# Quantum mechanics of the Schwarzschild–de Sitter black hole

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In the present paper, we apply the quantum general relativity theory to the Schwarzschild–de Sitter black hole. We start by writing down the super-Hamiltonian and supermomentum constraints for general, neutral, spherically symmetric, space-times, with a positive cosmological constant. Then we solve the constraints for the canonical momenta and canonically quantize the space-time using Dirac's formalism for constrained systems. The resulting operatorial equations are exactly solved in the WKB approximation, giving rise to a wave function. Finally, for a particular ansatz of the canonical variables, we compute the quantum mechanical density probability and show how it depends on the mass of the hole and on the cosmological constant. [S0556-2821(98)03214-7]

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## I. INTRODUCTION

Investigations of the classical and quantum properties of black holes in a cosmological background have been greatly increased in recent years. The cosmological background is characterized by the presence of a positive cosmological constant.

In the classical domain, the discovery that the Cauchy horizon of a Reissner-Nordström black hole in de Sitter space may be stable against gravitational perturbations [1,2] prompted a series of papers [3]. This result has important consequences for the cosmic censorship hypothesis [4], because an observer who is beyond the Cauchy horizon of such a black hole can look back and see a singularity [1].

Quantum mechanically, it is possible to obtain instantons, solutions to the Euclidean Einstein's equations, which describe the pair production of black holes in a de Sitter background [5,6]. Using these instantons, one may compute the saddle point approximation to the partition function $Z$. And from $Z$ one may compute the pair creation rate of black holes in a de Sitter background [6,7], as well as the entropy of the configurations [6].

It seems, therefore, interesting to study these black holes immersed in a de Sitter background in the light of quantum general relativity theory (for a review of the canonical quantization approach of this theory, see Ref. [8]). This will allow us to derive a wave function for these black holes and learn about their quantum properties.

In the present paper, we shall canonically quantize the Schwarzschild–de Sitter black hole. After finding a general WKB wave function, we shall specialize it by using a particular ansatz of the canonical variables. From this particular wave function, we derive the quantum mechanical density probability and study how it depends on the mass of the hole and on the cosmological constant.

In Sec. II, we shall start by writing down the Hamiltonian form of the action, with the constraints and a small discussion about the absence of boundary terms, following closely previous works [9,10].

In the following section, Sec. III, we canonically quantize the theory following Dirac's formalism for constrained systems. We avoid factor-ordering problems by first solving the classical constraints for the momenta and then quantizing the resulting equations, as in Refs. [11,12,13]. We find a WKB solution to the quantum equations and discuss some of its general properties.

Section IV is devoted to the study of a particular wave function of the Schwarzschild–de Sitter black hole. We start by identifying the cosmological constant with a classical, background, external field. Then we specialize the wave function found in the previous section by introducing the appropriate version of the ansatz proposed in Ref. [12]. After that, we numerically integrate the phase present in the quantum mechanical density probability ($P$) and construct some graphs. They show how $P$ depends on the mass of the hole and on the cosmological constant.

Finally, in Sec. V, we summarize the main points and results of the paper.

## II. CLASSICAL FORMALISM

As pointed out in the Introduction, we would like to study the quantum general relativity theory of the Schwarzschild–de Sitter black hole. Therefore, our starting point must be the Hamiltonian formalism for neutral, spherically symmetric, space-times, in the presence of a positive cosmological constant.

We start with the general, spherically symmetric metric, written in the Arnowitt-Deser-Misner (ADM) form:

$$ds^2 = -N^2 dt^2 + \Lambda^2 (dr + N^r dt)^2 + R^2 d\Omega^2, \tag{1}$$

where $d\Omega^2$ is the metric on the unit two-sphere, and $N$, $N^r$, $\Lambda$, and $R$ are functions of $t$ and $r$ only. Here we are using a unit system in which all physical constants are set to the identity.

Next, we must write the gravitational action for the space-times given by the metric (1). The gravitational action $S$, for

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space-times with generic boundary properties, is given by the sum of an hypersurface term $S_\Sigma$, plus a boundary term $S_{\partial\Sigma}$ [14]:

$$S=\frac{1}{16\pi}\int_M (R-2\lambda)(-g)^{1/2}dx^4+\frac{1}{8\pi}\int_{\partial M}Kh^{1/2}dx^3. \tag{2}$$

where $R$ is the curvature scalar, $g$ is the determinant of the four-dimensional metric, $K$ is the trace of the second fundamental form of the boundary, and $h$ is the determinant of the three-dimensional metric induced on the boundary.

Let us write down $S_\Sigma$ in terms of $R$ and $\Lambda$, the fields describing the gravitational degrees of freedom, and their conjugate momenta. In order to identify these momenta, one must first cast the hypersurface action in its ADM Lagrangian form [15]:

$$S_\Sigma[h_{ij};N,N^k]=-\int dr\int_\Sigma dt\ dx^2 L_\Sigma, \tag{3}$$

where

$$L_\Sigma=\frac{1}{16\pi}Nh^{1/2}[\overline{R}-(K_{ij}K^{ij}-K^2)-2\lambda]. \tag{4}$$

In Eq. (4), $\overline{R}$ is the intrinsic scalar of curvature and $K_{ij}$ is the second fundamental form of the hypersurfaces which foliate the space-time.

Now, using the results obtained in Ref. [9] for $\overline{R}$ and $K_{ij}$, we may easily write $S_\Sigma$ for the present situation:

$$\begin{aligned}S_\Sigma[R,\Lambda;N,N^r]=\int dt\int_{-\infty}^{\infty}dr\Bigg\{&\frac{1}{N}\Bigg(R[(\Lambda N^r)'-\dot{\Lambda}]\\&\times(N^rR'-\dot{R})+\frac{\Lambda}{2}(N^rR'-\dot{R})^2\Bigg)\\&+N\Bigg(\frac{\Lambda}{2}-\frac{RR''}{\Lambda}+\frac{RR'\Lambda'}{\Lambda^2}\\&-\frac{R'^2}{2\Lambda}-\frac{\lambda}{2}\Lambda R^2\Bigg)\Bigg\},\end{aligned} \tag{5}$$

where the overdots and primes mean differentiations in the time and radial parameters, respectively.

By functional differentiation of the above action, Eq. (5), with respect to the velocities $\dot{\Lambda}$ and $\dot{R}$, we obtain the momenta $P_\Lambda$ and $P_R$, respectively. Their expressions are identical to the ones given in Ref. [9].

Now we are prepared to write the canonical Hamiltonian

$$H_c=P_i\dot{Q}^i-L_\Sigma, \tag{6}$$

which has the explicit form

$$H_c=NH+N^rH_r, \tag{7}$$

such that

$$H=\frac{\Lambda P_\Lambda^2}{2R^2}-\frac{P_RP_\Lambda}{R}+\frac{RR''}{\Lambda}-\frac{RR'\Lambda'}{\Lambda^2}+\frac{R'^2}{2\Lambda}-\frac{\Lambda}{2}+\frac{\lambda}{2}\Lambda R^2, \tag{8}$$

$$H_r=P_RR'-\Lambda P_\Lambda', \tag{9}$$

where $H_r$ is the supermomentum and $H$ is the super-Hamiltonian.

The hypersurface action is promptly written in terms of the canonical Hamiltonian as

$$\begin{aligned}&S_\Sigma[\Lambda,R,P_\Lambda,P_R;N,N^r]\\&\quad=\int dt\int_{-\infty}^{\infty}dr(P_\Lambda\dot{\Lambda}+P_R\dot{R}-NH-N^rH_r).\end{aligned} \tag{10}$$

As we have said before, the other important ingredient in order to write the total action, consistently, is the boundary action.

The space-time we want to study at the quantum level is the Schwarzschild–de Sitter black hole [16,17]. It belongs to a set of space-times whose line element tend asymptotically, for large parameter $r$, to a de Sitter line element. The de Sitter space-time is topologically a $\mathcal{R}\times S^3$ manifold, which means that the spacelike hypersurfaces are compact [18]. Therefore, for all members of this set there is no asymptotic structure; in particular, the boundary action in Eq. (2) is nil. The total action is given solely by the hypersurface action, Eq. (10) [19].

## III. QUANTUM FORMALISM

### A. Preliminaries

We would like to quantize the theory using the Dirac's formalism for quantization of constrained systems.

First, we introduce a wave function which is a functional of the canonical fields in their operatorial form $\hat{R}$ and $\hat{\Lambda}$:

$$\Psi=\Psi[\hat{R},\hat{\Lambda}]. \tag{11}$$

Then, we impose the appropriated commutators between the fields operators and their conjugated momenta $\hat{P}_R$ and $\hat{P}_\Lambda$. Finally, we demand that the operatorial form of the constraints, Eqs. (8) and (9), annihilate the wave function, Eq. (11).

Observing the constraints, Eqs. (8) and (9), we notice that the above quantization prescription cannot be uniquely implemented in the present case due to factor-ordering ambiguities. One solution to this problem, which may be used here following Refs. [11,12,13], starts by solving the constraints for the momenta and only then quantizing the theory.

In order to obtain the expressions for the momenta $P_R$ and $P_\Lambda$, we write down the following expression involving the constraints:

$$-\Lambda^{-1}(R^{-1}P_\Lambda H_r+R'H)\approx 0. \tag{12}$$

After some calculations and an integration in the parameter $r$, we find

$$\frac{1}{2}R^{-1}P_\Lambda^2-\frac{1}{2}\Lambda^{-2}RR'^2-\frac{\lambda}{6}R^3+\frac{1}{2}R\approx C(t), \quad (13)$$

where $C(t)$ is an integration constant and may depend on $t$.

From Ref. [9], we notice that $C$ above may be identified with the canonical mass variable $M$, and it is easy to show that $C$ is an observable [20].

In order to show it, we first use the fact that $C$ does not depend on $r$. Since by definition we have

$$\{F(r),H_r(\overline{r})\}=F'(r)\delta(r,\overline{r}). \quad (14)$$

for a generic canonical variable $F$, we conclude that $C$ has vanishing Poisson bracket with $H_r$, Eq. (9).

The Poisson bracket between $C$, Eq. (13), and $H$, Eq. (8), may be easily computed if we rewrite these quantities in terms of the corresponding ones introduced by Kuchař in Ref. [9]:

$$H=H_K+\frac{\lambda}{2}\Lambda R^2 \quad \text{and} \quad C=M_K-\frac{\lambda}{6}R^3, \quad (15)$$

where $H_K$ and $M_K$ are, respectively, the super-Hamiltonian and the canonical mass variable for the eternal Schwarzschild black hole [9].

Therefore,

$$\{C,H\}=\{M_K,H_K\}+\frac{\lambda}{2}\{M_K,\Lambda R^2\}-\frac{\lambda}{6}\{R^3,H_K\}$$
$$-\frac{\lambda^2}{12}\{R^3,\Lambda R^2\}. \quad (16)$$

The last term on the right-hand side of Eq. (16) is nil; the second and third have the same value $(\lambda/2)RP_\Lambda$, with opposite signs. Then the Poisson bracket between $C$ and $H$ are given by $\{M_K,H_K\}$, and from Ref. [9] we obtain

$$\{C,H\}=-\Lambda^{-3}R'H_r. \quad (17)$$

This Poisson bracket weakly vanishes, and $C$ is an observable and also a constant of motion. From now on we shall write $M$ instead of $C$.

As we shall see below Sec. IV, the wave function for the Schwarzschild–de Sitter black hole will depend on $M$, as well as on $\lambda$, although the last quantity is not a canonical variable of the gravitational field.

Returning now to our task of determining the momenta, we see that it is straightforward to compute $P_\Lambda$ from Eq. (13):

$$P_\Lambda\approx\frac{R}{\Lambda}\Theta[R,\Lambda;M,\lambda], \quad (18)$$

where we took the + sign of the square root $\Theta$, which is given by

$$\Theta[R,\Lambda;M,\lambda]=\sqrt{R'^2-\left[1-\frac{2M}{R}-\frac{\lambda}{3}R^2\right]\Lambda^2}. \quad (19)$$

In order to clarify the nature of the quantities on which $\Theta$ and all other functionals given below depend, we shall follow the notation of Ref. [9].

Introducing the value of $P_\Lambda$, Eq. (18), in the super-Hamiltonian, Eq. (8), we may derive $P_R$:

$$P_R\approx\frac{G[R,\Lambda;M,\lambda)}{\Theta[R,\Lambda;M,\lambda)}, \quad (20)$$

where

$$G[R,\Lambda;M,\lambda)=R'^2+\frac{M\Lambda^2}{R}+RR'-\frac{RR'\Lambda'}{\Lambda}$$
$$-\Lambda^2+\frac{2}{3}\lambda\Lambda^2R^2. \quad (21)$$

### B. Wave function

We may now apply the Dirac's formalism to the theory using the new constraints, Eqs. (18) and (20), which are free of factor-ordering ambiguities.

Working in the field representation, the operators $\hat{R}$ and $\hat{\Lambda}$ are replaced by the fields themselves, and the conjugate momenta are defined as the following functional derivatives:

$$\hat{P}_R=-i\frac{\delta}{\delta R(r)}. \quad (22)$$

$$\hat{P}_\Lambda=-i\frac{\delta}{\delta\Lambda(r)}. \quad (23)$$

We rewrite the constraints, Eqs. (18) and (20), in their operatorial forms, with the aid of Eqs. (22) and (23), and demand that $\Psi$, Eq. (11), satisfy them:

$$-i\frac{\delta}{\delta\Lambda(r)}\Psi[R,\Lambda;M,\lambda)$$
$$=\frac{R}{\Lambda}\Theta[R,\Lambda;M,\lambda)\Psi[R,\Lambda;M,\lambda), \quad (24)$$

$$-i\frac{\delta}{\delta R(r)}\Psi[R,\Lambda;M,\lambda)=\frac{G[R,\Lambda;M,\lambda)}{\Theta[R,\Lambda;M,\lambda)}\Psi[R,\Lambda;M,\lambda), \quad (25)$$

where we have introduced explicitly $M$ and $\lambda$ among the variables of $\Psi$, following the discussion at the end of Sec. III A.

Inspired by previous works [11,12,13], we shall try a WKB solution to the system of equations (24),(25):

$$\Psi[R,\Lambda;M,\lambda)=\exp\{iA[R,\Lambda;M,\lambda)\}. \quad (26)$$

Introducing the above ansatz for $\Psi$, Eq. (26), in the system mentioned above, we obtain the following equations for $A$:

$$\frac{\delta}{\delta\Lambda(r)} A[R,\Lambda;M,\lambda] = \frac{R}{\Lambda}\,\Theta[R,\Lambda;M,\lambda], \tag{27}$$

$$\frac{\delta}{\delta R(r)} A[R,\Lambda;M,\lambda] = \frac{G[R,\Lambda;M,\lambda]}{\Theta[R,\Lambda;M,\lambda]}. \tag{28}$$

Observing Eq. (27), we notice that it does not involve any derivative of the field $\Lambda$ with respect to $r$. So we may integrate it as an ordinary line integral to find

$$A[R,\Lambda;M,\lambda] = \int dr\left[\Theta + \frac{R'}{2}\ln\left(\frac{R'-\Theta}{R'+\Theta}\right)\right] + \Gamma[R;M,\lambda]. \tag{29}$$

where $\Gamma[R;M,\lambda]$ is an arbitrary functional independent of $\Lambda$.

When we substitute $A$, Eq. (29), in the remaining equation of the system, Eq. (28), we conclude that it is a solution of the system if and only if $\Gamma$ is independent of $R$. Since $\Gamma$ can only depend on the constants $M$ and $\lambda$, for simplicity we shall discard it.

The wave function, Eq. (26), becomes, with the aid of Eq. (29) and the simplification mentioned above,

$$\Psi[R,\Lambda;M,\lambda] = \exp\left\{ i\int dr\left[\Theta + \frac{R'}{2}\ln\left(\frac{R'-\Theta}{R'+\Theta}\right)\right]\right\}. \tag{30}$$

Although we have quantized the theory using the constraints written in the forms (18) and (20), it is not difficult to show that the following operatorial versions of $H$, Eq. (8), and $H_r$, Eq. (9), annihilate $\Psi$, Eq. (30):

$$\hat{H} = \frac{1}{2}\frac{\Lambda}{R^2}\left(\frac{R}{\Lambda}\,\Theta\hat{P}_\Lambda\,\frac{\Lambda}{R}\,\Theta^{-1}\hat{P}_\Lambda\right) - \frac{1}{R}\left(\frac{R}{\Lambda}\,\Theta\hat{P}_R\,\frac{\Lambda}{R}\,\Theta^{-1}\hat{P}_\Lambda\right) + G[R,\Lambda;M,\lambda], \tag{31}$$

$$\hat{H}_r = R'\hat{P}_R - \Lambda\,\frac{\partial}{\partial r}\,\hat{P}_\Lambda. \tag{32}$$

Let us now analyze some of the general properties of the wave function $\Psi[R,\Lambda;M,\lambda]$, Eq. (30).

First of all, if we substitute in $\Psi$ the values of $R$, $\Lambda$, and $M$ associated with the classical solution, which from Ref. [17] are

$$R = r, \quad \Lambda = 1\Big/\sqrt{1-\frac{2m}{r}-\frac{\lambda}{3}r^2}, \tag{33}$$

and

$$M = m, \tag{34}$$

we obtain

$$\Psi[R,\Lambda;M,\lambda] = 1. \tag{35}$$

We may identify from $\Psi$, Eq. (30), sectors of its domain where the space-times described by $\Psi$ will be in classical allowed or forbidden regions [12,13,21].

One of this situations appears when we notice that if $\Theta$, Eq. (19), is imaginary, $A$, Eq. (29), becomes complex and $\Psi$, Eq. (30), develops an exponentially decreasing factor. This situation is generally associated with a classically forbidden region where the classical momenta are imaginary. Since the momenta, Eqs. (18) and (20), are written in terms of $\Theta$, we may see that they will become imaginary as long as the other quantities present in their expressions are real.

Another such situation occurs when the argument of the logarithm in $\Psi$, Eq. (30), is negative because then $\Psi$ may have a imaginary part. This happens when

$$R'^2 - \Theta^2 < 0; \tag{36}$$

more explicitly, it implies that, from Eq. (19),

$$\left(1-\frac{2M}{R}-\frac{\lambda}{3}R^2\right)\Lambda^2 < 0. \tag{37}$$

Observing the classical solution, Eqs. (33) and (34), we see that the set of configurations that satisfy the condition (37) is not classical in the sense that it does not contain the classical solution.

## IV. WAVE FUNCTION FOR THE SCHWARZSCHILD–DE SITTER BLACK HOLE

As discussed in Ref. [9], the wave function for a Schwarzschild black hole can only depend on the mass of the hole, the unique physical degree of freedom of the system. Therefore, from this wave function one cannot obtain information on how the black hole interacts with external fields or about the gravitational collapse.

In the present situation we are still treating a static system, and so we shall not be able to describe the gravitational collapse. On the other hand, as we briefly discussed in Sec. III A, we have, besides the mass of the black hole, another physical quantity: the positive cosmological constant. As it is clear form Secs. II and III, $\lambda$ is not a degree of freedom of the gravitational field. So it seems natural to try to associate it with an external field in interaction with the black hole. Of course, since we have not treated $\lambda$ as a canonical variable and later quantized it, we can only associate this constant to a classical, background, field.

This field will have very particular properties. They will be found by the requirement that its stress-energy tensor be given by the $\lambda$ term, present on the right-hand side of Einstein's equations. There is already in the literature a scalar field which has all the required properties, at least during a certain period of time. It is called an inflaton [22]. The inflaton is the driving force of the inflationary model of the Universe [22] and has already been used in connection with the quantum creation of primordial black holes [7].

Therefore, the wave function, Eq. (30), and the quantum density probability computed from it should give us information on how the black hole behaves in the presence of a classical, background, inflaton field.

In order to obtain predictions from our model comparable with the results already derived for the Schwarzschild black hole, we shall follow the ansatz proposed in Ref. [12], suitably adapted for the present case.

The ansatz chooses among all possible values of the fields $R$ and $\Lambda$ those given by the classical solution (33), associated with the Schwarzschild–de Sitter black hole with a fixed $\lambda$. The quantum aspect of the model is characterized by also fixing $M$ to have a value different from the classical mass $m$, Eq. (34). It means that we shall have many different quantum states, one for each distinct value of $m$, as long as $m \neq M$. Finally, the black hole is put in a box of radius $R > M$.

From the Penrose diagram for the Schwarzschild–de Sitter black hole [17], let us restrict our attention to the section including regions I, $\mathrm{II}_H$, $\mathrm{II}_C$, and $\mathrm{III}_C$. Region $\mathrm{II}_H$ contains the future, $r=0$, spacelike singularity and is separated from region I by the black hole event horizon located at $r=r_+$. Region I is the only one where the Killing vector $k=\partial/\partial t$ is timelike and future directed. This region is separated from regions $\mathrm{II}_C$ and $\mathrm{III}_C$ by the cosmological event horizon located at $r=r_{++}$, where $r_{++}>r_+$. Regions $\mathrm{II}_C$ and $\mathrm{III}_C$ incorporate, respectively, the spacelike infinities at $\mathcal{J}^+$ and $\mathcal{J}^-$.

Because of the above-mentioned property of the vector $k$ in region I, this region resembles the external region of the Schwarzschild black hole also called region I. Therefore, in order to compare our results with the ones in Ref. [12], we shall choose $R$ such that $r_+ < R < r_{++}$.

If we introduce all the requirements of our ansatz in the wave function, Eq. (30), we get

$$\Psi(m)=\exp\left(i\int_0^R dr\left[\frac{\sqrt{2M-2m}}{\sqrt{r-2m-\hat{\lambda}r^3}} + \frac{1}{2}\ln\left(\frac{\sqrt{r-2m-\hat{\lambda}r^3}-\sqrt{2M-2m}}{\sqrt{r-2m-\hat{\lambda}r^3}+\sqrt{2M-2m}}\right)\right]\right), \tag{38}$$

where $\Psi$ is now an ordinary function of $m$, with $M$ and $\hat{\lambda}$ fixed, $\hat{\lambda}$ is defined as $\lambda/3$, and the integral is computed from the singularity at $r=0$ until the radius of the box at $r=R$.

From $\Psi$, Eq. (38), we may directly evaluate the (unrenormalized) quantum probability density $P(m)$, given by

$$P(m)=[\Psi(m)]^*\Psi(m), \tag{39}$$

which is the quantity we shall actually be concerned with. It is important to mention, at this stage, that the integral in Eq. (38) cannot be expressed in terms of known functions, and so all results below were found numerically.

Our investigation of the quantum properties of the Schwarzschild–de Sitter black hole starts with the evaluation of $P(m)$, Eq. (39), for a fixed $\lambda=0.3$, and three different, but fixed values of $M$, 0.1, 0.135, and 0.17. The radius of the box is $R=1.2$, which satisfies all requirements.

The three curves of $P(m/M)\times m/M$, one for each value of $M$, are given in Fig. 1 for $m/M \geqslant 1$ and in Fig. 2 for $m/M \leqslant 1$, where we have introduced the new variable $m/M$, in order to draw all three curves together.

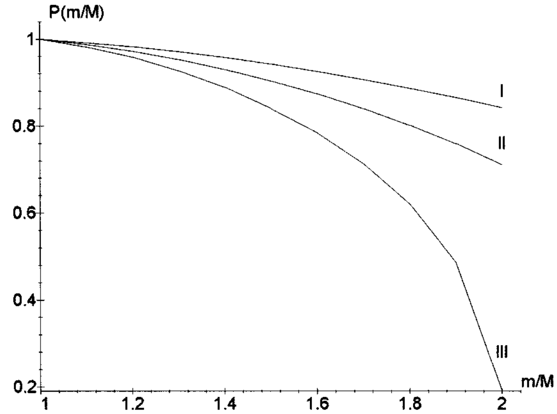


FIG. 1. Curves of $P(m/M)\times m/M$ for fixed $\lambda=0.3$, three different values of $M$, and $m/M \geqslant 1$. Curves I, II, and III are associated, respectively, with $M=0.1$, 0.135, and 0.17.

From Figs. 1 and 2, we may draw some conclusions about the quantum properties of the Schwarzschild–de Sitter black hole.

The integral in $P(m)$, Eq. (38), and consequently $P(m)$, did not show any problem when evaluated at $r=0$, and so the classical curvature singularity has disappeared at the quantum level. For all three curves, $P(m/M)$ is peaked when $m=M$, which means that the classical state is the most probable one. When we move away from the classical state, increasing or diminishing $m$, $P(m/M)$ decreases exponentially. Therefore, the more distant from the classical state, the less probable is the quantum state. The width of $P(m/M)$ decreases rapidly with increasing $M$. Finally, there is an asymmetry between Figs. 1 and 2, which implies that quantum black hole states with $m<M$ are less suppressed than the ones with $m>M$.

All the above conclusions are in agreement with the re-

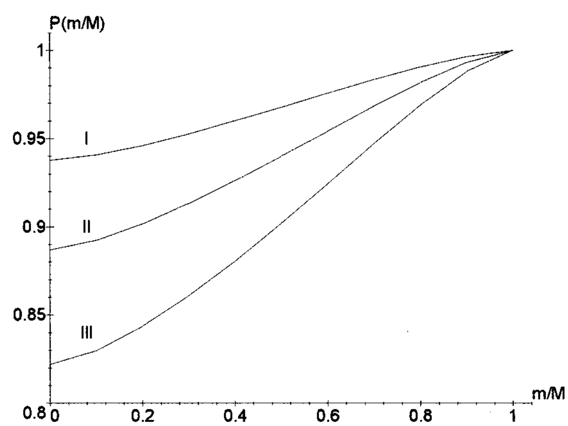


FIG. 2. Curves of $P(m/M)\times m/M$ for fixed $\lambda=0.3$, three different values of $M$, and $m/M \leqslant 1$. Curves I, II, and III are associated, respectively, with $M=0.1$, 0.135, and 0.17.

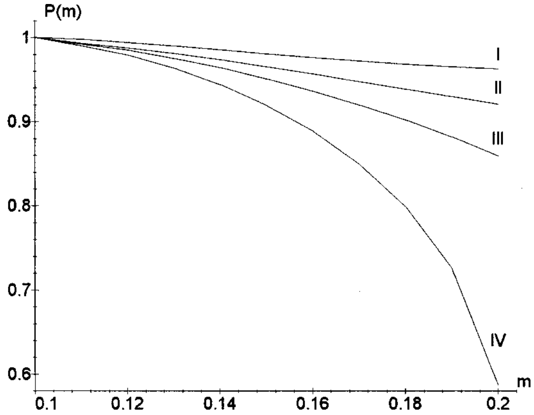


FIG. 3. Curves of $P(m)\times M$ for fixed $M=0.1$, four different values of $\lambda$, and $m\geqslant 0.1$. Curves I, II, III, and IV are associated, respectively, with $\lambda=0$, 0.3, 0.5, and 0.7.

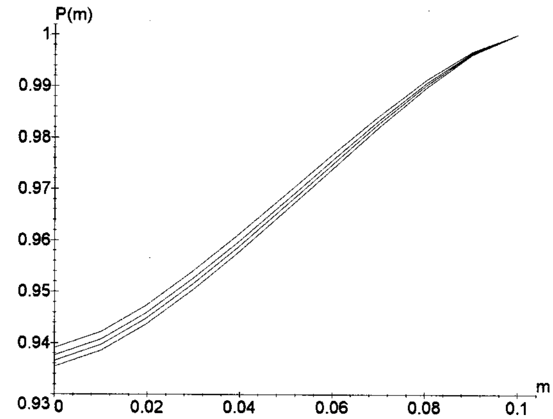


FIG. 4. Curves of $P(m)\times M$ for fixed $M=0.1$, four different values of $\lambda$, and $m\leqslant 0.1$. From the top to the bottom, the curves are associated, respectively, with $\lambda=0$, 0.3, 0.5, and 0.7.

sults found in Ref. [12] for the Schwarzschild black hole, which generalizes the situation when $\lambda=0$ for the case when $\lambda>0$.

In order to learn how the quantum black hole feels the presence of an external, classical, background, inflaton field, let us compute $P(m)$, Eq. (39), for a fixed $M=0.1$, and four different, but fixed values of $\lambda$, 0, 0.3, 0.5, and 0.7. The radius of the box is $R=0.88$, which satisfies all requirements.

The four curves of $P(m)\times m$, one for each value of $\lambda$, are given in Fig. 3 for $m\geqslant M$ and in Fig. 4 for $m\leqslant M$.

It is clear from Figs. 3 and 4 that the bigger the cosmological constant, the more suppressed are the quantum states. The same conclusion holding for both $m\geqslant M$ and $m\leqslant M$, despite the asymmetry between Figs. 3 and 4, which has been already noted in Figs. 1 and 2.

The fact that the bigger $\lambda$ the more suppressed are the quantum black hole states may be understood if we remember that, already classically, a positive $\lambda$ acts as a repulsive term, as noted by Gibbons and Hawking in Ref. [17]. It means that the bigger the inflaton field, the more difficult is the black hole formation and permanence, classically and quantum mechanically.

## V. CONCLUSIONS

In this paper we have computed the wave function for a Schwarzschild–de Sitter black hole, and from the quantum probability density we derived the quantum properties of the system.

The action for general, neutral, spherically symmetric, space-times, with a positive cosmological constant, including the Hamiltonian and momentum constraints, was computed in Sec. II, Eq. (10).

In Sec. III, we quantized the theory following Dirac's formalism, after solving the constraints for the momenta which avoided factor-ordering ambiguities. The result of this quantization was a wave function in the WKB approximation, Eq. (30).

Observing the formalism developed in previous sections and also in the literature, we concluded in Sec. IV, that $\lambda$ could be interpreted as a degree of freedom of a classical, background, external field. The field is the inflaton [22].

In order to obtain some information from our wave function, Eq. (30), and compare it with the case of a Schwarzschild black hole, we applied the ansatz introduced in Ref. [12] to the present situation.

From the density probability, Eq. (39), we learned that all properties obtained for the Schwarzschild black hole [12] are generalized for a positive $\lambda$. In addition to that, we also discovered that the presence of a positive cosmological constant or (which is equivalent) a classical, background, inflaton field causes the quantum density probability to be suppressed. The bigger the $\lambda$, the less probable is a given quantum black hole state. This result was not difficult to interpret when we remembered that, already classically, a positive cosmological constant acts as a repulsive term.

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