

Bulk charges in eleven dimensions

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(Received 9 December 1997; published 12 June 1998)

Eleven dimensional supergravity has electric type currents arising from the Chern-Simon and anomaly terms in the action. However the bulk charge integrates to zero for asymptotically flat solutions with topological trivial spatial sections. We show that by relaxing the boundary conditions to generalisations of the ALE and ALF boundary conditions in four dimensions one can obtain static solutions with a bulk charge. Solutions involving anomaly terms preserve between 1/16 and 1/4 of the supersymmetries but Chern-Simons fluxes generally break all of the remaining supersymmetry. One can introduce membranes with the same sign of charge into these backgrounds. This raises the possibility that these generalized membranes might decay quantum mechanically to leave just a bulk distribution of charge. Alternatively and more probably, a bulk distribution of charge can decay into a collection of singly charged membranes. Dimensional reductions of these solutions lead to novel representations of extreme black holes in four dimensions with up to four charges. We discuss how the eleven-dimensional Kaluza-Klein monopole wrapped around a space with non-zero first Pontryagin class picks up an electric charge proportional to the Pontryagin number. [S0556-2821(98)01914-6]

PACS number(s): 04.65.+e, 04.50.+h, 04.60.Ds, 11.25.Hf

I. INTRODUCTION

The only bosonic fields in eleven dimensional supergravity are the metric and a three form potential A for a four form field strength F . The gauge symmetry is Abelian and the gravitino couples to the four form field strength rather than the potential. Thus it might seem that there were no charged fields in the theory. However, the action contains a Chern-Simons term [1]

$$S_{CS} \propto \int (A \wedge F \wedge F), \quad (1)$$

which implies that the divergence of the four form field strength is nonzero,

$$d * F \propto F \wedge F. \quad (2)$$

In other words, there is an electric type bulk current $*(F \wedge F)$; the magnetic type bulk current is, however, zero since $dF=0$. This means that any magnetic charges $P_X = \int_X F$ where X is a four cycle are purely topological and are the same for all homologous X . On the other hand, the electric charges $Q_Y = \int_Y *F$ where Y is a closed seven surface can have both topological and bulk contributions. There is also a contribution to the electric current and charge from the term in the bulk action that is needed to cancel anomalies from the world volume of the five brane [2].

One can integrate the electric current over an eight surface K with $\partial K = Y$ to obtain the bulk contribution to the electric charge. If K were asymptotically Euclidean, one could conformally compactify by adding a point at infinity. The Chern-Simons part of the electric current is conformally

invariant. Thus its contribution to the charge at infinity must integrate to zero unless the fourth cohomology class $H^4(K, \mathbb{R})$ is non-trivial. In particular, it will be zero for $K = \mathbb{R}^8$ and the anomaly bulk contribution will also integrate to zero. This means that the ordinary asymptotically flat membrane cannot decay by losing its charge to a bulk distribution in space.

On the other hand, a bulk electric charge can occur if K is either not topologically trivial or not asymptotically Euclidean. As an example of the first kind, consider $S^4 \times S^4$ and send a point to infinity by a conformal transformation to obtain an asymptotically Euclidean surface K . One can choose the conformal factor, the scale of two extra flat spatial dimensions and the four form to satisfy the time symmetric constraint equations. This would give time symmetric initial data for a solution with an electric charge that arose solely from bulk contributions. The classical evolution of such a solution would be collapse to a membrane. However, we shall show that the situation may be different when K is not asymptotically Euclidean but obeys some generalization of the asymptotically locally Euclidean (ALE) or asymptotically locally flat (ALF) boundary conditions in four dimensions. In that case bulk charges can be classically stable though one would expect that they may decay quantum mechanically into a collection of membranes which would have more phase space.

We will show that there are static solutions with bulk distributions of electric charge even when the topology of K is $\mathbb{R}^8$. Such solutions can preserve a fraction of the supersymmetry provided that K admits covariantly constant spinors. If the anomaly form confined to K does not vanish, one must have such a bulk distribution of charge. Provided that K admits a self-dual harmonic 4-form which is finite on the boundary Y one can also have contributions from a Chern-Simons bulk current but such a current generally causes all the supersymmetry to be broken. Solutions for which the Chern-Simons term is non-zero have not been studied much, but an elegant framework for incorporating

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such an F in a fashion that preserves supersymmetry was developed in [3] and we extend this framework here. One can introduce generalized membranes with the same sign of charge into these backgrounds. This indicates that Chern-Simons bulk charges in these more general backgrounds could decay into membranes.

Several of the solutions which we construct have particularly interesting physical interpretations. For example, we find solutions describing eleven-dimensional Kaluza-Klein (KK) monopoles wrapped around topologically non-trivial world volumes; the anomaly induces an electric type bulk charge. This result is as one would expect: if one wraps any p -brane around a surface of a non-zero first Pontryagin class, one induces $(p-4)$ -brane charges [4].

When one wraps the KK monopoles around a torus or $K3 \times T^2$ one obtains extreme black holes in four dimensions, with up to four charges, some of which arise from Chern-Simons and anomaly terms. The resulting four-dimensional spacetimes resemble those of multi-center black hole solutions, although the two types of solutions differ by the presence of pseudo-scalar fields in the former, which originate from the part of the 4-form confined to the compact space. Black holes obtained by dimensional reduction of bulk distributions of charge break all the supersymmetry, and have more singular horizons and higher temperatures than those with the same charge in membranes. This again suggests that bulk charges will decay into membranes.

II. VACUUM SOLUTIONS

The bosonic part of the action of $d=11$ supergravity is given by [1]

$$S_{11} = \frac{1}{2} \int d^{11}x \sqrt{g} R - \frac{1}{2} \int \left(\frac{1}{2} F \wedge *F + \frac{1}{6} A \wedge F \wedge F \right) \quad (3)$$

where g_{MN} is the space time metric and A is a 3-form with field strength $F = dA$. We have set κ^2 to 1. The field strength obeys the Bianchi identity $dF = 0$ with its field equation being

$$d*F = -\frac{1}{2} F \wedge F. \quad (4)$$

In general, there will be gravitational Chern-Simons corrections to this equation associated with the σ -model anomaly on the $d=6$ five-brane [2]. The corrected 5-brane Bianchi identity takes the form

$$d*F = -\frac{1}{2} F \wedge F + (2\pi)^4 \beta X_8, \quad (5)$$

where β is related to the five-brane tension by $T_6 = \beta/(2\pi)^3$. In all that follows we shall take $\beta=1$ for simplicity. The 8-form anomaly polynomial can be expressed in terms of the curvature as [5]

$$X_8 = \frac{1}{(2\pi)^4} \left\{ -\frac{1}{768} (\text{Tr} R^2)^2 + \frac{1}{192} (\text{Tr} R^4) \right\}, \quad (6)$$

and there is thence an additional term in the action of the form

$$\Delta S_{11} = \frac{1}{2} \int A \wedge \left(-\frac{1}{768} (\text{Tr} R^2)^2 + \frac{1}{192} (\text{Tr} R^4) \right). \quad (7)$$

Motivated by the usual form of membrane solutions, we will start by looking for a solution of the form

$$ds^2 = H(x^m)^{-2/3} ds^2(B^3) + H(x^m)^{1/3} ds^2(B^8), \quad (8)$$

where we take coordinates x^μ , $\mu=0,1,2$, on the Lorentzian 3-fold, and coordinates x^m , $m=3, \dots, 11$, on the Euclidean 8-fold. We allow the scalar function to depend only on the latter and assume both that the anomaly form is non-trivial and that the Chern-Simons term does not vanish. Initially we will consider vacuum solutions; that is, we do not include membrane or five-brane source terms. Since it will turn out that the metrics on both B^3 and B^8 are Ricci-flat, we will refer to them as $g_{\mu\nu}$ and g_{mn} respectively. We will refer to the eleven-dimensional metric as $\bar{g}_{MN}$, to the associated covariant derivative as $\bar{D}$ and to the associated gamma matrices as $\bar{\Gamma}^M$.

It is natural to choose B^3 to be a symmetric space, and take F to be of the form

$$F_{\mu\nu\rho n} = \pm \epsilon_{\mu\nu\rho} \partial_n f(x^m), \quad (9)$$

where $f(x^m)$ is a scalar function that will be related to the scale factor $H(x^m)$. We will also allow for a general F on the 8-fold, the form of which will be fixed by the field equations. For clarity we express the 4-form as the sum

$$F = F_1 + F_2, \quad (10)$$

where F_1 takes the form of Eq. (9) and F_2 represents the part of F which is confined to the 8-fold.

The presence of the anomaly term in the Einstein equations makes it difficult to look for solutions of this form by solving the Einstein equations explicitly. Instead we look for a supersymmetric configuration satisfying this ansatz. Since the gravitino Ψ_M vanishes in the background, the only non-trivial constraint on a supersymmetric solution is that for some Majorana spinor η ; variations of the gravitino vanish:

$$\delta_\eta \Psi_M = \bar{D}_M \eta - \frac{1}{288} (\bar{\Gamma}_M^{PQRS} - 8 \delta_M^P \bar{\Gamma}^{QRS}) F_{PQRS} \eta = 0, \quad (11)$$

where $\bar{\Gamma}_M$ is an eleven-dimensional gamma matrix. Our conventions and notation for the gamma matrices are given in the Appendix.

Solutions of this type, for which the 8-fold is compact, were discussed in [3]; since the analysis of the field equations changes little when the 8-fold is non-compact, we give only a brief summary here. We decompose the eleven-dimensional gamma matrices by taking

$$\bar{\Gamma}_\mu = \bar{\gamma}_\mu \otimes \bar{\gamma}_9,$$

$$\bar{\Gamma}_m = 1 \otimes \bar{\gamma}_m, \quad (12)$$

where γ_μ and γ_m are gamma matrices of B^3 and B^8 respectively, and the barred quantities with overbars are the appropriately conformally rescaled matrices. γ_9 is the eight-dimensional chirality operator which anticommutes with all the γ_m and satisfies $\gamma_9^2 = 1$.

We then decompose the eleven-dimensional spinor η as

$$\eta = \epsilon \otimes \zeta(x^m), \quad (13)$$

where ϵ is a three-dimensional anticommuting spinor, and ζ is a commuting eight-dimensional Majorana-Weyl spinor. Then the μ components of the gravitino equation reduce to the requirements that

$$D_\mu \epsilon = 0, \quad (14)$$

which implies that the symmetric three space B^3 must be Minkowskian. In addition, ζ is of negative or positive chirality corresponding to the positive or negative signs in Eq. (9), and the function $f(x^m)$ is given by

$$f(x^m) = H^{-1}(x^m) \quad (15)$$

and the spinor ζ must satisfy the condition

$$F_{mnpq} \gamma^{mnpq} \zeta = 0. \quad (16)$$

This gives the first constraint on the four-form flux in the 8-fold; a stronger condition will be given by the other components of the gravitino equation which take the form

$$D_m \zeta + \frac{1}{24} H^{-1/2} F_m \zeta = 0, \quad (17)$$

where we have introduced the notation $F_m = F_{mnpq} \gamma^{nqp}$. This implies that $\zeta^\dagger \zeta$ is a constant which may be normalized to unity. One can then construct an almost complex structure in terms of ζ as

$$J_{mn} = i \zeta^\dagger \gamma_{mn} \zeta, \quad (18)$$

which is covariantly constant,

$$D_p J_{mn} = 0, \quad (19)$$

because $\zeta^\dagger \gamma^{a_1 \dots a_k} \zeta$ vanishes if k is odd since the spinor involved is Weyl and one can pull γ_9 through the expression. It is straightforward to show that J_{mn} is a complex structure and that B^8 is a Kähler manifold. In addition one can show that B^8 is a Calabi-Yau manifold, either following the approach given in [3] or by defining a one form from the Christoffel connection Γ_{mn}^p such that

$$\Gamma = \Gamma_{mp}^q J_q^p dx^m \quad (20)$$

and then showing that Eq. (17) implies that Γ is closed. Since Γ is the $U(1)$ part of the connection, closure of Γ implies that the first Chern class is trivial. Hence to satisfy the m components of the gravitino variation equation one imposes the condition that the holonomy of B^8 be contained

in $SU(4)$. [One needs to consider the case that B^8 has holonomy of precisely $Spin(7)$ separately as we will discuss below.]

In the former case, the only non-zero components of the 4-form within the 8-fold are the $F_{\bar{a}\bar{b}\bar{c}\bar{d}}$ components which must satisfy

$$F_{\bar{a}\bar{b}\bar{c}\bar{d}} J^{\bar{c}\bar{d}} = 0, \quad (21)$$

where J is the Kähler form and we use complex indices. On a compact Kähler space, we can express the solution for F_2 in terms of the harmonic 4-forms ω_4^i as

$$F_2 = \sum_{i=1}^{h_{22}} v^i \omega_4^i, \quad (22)$$

where h_{22} are components of the Hodge numbers. For our purposes it will be more useful to replace equation (21) by an equivalent constraint of the form (16) so that

$$F_{mnpq} \gamma^{nqp} \zeta = F_m \zeta = 0, \quad (23)$$

a condition which is apparent from (17) since on a Calabi-Yau 4-fold the spinor ζ is covariantly constant. Equation (23) is a very restrictive condition and severely limits the number of supersymmetric solutions with non-vanishing four form flux.

Quantization of the magnetic charge imposes a constraint on the v^i ; defining a four-form G which is related to F_2 by normalization factors, flux quantization requires that [6]

$$[G] - \frac{1}{4(2\pi)^2} P_1(B^8) \in H_4(B^8, \mathbb{Z}), \quad (24)$$

where P_1 is the first Pontryagin class of B^8 and $[G]$ is the cohomology class of G . Note that we are defining the Pontryagin classes without factors of (2π) . The existence of spinors on B^8 implies that $P_1/(2\pi)^2$ is canonically divisible by 2. Depending on whether it is also canonically divisible by 4, G will have integral or half-integral periods, and the coefficients v^i will be integers or half-integers.

Now the 32 component real spinor of the $d=11$ Lorentz group decomposes into representations of $SL(2, \mathbb{R}) \times SO(8)$:

$$32 \rightarrow (2, \mathbf{8}_s) \oplus (2, \mathbf{8}_c), \quad (25)$$

where $\mathbf{8}_s$ and $\mathbf{8}_c$ have opposite chiralities. If the holonomy is trivial, and F is zero, then each of the $\mathbf{8}$'s decomposes into eight singlets, and there are 32 covariantly constant spinors. If the holonomy is precisely $SU(4)$, then one of the spinor representations decomposes as

$$\mathbf{8}_c \rightarrow \mathbf{6} \oplus \mathbf{1} \oplus \mathbf{1}, \quad (26)$$

and there is thus a total of four covariantly constant spinors, of a defined chirality; only 1/8 of the supersymmetry is preserved. If the holonomy breaks down further, a greater fraction of the supersymmetry will be preserved.

Given that this solution is obtained by requiring a fraction of the supersymmetry to be preserved, it is natural to assume

that the field equations will also be satisfied. Let us first consider the equation for the 4-form. Then the equation of motion for F_2 is

$$d(*F_2) = -\frac{1}{2}F \wedge F = -F_1 \wedge F_2, \quad (27)$$

where the conformal invariance of the field equation implies that there is no X_8 contribution. Now it was incorrectly stated in [3] that this equation implies no further condition on the 4-form than closure of F_2 . Using the explicit form for F_1 [taking the sign in Eq. (9) to be positive] we find that

$$d(H^{-1}\eta_3 \wedge *F_2) = -\eta_3 \wedge dH^{-1} \wedge F_2, \quad (28)$$

where η_3 is the flat volume form on the transverse 3-fold and we take the dual $*$ in the Ricci-flat metric on the 8-fold. Since F_2 is harmonic, to satisfy this equation we must take F_2 to be anti-self-dual. If we reverse the sign in Eq. (9), the covariantly constant spinors on the 8-fold must have the opposite chirality, and the four-form F_2 must be self-dual. Thus we must include only self-dual–anti-self-dual forms in the summation (22).

It is interesting to note that this field equation for F is automatically satisfied if the gravitino variation equation $\delta\Psi_\mu = 0$ is obeyed. The required duality condition on F is implicit in Eq. (16); if one chooses the sign of F_1 such that ζ is of positive chirality, then Eq. (16) implies that F_2 is anti-self-dual. Of course Eq. (16) is a stronger condition than the four form equation of motion since one may not be able to find such a conserved spinor ζ .

The field equation for F_1 does include an anomaly term and imposes a constraint on the scalar function as

$$d(*_8 dH) = -\frac{1}{2}F \wedge F + (2\pi)^4 X_8, \quad (29)$$

where without loss of generality we have chosen the negative sign in Eq. (9) and $f(x^m) = H^{-1}(x^m)$. Note that we are again taking the dual in the Ricci-flat metric on the 8-fold. When the 8-fold is compact, there is a relationship between the anomaly form defined in Eq. (6) and the Euler class for any manifold which admits a nowhere vanishing spinor [7]. For a nowhere vanishing positive chirality spinor to exist, the Euler class $e(B^8)$ must be related to the first and second Pontryagin classes as

$$8e(B^8) = 4P_2 - P_1^2. \quad (30)$$

One can show explicitly that this constraint is satisfied by all eight-dimensional Calabi-Yau manifolds. Defining the Pontryagin classes as

$$P_1 = -\frac{1}{2}\text{Tr}R^2 \quad \text{and} \quad P_2 = -\frac{1}{4}\text{Tr}R^4 + \frac{1}{8}(\text{Tr}R^2)^2, \quad (31)$$

it is apparent that X_8 is related to the Euler class as

$$X_8 = -\frac{1}{4!(2\pi)^4}e(B^8) = -\frac{1}{4!}\text{Pf}(R), \quad (32)$$

where $\text{Pf}(R)$ is the Pfaffian of the curvature form. The volume contribution to the Euler number is given by the integral of $e(B^8)/(2\pi)^4$ over the manifold. Note that in [3] the factors of (2π) were omitted; one can easily verify that one needs to include these factors when the Pontryagin classes are defined without factors of (2π) .

Integrating Eq. (29) over a compact 8-fold with no boundary, we find that

$$\int_{B^8} F \wedge F = 2(2\pi)^4 \int_{B^8} X_8. \quad (33)$$

Using the relationship to the Euler number we find the topological constraint on the 8-fold:

$$\int_{B^8} F \wedge F + \frac{(2\pi)^4}{12} \chi = 0, \quad (34)$$

where χ is the Euler number. Thus the constants v^i are constrained, and the possible compactifications are restricted topologically. Since the Euler number will not vanish for topologically non-trivial compactifications, F_2 cannot vanish. One can obtain a natural interpretation of this topological constraint in terms of the quantization law for the 4-form [6].

Whenever the anomaly form, and hence the Euler number, vanishes, the 4-form F_2 must be zero; we can then have nowhere vanishing spinors of both chiralities on the 8-fold, and in the vacuum solution there will be up to 16 conserved spinors of each chirality.

If the anomaly does not vanish, one can also ensure that the net electric charge vanishes by including membrane point singularities which are localized within the 8-fold. The constraints on the 8-folds and the membrane charges are discussed in [8]. By replacing the Chern-Simons term in Eq. (29) by point membrane contributions, one can obtain the metric for such solutions.

We have discussed the analysis for positive chirality conserved spinors; let us now consider an 8-fold which admits covariantly constant spinors of negative chirality. Then the $\mathbf{8}_s$ spinor representation must admit a decomposition containing singlets. From the above we know that we must include only self-dual forms in F_2 while from [7] we know that the sign in Eq. (30) is reversed. Taking the minus sign in Eq. (9) we find that

$$d(*_8 dH_-) = \frac{1}{2}F_- \wedge F_- - (2\pi)^4 X_{(-)8} \quad (35)$$

$$= \frac{1}{2}F_- \wedge F_- - \frac{1}{4!(2\pi)^4}e(B_-^8), \quad (36)$$

where we include minus signs to indicate that we are taking quantities on an 8-fold which admits negative chirality conserved spinors. Now symmetry between the positive and negative chirality spinor solutions implies that there exist solutions for which the warp factor $H(x^m)$ is the same for both. Then the Euler numbers of the corresponding 8-folds will be the same but both anti-self-dual and self-dual forms

and the $\mathbf{8}_s$ and $\mathbf{8}_c$ spinor representations will be exchanged. In most of what follows we will assume that the conserved spinors are of negative chirality so that the corresponding four form is self-dual.

We said above that the other possible solution was an 8-fold of exceptional holonomy $Spin(7)$. Although we can certainly find such solutions in the absence of an anomaly term [9], a consequence of the constraints on F_2 is that compactification on a manifold of exceptional holonomy $Spin(7)$ may not in general be a solution of the field equations when we include an anomaly term. If supersymmetry is to be preserved, the harmonic 4-form must satisfy conditions (16) and (23) as well as the topological condition (34). As for the compact Calabi-Yau 8-folds, a generic $Spin(7)$ manifold may not admit such a harmonic form satisfying these constraints.

One can show that the unique $Spin(7)$ invariant anti-self-dual 4-form does not even satisfy the weaker condition (16); this is apparent if one defines it as [10]

$$\omega_{mnpq} = \bar{\zeta} \gamma_{mnpq} \zeta, \quad (37)$$

where ζ is conserved spinor on the 8-fold. Using the properties of Majorana-Weyl spinors, it is apparent that ω is anti-self-dual if and only if ζ is of negative chirality, and hence Eq. (16) is not satisfied. One may, however, be able to find appropriate four forms which permit the gravitino variations generated by ζ to vanish.

Given this form of the solution obtained, it is straightforward to confirm that the Einstein equations are also satisfied. Since the anomaly term is related to a closed 8-form, variations vanish within the 8-fold, but there will be contributions to the 11-dimensional Einstein equations whenever the anomaly form is non-trivial.

III. NON-COMPACT VACUUM SOLUTIONS

Let us now suppose that the 8-fold is non-compact; the net electric charge need not vanish, since there is a boundary to the 8-fold. This implies that there are no topological constraints imposed by the anomaly term on the 4-form.

For general non-compact 8-folds there may not be a well-defined cycle over which we can integrate a 4-form, and hence magnetic charge is not well defined. This problem has been discussed in the context of four dimensional Taub-Newman-Unti-Tamburino (Taub-NUT) manifolds with dyonic magnetic fields [11]; one cannot define charge by integrating the relevant components of the Maxwell 2-form over a sphere at infinity, since the topology of surfaces of constant radius is non-trivial. One can only define charges by considering the motion of point charges in the asymptotic region, or by taking analogies to asymptotically flat space-times. This will be a generic problem in the manifolds of non-trivial holonomy that we consider here.

Although for the compact manifolds the number of harmonic 4-forms is given by the related Betti number b_4 , for non-compact manifolds we can include all harmonic 4-forms whose norm is finite, and which satisfy the constraints (16) and (23). This can include those not counted in the Betti number, since they are non-zero on the boundary at infinity.

In fact, harmonic 4-forms for which the magnetic behavior is non-trivial cannot vanish on the boundary at infinity.

In the compact case, if the anomaly form is non-trivial, one must necessarily choose at least one of the v^i to be non-zero for a solution to exist. In the non-compact case, the magnitude of the 4-form is arbitrary; one can choose the integers to be as large or small as one requires, and adjust the solution of the scalar equation accordingly. In particular, one can choose the v^i to vanish, which has the advantage of giving trivial magnetic behavior. One can then certainly have generic $Spin(7)$ solutions when the 8-fold is non-compact.

Since for any $Spin(7)$ manifold one knows that there exists at least one conserved spinor with associated four form (37) satisfying Eq. (16), one can always include a non-zero flux through the 8-fold and still satisfy the equations of motion. However, generic solutions with non-zero flux will usually break all of the supersymmetry as we will discuss below.

If X_8 is trivial, then we can choose $H(x^m)$ to be harmonic, and, for vacuum solutions, to be a constant. The solutions we obtain this way are

$$ds^2 = ds^2(M^3) + ds^2(B^8), \quad (38)$$

which are widely known. However, if the holonomy of B^8 is non-trivial, the anomaly term may not vanish, and one cannot necessarily choose the scalar function to be constant. The physical interpretation is that there is a background charge distribution over the 8-fold. One would, however, expect that the anomaly term vanishes if the action of the holonomy group on one section of the tangent space of the 8-fold is trivial, and hence part of B^8 splits off as lines. Assuming a fraction of the supersymmetry is preserved, the anomaly will contribute only if the holonomy of B^8 is $Spin(7)$, $SU(4)$, $Sp(2)$ or $Sp(1) \times Sp(1)$. Even if the holonomy is contained in one of these groups, the anomaly can still vanish in special cases, which we will discuss in Sec. IV.

If the original 8-fold is non-singular, the anomaly form X_8 should be smooth and continuous over the 8-fold. Similarly, finiteness of the energy will require that the Chern-Simons term $F \wedge F$ have no singularities. Thus the background charge distribution implied by the equation for the scale factor (29) is smooth, non-zero, and finite at all points in the 8-fold. This implies in turn that a non-singular smooth solution for the scalar function will exist. We will give here three examples of such 8-folds; the first has holonomy $Sp(1) \otimes Sp(1)$, and the second is Calabi-Yau, while the third has holonomy of precisely $Spin(7)$.

The simplest example of a non-trivial solution of this type is to take B^8 as a Taub-NUT times Taub-NUT manifold. One would expect more general T^2 invariant hyper-Kähler metrics of this type, such as those constructed in [12], to give solutions of a very similar form. The holonomy is $Sp(1) \otimes Sp(1)$, and one-quarter of the supersymmetry is preserved. There is a single harmonic 4-form which is the wedge product of the 2-forms on the individual Taub-NUT manifolds. One can take the metric to be

$$ds^2 = H(x^m)^{-2/3} \{ ds^2(M^3) + H(x^m) \times [ds_{TN}^2(m_1, x_1) + ds_{TN}^2(m_2, x_2)] \} \quad (39)$$

where we take the metric on each manifold to be

$$ds_{TN}^2(m_1, x_1) = \left(1 + \frac{4m_1}{r_1}\right)^{-1} (d\psi_1 + \cos\theta_1)^2 + \left(1 + \frac{4m_1}{r_1}\right) (dr_1^2 + r_1^2 d\Omega_2^2), \quad (40)$$

so that the m_i are the NUT parameters and ψ_i is periodic with period $16\pi m_i$. For simplicity we consider the single-center metric on each manifold although this is an unnecessary restriction. Since B^8 is a direct product of two 4-folds, the $(\text{Tr}R^4)$ term in the anomaly form vanishes and we can show that

$$X_8 \propto - \prod_i \frac{r_i}{(r_i + 4m_i)^5} d\psi_i \wedge dr_i \wedge d\cos\theta_i \wedge d\phi_i, \quad (41)$$

where we will not need the constant of proportionality, but the absolute sign is important. Since the volume contribution to the Euler number of each Taub-NUT manifold is 1 [13], X_8 integrates over the 8-fold to $-1/24$.

If we include a non-trivial magnetic 4-form, we must take it to be

$$F = k^2 \prod_i \left\{ \frac{4m_i}{(r_i + 4m_i)^2} d\psi_i \wedge dr_i + \frac{4m_i r}{(r_i + 4m_i)} \sin\theta_i d\theta_i \wedge d\phi_i - \frac{(4m_i)^2}{(r_i + 4m_i)^2} \cos\theta_i dr_i \wedge d\phi_i \right\}, \quad (42)$$

with k a real constant, which has finite norm

$$\int_{B^8} F \wedge F = k^2 \prod_i (128\pi^2 m_i^2). \quad (43)$$

Note that the sign of $F \wedge F$ is positive, and F is self-dual on the 8-fold. The magnetic ‘‘charge’’ can be expressed as

$$\int \prod_i S_{r_i \rightarrow \infty}^2 F = k \prod_i (16\pi m_i). \quad (44)$$

Although one can show that the equations of motion are still satisfied if one includes a Chern-Simons form, the supersymmetry conditions are not satisfied and so such a solution breaks all of the supersymmetry.

It is easy to see that Eq. (23) cannot be satisfied on any product manifold $B^4 \times B'^4$ which satisfies the field equations. Let us write $F_2 = \omega \wedge \omega'$ and $\zeta = \zeta_{B^4} \otimes \eta_{B'^4}$. One can also decompose the gamma matrices as

$$\gamma_i = \sigma_i \otimes 1, \quad (45)$$

$$\gamma_{i'} = \sigma_5 \otimes \sigma_{i'}, \quad (46)$$

where $i = 1, \dots, 4$, $i' = 5, \dots, 8$ and σ are four-dimensional gamma matrices. Here $\sigma_5 = \epsilon_{ijkl} \sigma^{ijkl}/4!$ is the four-dimensional chirality operator.

Suppose that F_2 is self-dual and hence ζ is of negative chirality to satisfy Eq. (16); then without loss of generality one takes ω and ω' to be self-dual on B^4 and B'^4 respectively with ζ_{B^4} of negative chirality with respect to σ_5 and $\zeta_{B'^4}$ of positive chirality. On the product of two Taub-NUT manifolds, if one has defined the canonical two forms both to be self-dual, then the conserved spinors must have the same chirality. This implies that one cannot find an appropriate F_2 which preserves the supersymmetry.

Even if one can find appropriate self-dual 2-forms and conserved spinors of opposite chiralities, the strong supersymmetry condition is still not satisfied. The conditions in Eq. (23) take the form

$$(\omega_{ij} \sigma^j \zeta_{B^4}) \otimes (\omega_{i'j'} \sigma^{i'j'} \zeta_{B'^4}) = 0,$$

$$(\omega_{i'j'} \sigma^{j'} \zeta_{B'^4}) \otimes (\omega_{ij} \sigma^{ij} \zeta_{B^4}) = 0. \quad (47)$$

Given our choices of spinor chiralities we can see that

$$\omega_{i'j'} \sigma^{i'j'} \zeta_{B'^4} = 0, \quad (48)$$

which implies that the first equation in Eqs. (47) is satisfied. However, the second of the equations takes the form

$$(\omega_{i'j'} \sigma^{j'} \zeta_{B'^4}) \otimes \hat{\zeta} = 0, \quad (49)$$

where $\hat{\zeta}$ is a non-vanishing spinor. This equation cannot be satisfied for non-vanishing $\zeta_{B'^4}$ and therefore inclusion of non-zero F_2 breaks all remaining supersymmetry.

Irrespective of whether one includes a Chern-Simons form or not, the equation for the scalar function $H(x^m)$, Eq. (29), can be expressed in coordinate notation (in terms of the Ricci-flat metric on Taub-NUT cross Taub-NUT, g_{mn}) as

$$D_n \partial^n H(x^m) = g(r_i), \quad (50)$$

where $g(r_i)$ is a negative definite function on B^8 . Since the function depends only on the radii, the charge distribution is delocalized in the toroidal and angular directions. It is important to note that however positive or negative we choose the magnetic charge to be, the function in Eq. (50) will always be negative. If one ignores the anomaly and 4-form terms, then the field equations are solved provided that $H(x^m)$ is harmonic, as was found in [15]. The anomaly term will give only a small correction to the scalar function at infinity but cannot be neglected.

Supposing one defines the charge as

$$q = \int_{B^8} d^* F; \quad (51)$$

then the contribution to q from the background of Eq. (50) is negative since the charge density is negative definite throughout the manifold. The scalar function $H(x^m)$ satisfies a Poisson equation, with the negative charge density being concentrated about the origin of the two manifolds, and de-

caying at infinity. The form of the operator on the product manifold makes it difficult to solve the equation explicitly, but we would expect that there exists a non-singular solution of the form

$$H(x^m) = 1 + h(r_i), \quad (52)$$

where $h(r_i)$ is a function which is positive definite throughout the manifold B^8 , peaks at a finite value at the origin $r_i = 0$ and asymptotically vanishes.

We have been referring to this as a vacuum solution, but it is better interpreted in terms of eleven-dimensional Kaluza-Klein monopoles. When $m_1 = 0$ the solution can be described in terms of KK 6-branes [14] located at the origin $r = 0$. The anomaly and Chern-Simons terms vanish and one-half of the spacetime supersymmetry is preserved by the solution. If the 6-branes are located in the (123456) plane, then the condition for unbroken supersymmetry is [16]

$$\Gamma_{0123456} \eta = \eta, \quad (53)$$

which again gives us 16 conserved spinors. If we then add 6-branes located in the (12789(10)) plane, only spinors satisfying

$$\Gamma_{012789(10)} \eta = \eta \quad (54)$$

preserve supersymmetry. One half of the spinors satisfying Eq. (53) will also satisfy this condition, and so one-quarter of the supersymmetry is preserved, as we found above. Inclusion of a Chern-Simons form will break the remaining supersymmetry.

Suppose one reduces the dimension of the two monopole solution along closed orbits of the Killing vector ∂_{ψ_2} . Then the 6-branes lying in the (123456) plane are reduced to the D6-branes of type IIA theory [17], while those lying in the (12789(10)) plane are reduced to type IIA KK monopoles. Since the D6-brane charge is quantized, the ‘‘NUT’’ charges will determine the number of D6-branes in ten dimensions.

The effective ten-dimensional solution then describes D6-branes intersecting with type IIA monopoles, a configuration preserving 1/4 of the supersymmetry. The presence of a non-zero anomaly form implies that one should have electric charge corrections to such a solution. As mentioned in the Introduction, whenever a p -brane is wrapped around a surface of non-zero first Pontryagin class, one expects that the p -brane picks up a $(p-4)$ -brane charge. Our explicit construction of the spacetime solution demonstrates the eleven-dimensional origin of the membrane charge of the D6-brane induced when it wraps a topologically non-trivial space.

If one chooses B^8 to be a Taub-Nut manifold times any 4-fold of $Sp(1)$ holonomy B^4 , then the anomaly form is given by

$$X_8 = \frac{1}{4(2\pi)^4 4!} [P_1(B^4) \wedge P_1(TN)]. \quad (55)$$

Integrating over the 8-fold we find that the induced electric charge is

$$q = (2\pi)^4 \int_{B^8} X_8 = \frac{(2\pi)^2}{48} p_1(B^4), \quad (56)$$

where $p_1(B^4)$ is the first Pontryagin number of the 4-fold. Now in [8] the value of the charge induced on a p -brane wrapped around a surface of non-zero first Pontryagin class was given as 1/48 of the first Pontryagin number. Allowing for normalization differences in the 4-form and the Pontryagin classes, our result is consistent.

Our second example is the generalized Eguchi-Hanson solution in eight dimensions. This manifold was first discussed in [18,19] and the metric was given in the form that we shall use here in [20]. It is an ALE Ricci-flat Kähler manifold, which hence has holonomy $SU(4)$. The form of the metric is

$$ds_8^2 = \frac{dR^2}{\left(1 - \frac{a^8}{R^8}\right)} + \frac{R^2}{16} \left(1 - \frac{a^8}{R^8}\right) (d\tau + A)^2 + R^2 ds^2(CP^3), \quad (57)$$

where the metric g_{ij} on CP^3 is chosen with the scale $R_{ij} = 8g_{ij}$ and dA is the Kähler form on the complex projective space. The behavior of this solution is analogous to that of the more familiar four dimensional solution; in particular, the radial coordinate runs between a and infinity, and the CP^3 fixed point set of the isometry ∂_τ allows us to calculate the Euler number as 4 using the Lefschetz fixed point theorem.

The anomaly form for this manifold is

$$X_8 = -\frac{14}{\pi^4} \frac{a^{32}}{R^{33}} dR \wedge d\tau \wedge \eta_6, \quad (58)$$

where η_6 is the volume form on the complex projective space. By calculating the surface contribution to the Euler number, one can then verify that the anomaly form gives the correct volume contribution.

Assuming that the harmonic form vanishes the warp factor equation is then

$$D_n \partial^n H(x^m) = -896 \frac{a^{32}}{R^{40}}, \quad (59)$$

which has the regular solution

$$H(x^m) = 1 + 28 \left\{ \frac{1}{6R^6} + \frac{a^8}{14R^{14}} + \frac{a^{16}}{22R^{22}} + \frac{a^{24}}{30R^{30}} \right\}. \quad (60)$$

One can also take the harmonic form F_2 to be non-vanishing; if F_2 is proportional to the (self-dual) canonical four form, then there will be an additional contribution to Eq. (59) which is also negative definite and proportional to $1/R^{16}$. Solving for the scalar function then gives an additional $1/R^6$ term in Eq. (60). However, by constructing a complex structure or otherwise one can demonstrate that such a four form flux does not preserve the remaining supersymmetry.

As a third example, we mention an 8-fold with $Spin(7)$ holonomy which was discussed in [10]. The metric takes the form of a quaternionic line bundle over a 4-sphere:

$$ds_8^2 = \alpha^2(r)dr^2 + \beta^2(r)(\sigma^i - A^i)^2 + \gamma^2(r)ds_4^2, \quad (61)$$

where ds_4^2 is a suitably scaled metric on the base 4-sphere and σ^i are left-invariant one-forms on the $SU(2)$ fibers of the bundle over S^4 . The functions α , β and γ are given by

$$\alpha^2 = (1 - r^{-10/3})^{-1}, \quad \beta^2 = \frac{9}{100}r^2(1 - r^{-10/3})^{-1},$$

$$\gamma^2 = \frac{9}{20}r^2. \quad (62)$$

Asymptotically this solution tends to the metric on the cone,

$$ds^2 = d\rho^2 + \rho^2 ds_7^2(S^7), \quad (63)$$

where the metric on the seven-sphere is a homogeneous “squashed” Einstein metric. Given the curvature tensor for such a solution calculated in [21], we can calculate the anomaly form; again it describes a smooth negative charge distribution. Inclusion of an F_2 term, such as the $Spin(7)$ invariant 4-form, modifies the warp factor and increases the charge background but generically breaks the remaining supersymmetry.

IV. GENERALIZED MEMBRANES

Given vacuum solutions asymptotic to $M^3 \times B^8$ which preserve some or all of the supersymmetry, it is natural to ask whether we can include membranes. The backgrounds discussed above remain supersymmetric solutions if we include contributions to $H(x^m)$ which are harmonic on the 8-fold. Point singularities are naturally interpreted as the positions of parallel membranes, and these membranes do not necessarily break any more of the supersymmetries.

It is worth considering here the nature of harmonic solutions on the 8-fold. For the standard membrane on R^8 , the natural choice of harmonic function describes a single membrane localized at what can be chosen to be the origin of the 8-fold. The equation satisfied by the scalar function is

$$\partial^n \partial_n H(x^m) = -\alpha \delta^8(x^m), \quad (64)$$

where the delta function integrates over the manifold to give 1. Then we choose

$$H(x^m) = 1 + \frac{1}{6V_7} \frac{\alpha}{r^6}, \quad (65)$$

where V_7 is the volume of the seven sphere. This choice of scalar function gives the familiar membrane solution of [22]. The charge can be defined as

$$q = \int_{R^8} d*F = -\alpha, \quad (66)$$

as expected. Evidently inclusion of further point singularities in the harmonic function simply describes additional parallel membranes. Such solutions preserve 1/2 of the supersymmetry. Note that the scalar function is positive definite when α is positive.

Now let us consider harmonic functions on the product of two Taub-NUT manifolds. Suppose we look for a solution which depends neither on the position in one of the Taub-NUT manifolds nor on the circle direction in the other. An appropriate solution is given by

$$\delta H(x^m) \propto \frac{\alpha}{r_1}, \quad (67)$$

where our notation refers to the change in the scalar function induced by the inclusion of a point singularity. This describes a membrane of negative charge which is localized at the origin of one of the Taub-NUT manifolds, but which is delocalized in the other manifold, and along the circle direction. Such a solution preserves only 1/4 of the supersymmetry, provided of course that we include an appropriate anomaly term.

The ten-dimensional interpretation is as a delocalized membrane contained within a $D6$ -brane and type IIA monopole. As we would expect the charge determined by this harmonic function diverges, although the charge per unit volume of the second Taub-NUT manifold is finite; that is, the divergence is caused by delocalizing the membrane over a non-compact manifold.

If we take the function to be the sum of two harmonic functions on the individual 4-folds, the resulting solution will represent two parallel membranes, each of which is localized at the origin of one manifold, but delocalized in the other manifold. Such a solution still preserves 1/4 of the spacetime supersymmetry.

If we want the membrane to be localized at the origin of each Taub-NUT manifold, then we need to look for a solution to

$$\delta D_n \partial^n H(x^m) = -\alpha \delta^8(x^m), \quad (68)$$

where the delta function implies that the membrane is localized both in the circle directions and at the radial origin of each 4-fold. Note that we have chosen the sign so that the change in the scalar function is positive definite throughout the manifold. There are several reasons for this choice: α must be positive if $H(x^m)$ is not to pass through zero and if the mass of the membrane is to be positive. The solution to the equation above does not have a simple analytic form, and we will not discuss the explicit solution. We would expect the scalar function to be mildly singular at the membrane location, although the singularity may behave similarly to the horizon of the ordinary membrane [23].

We can find an explicit solution for the other 8-folds discussed in the previous section. For example, for the generalized Eguchi-Hanson solution, the change to the scalar function obtained by solving Eq. (68) is

$$\delta H(x^m) = \frac{3\alpha}{32\pi^4 a^6} \left\{ \ln \left(\frac{R^2 + a^2}{R^2 - a^2} \right) + 2 \tan^{-1} \left(\frac{R^2}{a^2} \right) - \pi \right\}. \quad (69)$$

Then the scalar function falls off as $1/R^6$ at infinity, and diverges logarithmically at the origin $R=a$. Unlike the ordinary membrane, for which the spacetime approaches the regular manifold $AdS_4 \times S^7$ [23] at the membrane, the size of the complex projective spaces will blow up logarithmically as we approach the membrane. Since the source is distributed over a six-dimensional complex projective space, one might regard such branes as being in some sense eight-dimensional.

Let us now consider whether we can interpret these types of solution as membranes. Since the equation for the warp factor on B^8 takes the form (50), a solution including point singularities in $H(x^m)$ can be interpreted as localized membranes within a background charge distribution. Now we have found that such point singularities must have a definite charge, implied by taking α to be positive in Eq. (68). If such a solution is to represent a membrane solution, the choice of signs in the spacetime equations must be consistent with the choices made to satisfy the membrane field equations.

The bosonic sector of the membrane action [22] is given by

$$S_M = T \int d^3 \xi \left(-\frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i X^M \partial_j X^N g_{MN} + \frac{1}{2} \sqrt{-\gamma} \right. \\ \left. \pm \frac{1}{3!} \epsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^P A_{MNP} \right), \quad (70)$$

where T is the membrane tension, and γ_{ij} is the metric on the membrane world volume, and ξ^i are world-volume coordinates. $X^M = (X^\mu, Y^m)$ are the spacetime coordinates, with $\mu = 0, 1, 2$ and $m = 3, \dots, 11$. There is a correction to the energy momentum tensor arising from the membrane source term:

$$\delta T_{MN} = T \int d^3 \xi \sqrt{-\gamma} \gamma_{ij} \partial^i X_M \partial^j X_N \frac{\delta^{11}(x-X)}{\sqrt{-g}}. \quad (71)$$

As in Sec. II we have taken $\kappa^2 = 1$. The corresponding equation of motion for the four-form then gives a correction to the scalar function:

$$\delta \tilde{D}_n \partial^n H(x^m) = -T \int d^3 \xi \epsilon^{ijk} \partial_i X^0 \partial_j X^1 \partial_k X^2 \delta^{11}(x-X). \quad (72)$$

From the membrane action, we have the membrane field equations

$$\partial_i (\sqrt{-\gamma} \gamma^{ij} \partial_j X^N g_{MN}) + \frac{1}{2} \sqrt{-\gamma} \gamma^{ij} \partial_i X^N \partial_j X^P \partial_M g_{NP} \\ \pm \frac{1}{3!} \epsilon^{ijk} \partial_i X^N \partial_j X^P \partial_k X^Q F_{MNPQ} = 0,$$

$$\gamma_{ij} = \partial_i X^M \partial_j X^N g_{MN}. \quad (73)$$

Now if one takes the static gauge choice and solution

$$X^\mu = \xi^\mu, \\ Y^m = \text{const}, \quad (74)$$

one can verify that with the choice of F_{MNPQ} corresponding to positive chirality conserved spinors the membrane field equations are satisfied provided that we choose the negative sign of the Wess-Zumino term in Eq. (70) and vice versa. The correction to the scalar function satisfies

$$\delta D_n \partial^n H(x^m) = -T \delta^8(x^m). \quad (75)$$

Comparison with the source term (68) then implies that $\alpha = T$, which is analogous to the relationship for the ordinary membrane [22].

We then need to determine the number of supersymmetries that are preserved when we include membrane point singularities. Preservation of the world-volume supersymmetries requires that the spinor η must satisfy the condition

$$\Gamma \eta = \eta, \quad (76)$$

where we have taken the sign to be negative in the Wess-Zumino term (70), and

$$\Gamma \equiv \frac{1}{3! \sqrt{-\gamma}} \epsilon^{ijk} \partial_i X^M \partial_j X^N \partial_k X^P \Gamma_{MNP}. \quad (77)$$

For our solutions we have $\Gamma = 1 \otimes \gamma_9$, so that the preserved world-volume supersymmetries are of the same chirality as the preserved spacetime supersymmetries. Thus the membrane does not break any more spacetime supersymmetries than the vacuum solution, and our solutions can indeed be interpreted as a membranes with T^2 invariant hyper-Kähler, Calabi-Yau and $Spin(7)$ transverse spaces preserving 1/4, 1/8 and 1/16 of the supersymmetries respectively.

For the monopole solutions, one can see why the addition of a membrane in the (12) plane to a (123456) and (12789(10)) brane configuration does not break any more supersymmetries. The above condition on the spinor does not impose any more restrictions than the two conditions (53) and (54).

There are several points to make about these generalized membrane solutions. All the solutions which we have discussed describe positive or negative charge membranes within a positively or negatively charged background. One might have thought that this indicates a possible instability of such membranes, in which the membranes decay into a background with a greater Chern-Simons term. The ordinary membrane could not of course decay in such a way, since

there exists no harmonic 4-form on R^8 which is finite at infinity. However, since the backgrounds with Chern-Simons charges are not supersymmetric, one would expect the Chern-Simons bulk charges to decay into membranes, a point that will be illustrated in the following section.

We should also mention the differences between the generalized membranes discussed here and those considered in [24]. The motivation for the construction of the latter was the observation that the solution for the ordinary membrane is asymptotic to $AdS_4 \times S^7$ at the membrane location. Since there exist other Einstein 7-folds which admit Killing spinors, one might expect that there exist analogous membranes which are asymptotic to $AdS_4 \times B^7$ where B^7 is a more general positive curvature Einstein manifold. The form of these solutions is

$$ds^2 = H(r)^{-2/3} ds^2(M^3) + H(r)^{1/3} [dr^2 + r^2 ds^2(B^7)], \quad (78)$$

where $H(r)$ is harmonic, and membranes contribute $1/r^6$ terms to the scale factor. Evidently as for the ordinary membrane such solutions are non-singular at the membrane location. The Ricci-flat metric on the 8-fold has holonomy contained in $Spin(7)$ when one chooses the manifold B^7 suitably, such as the squashed seven-sphere solution of [25]. Thus these membranes constitute one class of the solutions considered here, although we have allowed the metric to take a more general form.

Note that such a background is not complete in the absence of membranes; unless the metric on B^7 is the round metric on the sphere, there will be conical singularities at the origin $r=0$. In fact, the $Spin(7)$ 8-folds of this type, Ricci-flat metrics on cones, first constructed by [26] were all incomplete. The squashed seven-sphere solution mentioned above is closely related to the $Spin(7)$ manifold discussed in Sec. III; in the latter we smooth out the cone, with the singular “vertex” at $r=0$ being replaced by a smoothly embedded bolt.

Neither anomaly nor Chern-Simons terms were included in the analysis of [24]. Since the 8-fold is not flat, one might expect there to be corrections to the scalar function from an anomaly term; however, the form of the metric on the 8-fold indicates that the volume contribution to the Euler number vanishes, and no corrections are needed. In addition one cannot find a self-dual 4-form on the 8-fold which integrates to give a finite charge.

All of the 8-folds which we have considered admit at least a circle isometry group. As in [15], we could dimensionally reduce the vacuum and membrane solutions to ten dimensions, and then apply duality transformations to obtain new solutions. Supersymmetry is not preserved by the dimensional reduction unless the Killing spinors are also invariant under the action of the isometry, a condition which is non-trivial for general $Spin(7)$ 8-folds. However, supersymmetry will certainly be preserved if we take the 8-fold to be hyper-Kähler and T^2 invariant as in [15]. We will not consider such dimensional reductions here, although in the next section we will consider dimensionally reduced solutions of a different type.

V. MODIFIED KALUZA-KLEIN MONOPOLE SOLUTIONS

So far we have mostly been interested in solutions which are manifestly eleven-dimensional for which the anomaly form is non-vanishing. In this section we will consider the effects of including anomaly and Chern-Simons forms for solutions which can best be interpreted in lower dimensions.

When we compactify the solutions we need to be careful about the quantization condition on the 4-form. The vacuum solutions we consider are of the same form as those in Sec. III except that the 8-fold is conformal to the product $B_1 \times B_2$ where B_1 is compact (and one or more directions in B_2 is wrapped around a circle).

Preservation of any of the spacetime supersymmetry requires that B_1 be a torus or $K3$. The quantization condition on the four-form is evidently trivial for the former, and since the first Pontryagin class of $K3$ is canonically divisible by 4, G must have integral periods [27] on $K3$ also. One cannot find a self-dual four-form on B^8 of non-zero period over $K3$ which is finite on the boundary, and so G has a vanishing period over the compact 4-fold in the solutions considered here.

Directly dimensionally reducing an eleven-dimensional KK 6-brane gives a $D6$ -brane in ten dimensions and wrapping this $D6$ -brane around a torus gives an extreme four-dimensional black hole carrying a $U(1)_M$ magnetic charge, which preserves one-half of the supersymmetry. The formal temperature of the black hole as defined by the surface gravity is infinite, and thus it has a naked singularity which is protected by an infinite mass gap [28].

Now for a single KK 6-brane with a flat transverse space, say in the (123456) directions, the anomaly form necessarily vanishes. If the transverse space (3456) is non-compact, then one cannot find a Chern-Simons form F_2 on the 8-fold which would give a finite charge. If, however, we wrap the KK 6-brane around a four torus, we can find such a form although all of the supersymmetry is then broken.

If we further compactify the (12) directions on a two-torus, and take the circle direction in the Taub-NUT manifold to be small, we again obtain an extreme black hole in four dimensions. That is, the eleven-dimensional solution is

$$ds^2 = H(r)^{-2/3} [-dt^2 + ds^2(T^2)] + H(r)^{1/3} ds^2(T^4) + H(r)^{1/3} \{h(r) ds^2(R^3) + h(r)^{-1} (d\psi + 4m \cos \theta d\phi)^2\}, \quad (79)$$

where

$$h(r) = \left(1 + \frac{4m}{r}\right), \quad H(r) = \left(1 + \frac{4mk^2}{(r+4m)}\right). \quad (80)$$

The four-form is given by

$$F_1 = dt \wedge \eta(T^2) \wedge dH(r)^{-1},$$

$$F_2 = \omega_{T^4} \wedge \omega_{TN}, \quad (81)$$

where ω_{T^4} is the (constant) self-dual 2-form on T^4 and ω_{TN} is the self-dual 2-form on the Taub-NUT manifold, given in Eq. (42). Then the charge in eleven dimensions is given by

$$\int_{B^8} d(*F) = 64\pi^2 m V_4 (4mk^2), \quad (82)$$

where V_4 is the volume of the T^4 . Using the standard ansatz for dimensional reduction of the eleven-dimensional solution [29], we find that the effective four-dimensional solution (in the Einstein frame) is

$$ds_E^2 = -h(r)^{-1/2} H(r)^{-1/2} dt^2 + h(r)^{1/2} H(r)^{1/2} [dr^2 + r^2 d\Omega_2^2],$$

$$F^E = dt \wedge dH(r)^{-1} = \frac{4mk^2}{[r + 4m(1+k^2)]^2} dt \wedge dr, \quad (83)$$

$$F^M = 4m \sin\theta d\theta \wedge d\phi,$$

with the other effective fields in four dimensions being the dilaton and a pseudo-scalar originating from F_2 . The notation for the gauge fields indicates that the charges come from different gauge groups, $U(1)_M$ and $U(1)_E$.

From the form of the metric, the effective four-dimensional solution seems to describe an extreme $U(1)_M$ black hole of mass m with a (singular) horizon at $r=0$, plus an extreme $U(1)_E$ black hole of mass mk^2 with a (singular) horizon at $r=-4m$. We can, however, rewrite the solution as

$$ds_E^2 = -\left(1 + (1+k^2)\frac{4m}{r}\right)^{-1/2} dt^2 + \left(1 + (1+k^2)\frac{4m}{r}\right)^{1/2} [dr^2 + r^2 d\Omega_2^2], \quad (84)$$

which looks like the metric for an extreme black hole carrying only *one* charge, with horizon at $r=0$ and mass $m(1+k^2)$. Since the black hole again has a formal temperature which is infinite, it is protected by an infinite mass gap though none of the supersymmetry is preserved and there are two non-zero charges.

We could also add a membrane to the KK monopole solution; the form of the solution is the same as in Eq. (79) except that the scalar function $H(r)$ is now defined as

$$H = 1 + \frac{q}{r}, \quad (85)$$

where we choose F_2 to vanish and delocalize the charge over the internal torus. The charge in eleven dimensions is given by

$$\int_{B^8} d(*F) = 64\pi^2 m V_4 q, \quad (86)$$

while the effective fields in four dimensions are as in Eq. (83) with the only other scalar field being the dilaton. This solution describes a $U(1)_M \times U(1)_E$ black hole with horizon at $r=0$ and singularity at $r=-q$ or $r=-4m$ depending on the relative magnitudes of q and m . Since the formal temperature of the black hole is finite, it is protected from excitations by a finite mass gap. One-quarter of the supersymmetry is preserved since F_2 vanishes.

The most general eleven-dimensional solution of this type will have non-zero k and q preserving none of the supersymmetry. The four-dimensional solution can be interpreted in terms of electric black holes with singular horizons located at $r=0$ and $r=-4m$ and magnetic black holes with singular horizons located at $r=0$. The effective solution has a non-singular horizon at $r=0$ and a singularity at some $r<0$.

For a given electric charge, the masses of both types of black hole solutions are the same, although the temperatures and singularity structure differ. Actually the four-dimensional solutions differ only in the background fields and the location of the electric black hole. Although the most interesting membrane plus $D6$ -brane intersections are those for which the branes are localized at the same point in the transverse space, one can find a multi-center solution in which the membrane in ten dimensions is located at $r<0$ while the $D6$ -brane is located at $r=0$. The resulting four-dimensional black hole solution differs from Eq. (79) only by the absence of the pseudo-scalar field.

One might question how solutions preserving one-quarter and none of the supersymmetry respectively can have the same mass and charge. The answer requires a consideration of how the mass bound is derived. As usual one can define a global supersymmetry charge from a spinor η and the gravitino as

$$Q_\eta = \int_{\partial B^8} \bar{\eta} \Gamma^{MNP} \Psi_P d\Sigma_{MN}. \quad (87)$$

Here we are considering the supercharge per unit area of ‘‘membrane’’ and hence are considering a nine dimensional manifold with a boundary orthogonal to a timelike direction and also to an area coordinate. The corresponding anticommutator is then given by

$$\{Q_{\eta_1}, Q_{\eta_2}\} = \delta_{\eta_1 \eta_2} \int_{\partial B^8} N^{MN} d\Sigma_{MN}, \quad (88)$$

where the Nester form is defined as

$$N^{MN} = \bar{\eta}_1 \Gamma^{MNP} \delta_{\eta_2} \Psi_P. \quad (89)$$

Using the gravitino transformation law, one obtains the following for the Nester form:

$$N^{MN} = \bar{\eta}_1 \Gamma^{MNP} \bar{D}_P \eta_2 + \frac{1}{8} \bar{\eta}_1 \Gamma^{PQ} F_{PQ}^{MN} \eta_2 \quad (90)$$

$$+ \frac{1}{96} \bar{\eta}_1 \Gamma^{MNPQRS} F_{PQRS} \eta_2. \quad (91)$$

Now any spinor η which preserves supersymmetry in the absence of a Chern-Simons term will in the presence of such a term define a supersymmetry charge which anticommutes with itself. In fact,

$$\{Q_\eta, Q_\eta\} = \int_{B^8} d\Sigma^m \eta^\dagger (\partial_m H + F_{m012} \Gamma^{012} + F_{mnpq} \Gamma^{npq}) \eta. \quad (92)$$

The last term vanishes since $\bar{\zeta} \gamma_{i_1 \dots i_p} \zeta$ vanishes for odd p (as stated in Sec. II) and hence the flux through the 8-fold does not contribute to the anticommutator. By the usual techniques we can then prove that the mass per unit area m defined by

$$m = \int_{B^8} d(*_8 dH) \quad (93)$$

is greater than or equal to the charge per unit area, $q = \int_{B^8} *F$. Equality in the bound will require that both the spinor be conserved on B^8 and M^3 . Although the Chern-Simons term does not contribute to the diagonal terms in the Bogomolnyi matrix, one will have contributions from the cross terms arising from spinors of opposite chiralities in the eight-dimensional sense. That is, if we consider spinors $\eta_\pm = \epsilon \otimes \zeta_\pm$ where $\zeta_\pm$ are spinors of positive and negative chirality, then

$$\{Q_{\eta_+}, Q_{\eta_-}\} \neq 0, \quad (94)$$

even if one or both of $\zeta_\pm$ is covariantly constant on B^8 . It is these cross terms which cause the zero eigenvalues of the Bogomolnyi matrix to become non-vanishing in the presence of a four form flux and hence display the broken supersymmetry.

It is also interesting to consider KK 6-branes wrapped around $K3 \times T^2$. In the supergravity theory one can interpret such a solution in four dimensions as an extreme magnetic black hole preserving one-quarter of the supersymmetry. Since the anomaly form on a $K3 \times$ Taub-NUT manifold is nonzero, one needs to include a warp factor in the solution. So the metric is

$$ds^2 = H(r)^{-2/3} [-dt^2 + ds^2(T^2)] + H(r)^{1/3} ds^2(K3) + H(r)^{1/3} \{h(r) ds^2(R^3) + h(r)^{-1} (d\psi + 4m \cos \theta d\phi)^2\}, \quad (95)$$

where $h(r)$ is given in Eq. (79). Calculation of the anomaly form then implies that the scale factor satisfies

$$D_n \partial^n H(x^m) = -12c \frac{(4m)^2}{(r+4m)^6}, \quad (96)$$

where c is a real constant which could be calculated. This equation is straightforward because the Euler class of $K3$ is proportional to the volume form. Then the scalar function is

$$H(r) = 1 + \frac{c}{4m} \left\{ \frac{1}{(r+4m)} + \frac{4m}{(r+4m)^2} + \frac{(4m)^2}{(r+4m)^3} \right\}, \quad (97)$$

and the associated electric charge is given by

$$\int_{B^8} d*F = 16\pi^2 c V_{K3} = (2\pi)^4, \quad (98)$$

where V_{K3} is the volume of the $K3$ and a Euler number of $K3$ ($\chi=24$) is used in calculating the latter equality. Since the first Pontryagin number of $K3$ is $48(2\pi)^2$, one can also obtain this result from Eq. (56).

Reduction to four dimensions again gives us a black hole carrying two charges, which preserves one-quarter of the supersymmetry, but is singular with a formally infinite temperature. The presence of the anomaly form causes the black hole to carry an electric charge as well as a magnetic charge.

One can include several different Chern-Simons forms F_2 in these solutions; choosing the conserved spinors to be of positive chirality and defining F_2 as in Eq. (81) the 2-form on $K3$ must be anti-self-dual. There are 19 distinct anti-self-dual 2-forms on $K3$, but the only way in which the 2-form, ω_2 , will contribute to the equation for motion is via the wedge product $\omega_2 \wedge \omega_2$. As the latter is cohomologous to the unique harmonic volume four-form of $K3$, we know that

$$\omega_2 \wedge \omega_2 = C \eta_{K3} + d\omega_3, \quad (99)$$

where η_{K3} is the volume form and C is a constant. The requirement that ω_2 be anti-self-dual implies that the exact term vanishes [27], and so we can find solutions for which the correction to the scalar function is of the form (80). Chern-Simons terms modify the electric charge and break the supersymmetry. Again for a given charge one can find solutions of the same mass but larger Page charge, and hence the Chern-Simons charge should decay.

More generally, of course, we could wrap further membranes about holomorphic cycles in the torus or $K3$. One can have a $(2 \perp 2 \perp 2) \parallel 6_{KK}$ configuration in which each of the membranes is wrapped around a torus. What is novel about our solutions is that one can also include Chern-Simons contributions to the 4-form from the self-dual 4-form on the 8-fold transverse to each membrane, although in doing so we will break the remaining supersymmetry.

The effective four-dimensional black hole as usual has four charges and a finite horizon area, but the metric will be non-standard and pseudo-scalar fields will be non-zero. Depending on the relative sizes of the charges, there may be another horizon inside the horizon at $r=0$ before one reaches a physical singularity.

Evidently the usual further generalizations of intersecting brane solutions are possible. Instead of taking the membranes to intersect over a point, one can choose them to be at general angles, with the Chern-Simons forms chosen appropriately. One can also wrap additional membranes about holomorphic cycles in $K3$; solutions of this kind were discussed in [30]. A general feature of all these solutions is that black holes obtained by dimensional reduction of bulk dis-

tributions of charge have more singular horizons and higher temperatures than those with the same charge in membranes. Since the solutions involving Chern-Simons forms are non-supersymmetric, one would expect the bulk charges to decay into membranes.

VI. FIVE-BRANE SOLUTIONS

If one considers a five-brane within an eleven-dimensional background for which the anomaly form is non-zero, then one has to take account of electric charge corrections. For example, corrections will be required for the generalized five-brane solutions discussed in [31]. The simplest solution to consider is that for a single five-brane:

$$ds_{11}^2 = \mathcal{F}^{-1/3} [ds^2(M^2) + ds_4^2(B_1)] + \mathcal{F}^{2/3} [ds_4^2(B_2) + dz^2],$$

$$F = *_2 d\mathcal{F} \wedge dz, \quad (100)$$

where we take the dual in the last equation on the manifold B_2 . M^2 is 2-dimensional Minkowski space, while the manifolds B_i must be Ricci-flat to satisfy the field equations. $\mathcal{F}$ is a harmonic function on this manifold, and the magnetic charge is given by integrating F over the few-dimensional manifold defined as the boundary of B_2 times the line. Again point singularities in $\mathcal{F}$ represent localized five-branes.

Preservation of any of the spacetime supersymmetry requires that the holonomy of each of the B_i be contained in $SU(2) \cong Sp(1)$. Usually one assumes that the manifolds are flat, and one-half of the background supersymmetry is then preserved in the 5-brane solution.

If B_1 has trivial holonomy, but B_2 has holonomy $Sp(1)$, then the vacuum solution preserves 1/2 of the supersymmetry, and the 5-brane solution preserves 1/4 of the supersymmetry, with a suitable choice of charge sign. If both of the manifolds have holonomy $Sp(1)$, the vacuum solution preserves 1/4 of the supersymmetry, but the 5-brane solution preserves only 1/8 of the supersymmetry. This follows from the fact that the vacuum solution preserves eight Killing spinors, four of each chirality on the six-dimensional manifold $M^2 \times B_2$. If $\mathcal{F}$ has point singularities, then only spinors of one particular chirality on the world volume preserve the supersymmetry.

If both of the B_i are non-trivial, however, the anomaly form does not vanish. Using the conformal invariance of the field equation for the 4-form, we know that the anomaly polynomial is transverse to the conformally flat space. Note that if only one of the manifolds has non-trivial holonomy conformal invariance implies that the anomaly form vanishes.

Since a $D4$ -brane wrapped around a space for which the first Pontryagin class does not vanish picks up an induced 0-brane charge and the $M5$ -brane reduces to the $D4$ -brane on double dimensional reduction, one might wonder from where this charge originates in eleven dimensions. In fact, this charge originates from the self-dual 3-form field propagating on the world volume of the $M5$ -brane. However we

do not need to use the world-volume fields of the five-brane in what follows and can ignore the non-zero value of this field.

To find a solution which takes account of the anomaly, it is natural to add a correction of the type discussed in the previous sections. The corrected five-brane solution will take the form

$$ds_{11}^2 = \mathcal{F}^{-1/3} H^{-2/3} \{ [ds^2(M^2) + H ds_4^2(B_1)] + \mathcal{F}^{2/3} [H ds_4^2(B_2) + dz^2] \},$$

$$F = *_2 d\mathcal{F} \wedge dz + \eta_{M^2 \times R} \wedge dH^{-1}, \quad (101)$$

where η is the volume form on the flat space. $\mathcal{F}$ remains a function which is harmonic on B_2 , while H satisfies Eq. (29), with additional point singularities representing membranes.

One can verify that the addition of such scalar function terms to the metric and to the 4-form does not break any additional supersymmetries, provided that one chooses charge signs appropriately. Even if one includes point singularities in $H(x^m)$ representing membranes, 1/8 of the supersymmetry is preserved.

The physical interpretation of this result is that a 5-brane wrapped around a space of non-trivial holonomy, with a non-flat transverse space, receives electric charge corrections when one takes account of the anomaly. Note that just as for the single membrane one adds an arbitrary amount of harmonic 4-form on the 8-fold although this must break the remaining supersymmetry.

Furthermore, the solution representing two 5-branes intersecting on a string, which was discussed in [15], must in general be corrected to

$$ds_{11}^2 = (\mathcal{F}_1 \mathcal{F}_2)^{2/3} H^{-2/3} \{ (\mathcal{F}_1 \mathcal{F}_2)^{-1} ds^2(M^2) + \mathcal{F}_1^{-1} H ds_4^2(B_2) + \mathcal{F}_2^{-1} H ds_4^2(B_1) + dz^2 \}$$

$$F = (*_1 d\mathcal{F}_1 + *_2 d\mathcal{F}_2) \wedge dz \pm \epsilon_{M^2 \times R} \wedge dH^{-1}, \quad (102)$$

with $H(x^m)$ satisfying Eq. (29). Here M^2 is Minkowski space, and B_i are Ricci-flat four-dimensional manifolds with holonomy contained in $Sp(1)$. In the last line, $*$ implies that the duals are taken on the manifolds B_i . Such a solution is the same as in [32], except that the definition of $H(x^m)$ includes both anomaly and Chern-Simons terms.

In the absence of an anomaly term, the field equations are satisfied provided that the functions H_i are harmonic on the manifolds B_i . If the B_i are flat, then each five-brane preserves 1/2 of the supersymmetry, and the overlap preserves 1/4 of the supersymmetry. Inclusion of point singularities in $H(x^m)$ gives a solution preserving 1/8 of the supersymmetry.

If only one of the B_i is flat, then we obtain a solution when $H(x^m)$ is harmonic. If $H(x^m)$ is constant, then 1/8 of the supersymmetry is preserved. Inclusion of point singularities in $H(x^m)$, i.e. membranes wrapped around the conformally flat space, does not affect the amount of supersymmetry which is preserved. If both of the B_i are hyper Kähler, the scalar function is no longer harmonic, but 1/8 of the super-

symmetry is preserved, whether or not we include membranes, provided that we choose the signs of charges suitably.

Although the B_i can be any manifolds of $Sp(1)$ holonomy, one obtains the most interesting physical interpretations for T^4 , $K3$ and Taub-NUT manifolds. Suppose that both of the B_i are Taub-NUT manifolds. Then our general solution (102) describes two five-branes intersecting a membrane over a string. Each five-brane is parallel to one KK 6-brane and intersects the other over the common string. One could wrap further membranes around holomorphic cycles in the Taub-NUT manifolds to obtain solutions preserving smaller fractions of the supersymmetry.

If one of the B_i is a torus and the other is a Taub-NUT manifold, then compactification to four dimensions of the single five-brane plus monopole solution gives us a black hole with two magnetic charges which preserves 1/4 of the supersymmetry. Addition of a Chern-Simons term leads to a black hole carrying three charges which breaks the remaining supersymmetry. If we include membranes but no four-form flux, the four-dimensional interpretation of the solution is as a black hole carrying three charges, preserving 1/8 of the supersymmetry, which has zero temperature. Obviously wrapping the 5-branes around $K3 \times T^2$ gives analogous black hole solutions.

Solutions describing three overlapping five-branes are known; for example one can have the five-branes all overlapping on a string, and each pair overlapping on a three-brane [31]. As one would expect, the solutions generically preserve 1/8 of the background supersymmetry. However, the anomaly form necessarily vanishes, since the three-brane spaces must be conformally Ricci flat, and thence Riemann flat. Even if one wraps the branes around tori, there exists no finite Chern-Simons form. More generally, the anomaly form can only be non-zero when we have two transverse four dimensional manifolds with non-trivial holonomy or an eight-dimensional manifold with non-trivial holonomy.

There is a more general class of five-brane solutions related to the vacua $B^3 \times B^8$ where the holonomy of a manifold conformal to B^8 is a larger subgroup of $Spin(7)$. One can wrap a five-brane around $M^2 \times Y_4$, where Y_4 is a 4-fold within the 8-fold. A related discussion wrapping branes about cycles within such manifolds can be found in [30] and [33]. With a suitable choice of the 4-fold one may preserve a fraction of the supersymmetry. Since the anomaly form is non-vanishing, one gets electric charge corrections, and the resulting five-brane solutions are the generalization of that given above.

There is also a class of solutions obtained by including Brinkmann waves. One can include a wave to any intersection involving at least a common string. Thus one could for example add a wave to a KK 6-brane parallel to a five-brane intersecting a membrane along a string. Wrapping around a torus or $K3 \times T^3$, the boosting gives us a four charge black hole in four dimensions. Chern-Simons and anomaly terms change the mass, charge and singularity structure of the resulting black hole.

APPENDIX: CONVENTIONS

The different type of indices that we use are as follows. $M=0, \dots, 10$ represent eleven-dimensional space-time indices. $\mu=0, 1, 2$ represent three-dimensional space-time indices. $m=3, \dots, 10$ represent eight-dimensional space-time indices. We use $A=0, \dots, 10$ to indicate eleven-dimensional tangent space indices. When referring to 8-folds admitting a complex structure, we use indices $a, \bar{a}$ where $a=1, \dots, 4$.

The $d=11$ Dirac matrices Γ_M satisfy

$$\{\Gamma_M, \Gamma_N\} = 2g_{MN}, \quad (A1)$$

where g_{MN} has signature $(-, +, \dots, +)$. $\Gamma_{M_1 \dots M_n}$ is the anti-symmetrised product:

$$\Gamma_{M_1 \dots M_n} = \Gamma_{[M_1 \dots M_n]}, \quad (A2)$$

where the square brackets imply a sum over $n!$ terms with a $1/n!$ prefactor. The chirality operator is defined by

$$\gamma_9 = \frac{1}{8!} \epsilon_{mnpqrstu} \gamma^{mnpqrstu}, \quad (A3)$$

while our definition of the Hodge star is

$$\begin{aligned} &*(dx^{m_1} \wedge \dots \wedge dx^{m_p}) \\ &= \frac{1}{(d-p)!} \epsilon^{m_1 \dots m_p m_{p+1} \dots m_d} dx^{m_{p+1}} \wedge \dots \wedge dx^{m_d}. \end{aligned} \quad (A4)$$

We define n -forms with a factor of $1/n!$. For example, we take

$$F = \frac{1}{4!} F_{mnpq} dx^m \wedge dx^n \wedge dx^p \wedge dx^q. \quad (A5)$$

For further conventions and identities applying to the Dirac matrices, see the Appendix of [3]. A further convention used in the derivation of Eq. (14) is that $\epsilon_{012} = 1$ on M^3 .

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