

Reparametrization invariance to all orders in heavy quark effective theory

Raman Sundrum*

Department of Physics, Boston University, Boston, Massachusetts 02215

(Received 15 July 1997; published 4 December 1997)

Heavy quark effective theory splits heavy quark momentum into a large fixed momentum and a variable residual momentum $p_\mu = m_Q v_\mu + k_\mu$. It thereby suffers a redundancy of description corresponding to small changes in the choice of the fixed velocity v_μ . The fact that full QCD is manifestly v independent should lead to a nontrivial constraint on the form of the effective theory, known as reparametrization invariance. For spin- $\frac{1}{2}$ quarks, the precise form of the constraint and its solution at the level of the effective Lagrangian have proven to be rather subtle, and the original proposal by Luke and Manohar has been questioned. In this paper I employ a version of heavy quark effective theory containing the “antiparticle” field as a nonpropagating auxiliary field, which greatly simplifies keeping track of v dependence. This permits a very simple derivation of reparametrization invariance from first principles. The auxiliary field can also be integrated out to return to the standard formulation of the effective theory, but with the effective Lagrangian now satisfying the full reparametrization constraint. I compare this result with earlier proposals. [S0556-2821(98)03501-2]

PACS number(s): 12.39.Hg

I. INTRODUCTION

Heavy quark effective theory (HQET) provides a systematic expansion of QCD amplitudes in powers of (momentum transfer)/ m , where m is a heavy quark mass. Calculations are very conveniently done in terms of an effective Lagrangian which is also organized in powers of $1/m$. Reference [1] contains a review and references to the original literature. There are many interesting processes where the momentum transfers between a heavy quark and the light quarks and gluons are of order Λ_{QCD} . For such situations and for $m \gg \Lambda_{\text{QCD}}$, the heavy quark velocity is approximately constant. This is formally reflected in setting up the $1/m$ expansion by splitting the heavy quark momentum into two pieces:

$$p_\mu = m v_\mu + k_\mu, \quad (1)$$

where v_μ is a fixed four-velocity ($v^2 = 1$), and k_μ is a variable residual momentum of order Λ_{QCD} . While this split is very useful for book-keeping, clearly there is an arbitrariness of order Λ_{QCD}/m in the precise choice of v_μ for any process [2]. Formally we need just note the redundancy of the momentum decomposition under infinitesimal shifts in the fixed velocity:

$$v' = v + \delta v, \quad v \cdot \delta v = 0. \quad (2)$$

QCD is manifestly invariant under such infinitesimal reparametrizations, so a systematic approximation to full QCD such as HQET must also reflect this fact. This constraint on HQET is known as reparametrization invariance (RPI). Luke and Manohar [3] proposed that RPI should directly constrain the form of the Lagrangian, as with any other symmetry. Since the HQET Lagrangian must be determined order by order in α_s and $1/m$ by matching to full QCD, there is a

significant reduction of work and increase in understanding if one can first enforce RPI on the general form of the effective Lagrangian.

The precise form of the RPI constraint on the HQET Lagrangian for spin- $\frac{1}{2}$ quarks has been rather difficult to identify and prove. The original proposal by Luke and Manohar [3] for the form of the RPI-constrained Lagrangian was shown by Chen [4] to be inconsistent with the direct matching computation already at tree level. It is possible that the proposal is valid up to field redefinitions (see Ref. [5] for an application of RPI by this means) but this is less useful. Chen proposed another form for the RPI constraint consistent with tree-level matching, but its general validity has been unclear because the derivation effectively depends on the use of the classical equations of motion. Kilian and Ohl [6] obtained a RPI form for the HQET Lagrangian valid to all orders, and which reduces at tree level to Chen's version. I believe this to be correct and part of my present purpose is to give a simpler derivation and better understanding of this result. RPI in HQET has most recently been investigated by Finkemeier, Georgi, and McIrvin [7], who showed that the constraints from Chen's version of RPI are correct to order $1/m^2$. The framework of the present paper will clarify why this happens even though Chen's RPI is wrong in general (see Sec. VI). Reference [7] also showed that there is a field redefinition (which does not affect the S matrix) taking Chen's constraints to those predicted by Luke and Manohar's RPI, to order $1/m^2$.

In order to understand the nature of the difficulty and how to resolve it, let us briefly review the essentials of the HQET procedure. *A priori* the correct procedure for calculating an amplitude to some order in $1/m$ (and in α_s) is to compute the relevant Feynman diagrams of full QCD and to expand the result in powers of $1/m$. HQET works essentially in the reverse order, which is the key to its conceptual and computational advantages. Having separated out the fixed large momentum as in Eq. (1), the QCD vertices and propagators are first expanded in powers of $1/m$ and then used to compute Feynman diagrams. Because of the different ultraviolet divergence structures that arise depending on whether one ex-

*Email address: sundrum@budoie.bu.edu

pands in $1/m$ first or last, this naive version of HQET only agrees at tree level with full QCD. However, the two procedures can be made to agree (“match”) by adding local terms to the HQET Lagrangian. Once the extra terms in $\mathcal{L}_{\text{HQET}}$ are determined (to any particular order) by matching to some number of full QCD amplitudes, they are universal, and can be used to compute other amplitudes without further reference to full QCD. This procedure is quite general and applies even if the quarks were spinless. In the spinless case there is no difficulty in tracking v dependence, deriving the form of RPI for the effective theory, and enforcing the constraint on the form of the effective Lagrangian [3].

For spin- $\frac{1}{2}$ quarks there is an extra complication. When the quark propagator is expanded for $k_\mu \ll m$ in Eq. (1), one finds

$$\frac{1}{\not{p} - m + i\epsilon} = \frac{1 + \not{v}}{2} \frac{1}{k \cdot v + i\epsilon} + \mathcal{O}\left(\frac{1}{m}\right). \quad (3)$$

This shows that in the heavy quark limit $m \rightarrow \infty$, not all four spinor degrees of freedom propagate, but only the two components projected out by $(1 + \not{v})/2$. The usual practice is to interpolate these spinor modes in HQET with a heavy quark field ψ_{+v} ,

$$\not{v} \psi_{+v} = \psi_{+v}. \quad (4)$$

(For an interpolating field this makes sense even when m is large but not infinite.) Of course the expansion of Eq. (3) is invalid in loops, where k need not be small, and all four spinor degrees of freedom are important. Let us denote the remaining degrees of freedom by ψ_{-v} :

$$\not{v} \psi_{-v} = -\psi_{-v}. \quad (5)$$

We can refer to ψ_{+v} and ψ_{-v} as the “particle” and “antiparticle” fields, respectively, since Eqs. (4) and (5) are just the Dirac equations for positive-energy particles and antiparticles with fixed momentum mv .

We can understand the status of ψ_{-v} by observing that, for $k_\mu \ll m$ in Eq. (1),

$$\frac{1 - \not{v}}{2} \frac{1}{\not{p} - m + i\epsilon} \frac{1 - \not{v}}{2} = \frac{1 - \not{v}}{2} \frac{1}{2m} + \mathcal{O}\left(\frac{1}{m^2}\right). \quad (6)$$

We see that relative to the ground state of the heavy quark sector, ψ_{-v} is a massive mode, with mass gap $2m$. Like any heavy mode in field theory we can integrate out its virtual effects and omit the field from the effective theory. This is what is done in standard HQET, so $\mathcal{L}_{\text{HQET}}$ depends only on ψ_{+v} and the light quarks and gluons. Normally in field theory heavy and light particles transform independently under symmetries. Integrating out the heavy particles does not alter the symmetry properties of the light particles. RPI is different, in that under Eq. (2), $\psi_{\pm v}$ clearly mix. Integrating out the heavy mode ψ_{-v} necessarily complicates RPI in the effective theory.

In this paper, I derive RPI from first principles, in two versions of HQET. In Sec. II, I describe a version of HQET in which the antiparticle field ψ_{-v} is explicitly present, but

as a nonpropagating auxiliary field.¹ As in standard HQET, the $1/m$ expansion is performed *before* computing diagrams. The fact that both $\psi_{\pm v}$ are present makes RPI as simple as it is in the case of heavy spin-0 particles. One can in fact write the most general form of the RPI-constrained effective Lagrangian. This is done in Sec. III. In the second step of the program, described in Sec. IV, the ψ_{-v} field is completely integrated out, resulting in the general form of the RPI-constrained effective Lagrangian in the standard HQET formulation. In Sec. V, I check some of the consequences of RPI on $\mathcal{L}_{\text{HQET}}$ at the leading orders of $1/m$. Section VI contains some discussion and comparison with earlier proposals.

For simplicity, in this paper I only consider HQET applied to the case of a single heavy quark in interaction with light quarks and gluons. While the character of the arguments and results presented in this paper are nonperturbative in nature, the proofs are given within perturbation theory, in the context of minimal subtraction and dimensional regularization. I will address the topic of RPI in nonperturbative lattice HQET elsewhere.

II. HQET WITH AN AUXILIARY FIELD

To focus on the residual momenta k_μ as in Eq. (1), we make the standard field redefinition

$$\psi_v(x) \equiv e^{imv \cdot x} \psi(x), \quad (7)$$

where ψ is the full QCD quark field and ψ_v will be our HQET field, and m is the heavy quark mass. We can further decompose ψ_v :

$$\psi_{\pm v} \equiv \frac{1 \pm \not{v}}{2} \psi_v. \quad (8)$$

There is an ambiguity in just what is meant by the quark “mass.” The canonical choice is to choose m so that the *residual* mass term for ψ_{+v} vanishes in the HQET. In general though, any choice is allowed if it makes the residual mass $\ll m$ (see Ref. [8]). The minimal subtraction mass parameter of full QCD will be separately denoted by $\tilde{m}(\mu)$.

At tree level, expanding in $1/m$ before or after computing Feynman diagrams makes no difference, so matching the HQET to full QCD only amounts to accounting for Eq. (7):

$$\begin{aligned} \mathcal{L}_{\text{HQET}}^{\text{tree}} &= \overline{\psi_v} e^{imv \cdot x} (i\mathcal{D} - \tilde{m}) e^{-imv \cdot x} \psi_v \\ &= \overline{\psi_{+v}} (iD \cdot v + m - \tilde{m}) \psi_{+v} \\ &\quad - \overline{\psi_{-v}} (m + \tilde{m} + iD \cdot v) \psi_{-v} \\ &\quad + \overline{\psi_{+v}} i\mathcal{D}_\perp \psi_{-v} + \overline{\psi_{-v}} i\mathcal{D}_\perp \psi_{+v}, \end{aligned} \quad (9)$$

where $\mathcal{D}_\perp \equiv \mathcal{D} - D \cdot v \not{v}$. The gauge field and light quark terms have been suppressed. The canonical choice for m at this order is clearly $m = \tilde{m}$. In using this Lagrangian the HQET approach tells us to expand in $1/m$ ($1/\tilde{m}$) before com-

¹ ψ_{-v} plays a role in simplifying RPI analogous to the role of auxiliary fields in supersymmetric field theory.

puting diagrams, so we see that the ψ_{-v} propagator in a background gauge field is really completely local:

$$-\frac{1}{m+\widetilde{m}+iD\cdot v} \equiv -\frac{1}{m+\widetilde{m}} + \frac{iD\cdot v}{(m+\widetilde{m})^2} - \cdots \quad (10)$$

Order by order in $1/m$ this auxiliary field ψ_{-v} can be integrated out to yield interactions for the propagating ψ_{+v} field.

At the quantum level, matching corrections to $\mathcal{L}_{\text{HQET}}$ are nontrivial because in full QCD we expand in $1/m$ after computing diagrams while in the HQET we expand the Feynman vertices and propagators in $1/m$ before computing diagrams. The two steps do not commute inside loops because of ultraviolet divergences, but since such divergences are local, order by order in the loop expansion we can compensate by adding purely local terms to the HQET Lagrangian. To state the matching conditions precisely, we add sources to full QCD and to the HQET,

$$\begin{aligned} \delta\mathcal{L}_{\text{QCD}} &= \overline{\eta} e^{imv\cdot x} \psi + \text{H.c.}, \\ \delta\mathcal{L}_{\text{HQET}} &= \overline{\eta} \psi_v + \text{H.c.}, \end{aligned} \quad (11)$$

and demand that the corresponding Green functions agree order by order in $1/m$ and α_s .

The most general form for the HQET Lagrangian is

$$\mathcal{L}_{\text{HQET}} = \overline{\psi}_v \left[S + \not{V} + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \psi_v \quad (12)$$

$$\begin{aligned} &= \overline{\psi}_{+v} \left[S + \not{v} \cdot \not{V} + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \psi_{+v} \\ &+ \overline{\psi}_{-v} \left[S - \not{v} \cdot \not{V} + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \psi_{-v} \\ &+ \overline{\psi}_{+v} \left[\not{V}_\perp + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \psi_{-v} \\ &+ \overline{\psi}_{-v} \left[\not{V}_\perp + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \psi_{+v}, \end{aligned} \quad (13)$$

where $S, V_\mu, T_{\mu_1 \dots \mu_n}^{(n)}$ are local Hermitian operators constructed from the light fields, covariant derivatives, and v , and where $\sigma^{(n)}$ is the totally antisymmetrized product of $n(4+\epsilon)$ -dimensional γ matrices,

²These could be either connected or one particle irreducible (1PI) Green functions. Note that normally the relation between 1PI vertices in full and effective theories is complicated when a massive mode is completely integrated out. The reason is that a diagram in the full theory that can be cut in two by cutting an internal massive mode propagator can become 1PI in the effective theory where the massive mode is absent. In the present case, as discussed in the introduction, the massive mode is ψ_{-v} , but because we have left it in the effective theory as an auxiliary field, we have a simple equality of the 1PI effective actions of the full and effective theories after matching.

$$\sigma_{\mu_1 \dots \mu_n}^{(n)} \equiv \frac{1}{(n!)} \gamma_{[\mu_1} \gamma_{\mu_2} \cdots \gamma_{\mu_n]}; \quad n=4k-1, 4k-2,$$

$$\sigma_{\mu_1 \dots \mu_n}^{(n)} \equiv \frac{i}{(n!)} \gamma_{[\mu_1} \gamma_{\mu_2} \cdots \gamma_{\mu_n]}; \quad n=4k, 4k-3. \quad (14)$$

In four dimensions $\sigma^{(3)}$ and $\sigma^{(4)}$ can be reduced by introducing γ_5 , but I have refrained from doing this to avoid problems of dimensionally continuing γ_5 . In four dimensions we also have $\sigma^{(n)}=0$ for $n \geq 5$, so they correspond to *evanescent* operators, only occurring in the counterterm Lagrangian of the effective theory.³ The absence of operators higher than bilinear in the heavy quark field is because such vertices make no contribution to (dimensionally regulated) amplitudes in the effective theory describing a single heavy quark.

The different Dirac structures are of different orders in $1/m$, which can be worked out by finding the lowest dimension operators that can occur in HQET with these structures, and respecting QCD symmetries. However one of these symmetries is RPI itself, which is discussed next. This power-counting exercise is therefore deferred till the end of the next section.

III. REPARAMETRIZATION INVARIANCE

The HQET with the auxiliary antiparticle field satisfies a very simple form of RPI, $\mathcal{L}_{\text{HQET}}$ being invariant under

$$v \rightarrow v + \delta v; \quad \psi_v(x) \rightarrow e^{im\delta v \cdot x} \psi_v(x). \quad (15)$$

At tree level for example, this invariance is manifest in the first line of Eq. (9). Beyond tree level the proof of RPI goes as follows. By Eq. (7), the quark field of full QCD $\psi(x)$ is invariant under Eq. (15). This means $\mathcal{L}_{\text{QCD}}$ is invariant except for the source term, Eq. (11). Clearly the QCD partition functional then satisfies

$$Z[\eta, \overline{\eta}, J_\mu, v] = Z[e^{im\delta v \cdot x} \eta, \overline{\eta} e^{-im\delta v \cdot x}, J_\mu, v + \delta v], \quad (16)$$

where J_μ is the gauge field source. This relation holds even when the functional integral is dimensionally regulated and minimal subtraction counterterms added. [Dimensional regularization is a “good” regulator for reparametrizations, Eq. (15).] After matching, this RPI and renormalized partition functional is equated (to whatever order in α_s and $1/m$ one is working) to the renormalized HQET partition functional,

³The reason the evanescent operators *only* appear among the minimal subtraction counterterms is just the generalization of the arguments of Ref. [9]. It is possible that the technical problem of evanescent operators can be avoided by using *dimensional reduction* as the regulator. See, for example, the discussion in Ref. [10]. I have not investigated this possibility in the present context.

$$\begin{aligned}
& Z[\eta, \bar{\eta}, J_\mu, v] \\
&= \int \mathcal{D}A_\mu \mathcal{D}\psi_v \mathcal{D}\bar{\psi}_v e^{i \int d^4x [\mathcal{L}_{\text{HQET}} + \mathcal{L}_{\text{CT}} + J \cdot A + \bar{\eta} \psi_v + \bar{\psi}_v \eta]},
\end{aligned} \tag{17}$$

where $\mathcal{L}_{\text{CT}}$ denotes the HQET minimal subtraction counter-terms, and the Fadeev-Popov ghost determinant is subsumed into the gauge field measure. Noting that the source term, Eq. (11), and the regulated measure are both invariant under $\eta \rightarrow e^{im\delta v \cdot x} \eta$, $\psi_v \rightarrow e^{im\delta v \cdot x} \psi_v$, we see that

$$\begin{aligned}
& Z[e^{im\delta v \cdot x} \eta, \bar{\eta} e^{-im\delta v \cdot x}, J_\mu, v + \delta v] \\
&= \int \mathcal{D}A_\mu \mathcal{D}\psi_v \mathcal{D}\bar{\psi}_v \\
&\quad \times e^{i \int d^4x [\mathcal{L}_{\text{HQET}} + \delta \mathcal{L}_{\text{HQET}} + \mathcal{L}_{\text{CT}} + \delta \mathcal{L}_{\text{CT}} + J \cdot A + \bar{\eta} \psi_v + \bar{\psi}_v \eta]},
\end{aligned} \tag{18}$$

where

$$\begin{aligned}
\delta \mathcal{L} &\equiv \mathcal{L}(e^{im\delta v \cdot x} \psi_v, \bar{\psi}_v e^{-im\delta v \cdot x}, A_\mu, v + \delta v) \\
&\quad - \mathcal{L}(\psi_v, \bar{\psi}_v, A_\mu, v).
\end{aligned} \tag{19}$$

Equation (16) therefore reads

$$\begin{aligned}
& \int \mathcal{D}A_\mu \mathcal{D}\psi_v \mathcal{D}\bar{\psi}_v \int d^4x (\delta \mathcal{L}_{\text{HQET}} + \delta \mathcal{L}_{\text{CT}}) \\
&\quad \times e^{i \int d^4x \mathcal{L}_{\text{HQET}} + \mathcal{L}_{\text{CT}} + J \cdot A + \bar{\eta} \psi_v + \bar{\psi}_v \eta} = 0.
\end{aligned} \tag{20}$$

It follows that

$$\int d^4x (\delta \mathcal{L}_{\text{HQET}} + \delta \mathcal{L}_{\text{CT}}) = 0, \tag{21}$$

since all insertions of this operator into Green functions vanish. In minimal subtraction we separately have $\delta \mathcal{L}_{\text{HQET}} = 0$ and $\delta \mathcal{L}_{\text{CT}} = 0$ since they have distinct $1/\epsilon$ dependence. This proves the RPI of $\mathcal{L}_{\text{HQET}}$.

The RPI constraint on $\mathcal{L}_{\text{HQET}}$ has a simple solution: v_μ and D_μ can only occur in the Lagrangian with $\psi_v, \bar{\psi}_v$ [see Eq. (12)], in the combination $v_\mu + iD_\mu/m$. (This is only true for covariant derivatives acting on the heavy fields, not on the light fields, which are trivially RPI.) That is, $S, V_\mu, T^{(n)}$ are constructed out of $v_\mu + iD_\mu/m$. The proof is straightforward. Instead of working with $\psi_v, \bar{\psi}_v$ directly, we can write any effective Lagrangian in terms of the combinations $\psi_v e^{imv \cdot x}$ and $e^{-imv \cdot x} \bar{\psi}_v$ which, as we noted earlier, are invariants under Eq. (15). A general term of $\mathcal{L}_{\text{HQET}}$ has the form $\psi_v e^{imv \cdot x} f(v_\mu, D_\mu) e^{-imv \cdot x} \bar{\psi}_v$, where f is some monomial in its arguments (and can be a matrix in spinor, Lorentz, or color spaces), multiplied by a term involving only light fields and their covariant derivatives. In this decomposition only the v_μ argument of f transforms under Eq. (15). Furthermore the only invariant combination of v_μ 's alone is trivial, $v^2 = 1$. Therefore imposing RPI just translates into the v independence of f . The phase factors can be cancelled against each other by noting that $D_\mu e^{imv \cdot x} \dots$

$= e^{imv \cdot x} (imv_\mu + D_\mu) \dots$. Thus, the general RPI term in $\mathcal{L}_{\text{HQET}}$ has the form $\psi_v f(imv_\mu + D_\mu) \bar{\psi}_v$, multiplied by a factor consisting of light fields, thereby completing the proof. We see that in the auxiliary field formulation, RPI is as simple as in theories with spin-0 heavy quarks [3]. As emphasized by Luke and Manohar, the central reason for the complication of RPI in the standard formulation of HQET is that the heavy quark field must satisfy a v -dependent constraint. By including the auxiliary field in the present formulation, there is no constraint on $\psi_v = \psi_{+v} + \psi_{-v}$.

We are now in a position to give the power counting for the operators of the various Dirac structures, described at the end of the last section. They are restricted by conventional QCD symmetries, and by RPI. The $T^{(n)}$ must have at least one pair of antisymmetrized Lorentz indices to be contracted with the $\sigma^{(n)}$. These indices can only arise from a pair of reparametrization-covariant derivatives $v_\mu + iD_\mu/m$. That is, $T^{(n)} = \dots (v + iD/m)_{[\mu} \dots (v + iD/m)_{\nu]} \dots$. However, terms in which any v 's are pulled out of the parentheses do not survive the antisymmetrization unless there are also derivatives between the two sets of parentheses. It follows that the $T^{(n)}$ operators must contain at least two derivatives D_μ . So by dimensional analysis the $T^{(n)}$ can be at most $\mathcal{O}(1/m)$. The lowest dimension operator that can appear in S is the unit operator, with a coefficient of order m by dimensional analysis. Similarly the lowest dimension operator that can appear in V_μ is $v_\mu + iD_\mu/m$, again with a coefficient of order m . There is just one subtlety which can already be seen at tree level. Though S and V_μ are both of order m , $S + v \cdot V \sim \mathcal{O}(1)$ if we choose the residual mass to vanish [or be at most $\mathcal{O}(\Lambda_{\text{QCD}})$]. To summarize,

$$\begin{aligned}
S - v \cdot V &\sim \mathcal{O}(m); \quad S + v \cdot V \sim \mathcal{O}(1); \quad \Psi_\perp \sim \mathcal{O}(1); \\
T^{(n)} &\leq \mathcal{O}(1/m).
\end{aligned} \tag{22}$$

IV. INTEGRATING OUT THE ANTIPARTICLE FIELD

The auxiliary field formulation of HQET is a valid calculational scheme in its own right, once matched to full QCD. However, having solved the RPI constraint in this formulation we are free to integrate out the auxiliary field and return to the standard formulation of HQET in terms of ψ_{+v} (once we turn off the source for ψ_{-v}). The result will be the general RPI-constrained form of the standard HQET Lagrangian. Since ψ_{-v} occurs at most quadratically in Eq. (13) we can easily integrate it out to get

$$\begin{aligned}
\mathcal{L}_{\text{HQET}} + \mathcal{L}_{\text{CT}} &= \bar{\psi}_{+v} \left\{ S + v \cdot V + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right. \\
&\quad \left. - \left[\Psi_\perp + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \right. \\
&\quad \left. \times \left[S - v \cdot V + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right]^{-1} \right\}_{(-v)} \\
&\quad \times \left[\Psi_\perp + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \psi_{+v}.
\end{aligned} \tag{23}$$

This expression requires some explanation. The $(-v)$ subscript appearing in the operator inverse is an instruction to

perform the inversion completely within the $(1-\not{v})/2$ subspace, not the full spinor space, thereby getting a result that also lives in the subspace. Secondly, this inverse is really local. The reason is that the dominant part of $S-v \cdot V$ is a constant of order m . Therefore expanding the operator inverse in powers of $1/m$ yields a series of local operators, similarly to Eq. (10). The last observation concerns renormalization. When integrating ψ_{-v} out of the auxiliary field formulation, the effective Lagrangian in the functional integral contains counterterms. Therefore each operator $\mathcal{O}_0 = S, V, T^{(n)}$ appearing in Eq. (23) has the form

$$\mathcal{O}_0 = \mathcal{O}(\mu) + \mathcal{O}_{\text{poles}}, \quad (24)$$

where $\mathcal{O}_{\text{poles}}$ is a sum of $1/\epsilon^n$ pole counterterms obtained in the auxiliary field formulation of HQET. The standard HQET Lagrangian is just the finite part of Eq. (23), the poles providing the counterterms for the standard HQET formulation. Clearly, given Eq. (24), to all orders in α_s and $1/m$ this finite part must also have the form of Eq. (24), where the operators are the⁴ $\mathcal{O}(\mu)$. That is, both the renormalized and unrenormalized standard HQET Lagrangians have the form of Eq. (23). RPI restricts the $\mathcal{O}(\mu)$ and the $\mathcal{O}_{\text{poles}}$ to be constructed from $v_\mu + iD_\mu/m$. This result agrees with Ref. [6].

We can view the form of Eq. (23) as the solution to demanding invariance under a reparametrization transformation of ψ_{+v} . We can derive this transformation by noting that the Gaussian integral performed to eliminate ψ_{-v} is the same as using the ψ_{-v} “equation of motion” following from Eq. (13):

$$\begin{aligned} \psi_{-v} = & - \left[S - v \cdot V + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right]_{(-v)}^{-1} \\ & \times \left[\not{V}_\perp + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \psi_{+v}. \end{aligned} \quad (25)$$

[See the remarks after Eq. (23) regarding the interpretation of the operator inverse.] Thus the ψ_v reparametrization transformation in the auxiliary field formulation corresponds to the following transformation in standard HQET:

⁴The products of $(4+\epsilon)$ -dimensional γ matrices encountered in Eq. (23) can be reexpressed as linear combinations of the fully antisymmetrized products $1, \gamma_\mu, \sigma^{(n)}$, and simplified using $(1+\not{v})/2$ -projected identities to arrive at a canonical form for the standard HQET Lagrangian. These particular manipulations do not introduce any explicit powers of ϵ , which would have changed the separation of the effective Lagrangian into the finite part and minimal subtraction counterterms.

$$\begin{aligned} \psi_{+v'} &= \frac{1+\not{v}'}{2} e^{im\delta v \cdot x} \psi_v \\ &= e^{im\delta v \cdot x} \frac{1+\not{v}'}{2} \left\{ 1 - \left[S - v \cdot V + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right]_{(-v)}^{-1} \right. \\ &\quad \times \left. \left[\not{V}_\perp + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \right\} \psi_{+v} \\ &= \left\{ 1 + im \delta v \cdot x + \frac{\delta \not{v}}{2} - \frac{\delta \not{v}}{2} \right. \\ &\quad \times \left. \left[S - v \cdot V + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right]_{(-v)}^{-1} \right. \\ &\quad \times \left. \left. \left[\not{V}_\perp + \sum_{n \geq 2} T^{(n)} \cdot \sigma^{(n)} \right] \right\} \psi_{+v}, \end{aligned} \quad (26)$$

where $\mathcal{O}(\delta v^2)$ terms have been dropped in the last line. This general form also agrees with Kilian and Ohl [6]. However, the *specific* forms of $S, V, T^{(n)}$ must be determined by matching the auxiliary field formulation of HQET to full QCD. It is straightforward to see that the operators appearing in Eq. (26) are either the $\mathcal{O}(\mu)$ or $\mathcal{O}_0$ depending on whether one is considering RPI of $\mathcal{L}_{\text{HQET}}$ or $\mathcal{L}_{\text{HQET}} + \mathcal{L}_{\text{CT}}$, respectively.

V. EXAMPLES IN STANDARD HQET

It is a well-believed expectation that RPI should constrain the term in the standard HQET Lagrangian $-(1/2m)\psi_{+v}D^2\psi_{+v}$, to have unit coefficient to all orders in α_s (for example, the analogous statement is straightforward to prove for heavy scalars [3]). Let us see how this emerges from the general form, Eq. (23). One can easily check that the $T^{(n)}$ can have no effect on the $\mathcal{O}(1/m)$ operator of interest. Thus the part of the Lagrangian with the right Dirac structure to this order is given by

$$\mathcal{L}_{\text{HQET}} = \overline{\psi_{+v}} \{ S + v \cdot V - \not{V}_\perp (S - v \cdot V)^{-1} \not{V}_\perp \} \psi_{+v} + \dots, \quad (27)$$

where S is a function $f((v+iD/m)^2) = f(1+2iv \cdot D/m - D^2/m^2)$, and $V_\mu = g(1+2iv \cdot D/m - D^2/m^2)(v_\mu + iD_\mu/m)$. Actually, RPI permits more general forms for S and V which can be reduced to the above by using the fact that $v_\mu + iD_\mu/m$ commutes with $v_\nu + iD_\nu/m$ up to terms involving the gauge field strength, such terms being discarded into the ellipsis above. To order $1/m$ the general forms of S and V_μ are

$$\begin{aligned} S &= -m_1 + Z_1 \left(iv \cdot D - \frac{D^2}{2m} \right) + k_1 \frac{(v \cdot D)^2}{m}, \\ V_\mu &= \left[m_2 + Z_2 \left(iv \cdot D - \frac{D^2}{2m} \right) + k_2 \frac{(v \cdot D)^2}{m} \right] \left(v_\mu + i \frac{D_\mu}{m} \right). \end{aligned} \quad (28)$$

Substituting into Eq. (27),

$$\mathcal{L}_{\text{HQET}} = \overline{\psi}_{+v} \left\{ m_2 - m_1 + (Z_1 + Z_2) \left(i v \cdot D - \frac{D^2}{2m} \right) + \frac{m_2}{m} i v \cdot D - \frac{m_2^2 D^2}{m^2 (m_1 + m_2)} \right\} \psi_{+v} + \dots \quad (29)$$

It remains to put $\mathcal{L}_{\text{HQET}}$ into canonical form. The $m_2 - m_1$ term is the “residual mass” term. We choose m to make the residual mass vanish, $m_1 = m_2$. Next we perform wave function renormalization to ensure that $\overline{\psi}_{+v} i v \cdot D \psi_{+v}$ has canonical unit coefficient. The result is

$$\mathcal{L}_{\text{HQET}} = \overline{\psi}_{+v} \left(i v \cdot D - \frac{D^2}{2m} \right) \psi_{+v} + \dots, \quad (30)$$

as we wished to prove.

A second example involves spin-dependent terms in the standard effective Lagrangian. Both Chen’s and Luke and Manohar’s versions of RPI predict that the coefficients of the operators

$$O_{\text{mag}} = \frac{g_s}{4m} \overline{\psi}_{+v} \sigma^{\mu\nu} G_{\mu\nu} \psi_{+v},$$

$$O_2 = \frac{i g_s}{8m^2} \overline{\psi}_{+v} \sigma^{\alpha\mu} v^\nu \{ D_\alpha, G_{\mu\nu} \} \psi_{+v}, \quad (31)$$

are related by

$$2C_{\text{mag}} = C_2 + 1 \quad (32)$$

(when the heavy quark kinetic term and residual mass terms are put into canonical form) [3,4,7]. Reference [7] showed this to be true independently of the earlier RPI proposals. It is therefore of interest to know whether Eq. (32) follows from our general form, Eq. (23). The answer is yes, as a straightforward but tedious calculation (along similar lines to the last example) shows.

VI. DISCUSSION

At tree level, $S = -\widetilde{m}$, $V_\mu = m v_\mu + i D_\mu$, $T^{(n)} = 0$, and if we choose $m = \widetilde{m}$, Eq. (26) becomes the transformation proposed by Chen [4]. Beyond tree level, there are definitely

matching corrections to $S, V, T^{(n)}$ in the auxiliary field formulation of HQET, so the reparametrization transformation of standard HQET, Eq. (26), is necessarily corrected beyond Chen’s tree-level form. This means that Chen’s proposed form of the RPI transformation must be wrong. Therefore at first sight, it is surprising that Finkemeier, Georgi, and McIrvin [7] could prove all the predictions of Chen’s form of RPI to $\mathcal{O}(1/m^2)$ in $\mathcal{L}_{\text{HQET}}$, such as Eq. (32), for example. We are in a position to understand this. The results to this order in the effective Lagrangian are completely determined by the form of the reparametrization transformation to order $1/m$, so that the $T^{(n)}$ are irrelevant inside Eq. (26), by Eq. (22), and we can use the forms in Eq. (28) for S and V . When we enforce the vanishing of the residual mass $m_1 = m_2$ we find

$$\psi_{+v}' = \left\{ 1 + i m \delta v \cdot x + \frac{\delta \not{v}}{2} + \frac{\delta \not{v} i \not{D}_\perp}{2m} \right\} \psi_{+v} + \mathcal{O}(1/m^2). \quad (33)$$

This is precisely the same as Chen’s transformation to order $1/m$, thereby explaining the success of Chen’s proposal to order $1/m^2$ in the effective Lagrangian. Beyond this order, Eq. (26) and Chen’s transformation no longer coincide, and only Eq. (26) is correct.

Equation (26) is also not the RPI transformation proposed by Luke and Manohar [3]. However, to order $1/m^2$, Ref. [7] has shown that there is a field redefinition (which does not affect the S matrix) which, when compounded with the Luke-Manohar transformation, yields Eq. (33). It is possible this agreement with Eq. (26) up to field redefinitions persists at higher orders in HQET. See Ref. [5] for some more discussion.

The auxiliary field formulation of HQET is a viable calculational scheme which makes RPI extremely simple, given by Eq. (15). If one prefers the standard formulation of HQET, the correct form of RPI is the one obtained by Kilian and Ohl [6] and rederived in this paper.

ACKNOWLEDGMENTS

This research was supported by the U.S. Department of Energy under Grant No. DE-FG02-94ER40818. I am grateful for useful discussions with Markus Finkemeier, Howard Georgi, Markus Luty, and Matt McIrvin.

-
- [1] B. Grinstein, Annu. Rev. Nucl. Part. Sci. **42**, 101 (1992).
 - [2] M. J. Dugan, M. Golden, and B. Grinstein, Phys. Lett. B **282**, 142 (1992).
 - [3] M. Luke and A. V. Manohar, Phys. Lett. B **286**, 348 (1992).
 - [4] Y.-Q. Chen, Phys. Lett. B **317**, 421 (1993).
 - [5] A. Manohar, Phys. Rev. D **56**, 230 (1997).
 - [6] W. Kilian and T. Ohl, Phys. Rev. D **50**, 4649 (1994).

- [7] M. Finkemeier, H. Georgi, and M. McIrvin, Phys. Rev. D **56**, 6933 (1997).
- [8] A. Falk, M. Luke, and M. Neubert, Nucl. Phys. **B388**, 363 (1992).
- [9] M. J. Dugan and B. Grinstein, Phys. Lett. B **256**, 239 (1991).
- [10] V. Gimenez, Nucl. Phys. **B401**, 116 (1993).