Neutrino oscillations in the dualized standard model

José Bordes*
Deptartament Fisica Teorica, Universitat de Valencia, calle Dr. Moliner 50, E-46100 Burjassot (Valencia), Spain

Chan Hong-Mo†
Rutherford Appleton Laboratory, Chilton, Didcot, Oxon, OX11 0QX, United Kingdom

Jakov Pfaudler‡
Department of Physics, Theoretical Physics, University of Oxford, 1 Keble Road, Oxford, OX1 3NP, United Kingdom

Tsou Sheung Tsun§
Mathematical Institute, University of Oxford, 24-29 St. Giles', Oxford, OX1 3LB, United Kingdom

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A method developed from the dualized standard model for calculating the quark CKM matrix and masses is applied to the parallel problem in neutrino oscillations. Taking the parameters determined from quarks and the masses of two neutrinos: $m_3^2 \sim 10^{-2}$–10^{-3} eV^2 suggested by atmospheric neutrino data and $m_2^2 \sim 10^{-10}$ eV^2 suggested by the long-wavelength oscillation solution of the solar neutrino problem, one obtains from a parameter-free calculation all the mixing angles in reasonable agreement with existing experiments. However, the scheme is found not to accommodate comfortably the mass values $m_2^2 \sim 10^{-5}$ eV^2 suggested by the MSW solution for solar neutrinos. [S0556-2821(98)05715-4]

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Experiments of recent years have accumulated an increasing amount of quite convincing evidence for the existence of neutrino oscillations which is beginning to constrain seriously the theoretical models invented for their explanation [1]. The problem thus offers, on the one hand, a possible window into a region of physics which is so far unexplored and, on the other, a challenge and a valuable testing ground for any theory which attempts to understand the many intriguing features of the standard model as we know it today. In particular, it would be interesting to ask whether the mass and mixing patterns we see in the quarks and the charged leptons are reflected in some way in the neutrinos, and if so, why it is that the neutrinos should appear nevertheless to be so very different, for example, in the extreme smallness of their mass and in the apparent absence of their right-handed partners.

We have recently suggested a scheme called the dualized standard model [2] which purports to have explained with some success the mass and mixing patterns of the quarks and the masses of the charged leptons [3]. Thus, it would seem incumbent upon us to make an attempt also at explaining neutrino oscillations with the same methodology. The purpose of the present article is to make a start in doing so.

The dualized standard model (DSM) is based on a recent theoretical result that non-Abelian Yang-Mills theory has an analogue of the electric-magnetic duality of the Abelian theory via a generalized dual transform [4]. This implies in particular that in addition to the (electric) $SU(3)$ color symmetry the standard model has also a dual (magnetic) $\widetilde{SU}(3)$ symmetry. The $SU(3)$ color symmetry being, as we know, in the confined phase, it then follows from the well-known result of 't Hooft [5] that the $\widetilde{SU}(3)$ dual color symmetry is in the Higgs phase and broken. Fermions occurring in the triplet representation of $\widetilde{SU}(3)$ (which are actually monopoles of color) would then carry a broken dual color index which would be similar in appearance to the generation index. If we choose to identify the two indices, as we did in the DSM scheme [2], then it follows that there are three and only three generations, a fact which seems strongly supported by present experiments.

The scheme further predicts that, at the tree level, only the highest generation fermion has a mass and that the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix between the U-type and D-type quarks is the identity, which is already not a bad zeroth order approximation to the experimental picture. Moreover, it goes on to predict that loop corrections will lift this degeneracy, and even suggests a method whereby such loop corrections can be perturbatively calculated [2]. A calculation to one-loop order has already been performed, which shows that with only a small number of parameters, one gets a good fit to the experimental CKM matrix and sensible values also for the quarks and charged leptons masses [3]. It seems therefore natural, perhaps even unavoidable, to ask whether the same procedure would apply also to neutrinos.

To set up the inquiry, let us first recall that in the DSM scheme, the fermion mass matrix remains factorizable to all orders, namely, that

$$M' = M_T \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} (x', y', z'), \tag{1}$$

*E-mail address: bordes@evalvx.ific.uv.es
†E-mail address: chanhm@v2.rl.ac.uk
‡E-mail address: jakov@thphys.ox.ac.uk
§E-mail address: tsou@maths.ox.ac.uk

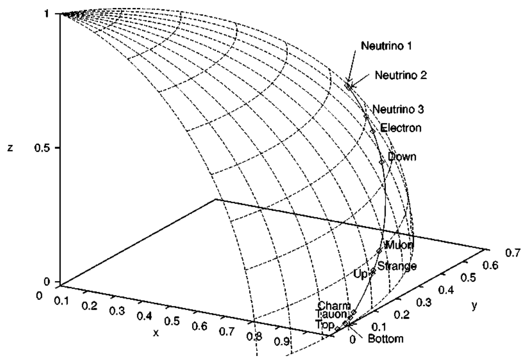

FIG. 1. The trajectory traced out by (x',y',z').

where M_T is the mass of the highest generation. Everything we need to know for calculating the CKM mixings and the fermion masses is encoded in the vector (x',y',z'), which for the questions we ask here we may take to be normalized thus:

$$x'^2+y'^2+z'^2=1. \tag{2}$$

This vector (x',y',z') rotates with the energy scale, and thus traces out a trajectory on the unit sphere, starting from near a fixed point $(1,0,0)$ at high energy scales to near another fixed point $(1/\sqrt{3})(1,1,1)$ at low energies, as illustrated in Fig. 1. The actual trajectory it traces out depends on (x,y,z), the tree-level values of (x',y',z') which are also the vacuum expectation values of the Higgs fields which break the dual color symmetry, and also weakly on the strength ρ of the Yukawa coupling of these Higgs fields to the fermion under consideration. The vacuum expectation values (VEV's) (x,y,z) are common to all fermion types but, as far as we understand it at present, the ρ's can in principle be different for different fermion types. However, for some yet unknown reason, the three values obtained by fitting the quark CKM matrix and the two higher generation quark and charged lepton masses turned out to be equal to a high accuracy, so much so that we suspect that there is a hidden symmetry in the problem which we have not yet understood. The result in practical terms is that all three fermion types (U,D,L) run on the same trajectory, and they differ only in the positions where the actual physical states of each type are located on that trajectory. The trajectory shown in Fig. 1 is in fact the one determined in [3] by fitting the quark CKM matrix and masses. Also shown are the locations of the various quark and charged lepton states on the trajectory obtained in that calculation. In this scenario, the masses and state vectors of all three generations of each fermion type are obtained once the mass M_T of the highest generation appearing in Eq. (1) is specified. For more details, the reader is referred to our earlier paper [3].

Let us turn now to the problem of neutrinos. Since neutrinos seem to exist also in three generations [6] which in the DSM scheme would be identified with dual color, it would appear that nothing is changed compared with the other three fermion types as far as their Dirac mass matrix is concerned. Hence, once given the mass $M_3=M_T$ of the highest generation and assuming the same ρ as for the other fermions, our prescription will allow us to calculate the Dirac masses M_2 and M_1 of the two lower generations as well as the state vectors of all three generations. And since the state vectors of the charged leptons are already known from our earlier work [3], one can then calculate also the leptonic CKM matrix and hence the mixing angles appearing in neutrino oscillations. However, the Dirac masses M_i specified above are not yet the physical masses of the neutrino states, for, as is well known, right-handed neutrinos can have Majorana masses, so that for each generation i one has yet to diagonalize a 2×2 submatrix of the form

$$\mathbf{M}_i=\begin{pmatrix} 0 & M_i \\ M_i & B \end{pmatrix}, \tag{3}$$

giving, for the physical masses of the neutrinos,

$$m_i=M_i^2/B, \tag{4}$$

where for the DSM as formulated in [2] B has to be the same for all i for consistency. For B large, this way of determining the physical masses of neutrinos is the famous seesaw mechanism [7] which can give very small physical masses m_i for the neutrinos with not too small Dirac masses M_i. The parameter B, which can be interpreted as the mass of the yet undiscovered neutrinos with a large right-handed component (henceforth referred to as ''right-handed neutrinos'' for short), is unknown, so that in contrast with the other fermion types, depending on only one mass scale, the neutrino calculation involves two mass scales which have still to be specified. This we can do once we know the masses of any two of the neutrinos.

We turn then to experiment to see whether we can find enough information to determine two of the neutrino masses. Attempts at direct measurements of neutrino masses have yielded up to now the following sort of upper limits [8]:

$$m_{\nu_\tau}<24 \text{ MeV}, \quad m_{\nu_\mu}<0.17 \text{ MeV}, \quad m_{\nu_e}<10 \text{ eV}, \tag{5}$$

which one suspects to be rather weak. On the other hand, information from other sources, such as the depletion effects of solar and atmospheric neutrinos, when interpreted as being due to neutrino oscillations is much more stringent. For example, to explain the solar neutrino puzzle as neutrino oscillations, we are offered two solutions: (i) the so-called long-wavelength oscillation (LWO) solution which corresponds to oscillations over distance scales of the order of the radius of Earth's orbit and requires $\Delta m_{12}^2 \sim 10^{-10}$ eV^2 [9,10] and (ii) the Mikheyev-Smirnov-Wolfenstein (MSW) solution [11,12] which corresponds to oscillations over distance scales of the order of the Sun's radius and requires $\Delta m_{12}^2 \sim 10^{-5}$ eV^2. On the other hand, to explain the atmospheric neutrino anomaly observed by the Kamiokande [13], IMB [14], and Soudan experiments [15], $\Delta m_{23}^2 \sim 10^{-2}$–$10^{-3}$ eV^2 is required [16]. Here Δm_{ij}^2 represents the mass-squared difference between the ith and jth generation neutrinos.

As far as we are concerned within the framework of the DSM, where masses for the lower two generations of fermions are obtained by the ''leakage'' mechanism of a rotating mass matrix as explained in [3], the Dirac masses M_i of the neutrinos, like other fermions, have to be hierarchical, implying thus, by Eq. (4), $m_1 \ll m_2 \ll m_3$. Hence, from the above estimates for mass differences, we conclude that we should put for the mass of the heaviest neutrino $m_3^2 \sim 10^{-2}-10^{-3}$ eV^2, and for the mass of the second heaviest neutrino either (i) $m_2^2 \sim 10^{-10}$ eV^2 (LWO) or (ii) $m_2^2 \sim 10^{-5}$ eV^2 (MSW). We have thus the two mass scales we seek as input information for proceeding with our calculation, which, apart from the two masses here determined from experiment, will be parameter free, and will give us as predictions all the mixing angles in the leptonic CKM matrix, as well as the masses of the lightest neutrino and the right-handed neutrinos.

The calculation follows along exactly the same lines as for the quarks and charged leptons given in [3] and uses essentially the same numerical programs. Starting with some assumed value for M_3, the Dirac mass, one runs the scale down until it becomes equal to the Dirac mass M_2 of the second generation obtained by the ''leakage'' mechanism from the rotating mass matrix. From these values of M_3 and M_2, one can then evaluate the corresponding values of $m_2/m_3 = M_2^2/M_3^2$, the ratio of the actual masses of the two heaviest neutrinos as given by the seesaw mechanism (4). This value of the mass ratio, of course, need not agree with either of the values obtained from the estimates given in the above paragraph deduced from experiment. One then varies the assumed value for M_3 until the output ratio for m_2/m_3 agrees with the value favored by experiment. For each assumed value of M_3, our program automatically calculates the corresponding value for m_1, the mass of the lightest neutrino, and the predicted mass of the right-handed neutrino B. Furthermore, the orientation in dual color space of the state vectors for all three generations of neutrinos is also given. Hence, combining this with the result previously obtained in [3] for the state vectors of the charged leptons, the whole leptonic CKM matrix can be evaluated.

Consider first the case corresponding to the long-wavelength solution (i) for the solar neutrino puzzle, namely, $m_2^2 \sim 10^{-10}$ eV^2. We obtain for input values for $M_3 = 2.0, 3.7$ MeV, respectively, the following results:

$$m_3^2 = 10^{-2}\ \text{eV}^2, \quad m_1 = 5\times 10^{-17}\ \text{eV}, \quad B = 40\ \text{TeV}, \tag{6}$$

$$|\text{CKM}|_{lepton} = \begin{pmatrix} 0.9671 & 0.2417 & 0.0795 \\ 0.2277 & 0.6823 & 0.6947 \\ 0.1136 & 0.6899 & 0.7149 \end{pmatrix}, \tag{7}$$

and:

$$m_3^2 = 10^{-3}\ \text{eV}^2, \quad m_1 = 10^{-15}\ \text{eV}, \quad B = 430\ \text{TeV}, \tag{8}$$

$$|\text{CKM}|_{lepton} = \begin{pmatrix} 0.9694 & 0.2355 & 0.0700 \\ 0.2215 & 0.7142 & 0.6640 \\ 0.1063 & 0.6591 & 0.7445 \end{pmatrix}. \tag{9}$$

We notice first that it is possible to obtain values of m_3 within the range required by the atmospheric neutrino experiments. Second, we note that the mass obtained for the lightest neutrino is extremely small. This is because the value of (x', y', z') for this neutrino, as seen in Fig. 1, is already getting very close to the fixed point $(1/\sqrt{3})(1,1,1)$ so that the leakage mechanism hardly operates and thus gives it very little mass. Needless to say, our calculation in this mass region is far from reliable and gives at best just a rough order of magnitude. The estimate for the mass of the right-handed neutrino B depends only on the two heaviest neutrinos and should be more reliable. Interestingly, its value turns out to be of the same order as the value of the VEV's of the Higgs fields responsible for breaking the dual color or generation symmetry as estimated from the experimental bounds on the $K^0 - \bar{K}_0$ mass difference and on flavor-changing neutral current decays [17]. Notice that our estimate for B is considerably lower than what is usually assumed, in grand unified theories, for example [18]. The reason is that one usually uses a Dirac mass for the neutrino similar to that for the charged lepton, namely, for the highest generation a mass of around 1 GeV. On the other hand, for our calculation here we want for the Dirac mass M_3 a value of only a few MeV, and for fixed m_3, B according to Eq. (4) is proportional to M_3^2. That neutrinos and charged leptons can have widely different Dirac masses one need not find disturbing if one recalls that even for the quarks, Dirac masses between the U and the D types differ by as much as a factor of 50, as witnessed by the masses of the t and the b. The fact that our estimate for B is of the order of 10^3 TeV means that the implied limits for neutrinoless double beta decay and for neutrino-antineutrino oscillations, while compatible with existing experimental bounds, may be much more accessible than previously anticipated. A detailed analysis of the experimental situation has not, however, been done and is beyond the scope of the present work.

In this paper, we focus on the mixing angles in the leptonic CKM matrix for comparison with existing data. To the order of one-loop corrections in the DSM scheme, the CKM matrix whether for quarks or leptons is real [3], so that there are only three independent parameters to consider, which we can take to be the three elements in the upper right corner of the matrix, namely, $U_{\mu 3}$, U_{e3}, and U_{e2}. We shall examine each in turn.

Consider first the element $U_{\mu 3}$ which plays the central role in atmospheric neutrinos, where, to explain the muon puzzle, one needs a sizable value for $U_{\mu 3}$. Indeed, according to the recent analysis in [16], for example, $|U_{\mu 3}|$ has to have a value roughly between 0.45 and 0.85 to explain the Kamiokande data [13]. One notes that the values we obtained in Eqs. (7) and (9) fall right in the middle of the permitted range.

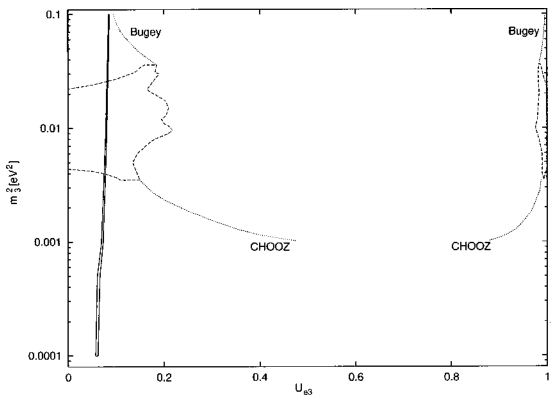

FIG. 2. Ninety percent C.L. limits on the CKM element U_{e3} compared with the result from our calculation.

Next, consider the element U_{e3}. Its value is constrained not only by atmospheric neutrino data but also by terrestial experiments such as Bugey [19] and CHOOZ [20]. The absence of any observed effects in the latter type of experiments puts an upper bound on $|U_{e3}|$ of around 0.15 at, for example, $m_3^2 \sim 10^{-2}$ eV^2. Again, one notes that the value we calculated, as quoted in Eqs. (7) and (9) above, satisfy this bound.

In Figs. 2 and 3, we reproduce in terms of the CKM matrix elements the individual 90% C.L. limits on U_{e3} and $U_{\mu 3}$ obtained by [16] with the data from the Kamiokande, Bugey, and CHOOZ experiments and compare them with the result of our calculation for a range of m_3 values. Also shown in Figs. 4 and 5 are the correlated bounds on these elements quoted from the same source. The width of our curves represents the range of values calculated from m_2^2 values lying within the admissible range 5×10^{-11}–1.1×10^{-10} eV^2 obtained from the analysis of solar neutrino data by [9,10]. One notes first that the calculated values of the mixing parameters are quite insensitive both to the input value of m_2^2 and to the value of m_3^2. Second, one sees that in each case our curve lies comfortably within the experimental limits for all reasonable values of m_3^2 except for Fig. 5 where it passes near the edge. It seems thus that the agreement of our calculated $U_{\mu 3}$ and U_{e3} with experiment is rather good. Notice that the analysis in [16] did not take into account the new SuperKamiokande [21] data which is thought to lower the estimate for m_3^2 to around 10^{-3} eV^2. For this reason we have given the result of our calculation also for m_3^2 values outside the quoted limits from [16] in anticipation of a comparison with future analyses of the SuperKamiokande data.

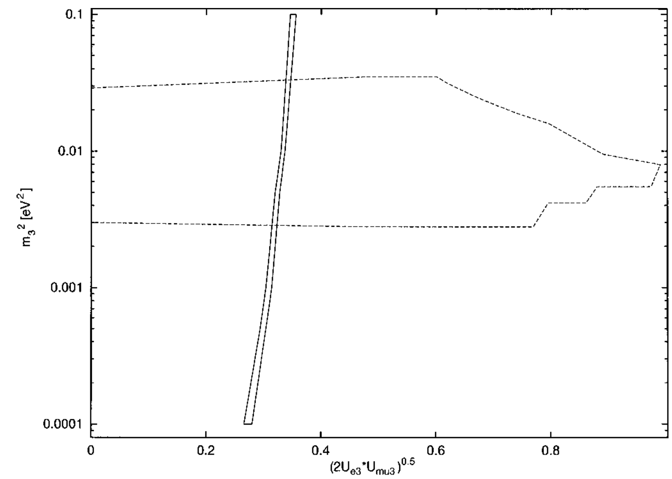

FIG. 4. Ninety percent C.L. limits on the quantity $(2U_{e3}U_{\mu 3})^{1/2}$ compared with the result from our calculation.

For the remaining CKM element U_{e2} which enters mainly in the solar neutrino problem, we cannot as yet make a clear comparison with experiment. Detailed analyses of the solar neutrino data for the LWO scenario have, as far as we know, been performed only for two flavors [9,10]. If we compare our calculated U_{e2} with these two-flavor analyses, then our values for the mixing angle of around 14° lie outside the bound obtained of $>27°$. However, a full three-flavored analysis of the solar neutrino data where account is taken of both long-wavelength and Mikheyev-Smirnov-Wolfenstein (MSW) oscillations has to be done before a meaningful comparison can be made, since for m_3^2 approaching 2×10^{-4} eV^2, a value possibly compatible with SuperKamiokande [21], an adiabatic MSW transition $e\rightarrow 3$ may lead to

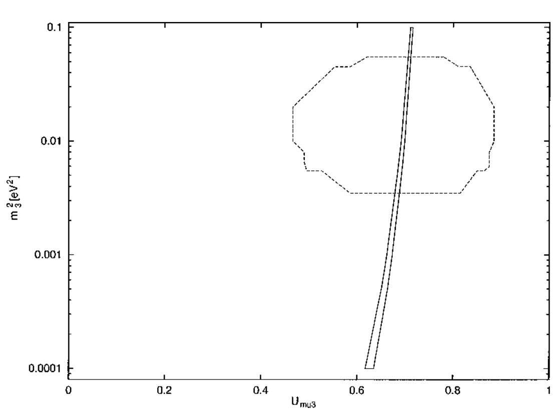

FIG. 3. Ninety percent C.L. limits on the CKM element $U_{\mu 3}$ compared with the result from our calculation.

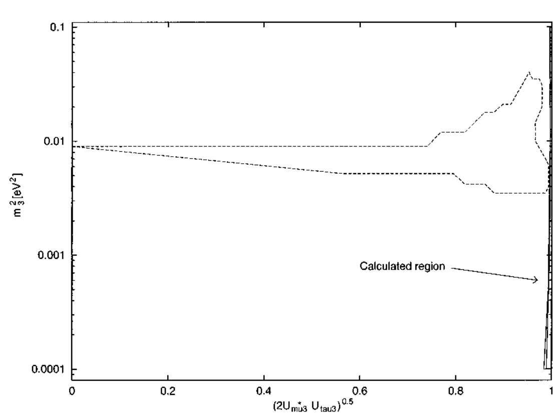

FIG. 5. Ninety percent C.L. limits on the quantity $(2U_{\mu 3}U_{\tau 3})^{1/2}$ compared with the result from our calculation.

significant depletion. Even if it turns out that the quoted estimate from the two-flavored analyses is not appreciably affected with three flavors taken into account so that the discrepancy with our calculation remains, one should perhaps still not be too disappointed, given that the value is obtained from a parameter-free calculation in which fairly crude approximations have been made, and that in our scheme U_{e2} is particularly sensitive to details [22].

It is instructive to compare the lepton CKM matrix (7) or (9) obtained above with the quark CKM matrix calculated in [3] by the same method with a common set of parameters:

$$|\mathrm{CKM}|_{quark}=\begin{pmatrix} 0.9755 & 0.2199 & 0.0044 \\ 0.2195 & 0.9746 & 0.0452 \\ 0.0143 & 0.0431 & 0.9990 \end{pmatrix}, \qquad (10)$$

which was seen to fit very well with that obtained in experiment.

One notices first the striking fact that the upper right corner (i.e., the 13) elements in both matrices are particularly small and much smaller than the 12 elements (i.e., the Cabibbo angle) or the 23 elements. For the quark case, the smallness of the 13 element is needed for explaining b decays, while for the lepton case, we recall, it is needed to satisfy the bounds imposed by the CHOOZ [20] oscillation experiment, at least for $m_3^2 > 10^{-3}$ eV^2. It would thus be interesting to understand why this feature should emerge correctly from our calculations with the DSM scheme. The answer turns out to be quite intriguing, giving the above feature as a consequence of the general differential geometric properties of space curves. As already explained, the nondiagonal CKM matrix elements arise from loop corrections which forces the vector (x',y',z') to run along a trajectory on a sphere, as depicted in Fig. 1 above. Recalling then from [3] in detail how the state vectors of the three physical states of each fermion type are defined and how the CKM matrix is constructed from these, it can be shown [22] that the 12 and 23 elements of the CKM matrix are associated with the curvatures of the trajectory on the sphere while the element 13 is associated with its torsion. It then follows that the 13 element is necessarily small compared with the 12 and 23 elements.

Second, we note that the 23 element is much larger in the lepton than in the quark CKM matrix. This is physically important, or otherwise, as explained above, one would not be able to explain the large muon anomaly observed in atmospheric neutrinos. Within the scheme employed here, this enhancement of the 23 element for leptons over quarks can again be easily understood in differential geometrical terms. Indeed, it can be shown [22] that the 23 element of a CKM matrix is associated with the so-called normal curvature of the trajectory which on a sphere is constant. This element is thus roughly proportional to the separation on the trajectory between the locations of the two fermion types to which it refers. Now, as can be seen in Fig. 1, the leptons have a much longer distance to run from τ to ν_3 than the quarks from t to b, so that it follows that $U_{\mu 3}$ is necessarily much larger than V_{cb}, as is experimentally observed.

The fact that these important empirical features can be traced through some simple differential geometry directly back to the intrinsic properties of the dualized standard model we consider to be a nontrivial and encouraging check both of the scheme itself and of our calculations.

So far, we have considered only the case (i) with $m_2^2 \sim 10^{-10}$ eV^2 corresponding to the so-called LWO solution to the solar neutrino problem. What about the case (ii) with $m_2^2 \sim 10^{-5}$ eV^2 corresponding to the so-called MSW solution? This is not as easy for the present scheme to accommodate. We recall that in the DSM scheme, the second generation acquires a mass only through ''leakage'' from the highest generation, and this ''leakage'' is limited, as explained above, by the curvature of the trajectory. Given that the value of m_2 required by the MSW solution (ii) is so much larger than that required by the LWO solution (i), the former will require a much larger ''leakage'' and this is not easily available on our trajectory. Indeed, taking the same value of ρ for neutrinos as for the other fermions as we have done above, one can easily find the maximum value for M_2/M_3 to be about 0.11, which for m_2^2 of order 10^{-5} eV^2 gives necessarily $m_3^2 > 7\times 10^{-2}$ eV^2, which is some way above the range preferred by Kamiokande [13] and SuperKamiokande [21]. If one relaxes the condition that ρ should be the same as for the other fermions, for which after all there is as yet no theoretical justification, then one can obtain enough ''leakage'' to move m_3^2 into the 10^{-2}–10^{-3} eV^2 range, but only at the cost of a large $\rho > 5$ and a very large Dirac mass $M_3 \sim 1$ TeV. Besides, it requires further struggle to get the mixing angles within the experimental bounds set by, e.g., [16] since the sizable value for $U_{\mu 3}$ required by the atmospheric neutrino data necessitates, in our present framework, also a sizable separation on the trajectory between the locations of the two highest generation neutrinos. Indeed, in all the attempts we have made so far, it is only by choosing values as high as $\rho \sim 18$, $M_3 \sim 16$ TeV that we manage to get $m_3^2 \sim 10^{-2}$ eV^2 and $U_{\mu 3} \sim 0.44$ just within the 90% limits set by [16] from the Kamiokande data. It thus seems that although one can still possibly obtain some fits at the cost of one more parameter ρ than the case above for $m_2^2 \sim 10^{-10}$ eV^2, this scenario for $m_2^2 \sim 10^{-5}$ eV^2 is far less comfortably accommodated.

It is clear also that the DSM scheme would have difficulty accommodating neutrinos with masses of the order of several eV's as those wanted by astrophysicists for hot dark matter [23] or the neutrinos possibly indicated by the LSND [24] experiment. One can, of course, introduce here by hand, as one does in other schemes, extra sterile neutrinos to foot the bill, but that would be against the spirit of the whole idea which is, perhaps overambitiously, to aim at an overall explanation for the quark and lepton spectrum as we know it today.

However, in the case with $m_2^2 \sim 10^{-10}$ eV^2 with which the DSM scheme is most compatible, one is able to predict with no free parameter all the mixing angles which appear to be consistent with what is known so far in experiment. And these results are obtained with the same method as that applied before to calculate the quark CKM matrix using exactly the same values of the common parameters. It is this possibility of a consistent treatment of the two related problems that we find most encouraging.

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