν_s with $\delta m_{es}^2 \sim 10^{-5} \text{ eV}^2$ and $\sin^2 2\theta \sim 10^{-2}$ [14], while the atmospheric neutrino deficit is explained by $\nu_\tau - \nu_\mu$ oscillations [15] with $\delta m_{\tau\mu}^2 \sim 10^{-2} - 10^{-3} \text{ eV}^2$ and $\sin^2 2\theta \approx 1$. The LSND data requires a $\nu_\mu - \nu_e$ solution with $\delta m_{\mu e}^2$ in the range $0.2 - 10 \text{ eV}^2$ and $\sin^2 2\theta \approx 3 \times 10^{-2} - 10^{-3}$ [16]. It has been argued [17] from considerations of structure formation in the early universe that the optimal value for the degenerate ν_μ and ν_τ masses is 2.4 eV . Note, however, that the creation of a large L_μ and L_τ neutrino asymmetry should impact on the favored neutrino mass for structure formation and dark matter. For this reason it may well be necessary for the optimal

mass of approximately 2.4 eV to be recalculated, taking into account the modified neutrino number densities.

III. NEUTRINO ASYMMETRIES AND CONSEQUENCES FOR BBN

For oscillations between ν_α and ν_s , the weak eigenstates are linear combinations of the mass eigenstates ν_1 and ν_2 ,

$$\begin{aligned} \nu_\alpha &= \cos \theta_0 \nu_1 + \sin \theta_0 \nu_2, \\ \nu_s &= -\sin \theta_0 \nu_1 + \cos \theta_0 \nu_2, \end{aligned} \quad (5)$$

where θ_0 is the vacuum mixing angle (we assume that θ_0 is defined such that $\cos 2\theta_0 > 0$). The mass-squared difference between the two eigenstates is defined as $\delta m_{\alpha s}^2 = m_2^2 - m_1^2$. The neutrinos interact with matter, so a matter mixing angle θ_m is defined, which is related to θ_0 by [18],

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta_0}{\sin^2 2\theta_0 + (b \pm a - \cos 2\theta_0)^2}. \quad (6)$$

The term $(b \pm a)$ is related to the effective potential due to interactions of the neutrinos with matter:

$$V = \frac{\delta m_{\alpha s}^2}{2p} (b \pm a), \quad (7)$$

where $-/+$ corresponds to $\nu/\bar{\nu}$ and p is the neutrino momentum. The functions a and b are given by

$$\begin{aligned} a &= -\frac{4\sqrt{2}\zeta(3)G_F T^3 L^{(\alpha)} p}{\pi^2 \delta m_{\alpha s}^2}, \\ b &= -\frac{4\sqrt{2}\zeta(3)G_F T^4 A_\alpha p^2}{\pi^2 \delta m_{\alpha s}^2 M_W^2}, \end{aligned} \quad (8)$$

where $A_e \approx 17$ and $A_{\tau,\mu} \approx 4.9$ [19].

The function $L^{(\alpha)}$ is defined as

$$L^{(\alpha)} = L_\alpha + L_\tau + L_\mu + L_e + \eta, \quad (9)$$

where η is approximately equal to the baryon to photon ratio, and the L_ν 's are the lepton asymmetries which are given by

$$L_{\nu_\alpha} = \frac{n_{\nu_\alpha} - n_{\bar{\nu}_\alpha}}{n_\gamma}, \quad (10)$$

where n_ν and n_γ are the neutrino and photon number densities, respectively.

The neutrinos have a distribution of momenta and only neutrinos with momentum of a certain value will be resonant at a given time. The momentum of the resonance will move through the neutrino distribution as the temperature decreases, creating a neutrino asymmetry at the particular value of the resonance momentum at that temperature. The evolving neutrino asymmetry also non-linearly affects the evolution of the resonance momentum. The asymmetry is distributed across the momentum distribution as weak interactions keep the distributions in thermal equilibrium.

The resonance occurs for

$$(b \pm a) = \cos 2\theta_0 \approx 1, \quad (11)$$

for $\sin^2 2\theta_0 \ll 1$. The a and b terms dominate this condition at different temperatures. We may obtain a qualitative understanding of the behavior by neglecting the thermal spread of momentum and using the average value $\langle p \rangle \approx 3.15T$. At high temperatures the $\langle b \rangle$ term dominates, and in [4] it was found that for $\langle b \rangle < 1$ oscillations create lepton number while for $\langle b \rangle > 1$ the oscillations are lepton number destroying. The dependence of $\langle b \rangle$ on temperature is $\langle b \rangle \sim T^6$, so when the temperature drops to where $\langle b \rangle \sim 1$, which we denote by T_C , the oscillations creating lepton asymmetry dominate and lead to an exponential growth in lepton number. In Ref. [4] it was shown that for $\delta m^2 < 0$ (and $|\delta m^2| \gtrsim 10^{-4} \text{ eV}^2$) large lepton asymmetries are created provided that

$$\sin^2 2\theta_0 \gtrsim 5 \times 10^{-10} \left(\frac{\text{eV}^2}{|\delta m^2|} \right)^{1/6}. \quad (12)$$

This result is independent of the initial value of the asymmetry, provided that $|L_\alpha| \lesssim 10^{-5}$ [4]. For δm^2 in the range $1-10 \text{ eV}^2$, we have $T_C \approx 16-25 \text{ MeV}$, obtained from the condition $\langle b \rangle = 1$. The rapid growth drops off as the temperature decreases and when the temperature has decreased to below about $T_C/2$, the $\langle a \rangle$ term has become larger and we can now neglect the $\langle b \rangle$ term so that the resonance condition is approximately given by [6,4]

$$\langle a \rangle \approx \cos 2\theta_0 \approx 1. \quad (13)$$

Collisions and oscillations both affect the generation of lepton asymmetries. At high temperatures ($T \gtrsim T_C/2$) collisions affect neutrinos to a greater extent than oscillations, and these in turn affect lepton number because the ν 's and $\bar{\nu}$'s interact at different rates in a CP asymmetric background medium. (A CP asymmetric medium is required by the cosmological baryon-antibaryon asymmetry.) At lower temperatures, the collision rates are not as rapid and oscillations become coherent, so that oscillations take over from collisions to control the growth of lepton number. Thus, for $T \lesssim T_C/2$, MSW [18] transitions become important.

We have two oscillation resonances to consider, $\nu_{\tau,\mu} - \nu_s$, which will generate a large $L_{\tau,\mu}$ asymmetry and $\nu_{\tau,\mu} - \nu_e$, which will then transform some of the asymmetry into the ν_e sector to produce L_e . Oscillations between ν_τ and ν_μ or ν_e and ν_s may be neglected as they have much smaller mass-squared differences.

The asymmetries are calculated, starting from an initial temperature of $T = T_C/2$ where $|L_\tau| \ll 1$ and T is low enough for oscillations and MSW transitions to dominate over collisions. The initial number density of sterile neutrinos is taken to be negligible (which will occur for a range of parameters) in accord with the results of the numerical calculation in Ref. [4] for the initial creation of lepton number at $T = T_C$. We study how the asymmetries evolve down to temperatures where they become important for BBN. Note that for the moment we will assume that $L_\tau > 0$ for definiteness.

At $T \gtrsim T_C$ where the b term dominates, $\nu_\tau \rightarrow \nu_s$ and $\bar{\nu}_\tau \rightarrow \bar{\nu}_s$ resonances both occur at the same momentum, but as

the temperature falls to $T \approx T_C/2$ the $\nu_\tau \rightarrow \nu_s$ and $\bar{\nu}_\tau \rightarrow \bar{\nu}_s$ oscillations no longer have the same resonance momenta. This allows the asymmetry to grow very large because when most of the $\bar{\nu}$'s have approximately the right momentum to be near a resonance, the bulk of the ν 's are far away from a resonance. It was found in [6] that if the created L_α is positive the antineutrino resonance momentum has generally low values $p_{res}/T \sim 0.2-0.8$ at $T \approx T_C/2$ while the neutrino resonance momentum is toward the tail of the neutrino momentum distribution (see Fig. 2 of Ref. [6] for an illustration). Hence the neutrino resonance is neglected and we will only be concerned with the antineutrino resonance.

We will neglect the $\nu_\tau - \nu_\mu$ mass difference and thus make the approximation that the $\nu_\tau - \nu_s$ and $\nu_\mu - \nu_s$ oscillations have exactly the same resonance momentum, p_1 , and the $\nu_{\tau,\mu} - \nu_e$ resonances have momentum p_2 . These are obtained from the resonance condition

$$a \approx \cos 2\theta_0 \approx 1, \quad (14)$$

where a is given by Eq. (8). The resonance momenta are given by

$$p_1 \approx - \frac{\pi^2 \delta m^2}{4\sqrt{2}\zeta(3)G_F} \frac{1}{T^3 L^{(\tau)}},$$

$$p_2 \approx - \frac{\pi^2 \delta m^2}{4\sqrt{2}\zeta(3)G_F} \frac{1}{T^3 (L^{(\tau)} - L^{(e)})}. \quad (15)$$

Since ν_τ and ν_μ are nearly degenerate and their matter effects are identical, the calculation is symmetric with respect to ν_μ and ν_τ , so $L_\tau = L_\mu$. We therefore find that

$$L^{(\tau)} = 3L_\tau + L_e \approx 3L_\tau,$$

$$L^{(\tau)} - L^{(e)} = L_\tau - L_e \approx L_\tau. \quad (16)$$

This means that the $\nu_{\tau,\mu} - \nu_e$ resonance occurs for a momentum which is approximately a factor of three higher than for the $\nu_{\tau,\mu} - \nu_s$ resonance,

$$p_2 \approx 3p_1. \quad (17)$$

Because the $\nu_\tau - \nu_s$ and $\nu_\mu - \nu_s$ oscillations have the same resonance momenta, instead of having two separate resonances, the $\nu_\tau - \nu_s$ and $\nu_\mu - \nu_s$ resonances completely overlap. Therefore we must consider the three-flavor system consisting of ν_τ , ν_μ and ν_s . To take into account that there is only one sterile neutrino state for both ν_τ and ν_μ to couple to, we may form two linear combinations of ν_τ and ν_μ . One of these oscillates with ν_s , and undergoes MSW conversion at the resonance, while the other ν_τ/ν_μ linear combination does not oscillate with ν_s . This state is decoupled, and is not converted into the sterile state at the resonance. Our equations include factors of $\frac{1}{2}$ to account for this situation. The mass eigenstate which is dominantly a ν_s at high density, becomes a ν_τ/ν_μ linear combination at low density, so that a ν_s which goes through the resonance has a 50% chance of

being converted to ν_τ and a 50% chance of being converted to ν_μ . Similarly, since one linear combination of ν_τ and ν_μ only will be converted to a ν_s upon passing through the resonance, both ν_τ or ν_μ have only a 50% probability of being converted to the sterile state.

Naively one might believe that by making ν_μ and ν_τ degenerate, so as to have two heavier neutrinos oscillating with ν_s instead of only one heavier neutrino oscillating with the sterile neutrino, that twice the electron-neutrino asymmetry would be produced. However, this will not occur, because there is only one sterile eigenstate for the two linear combinations of ν_τ and ν_μ to oscillate with. Because of the factors of $\frac{1}{2}$ arising from this three-flavor nature, we would not expect the electron-neutrino asymmetry created to be any larger than for the model of Ref. [6], where only a heavy ν_τ oscillates with ν_s (in fact it turns out to be smaller—see later).

Numerically integrating the quantum kinetic equations, we find that the adiabatic limit of the MSW effect holds provided

$$\sin^2 2\theta_0 \gtrsim 10^{-9} - 10^{-10} \quad (18)$$

for $\delta m^2 \sim 10 \text{ eV}^2$. The adiabatic limit implies all neutrinos that pass through the resonance will undergo MSW conversion. This means the lepton number created is basically equal to the number of antineutrinos minus the number of sterile antineutrinos that pass through the resonance, and the growth of lepton number is governed by how quickly the resonance momentum moves through the neutrino momentum distribution.

The rate of change of the lepton numbers is therefore given by

$$\frac{dL_{\nu_\tau}}{dT} = -X_1 \left| \frac{d}{dT} \left(\frac{p_1}{T} \right) \right| - X_2 \left| \frac{d}{dT} \left(\frac{p_2}{T} \right) \right|, \quad (19)$$

$$\frac{dL_{\nu_e}}{dT} = 2X_2 \left| \frac{d}{dT} \left(\frac{p_2}{T} \right) \right|, \quad (20)$$

where $X_1(X_2)$ expresses the difference between the number of $\bar{\nu}_\tau$ and $\bar{\nu}_s(\bar{\nu}_e)$ with the right momentum to pass through the resonance and is given by

$$X_1 = \frac{1}{2} \frac{T}{n_\gamma} (N_{\bar{\nu}_\tau} - N_{\bar{\nu}_s}) \bigg|_{p=p_1}, \quad X_2 = \frac{1}{2} \frac{T}{n_\gamma} (N_{\bar{\nu}_\tau} - N_{\bar{\nu}_e}) \bigg|_{p=p_2}. \quad (21)$$

The factor of 2 in Eq. (20) and the factors of $\frac{1}{2}$ in Eqs. (21) arise from the three-flavor nature of the problem as discussed above.

The N 's are the momentum distributions of the neutrinos, so that

$$n = \int N dp, \quad (22)$$

which in thermal equilibrium are just given by a Fermi distribution

$$N_{\bar{\nu}} = \frac{1}{2\pi^2} \frac{p^2}{1 + \exp\left(\frac{p + \mu_{\bar{\nu}}}{T}\right)}, \quad (23)$$

where $\mu_{\bar{\nu}}$ is the antineutrino chemical potential. The $T|d/dT(p_{res}/T)|$ terms describe how quickly the resonance momenta move, and are expressed in terms of the variable p_{res}/T because we want to distinguish the roles of expansion and oscillations, and unlike p , the quantity p/T does not change due to the direct effect of the expansion of the universe. Evaluating the derivatives, we obtain

$$\frac{dL_{\nu_e}}{dT} = \frac{2DA + CE}{A + E(A - B)}, \quad (24)$$

$$\frac{dL_{\nu_\tau}}{dT} = \frac{C}{A} + \frac{B}{A} \frac{dL_{\nu_e}}{dT}, \quad (25)$$

where

$$A = -\left(1 + \frac{3X_1 p_1}{TL^{(\tau)}}\right), \quad B = \frac{1}{2} + \frac{X_1 p_1}{TL^{(\tau)}},$$

$$C = \frac{4X_1 p_1}{T^2}, \quad D = \frac{4X_2 p_2}{T^2},$$

$$E = \frac{2X_2 p_2}{T} \frac{1}{L^{(\tau)} - L^{(e)}}. \quad (26)$$

In deriving Eqs. (24), (25) we have used $d/dT(p_{1,2}/T) < 0$, because the resonances (which start at low values of approximately $p_1/T \sim 0.2-0.8$ and $p_2/T \sim 3p_1/T$) move through the momentum distributions to higher values as the temperature decreases.

Numerically integrating Eqs. (24), (25), we find that the final lepton asymmetries generated are

$$L_\mu/h = L_\tau/h \simeq 0.16, \quad L_e/h \simeq 6.7 \times 10^{-3} \quad (27)$$

for δm^2 in the range $6-8 \text{ eV}^2$, where $h = T_\nu^3/T_\gamma^3$.

The L_e asymmetry then affects BBN rates via the modification of the neutrino momentum distributions. Weak interactions keep the neutrino distribution in thermal equilibrium for $T \gtrsim 1 \text{ MeV}$, however for $\delta m^2 \sim 10 \text{ eV}^2$ the neutrino asymmetries only become large at temperatures where the distributions are beginning to go out of equilibrium. Because the neutrino distributions are not completely thermalized, we cannot simply describe them in terms of chemical potentials, so for this reason we keep track of the neutrino number densities numerically in momentum cells. We assume complete MSW conversion at the resonance, which is a good approximation for the large range of parameters given by Eq. (18), so that numbers densities of sterile and active neutrinos are interchanged at the resonance momentum such that

$$N_{\bar{\nu}_s}(p_1) \rightarrow \frac{1}{2}(N_{\bar{\nu}_\tau}(p_1) + N_{\bar{\nu}_\mu}(p_1)) = N_{\bar{\nu}_{\tau,\mu}}(p_1),$$

$$N_{\bar{\nu}_{\tau,\mu}}(p_1) \rightarrow \frac{1}{2}(N_{\bar{\nu}_s}(p_1) + N_{\bar{\nu}_{\tau,\mu}}(p_1)), \quad (28)$$

and similarly for the $\bar{\nu}_{\tau,\mu} - \bar{\nu}_e$ resonance. Cells are refilled according to the interaction rates

$$\frac{d}{dt} \left(\frac{N_{actual}}{N_0} \right) = \Gamma(p) \left(\frac{N_{equilib}}{N_0} - \frac{N_{actual}}{N_0} \right), \quad (29)$$

where

$$N_0 = \frac{1}{2\pi^2} \frac{p^2}{1 + \exp\left(\frac{p}{T}\right)} \quad (30)$$

and $N_{equilib}$ is given by the equilibrium distribution Eq. (23). The chemical potential is obtained from the lepton number via the equation

$$L_\alpha \approx -\frac{1}{24\zeta(3)} [\pi^2(\tilde{\mu}_\nu - \tilde{\mu}_{\bar{\nu}}) - 6(\tilde{\mu}_\nu^2 - \tilde{\mu}_{\bar{\nu}}^2) \ln 2 + (\tilde{\mu}_\nu^3 - \tilde{\mu}_{\bar{\nu}}^3)] \quad (31)$$

where $\tilde{\mu}_i \equiv \mu_i/T$. Equation (31) is an exact equation for $\tilde{\mu}_\nu$, $= -\tilde{\mu}_{\bar{\nu}}$, otherwise it holds to a good approximation provided that $\tilde{\mu}_i \lesssim 1$. For $\delta m^2 \sim 8 \text{ eV}^2$ a significant lepton number (here significant means larger than about 0.01) is not created until $T \lesssim 2-3 \text{ MeV}$. At these temperatures, the neutrinos and anti-neutrinos have already chemically decoupled. (Note that $T_{dec}^e \approx 3 \text{ MeV}$ and $T_{dec}^{\mu,\tau} \approx 5 \text{ MeV}$ are the chemical decoupling temperatures.) Because of this, $\tilde{\mu}_\nu \approx 0$, while the anti-neutrino chemical potential $\tilde{\mu}_{\bar{\nu}}$ continues increasing as per Eq. (31).

The interaction rate $\Gamma(p)$ in Eq. (29) is

$$\Gamma(p) \approx \frac{\langle \Gamma \rangle p}{\langle p \rangle}, \quad (32)$$

where $\langle \Gamma \rangle$ is the thermal average of the total neutrino collision rate given by [1]

$$\langle \Gamma_{\nu_e} \rangle = 4.0 G_F^2 T^5,$$

$$\langle \Gamma_{\nu_{\tau,\mu}} \rangle = 2.9 G_F^2 T^5. \quad (33)$$

Integrating the rate equations for the processes given in Eq. (3), using the modified neutrino momentum distributions ($N_{actual}(p)$), the change in the neutron to proton ratio and hence Y_p is determined. We find the size of the effect in this case is given by $\delta Y_p \approx -0.0023$ which corresponds to a change in the effective number of neutrinos of $\delta N_\nu^{eff} \approx -0.19$.

Excitation of the sterile neutrino, and the modified momentum distribution, will change the energy density of the universe, and this will also change the effective number of neutrinos. The number density and average energy of the

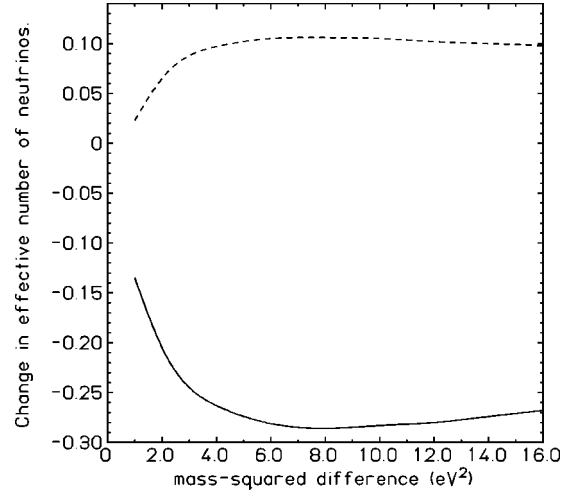


FIG. 1. Change in the effective number of neutrinos as a function of the mass-squared difference. The solid line corresponds to a positive electron neutrino asymmetry, and the dashed line a negative asymmetry.

neutrinos were calculated from the modified momentum distributions. For δm^2 in the range $6-8 \text{ eV}^2$ we get

$$\begin{aligned} \frac{n_{\bar{\nu}_e}}{n_0} &= 0.99, & \frac{\langle E_{\bar{\nu}_e} \rangle}{\langle p \rangle} &= 0.99, \\ \frac{n_{\bar{\nu}_\tau}}{n_0} &= \frac{n_{\bar{\nu}_\mu}}{n_0} = 0.53, & \frac{\langle E_{\bar{\nu}_\tau} \rangle}{\langle p \rangle} &= \frac{\langle E_{\bar{\nu}_\mu} \rangle}{\langle p \rangle} = 0.91, \\ \frac{n_{\bar{\nu}_s}}{n_0} &= 0.91, & \frac{\langle E_{\bar{\nu}_s} \rangle}{\langle p \rangle} &= 0.97, \end{aligned} \quad (34)$$

where n_0 is the number density with zero chemical potential and $\langle p \rangle = 3.15T$ is the average momentum for a Fermi-Dirac distribution. (The chemical potentials for the neutrinos are very small and so the distributions are not significantly modified.) These changes in the energy density correspond to $\delta N_\nu^{eff} \approx -0.09$. As we have an ambiguity concerning the sign of the lepton asymmetries and hence also the sign of δN_ν^{eff} due to modification of the nuclear reaction rates, we have two possible values for the overall change in the effective number of neutrinos.

In Fig. 1 δN_ν^{eff} is plotted as a function of the mass squared difference. For the mass range $\delta m^2 \approx 6-8 \text{ eV}^2$, we find:

$$\begin{aligned} \delta N_\nu^{eff} &\approx -0.3, & \text{positive asymmetry,} \\ \delta N_\nu^{eff} &\approx +0.1, & \text{negative asymmetry.} \end{aligned} \quad (35)$$

This remains approximately constant to above where the NOMAD experiment cuts off the LSND data at $\delta m^2 \approx 10 \text{ eV}^2$ [20], while for small δm^2 we have $\delta N_\nu^{eff} \rightarrow 0$.

The L_e asymmetry and hence the δN_ν^{eff} obtained is smaller than that obtained for the model of Ref. [6]. The $L_{\tau,\mu}$ asymmetry created in the present model is roughly half as

big as the L_τ asymmetry obtained in Ref. [6], because only one of the two linear combinations of ν_μ and ν_τ oscillates with ν_s , as discussed above. Naively one might expect that with both $\nu_\tau - \nu_e$ and $\nu_\mu - \nu_e$ oscillations producing L_e that an effect of comparable size to Ref. [6] would be obtained, however, this does not turn out to be correct. The crucial difference between the two cases is the separation of the two resonances. The separation of the resonances is given by Eq. (17), and whereas we have $p_2 \approx 3p_1$, in the previous case the resonances were closer together with $p_2 \approx 2p_1$. This is important because the resonance which creates L_e is at a higher momentum than the resonance which creates L_τ , so that by the time L_τ has grown sufficiently large to start producing significant L_e , the p_2 resonance has already moved a considerable way through the neutrino momentum distribution. For the present model, p_2/T has already moved past the average value of $p/T \approx 3.15$ and is out toward the tail of the distribution, where the number density is much lower, so that the L_τ asymmetry cannot be transferred to L_e as efficiently.

IV. CONCLUSION

We conclude that the neutrino oscillations in the four neutrino model of Ref. [7] can modify the effective number of

neutrinos during nucleosynthesis and give $N_\nu^{eff} < 3$. However, it turns out that the effect is relatively small, $\delta N_\nu^{eff} \approx -0.3$ (if $L_e > 0$). If future measurements confirm the low D/H result as well as the Y_p value used in Ref. [10], then this model will become cosmologically disfavored. On the other hand, if future measurements favor $N_\nu^{eff} \approx 3$ then this model will remain viable.

There is also the interesting possibility that the neutrino mass favored for dark matter [17] will have to be recalculated. In fact, from Eq. (34) the number of heavy neutrinos is actually about 1.5 because about 50% of the $\bar{\nu}_\tau$ and $\bar{\nu}_\mu$ have been converted into light $\bar{\nu}_s$. This would suggest that the optimal mass for hot+cold dark matter scenarios is somewhat heavier, about $m_{\nu_\tau} = m_{\nu_\mu} = 3.1$ eV rather than 2.4 eV, for this particular model. This implies that the δm^2 relevant for LSND is about 9 eV^2 rather than 6 eV^2 [21].

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