

Evolution of near-extremal black holes

S. W. Hawking^{*} and M. M. Taylor-Robinson[†]

*Department of Applied Mathematics and Theoretical Physics, University of Cambridge, Silver Street,
Cambridge CB3 9EW, United Kingdom*

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Near-extreme black holes can lose their charge and decay by the emission of massive Bogomol'ni-Prasad-Sommerfield charged particles. We calculate the greybody factors for low-energy charged and neutral scalar emission from four- and five-dimensional near extremal Reissner-Nordström black holes. We use the corresponding emission rates to obtain ratios of the rates of loss of excess energy by charged and neutral emission, which are moduli independent, depending only on the integral charges and the horizon potentials. We consider scattering experiments, finding that evolution towards a state in which the integral charges are equal is favored, but neutral emission will dominate the decay back to extremality except when one charge is much greater than the others. The implications of our results for the agreement between black hole and D -brane emission rates and for the information loss puzzle are then discussed. [S0556-2821(97)02612-X]

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I. INTRODUCTION

In the last year, there has been rapid progress in the use of D -branes to describe and explain the properties of black holes. In a series of papers, starting with [1], the Bekenstein-Hawking entropies for the most general five-dimensional Bogomol'ni-Prasad-Sommerfield (BPS) black holes in string theory were derived by counting the degeneracy of BPS-saturated D -brane bound states. Later these calculations were extended to near-extremal states [2], in the particular sector of the moduli space accessible to string techniques described by Maldacena and Strominger as the "dilute gas" region. There is some evidence, though no rigorous derivation as yet, that the agreement can be extended throughout the moduli space of the near-extremal black holes [3].

These ideas were then extended to supersymmetric four-dimensional black holes with regular horizons [4,5]. In [6–8], it was argued that it is useful to view the four-dimensional black holes as dimensionally reduced configurations of intersecting branes in M theory. Such configurations again permit the derivation of the entropy of the four-dimensional state in terms of the degeneracy of the brane bound states.

More recently, attention has been focused on the calculation of decay rates of five-dimensional black holes and the corresponding D -brane configurations. It was first pointed out in [9] that the decay rate of the D -brane configuration exhibits the same behavior as that of the black hole [10], when we assume that the number of right-moving oscillations of the effective string is much smaller than the number of left-moving ones. In a surprising paper by Das and Mathur [11], the numerical coefficients were found to match and it has recently been shown [12] that the string and semiclassical calculations also agree when we drop the assumption on the right-moving oscillations. For four-dimensional black holes intersecting brane models of four-dimensional black

holes also give agreement between M theory and semiclassical calculations of decay rates [13,14]. In the last month, a rationale for the agreement between the properties of near-extremal D -brane and corresponding black hole states in the dilute gas region has been proposed [22].

These D -brane and M -theory calculations are restricted to certain limited regions of the black hole parameter space. In this paper, we calculate the semiclassical emission rates in a sector of the moduli space which is out of the reach of D -brane and M -theory techniques (at present). We then obtain moduli-independent quantities describing the ratio of charged and neutral scalar emission rates and confirm that they are in agreement with the rates calculated in the dilute gas region of the moduli space. Thus scattering from black holes displays a certain universal structure for states not too far from extremality.

One can get an idea of when charged emission will be important compared to neutral emission by considering the expression for the entropy. For the five-dimensional extreme black hole this is

$$S = 2\pi \sqrt{n_1 n_5 n_K}, \quad (1)$$

where n_1, n_5, n_K are integers that give the one brane, five brane, and Kaluza-Klein charges, respectively. The emission of a massive charged BPS particle will reduce at least one of the integers (say n_K) by at least 1. This will cause a reduction of the entropy of

$$\Delta S = \sqrt{\frac{n_1 n_5}{n_K}}. \quad (2)$$

The emission of Kaluza-Klein charge will be suppressed by a factor of $\exp(\Delta S)$ and will be small unless

$$n_K > n_1 n_5. \quad (3)$$

Thus it seems that charged emission will occur most readily for the greatest charge and will tend to equalize the charges. However, when the charges are nearly equal, charged emission of any kind will be heavily suppressed. On the other

^{*}Electronic address: swh1@damtp.cam.ac.uk

[†]Electronic address: mmt14@damtp.cam.ac.uk

hand, neutral emission can take place at very low energies and so will not cause much reduction of entropy. One would therefore expect it to be limited only by phase space factors and to dominate over charged emission except when one charge is much greater than the others. The situation with four-dimensional black holes is similar except that there are four charges. Again charged emission will tend to equalize the charges but neutral emission will dominate except when one charge is much greater than the others. In what follows we shall consider the five-dimensional case and treat four-dimensional black holes in the Appendix.

In Sec. II we start by calculating the rates of emission of neutral and charged scalars from near-extremal five-dimensional Reissner-Nordström black holes. We find that the ratio of the rates of energy loss by charged and neutral emission are moduli independent; they depend only on the integral charges¹ and the horizon potentials. Neutral emission always dominates charged emission, unless one of the integral charges is much greater than the product of the other two.

We then discuss the implications for scattering from the black hole; it was suggested in [12] that under some circumstances the black hole will decay before we can measure its state. We point out an error in their analysis, and show that it should be possible to obtain entropy in the outgoing radiation equal to that of the black hole state without the black hole decaying.

Finally, in Sec. IV, we discuss the implications of our results for the information loss question. It has been explicitly shown that the emission rates from near-extremal black holes and D -branes agree in the sectors of the moduli space accessible to string calculations. One would expect that this agreement between the D -brane and black hole emission rates would continue throughout the entire moduli space of near BPS states, although a verification is not yet possible. Now for the D -brane configuration we can determine the microstate when the entanglement entropy in the radiation is equal to that of the D -brane system. Since it is possible to obtain such an entropy in the outgoing radiation from the black hole before it decays, it might seem as if we can extract enough information to determine the black hole microstate without it decaying. That is, there would seem to be no obstruction to scattering radiation from the black hole and obtaining information from the outgoing radiation. One might then expect any further scattering to be unitary and predictable.

This, however, by no means settles the information question. Although scattering off a D -brane regarded as a surface in flat space is unitary, it is not so obvious that information cannot be lost if one takes account of the geometry of the D -brane. The causal structure may have past and future singular null boundaries like horizons and, as with horizons, there is no reason that what comes out of the past surface should be related to what goes into the future surface. In the case of a static brane of one kind, there will be no information loss and the scattering will be unitary because this cor-

responds under dimensional reduction to a black hole of zero horizon area. However, in the case of four- and five-dimensional black holes with four and three nonzero charges, respectively, the effects of the charges balance to give a non-singular horizon of finite area and one might expect nonunitary scattering with information loss.

II. FIVE-DIMENSIONAL SCATTERING

In this section, following [9], [11], and [12], we consider scattering from a five-dimensional black hole carrying three electric charges; such black hole states were first constructed in [3] and [15]. We will work with a near extremal solution which is a solution of the low-energy action of type-IIB string theory compactified on a torus. Then, the five-dimensional metric in the Einstein frame is

$$ds^2 = -h f^{-2/3} dt^2 + f^{1/3} (h^{-1} dr^2 + r^2 d\Omega_3^2), \quad (4)$$

where

$$h = \left(1 - \frac{r_0^2}{r^2}\right), \quad f = \left(1 + \frac{r_1^2}{r^2}\right) \left(1 + \frac{r_5^2}{r^2}\right) \left(1 + \frac{r_K^2}{r^2}\right), \quad (5)$$

and the parameters r_i are related to r_0 by

$$r_1^2 = r_0^2 \sinh^2 \sigma_1, \quad r_5^2 = r_0^2 \sinh^2 \sigma_5, \quad r_K^2 = r_0^2 \sinh^2 \sigma_K. \quad (6)$$

We require here only the metric in the Einstein frame; the other fields in the solution may be found in [12]. The extremal limit is $r_0 \rightarrow 0$, $\sigma_i \rightarrow \infty$ with r_i fixed; we shall be interested in the sections of the moduli space where the BPS state is the extreme Reissner-Nordström solution, where the limiting values of r_i are equal to r_e , the Schwarzschild radius.

We may regard the black hole as the compactification of a six-dimensional black string carrying momentum about the circle direction; we will be using this six-dimensional solution in the following sections, and the metric (in the Einstein frame) is given by

$$\begin{aligned} ds^2 = & \left(1 + \frac{r_1^2}{r^2}\right)^{-1/2} \left(1 + \frac{r_5^2}{r^2}\right)^{-1/2} \\ & \times \left[-dt^2 + dx_5^2 + \frac{r_0^2}{r^2} (\cosh \sigma_K dt + \sinh \sigma_K dx_5)^2 \right] \\ & + \left(1 + \frac{r_1^2}{r^2}\right)^{1/2} \left(1 + \frac{r_5^2}{r^2}\right)^{1/2} \\ & \times \left[\left(1 - \frac{r_0^2}{r^2}\right)^{-1} dr^2 + r^2 d\Omega_3^2 \right]. \end{aligned} \quad (7)$$

We assume that we are in the very near-extremal region where $r_0 \ll r_e$, and moreover will consider all three hyperbolic angles to be finite. It is here that our analysis differs from previous work; with this choice of parameters, we move away from the dilute gas region and a straightforward D -brane analysis of emission rates is not possible.

The entropy is

¹We distinguish here between charges normalized to be integers, which we call *integral* charges, and the *physical* charges, which depend also on moduli.

$$S = \frac{A_h}{4G_5} = \frac{2\pi^2 r_0^3 \Pi_i \cosh \sigma_i}{4G_5}, \quad (8)$$

while the Hawking temperature is defined by

$$T_H = \frac{1}{2\pi r_0 \Pi_i \cosh \sigma_i}. \quad (9)$$

We may define symmetrically normalized charges by

$$\frac{1}{2} r_0^2 \sinh 2\sigma_i = Q_i. \quad (10)$$

For simplicity of notation, we assume throughout the paper that all charges are positive; obviously for negative charges we simply insert appropriate moduli signs. Our notation for the three charges Q_1 , Q_5 , Q_K indicates their origin in D -brane models, from 1D-branes, 5D-branes, and Kaluza-Klein charges, respectively. The energy in the BPS limit is

$$E = \frac{\pi}{4G_5} [Q_1 + Q_5 + Q_K], \quad (11)$$

where G_5 is the five-dimensional Newton constant, with the excess energy for a near-extremal state being

$$\Delta E = \frac{\pi r_0^2}{4G_5} \sum_i e^{-2\sigma_i}. \quad (12)$$

It was stated in [3] that the near-extremal solution is specified by six independent parameters, which we may take to be the mass, three charges, and two asymptotic values of scalar fields. However, there are in fact only *five* independent parameters; once we fix the three charges, as well as r_0 and one hyperbolic angle, the other two hyperbolic angles are fixed. So we specify the state of the black hole by its mass, three charges, and only *one* extremality parameter. If the BPS state is Reissner-Nordström, then excitations away from extremality leave the geometry Reissner-Nordström, since the three hyperbolic angles are the same. For small excitations, the relationship between the temperature and the excess energy is

$$T_H = \frac{2}{\pi r_e} \sqrt{\frac{G_5 \Delta E}{\pi r_e^2}}, \quad (13)$$

which will be useful in the following. With appropriate normalizations, we can define the potentials associated with the charges as

$$A_i = \frac{Q_i dt}{(r^2 + r_i^2)}, \quad (14)$$

with the potentials on the horizon $r = r_0$ being

$$A_i = \frac{Q_i dt}{(r_0^2 + r_i^2)}. \quad (15)$$

For perturbations which leave the compactification geometry passive, we obtain the standard Reissner-Nordström solution by the rescaling $\bar{r}^2 = (r^2 + r_i^2)$ which gives the solution in the familiar form

$$ds^2 = - \left(1 - \frac{r_+^2}{\bar{r}^2} \right) \left(1 - \frac{r_-^2}{\bar{r}^2} \right) dt^2 + \frac{1}{(1 - r_+^2/\bar{r}^2)(1 - r_-^2/\bar{r}^2)} d\bar{r}^2 + \bar{r}^2 d\Omega_3^2, \\ T_H = \frac{1}{2\pi} \left(\frac{r_+^2 - r_-^2}{r_+^3} \right), \quad (16)$$

$$A_i = \frac{Q dt}{\bar{r}^2},$$

where in the extremal limit $r_\pm^2$ are equal to Q .

A. Neutral scalar emission

In this section we compute the absorption probability for neutral scalars by the slightly nonextremal black hole. Our discussion parallels that in [12], and we hence give only a brief summary of the calculation. We solve the Klein-Gordon equation for a massless scalar on the fixed background; taking the field to be of the form $\Phi = e^{-i\omega t} R(r)$, we find that

$$\left[\frac{h}{r^3} \frac{d}{dr} \left(h r^3 \frac{d}{dr} \right) + \omega^2 f \right] R = 0, \quad (17)$$

where we have taken $l=0$ since we will be interested in very low-energy scalars. We assume the low-energy condition

$$\omega r_e \ll 1, \quad (18)$$

where we treat the ratios r_i/r_e as approximately 1.

Solutions to the wave equation may be approximated by matching near and far zone solutions. We divide the space into two regions: the far zone $r > r_f$ and the near zone $r < r_f$, where r_f is the point where we match the solutions. r_f is chosen so that

$$r_0 \ll r_f \ll r_1, r_5, r_K, \quad \omega r_e \left(\frac{r_e}{r_f} \right) \ll 1. \quad (19)$$

Now in the far zone, after setting $R = r^{-3/2} \psi$ and $\rho = \omega r$, Eq. (17) reduces to

$$\frac{d^2 \psi}{d\rho^2} + \left(1 - \frac{3}{4\rho^2} \right) \psi = 0, \quad (20)$$

which has the solution for small r , $r \approx r_f$:

$$R = \sqrt{\frac{\pi}{2}} \omega^{3/2} \left[\frac{\alpha}{2} + \frac{\beta}{\omega} \left(c + \ln(\omega r) - \frac{2}{\omega^2 r^2} \right) \right], \quad (21)$$

where α , β , and c are integration constants, to be determined by the matching of the solutions. The solution for large r is

$$R = \frac{1}{r^{3/2}} \left[e^{i\omega r} \left(\frac{\alpha}{2} e^{-i3\pi/4} - \frac{\beta}{2} e^{-i\pi/4} \right) + e^{-i\omega r} \left(\frac{\alpha}{2} e^{i3\pi/4} - \frac{\beta}{2} e^{i\pi/4} \right) \right]. \quad (22)$$

However, in the near zone, we have the equation

$$\frac{h}{r^3} \frac{d}{dr} \left(hr^3 \frac{dR}{dr} \right) + \left[\frac{(\omega r_1 r_K r_5)^2}{r^6} + \frac{\omega^2 (r_1^2 r_5^2 + r_1^2 r_K^2 + r_5^2 r_K^2)}{r^4} \right] R = 0. \quad (23)$$

Defining the variable $v = r_0^2/r^2$, the equation becomes

$$(1-v) \frac{d}{dv} (1-v) \frac{dR}{dv} + \left(D + \frac{C}{v} \right) R = 0, \quad (24)$$

where

$$D = \left(\frac{\omega r_1 r_5 r_K}{2r_0^2} \right)^2, \quad C = \left(\frac{\omega^2 (r_1^2 r_5^2 + r_1^2 r_K^2 + r_5^2 r_K^2)}{4r_0^2} \right). \quad (25)$$

Equation (24) is the same near zone equation as in [12], but with different definitions of the quantities C , D . We can hence write down the solution for R in the near zone as

$$R = A(1-v)^{-i(a+b)/2} \frac{\Gamma(1-ia-ib)}{\Gamma(1-ia)\Gamma(1-ib)}, \quad (26)$$

with A a constant to be determined and

$$a = \sqrt{C+D} + \sqrt{D}, \quad b = \sqrt{C+D} - \sqrt{D}. \quad (27)$$

By matching R and R' at $r=r_f$, we may determine the constants α and A , and then find the absorption probability for the S wave by

$$\sigma_{\text{abs}}^S = \frac{[R^* hr^3 (dR/dr) - \text{c.c.}]_\infty}{[R^* hr^3 (dR/dr) - \text{c.c.}]_{r_0}}. \quad (28)$$

That is, we take the ratio of the flux into the black hole at the horizon to the incoming flux from infinity. Using the values of integration constants determined by matching, we find

$$\sigma_{\text{abs}}^S = \pi^2 r_0^2 \omega^2 ab \frac{(e^{2\pi(a+b)} - 1)}{(e^{2\pi a} - 1)(e^{2\pi b} - 1)}. \quad (29)$$

The values of a and b are

$$a = \frac{\omega}{r_0^2} (r_1 r_5 r_K), \quad (30)$$

$$b = \frac{\omega}{4} \left(\frac{r_1 r_5}{r_K} + \frac{r_1 r_K}{r_5} + \frac{r_5 r_K}{r_1} \right). \quad (31)$$

If we now impose the conditions that the BPS state is Reissner-Nordström, then for small deviations away from extremality,

$$a = \frac{\omega}{2\pi T_H}, \quad b = \frac{3\omega r_e}{4}. \quad (32)$$

Since the Hawking temperature T_H is much smaller than $1/r_e$ in the near-extremal limit, $a \gg b$ and the low-energy condition (18) implies that $b \ll 1$. From Eq. (29) we find

$$\sigma_{\text{abs}}^S = \frac{1}{2} \pi \omega^3 r_1 r_5 r_K = \frac{1}{4\pi} A_h \omega^3. \quad (33)$$

In fact, the low-energy condition on ω implies that the absorption cross section exhibits the universal behavior discussed in [16]; however, we will use the more general solutions to Eq. (17) in the following sections (when we impose different conditions on the relative sizes of the r_i).

We can obtain the emission rate by converting the S -wave absorption probability to the absorption cross section using

$$\sigma_{\text{abs}} = \frac{4\pi}{\omega^3} \sigma_{\text{abs}}^S, \quad (34)$$

and then using the formula for the Hawking emission rate

$$\Gamma = \sigma_{\text{abs}} \frac{1}{(e^{\omega/T_H} - 1)} \frac{d^4 k}{(2\pi)^4}, \quad (35)$$

to obtain

$$\Gamma = A_h \frac{1}{(e^{\omega/T_H} - 1)} \frac{d^4 k}{(2\pi)^4}. \quad (36)$$

B. Charged scalar emission

We now turn to the problem of calculating the corresponding S -wave absorption cross section for charged scalars; for simplicity, we consider particles carrying only one type of charge. Let us consider a scalar carrying the Kaluza-Klein charge; such a particle is massive in five dimensions, with its mass satisfying a BPS bound, but in six dimensions the particle is massless, carrying quantized momentum in the circle direction. We can hence obtain the equation of motion by solving the massless Klein-Gordon equation for a minimally coupled scalar in the six-dimensional background (7). Considering only the S -wave component, and taking a field of the form $\Phi = e^{-i\omega t - imx^5} R(r)$, we obtain the radial equation

$$\frac{h}{r^3} \frac{d}{dr} \left(hr^3 \frac{dR}{dr} \right) + \left(1 + \frac{r_1^2}{r^2} \right) \left(1 + \frac{r_5^2}{r^2} \right) \times \left[\omega^2 - m^2 + (\omega \sinh \sigma_K - m \cosh \sigma_K)^2 \frac{r_0^2}{r^2} \right] R = 0, \quad (37)$$

where m is the BPS mass of the particle. We obtain the same equation, with the appropriate permutations of r_i and σ_i , for the propagation of BPS scalars carrying charges with respect to A_1 and A_5 from the coupled Klein-Gordon equation.

By defining new variables

$$\omega'^2 = \omega^2 - m^2, \quad r'_K = r_0 |\sinh \sigma'_K|, \quad e^{\pm \sigma'_K} = e^{\pm \sigma_K} \frac{(\omega \mp m)}{\omega'}, \quad (38)$$

we bring the equation into the form (17), and we can hence obtain the S -wave absorption fraction from Eq. (29), replacing the variables with primed variables. Expressed in the primed variables, the low-energy condition becomes

$$\omega' r_e \ll 1, \quad \omega' r'_K \ll 1, \quad (39)$$

that is, the momentum of the emitted particle must be much smaller than the reciprocal of both the Schwarzschild radius and the effective radius r'_K . In calculating the absorption probability for neutral scalars, we assumed that in the near extremal solution the ratios r_e/r_i are of order one for each of the radii. However, if we rewrite r'_K in terms of r_K , we find that

$$r'_K = r_K \frac{|\omega - m/\phi_K|}{\omega'}, \quad (40)$$

where $\phi_K = \tanh \sigma_K$ is the Kaluza-Klein electrostatic potential on the horizon. Let us take the low-energy limit, assuming that the emitted particles are nonrelativistic, with kinetic energy δ ; the near extremality condition implies that $\phi_K = 1 - \mu_K$ with $\mu_K \ll 1$. Under these conditions,

$$r'_K = r_K \frac{|\delta - m\mu_K|}{\sqrt{2m\delta}}. \quad (41)$$

There are two regions of interest. If the kinetic energy is of the same order or greater than $m\mu_K^2$, then $r'_K \leq r_e$, and in solving Eq. (17) we must impose this condition. As before, the low-energy condition implies that the momentum of the emitted particle is small compared to the scale set by the Schwarzschild radius.

The other region of interest is when the kinetic energy is very small, that is, $\delta \ll m\mu_K^2$; we then find that r'_K is of the same order or greater than the Schwarzschild radius. Since the thermal factor in the emission rate is large at small kinetic energies, it is important to consider carefully the behavior of the absorption probability in this limit. Note that in this region the enforcement of the low-energy condition requires that

$$mr_e \ll \frac{1}{\mu_K}. \quad (42)$$

We consider first the region where the kinetic energy is of the same order or greater than the potential term; the solution (29) applies, using the primed variables, where a and b are determined under the condition $r'_K \leq r_e$ as

$$a = \frac{\omega' r_1 r_5}{2r_0} e^{\sigma'_K} = \frac{(\omega - m)}{2\pi T_H}, \quad (43)$$

$$b = \frac{\omega' r_1 r_5}{2r_0} e^{-\sigma'_K} = \frac{(\omega + m)r_e}{4}, \quad (44)$$

and we assume that deviations from the extreme Reissner-Nordström state are small. In addition,

$$(a + b) = \frac{(\omega - m\phi_K)}{2\pi T_H}, \quad (45)$$

$$ab = \frac{(\omega^2 - m^2)r_e^4}{4r_0^2}, \quad (46)$$

so that substituting into Eq. (29) we find that

$$\sigma_{\text{abs}}^S = \frac{1}{8} A_h (\omega^2 - m^2)^2 r_e \times \frac{(e^{(\omega - m\phi_K)/T_H} - 1)}{(e^{(\omega - m)/T_H} - 1)(e^{(1/2)\pi(\omega + m)r_e} - 1)}. \quad (47)$$

This is the general expression for the absorption probability, and applies even when the mass is of the order of $1/r_e$, provided that the kinetic energy is greater than $m\mu_K^2$. It is interesting to consider the limiting expression when the kinetic energy is much smaller than the temperature. Now, the Hawking temperature is

$$T_H = \frac{\mu}{\pi r_e}, \quad (48)$$

where we have used the fact that for the Reissner-Nordström solution $\mu_i \equiv \mu$. The condition on the kinetic energy implies that δ is only smaller than the temperature when $mr_e \ll 1$. That is, the mass must be small on the scale of the Schwarzschild radius. We can then expand out the exponentials in Eq. (47) to obtain

$$\sigma_{\text{abs}}^S = \frac{1}{4\pi} A_h (\omega - m)(\omega + m)(\omega - \phi_K m). \quad (49)$$

The corresponding probabilities for BPS particles carrying the other two charges are given by the same expression, with appropriate masses and potentials. Since the horizon potentials for all three fields are the same, under the conditions that the extreme geometry is Reissner-Nordström, the probabilities for the three types of charges differ only in the BPS masses. In the limit that the $m\mu_K \ll \delta$, we find that

$$\sigma_{\text{abs}}^S = \frac{1}{4\pi} A_h (\omega - m)^2 (\omega + m). \quad (50)$$

We now find the absorption probability in the limit that the kinetic energy is very small, $\delta \ll m\mu_K^2$. With these conditions, we find that the absorption probability is given by Eq. (29) with a and b given by

$$a = \frac{\omega' r_e^2 r'_K}{r_0^2}, \quad b = \frac{\omega'}{4} \left(\frac{r_e^2}{r_K^2} + 2r'_K \right). \quad (51)$$

Now the condition $\omega' r'_K$ implies that $b \ll 1$, and so we find that

$$\sigma_{\text{abs}}^S = \frac{1}{4\pi} \omega'^2 A_h m \mu_K, \quad (52)$$

where the low-energy condition implies that $m \ll 1/r_e \mu_K$. We obtain the absorption cross section from the S -wave absorption probability using

$$\sigma_{\text{abs}} = \frac{4\pi}{\omega'^3} \sigma_{\text{abs}}^S, \quad (53)$$

and then obtain the emission rate from the expression

$$\Gamma = \nu \sigma_{\text{abs}} \frac{1}{(e^{(\omega-m\phi_K)/T_H} - 1)} \frac{d^4 k}{(2\pi)^4}. \quad (54)$$

Now from Eq. (47) we see that the general expression for the emission rate (assuming that $\delta \gg m \mu_K^2$) is

$$\begin{aligned} \Gamma &= \frac{\pi}{2} A_h \left(\frac{\omega^2 - m^2}{\omega} \right) r_e \frac{1}{(e^{(\omega-m)/T_H} - 1)} \\ &\times \frac{1}{(e^{(1/2)\pi(\omega+m)r_e} - 1)} \frac{d^4 k}{(2\pi)^4}. \end{aligned} \quad (55)$$

In the limit of small kinetic energy, we find that

$$\Gamma = A_h \mu_K \frac{1}{e^{\pi m r_e} - 1} \frac{d^4 k}{(2\pi)^4}, \quad (56)$$

where $m r_e \ll 1/\mu_K$. This holds not only for $\delta \gg m \mu_K^2$, but also for smaller kinetic energies, since we find the same emission rate from the absorption probability (52). So, although it was important to consider carefully the behavior of the cross section for very small kinetic energy, the emission rate (55) in fact holds for all low-energy emission.

It is interesting to look at the relative values of the neutral and charged emission rate at very small (kinetic) energy. At small energy, $k^3 dk = 2m^2 \delta d\delta$, and so assuming that the mass is small on the scale set by the Schwarzschild radius, we find that

$$\Gamma_{\text{neut}} = \frac{1}{4\pi^2} A_h T_H m \delta d\delta, \quad (57)$$

where we have integrated out the angular dependence. Now the emission rate of neutral scalars at very low energy such that $k^3 dk = \delta^3 d\delta$ is

$$\Gamma = \frac{1}{8\pi^2} A_h T_H \delta^2 d\delta, \quad (58)$$

and then the ratio of emission rates is

$$\frac{\Gamma_{\text{char}}}{\Gamma_{\text{neut}}} = \frac{2m}{\delta}. \quad (59)$$

Since the charged particles are emitted nonrelativistically, emission of light charged particles dominates the emission of neutral scalars at very small energy. Since the density of states factor in Eq. (55) peaks for small kinetic energy, this indicates that the total rate of emission of light charged par-

ticles dominates that of neutrals. When we integrate the differential emission rate for neutrals, we find that the total rate of emission is

$$\Gamma_{\text{neut}}^{\text{tot}} = \frac{\pi^2}{120} A_h T_H^4. \quad (60)$$

The total emission rate of light charged particles is approximated by

$$\Gamma_{\text{char}}^{\text{tot}} = \frac{\zeta(3)}{2\pi^2} A_h m T_H^3, \quad (61)$$

and we find that most of the particles are emitted with kinetic energies of the order of $m \mu_K$. So comparing the total neutral and charged emission rates we find that

$$\frac{\Gamma_{\text{char}}^{\text{tot}}}{\Gamma_{\text{neut}}^{\text{tot}}} = \frac{60\zeta(3)}{\pi^4} \left(\frac{m}{T_H} \right). \quad (62)$$

Very close to extremality, the Hawking temperature is much smaller than the BPS masses of emitted particles, and thus emission of light charged particles dominates.

If we now compare the rate of emission of higher mass particles to that of neutral scalars, at very low kinetic energy, we find

$$\frac{\Gamma_{\text{char}}}{\Gamma_{\text{neut}}} = \frac{2\pi m^2 r_e}{\delta} e^{-\pi m r_e}. \quad (63)$$

So the rate of emission of high mass particles is comparable to the rate of emission of neutrals only over a very small range of kinetic energies. The total rates of emission from the black hole are dominated by emission of particles of higher (kinetic) energy, and we would expect neutral emission to dominate.

This is evident from the total emission rate of higher mass particles, which we approximate by integrating the rate (55)

$$\Gamma_{\text{char}}^{\text{tot}} = \frac{\zeta(3)}{2\pi} A_h r_e m^2 T_H^3 e^{-\pi m r_e}. \quad (64)$$

So comparing the total neutral and charged emission rates, for high mass particles, we find that

$$\frac{\Gamma_{\text{char}}^{\text{tot}}}{\Gamma_{\text{neut}}^{\text{tot}}} = \frac{60\zeta(3)}{\pi^3} \frac{(m r_e)^2}{\mu_K} e^{-\pi m r_e}. \quad (65)$$

Then the neutral emission rate always dominates the charged emission rate, except at extremely low temperature.

Thus, for a Reissner-Nordström black hole, very close to the BPS state, we expect that the dominant decay mode is via charged emission provided that the minimum BPS mass of the charged particles is small on the scale of the Schwarzschild radius. Emission of charged scalars with a mass large compared to this scale is exponentially suppressed with respect to neutral emission.

In passing we mention that although we have been discussing emission of particles carrying a single type of charge the calculation applies also to BPS particles carrying all three types of charge, such that

$$m = m_1 + m_5 + m_K. \quad (66)$$

The equation of the motion of the particle is the coupled Klein-Gordon equation, where we consider coupling to all three fields. The emission rate is Eq. (55), implying that the rate of emission of particles of the same BPS mass is equal, whatever the distribution of the three charges, as we would expect for a Reissner-Nordström state. It might seem as though the emission of charged particles carrying several types of charges would be significant in determining the total charge emission rates. However, as we shall see in the following section, the relationships between the three (quantized) BPS masses are such that at most only one type of charged particle can be light on the scale of the Schwarzschild radius.

III. IMPLICATIONS FOR MEASUREMENTS

Our discussion so far has involved only the effective five-dimensional solution, which is a solution of the low-energy action of type-IIB theory compactified on a torus. Following the notation of [12], we can express the energy of the BPS state in terms of charges normalized to be integers n_1 , n_5 , and n_K as

$$E = \frac{Rn_1}{g} + \frac{RVn_5}{g} + \frac{n_K}{R}, \quad (67)$$

where R is the circle radius, V is the volume of the four torus, and g is the string coupling. In D -brane models, the integers n_1 , n_5 , and n_K are interpreted as the number of 1D-branes wrapping the Kaluza-Klein circle, the number of 5D-branes wrapping the five torus, and the momentum in the circle direction, respectively. We adopt the conventions of [3], including $\alpha' = 1$, so that all dimensional quantities are measured in string units and the five-dimensional Newton constant is given in terms of the moduli by $G_5 = \pi g^2/4VR$. In terms of the integral charges, the entropy of the BPS state takes the moduli-independent form (1) and, as we discussed in the Introduction, this formula immediately implies that charged emission is in general suppressed. We find precisely such suppression is implied by the rates we have calculated.

We first, however, address an issue that we have so far neglected. In the previous section, we have implicitly assumed that we can take the energy of the neutral scalar, and the kinetic energy of the emitted scalar to be arbitrarily small compared to all other energy scales. In [17], Maldacena and Susskind found that the low-lying excitations of D -brane configuration in which the Kaluza-Klein radius is large were quantized in units of

$$\Delta E = \frac{1}{n_1 n_5 R} \approx \frac{G_5}{r_e^4}. \quad (68)$$

For more general conditions on the moduli, one would expect there to be light excitations of the BPS D -brane configuration of the same scale. It has been suggested that the existence of such a mass gap, for which there is no analogue for the Schwarzschild black hole, can be justified even at the level of the classical black hole solution.

It was first pointed out in [18] that the statistical description of a near-extremal black hole breaks down as the temperature approaches zero. As the heat capacity approaches 1, gravitational back reaction must be included; the scale at which such effects become important is an excitation energy of G_5/r_e^4 . This excitation energy is of the same order as the kinetic energy of the black hole according to the uncertainty principle.

In [19], it was suggested that small perturbations about extreme black holes for which the entropy vanishes, but the formal temperature does not, are protected by mass gaps which remove them from thermal contact with the outside world. The particular class of black holes discussed was electrically charged dilaton black holes in four dimensions; a parameter a describes the dilaton coupling to the gauge fields with $a = 0$ describing the usual Reissner-Nordström solution, and $a = 1$ describing a solution of particular interest in string theory. In the case that $a > 1$, the entropy of the extreme state vanishes, with the formal temperature diverging; the existence of mass gaps was then suggested to prevent radiation at the extreme. For extreme states in which the entropy is finite and the temperature is zero—the type of states which we are analyzing here—there are no such objections to the black hole absorbing or emitting arbitrarily small amounts of energy, and no such justifications for introducing mass gaps in the classical solutions.

In [20], and more recently in [21], the thermal factors in black hole emission rates were derived taking account of self-interaction. This approach gives the appropriate thermal factors for both the high-energy tail of the emission spectrum of a nonextremal black hole and also for the emission spectrum of a very near-extremal black hole, and it is found that they differ significantly from those in the free field limit. There are, however, no physical reasons for requiring the excitation spectrum to be quantized in the very near-extremal limit in the semiclassical theory.

One would expect the spectrum of the classical black hole to be continuous with arbitrarily small amounts of energy being emitted and absorbed. In the parametrization of the previous section, this implies that the potentials μ_i are continuous and not discrete. Our emission rates will only be valid provided that the total excitation energy above the extremal state is greater than the uncertainty in the kinetic energy of the state according to the uncertainty principle; below this temperature our rates should be modified in the ways suggested in [20] and [21].

It is important to note that individual μ_i can correspond to excitation energies which are smaller than the uncertainty in the kinetic energy provided that the total excitation energy is much greater. This will occur if one physical charge, let us say the Kaluza-Klein charge, is much smaller than the other two. It may at first appear as though this implies that the kinetic energy of emitted scalars carrying the other two charges must be smaller than the uncertainty in kinetic energy, and much smaller than the temperature. However, the division of the excitation energy into three sectors is artificial in the sense that charged emission processes reduce the excitation energies in all three sectors. So we should still allow for the emission of scalars with kinetic energies up to the total excitation energy in integrating to find total emission rates.

Let us first assume that the BPS masses of the emitted particles are quantized in equal units, that is, $R/g = RV/g = 1/R$; then we can express the mass of the extreme Reissner-Nordström black hole as

$$E = \frac{3n\sqrt{n}}{r_e}, \quad (69)$$

where r_e is the Schwarzschild radius and $n \equiv n_i$. The masses of the emitted BPS charged particles are quantized as

$$m = \frac{c\sqrt{n}}{r_e}, \quad (70)$$

with c integral. When n is a large integer, the emission of all charged particles must be suppressed at low energy as $mr_e \gg 1$; from Eq. (55), we find that emission of particles of minimum BPS mass is suppressed as $e^{-\pi\sqrt{n}}$. This is precisely the factor we would expect; the entropy loss of the black hole when it loses a single particle of minimum BPS mass is $\Delta S = \pi\sqrt{n}$ and the emission rate is suppressed as $e^{-\Delta S}$. Under these conditions, we would expect the black hole to decay back to extremality by emission of low-energy neutral scalars, except when the Hawking temperature is very low.

There is a subtlety that we will mention briefly here and then ignore; if the integral charges are small, a significant fraction of the mass of the black hole will be lost when any charged particle is emitted and we must be more careful about the thermal factor. Following [21], we find that the emission rate is suppressed as

$$\Gamma = v \sigma_{\text{abs}} e^{[S_{\text{final}} - S_{\text{orig}}]} \frac{d^4 k}{(2\pi)^4} \quad (71)$$

which gives an exponential factor

$$\Gamma \propto [e^{-2\pi n[n^{1/2} - (n-1)^{1/2}]}], \quad (72)$$

where we assume that a particle of minimum BPS mass is emitted. So for very small n it is possible that a significant fraction of the charge of the black hole is lost as the black hole decays back towards extremality. We shall not attempt further analysis of such states, for which the techniques of [21] would be required.

If a Reissner-Nordström BPS state for which $n_k \gg n_i$ is slightly excited from extremality, it will decay predominantly via emission of particles carrying the Kaluza-Klein charge, since such particles have a mass small on the Schwarzschild radius. However, as it decays towards a state in which $n_k \sim n_i$, the emission starts to be suppressed by the unfavorable entropy loss from the black hole when each unit of charge is lost [Eq. (2)]. This behavior depends only on the integral charges. For the analysis in the dilute gas region of [12] factors of e^{-RT_L} appear in the rates, where T_L is the temperature of the left-moving excitations of the effective string. Since $RT_L \sim \Delta S$, this is precisely the behavior we would expect.

The authors of [12] suggested that the Kaluza-Klein charge of the hole could be lost before the entropy in the emitted radiation was sufficient to determine the state of the hole, but their analysis failed to take note of the fact that as

the Kaluza-Klein charge is reduced, further emission is suppressed. Expressed in terms of the temperature of the left-moving excitations, even if this temperature is initially large compared to the scale set by the Kaluza-Klein radius, it is reduced by the emission. As the temperature approaches $1/R$, further charged emission is suppressed. More generally, what we would expect to happen is that the black hole evolves towards a state in which all three charges are comparable. Thereafter, emission of even the lightest charged state will be exponentially suppressed relative to neutral emission.

For the Reissner-Nordström state in which all the integral charges are equal, as the black hole decays back towards extremality by neutral emission, the Hawking temperature decreases and the rate at which the excess energy is lost by the hole becomes very small. So, very close to extremality, the rates of loss by neutral and charged emission may become comparable despite the entropy loss involved in charged emission. This is apparent from looking at ratio of the emission rates in Eq. (65) but for later convenience we compare instead the approximate rates of energy loss for neutrals,

$$\left(\frac{d\Delta E}{dt} \right)_{\text{neut}} \approx \int \Gamma \omega, \quad (73)$$

with the corresponding rate for charged particles,

$$\left(\frac{d\Delta E}{dt} \right)_{\text{char}} \approx \int \Gamma(\omega - m), \quad (74)$$

where we will assume only particles of minimum BPS mass are emitted. Now the energy loss rate by neutral emission is

$$\left(\frac{d\Delta E}{dt} \right)_{\text{neut}} \approx \frac{3\zeta(5)}{\pi^2} A_h T_H^5. \quad (75)$$

For a Reissner-Nordström state with the integral charges equal we find that

$$\left(\frac{d\Delta E}{dt} \right)_{\text{char}} \approx \frac{\pi^3}{60} A_h T_H^4 \left(\frac{n}{r_e} \right) e^{-\pi\sqrt{n}}, \quad (76)$$

and so we find the ratio of energy loss rates to be, using Eq. (48),

$$\frac{(d\Delta E/dt)_{\text{char}}}{(d\Delta E/dt)_{\text{neut}}} \approx \frac{\pi^6}{180\zeta(5)} \frac{n}{\mu} e^{-\pi\sqrt{n}}, \quad (77)$$

where μ is the deviation of the potential from one on the horizon, and the rate of loss of all three charges is the same.

The relative rates of energy loss are independent of the values of the moduli. Using the results of [12], obtained under the condition that the momentum modes are light, setting the charges to be equal, we find that the rates of loss of energy by emission of Kaluza-Klein (KK) charged particles and neutrals compare as

$$\frac{(d\Delta E/dt)_{\text{KK}}}{(d\Delta E/dt)_{\text{neut}}} \approx \frac{\pi^6}{180\zeta(5)} \frac{n}{\mu_K} e^{-\pi\sqrt{n}}, \quad (78)$$

where μ_K is the Kaluza-Klein potential on the horizon. That is, we find the same ratio for Kaluza-Klein charged and neutral emission in this sector of the moduli space, confirming the modular independence of the result.

However, in [12], it was assumed that only μ_K was non-zero, which would imply that only Kaluza-Klein charge is lost. Under the condition that Q_K is much smaller than the other two charges, this is a reasonable approximation, since Eq. (6) implies that μ_K is much larger than the other μ_i for any given r_0 . When the integral charges are equal, then from Eqs. (6) and (8), assuming that $V=1$, we find that

$$\frac{\mu_K}{\mu_1} = \frac{R^2}{g}, \quad (79)$$

and so $\mu_1 (= \mu_5)$ is much smaller than μ_K under these conditions on the moduli.

From the point of view of the five parameter classical black hole solution, it is not consistent to set $\mu_i \equiv 0$ when $r_0 \neq 0$, as we pointed out above. In fact, for $n_K \gg n_i$, the physical charges can be comparable, and we would expect the nonextremality parameters in each sector to be comparable also. Since the authors of [12] imposed the condition that $Q_K \ll Q_i$, even for $n_K \gg n_1 n_5$ (corresponding to their condition $RT_L \gg 1$), their calculations are unaffected by taking μ_i to be finite. That is, μ_K will always be much greater than μ_i and for most purposes we can set μ_i to zero, although the deviation of μ_i from zero in the black hole solution is significant, as we shall see below.

It is not difficult to extend the analysis of the section above to show that, under the conditions $R^2 \gg g$ and $n_i \equiv n$, for emission of the other two types of charges, the energy loss rates compare as

$$\frac{(d\Delta E/dt)_i}{(d\Delta E/dt)_{\text{neut}}} \approx \frac{\pi^6}{180\zeta(5)} \frac{n}{\mu_i} e^{-\pi\sqrt{n}}, \quad (80)$$

where we assume that the μ_i are very small, but nonzero. There are two important points to notice. First, this is the same ratio as we get in the Reissner-Nordström sector of the moduli space above. If we assume that $\mu_i \equiv 0$ in this sector of the moduli space, the ratios are not moduli independent. Second, with these conditions on the moduli, we expect that μ_i is smaller for the heavier modes. So the rate of loss of energy by the heavier particles is actually greater, and will dominate emission by Kaluza-Klein charged particles

$$\frac{(d\Delta E/dt)_{\text{KK}}}{(d\Delta E/dt)_1} \approx \frac{\mu_1}{\mu_K} = \frac{g}{R^2} \ll 1. \quad (81)$$

The black hole loses the same amount of entropy in emitting a unit of each charge, so the exponential suppression factor is the same, but the physical Kaluza-Klein charge is smaller, and is less likely to be reduced.

For general integral charges the relative rates of loss of energy are

$$\frac{(d\Delta E/dt)_{\text{KK}}}{(d\Delta E/dt)_{\text{neut}}} \approx \frac{\pi^6}{180\zeta(5)} \frac{n_1 n_5}{n_K \mu_K} \exp\left(-\pi \sqrt{\frac{n_1 n_5}{n_K}}\right), \quad (82)$$

with the ratio for emission of the other two particles being given by the same expression with appropriate permutations of indices. If $n_K \ll n_i$, then we see that loss of the Kaluza-Klein charge is suppressed, and that the rate of loss of the other two charges dominates the neutral emission rate at higher temperature. So decay towards a state in which the n_i are equal is indeed favored although the rate of loss of charge will be slow compared to the loss of neutrals except at low temperature.

If $n_K \gg n_1 n_5$ we must allow for emission of particles of greater than the minimum BPS mass. For a Reissner-Nordström solution, the mass of Kaluza-Klein charged particles is quantized as

$$m = \frac{c}{r_e} \sqrt{\frac{n_1 n_5}{n_K}}, \quad (83)$$

where c is an integer; the mass is small on the scale of the Schwarzschild radius, and charged emission will dominate neutral emission. We calculate the rate of energy emission for a particle of general mass m , using Eqs. (54) and (74) as

$$\left(\frac{d\Delta E}{dt}\right)_{\text{char}} \approx \frac{\pi^3}{60} A_h T_H^4 \frac{m^2 r_e}{(e^{\pi m r_e} - 1)}, \quad (84)$$

and integrate over all masses to find that

$$\left(\frac{d\Delta E}{dt}\right)_{\text{KK}} \approx \frac{\zeta(3)}{30} A_h T_H^4 \frac{1}{r_e} \sqrt{\frac{n_K}{n_1 n_5}}. \quad (85)$$

Comparing this to the energy loss by neutral emission we find that

$$\frac{(d\Delta E/dt)_{\text{KK}}}{(d\Delta E/dt)_{\text{neut}}} \approx \frac{\pi^3 \zeta(3)}{90 \zeta(5)} \sqrt{\frac{n_K}{n_1 n_5}} \frac{1}{\mu_K}, \quad (86)$$

which is the same ratio as we obtain from [12]. Thus we find that emission of KK charged scalars dominates neutral emission, independently of the moduli, for any near extremal state with n_K very large.

Thus we find that, although the absolute rates of energy emission by the black hole are moduli dependent, the relative rates of neutral and charged emission depend only on the integral charges and horizon potentials. It is straightforward to demonstrate this explicitly by extending the scattering calculations to the most general near-extremal black holes. So the scattering rates from black holes exhibit a certain universality which follows from the modular independence of the BPS entropy. Let us assume that the agreement between D -brane and black hole emission rates extends throughout the moduli space of near-extremal states and then consider the implications of our results for scattering experiments.

Suppose that we excite a BPS state slightly above extremality with low-energy radiation and measure the outgoing radiation resulting from the decay. Whatever the value of the moduli, the black hole will decay towards a state in which the integral charges are equal, but such a decay will only proceed rapidly if one charge is much greater than the other two. Under the latter conditions, the black hole will lose a significant fraction of its charge before there is enough information in the outgoing radiation to measure its state. It is

simple to show that, as the black hole decays from a state with $n_K \gg n_1 n_5$ towards a state in which the charges are comparable, the entropy in the outgoing charged radiation is given by

$$\frac{\delta S_{\text{out}}}{S_{\text{BH}}} \sim \frac{1}{n_1 n_5}. \quad (87)$$

Now in the string picture one can measure the state of the black hole once the entanglement entropy in the outgoing radiation is equal to that of the black hole. So here the entropy in the outgoing radiation is certainly insufficient to determine the initial state of the black hole, and the state changes before we can measure it. However, as the black hole decays towards a more stable charge configuration, we might hope to be able to measure the state after neutral emission starts to dominate.

Suppose that we start with an extreme Reissner-Nordström state in which all the integral charges are equal; the maximum excitation energy we can add and still leave the black hole in a near extremal state is $\Delta E \sim \sqrt{n}/r_e$, i.e., an energy equal to that of the minimally charged BPS particle. We can then estimate the total amount of entropy in the outgoing neutral radiation as

$$\delta S_{\text{out}} \sim \int_0^{\sqrt{n}/r_e} \frac{d(\Delta E)}{T_H}, \quad (88)$$

and, from Eq. (13), expressing the temperature as a function of the excess energy, we find that $\delta S_{\text{out}} \sim n$. So in order to obtain an entropy in the outgoing radiation equal to that of the black hole we will need of the order of $\sqrt{n}$ experiments. In fact, for general charges and moduli, we can show that the maximum amount of entropy in the outgoing radiation is

$$\frac{\delta S_{\text{out}}}{S_{\text{BH}}} = \left[\frac{1}{E_1} + \frac{1}{E_5} + \frac{1}{E_K} \right]^{1/2} \Delta E_{\text{max}}^{1/2}, \quad (89)$$

where the energy of the BPS state is $E = \sum_i E_i$. Since by definition very close to extremality the excitation energy is much less than the smallest of the E_i , a large number of experiments will be required. After these experiments we let the black hole decay right back to the BPS state, which takes an infinitely long time. As the temperature becomes very small, charged emission dominates neutral emission, and we might expect the final excess energy of the black hole to be emitted in the form of charged radiation.

Working in the Reissner-Nordström sector, neutral emission will dominate until the ratio of rates in Eq. (77) is approximately 1; but when this happens, the remaining excess energy is

$$\Delta E \sim \frac{n^{5/2} e^{-2\pi\sqrt{n}}}{r_e}, \quad (90)$$

which compares to an energy scale set by the uncertainty principle of

$$E_{\text{uncert}} \sim \frac{1}{n^{3/2} r_e}, \quad (91)$$

which is much larger. That is, before charged emission can become significant, the excess energy falls below the uncertainty in energy of the BPS state (and the statistical approximations implicit in our rates break down).

We now suggest a resolution to a paradox discussed in [12]. If we have a state for which the momentum modes are light, and all three integral charges are comparable, then emission of any charge is suppressed. So we might expect that we could excite the black hole by an energy $\Delta E \gg n/R$, still remaining in the near-extremal state since the Kaluza-Klein radius is taken to be large. Neutral emission will dominate the decay, and the entropy in the outgoing radiation is

$$\delta S_{\text{out}} \sim n \sqrt{R \Delta E} \gg n^{3/2}. \quad (92)$$

That is, the entropy in the outgoing radiation is greater than that of the black hole, which presents a contradiction in the string picture.

However, if we attempt to excite the black hole with such a large excitation energy, we find that $r_K \ll r_0$ and $r_1 \sim r_0$, where we use the relationship between the extremality parameters. This implies that the black hole is very nonextremal, and its decay lies outside the range of the near-extremal calculations. For a near-extremal solution, we require $r_1, r_5 \gg r_0$, and hence we must restrict our excitation energies to $\Delta E \leq 1/R$. Under this condition, $\delta S_{\text{out}} \sim n$ is the maximum amount of entropy in the outgoing radiation, much smaller than the entropy of the black hole, as required. So it is important to take account of all three potentials; it is straightforward to show that the near-extremal calculations are valid only when the largest of the μ_i is less than or of the order of $1/n_i$.

Scattering from analogous four-dimensional black holes carrying four U(1) charges is also found to exhibit a universal structure which is implied by the modular independence of the BPS entropy. The analysis differs little from that in the five-dimensional system and we include a brief summary in the Appendix.

IV. CONCLUSIONS

We have shown that by repeated scattering from the black hole we can obtain an entropy in the outgoing radiation equal to that of the black hole before the BPS state changes. By careful experimentation we might then think that information about the actual microstate could be deduced from the absorption or scattering process. However, we must be more careful about extrapolating from the D -brane limit of the moduli space in which

$$gn_1 < 1; \quad gn_5 < 1; \quad g^2 n_K < 1, \quad (93)$$

to the black hole limit in which

$$gn_1 > 1; \quad gn_5 > 1; \quad g^2 n_K > 1. \quad (94)$$

In the former case, we have a discrete excitation spectrum. We can use D -brane models of near-extremal black holes to describe excitations in the “dilute gas” region of the moduli space; that is, we consider states in which the Kaluza-Klein radius is very large and the physical Kaluza-Klein charge is

small. In this region we can describe the near-extremal state in terms of excitation modes of an effective string of length Rn_1n_5 with the excitation energy being quantized in units of the reciprocal of the length. For a large black hole solution in which the Kaluza-Klein radius is large then in terms of the nonextremality parameters of the black hole solution, the excitation energy in the Kaluza-Klein sector is

$$\Delta E_{\text{KK}} = \frac{\pi r_0^2}{4G_5} e^{-2\sigma_K} = \frac{n_K \mu_K^2}{R}, \quad (95)$$

where μ_K is a continuous parameter. However, we will also have nonzero excitation energies in the other two sectors, which, assuming for simplicity that $V=1$, are given by

$$\Delta E_1 = \Delta E_5 = \frac{gn_K}{R^2 n_1} \Delta E_{\text{KK}}. \quad (96)$$

In the limit that $r_K \ll r_1$, then $\Delta E_1 \ll \Delta E_{\text{KK}}$ and by taking the radius to be sufficiently large we can choose $\Delta E_1 < 1/n_1 n_5 R$. For the classical solution, this means that the excitation energy in this sector is smaller than the scale set by the uncertainty principle, but is still finite because we have taken the nonextremality parameter to be continuous.

What this implies physically is that the large black hole can emit BPS charged particles with all three types of charges provided that the temperature is finite. According to the D -brane calculations, only BPS particles carrying the Kaluza-Klein charge can be emitted. That is, the agreement between the D -brane and black hole emission rates breaks down when the excitation energy of the near-extremal black hole is very small in one of the sectors. This will be particularly significant if, for example, $n_1 \gg n_5 n_K$ and ΔE_1 in the black hole solution is smaller than the uncertainty energy. Then according to the black hole calculations, the dominant decay mode should be via emission of particles carrying this charge whereas according to the D -brane model no such emission is possible.

Since from general duality arguments we would expect the agreement between black hole and D -brane emission rates to hold throughout the moduli space, the interpretation we give to this disagreement is that in this limit the effective string model breaks down. In terms of the moduli space analysis proposed recently in [22], the probability for the system to wander into the vector moduli space, corresponding to D -brane emission, becomes significant in this limit. Of course as the physical charges in the BPS state become comparable, the effective string approximation certainly breaks down; we require the D -brane model to describe emission of all three charges.

Now in the D -brane limit, if we do scattering experiments we will indeed know in which microstate the branes are. Suppose we examine the absorption of a (neutral) scalar of energy $2c/n_1 n_5 R$ by a D -brane configuration whose excitations are described by those of an effective string of length $n_1 n_5 R$. The absorption creates a pair of open strings moving on the string and the absorption probability depends on the quantum microstate of the configuration. More generally, there will be many distinct types of excitations of the BPS state which can be interpreted in terms of, for example, brane

or antibrane pairs, and the absorption spectrum will depend on the moduli and charges of the BPS state.

Then we can see that repeated absorption or emission processes will give us information about the microstate. In the black hole limit, if the spectrum is continuous, repeated scattering processes will simply produce an ever-increasing amount of entropy in the outgoing radiation which does not encode the state of the black hole. Another way of describing this would be to say that classical large black holes behave as complex extended objects with a continuous spectrum of low-lying excitations whereas in the D -brane limit the system behaves as an elementary particle with discrete excitation levels.

This picture was suggested in [23] where the excitation spectrum of an isolated D string was shown to change from one with discrete levels to one that has no sharp levels as we go towards an appropriate black hole type limit. This is what we have assumed in taking the μ_i to be continuous parameters, and such a spectrum change is of course implicit in the picture of black hole to D -brane transition discussed in [24]. It would be interesting if this change of spectrum could be demonstrated explicitly for bound states of D -branes.

The aim of this paper was to attempt to reconcile the nonunitary behavior of black holes with the unitary behavior of the corresponding D -brane configuration by demonstrating that systems decay before one can measure their states. We have, however, found that this is not the case. Since this work was completed, a correspondence principle between black holes and strings has been proposed [25] which highlights the apparent contradictions between the pictures still further. There have been suggestions that information may be lost from the D -brane configuration in subtle ways, such as by recoil effects involved in scattering [26]. However, as we discussed in the Introduction, we believe that if information is lost it is because one cannot neglect the causal structure and treat the system as though it is in flat space. This is a subject to which we hope to return in the near future.

APPENDIX: SCATTERING FROM FOUR-DIMENSIONAL BLACK HOLES

In this appendix we show that the same modular independence of ratios of scattering rates is found in analogous four-dimensional black hole systems. Following [13] and [14], we consider a four-dimensional black hole with four $U(1)$ charges described by the metric

$$ds_4^2 = -F^{-1/2} H dt^2 + F^{1/2} (H^{-1} dr^2 + r^2 d\Omega^2) \quad (A1)$$

with

$$H = \left(1 - \frac{r_0}{r}\right), \quad F = \left(1 + \frac{r_1}{r}\right) \left(1 + \frac{r_2}{r}\right) \left(1 + \frac{r_3}{r}\right) \left(1 + \frac{r_4}{r}\right), \quad (A2)$$

where, for each r_i ,

$$r_i = r_0 \sinh^2 \sigma_i \quad (A3)$$

with the r_0 and σ_i being extremality parameters, such that in the BPS limit $r_0 \rightarrow 0$ and $\sigma_i \rightarrow \infty$ with r_i fixed. The physical charges are given by $Q_i = r_0 \sinh \sigma_i \cosh \sigma_i$ and the energy is

$$E = \frac{1}{4G_4} \sum_i Q_i + \frac{r_0}{4G_4} \sum_i e^{-2\sigma_i}, \quad (\text{A4})$$

where G_4 is the four-dimensional Newton constant. The solution may be described by six independent parameters—the mass, four charges, and one nonextremality parameter.

In [13] and [14], scalar emission rates were calculated for this metric using semiclassical and effective string model approaches in the limit that the Kaluza-Klein parameter r_4 was much smaller than the other r_i . It is straightforward to extend their semiclassical calculations to general near-extremal black hole solutions for which $r_0 \ll r_i$ and we do not repeat the details of the scattering calculation here. We find that the neutral scalar emission rate at low energies is

$$\Gamma = A_h \frac{1}{(e^{\omega/T_H} - 1)} \frac{d^3k}{(2\pi)^3}, \quad (\text{A5})$$

with the emission rate of Kaluza-Klein charged scalars of mass m being

$$\begin{aligned} \Gamma = & \pi A_h \frac{(\omega^2 - m^2)^{3/2}}{m(\omega - m\phi_4)} \sqrt{\frac{r_1 r_2 r_3}{r_4}} \\ & \times \frac{1}{(\exp 2\pi m \sqrt{r_1 r_2 r_3 / r_4} - 1)} \frac{1}{(e^{(\omega - m)/T_H} - 1)} \frac{d^3k}{(2\pi)^3}, \end{aligned} \quad (\text{A6})$$

where ϕ_4 is the Kaluza-Klein potential on the horizon and corresponding expressions hold for the emission of the other charges. These rates are valid provided that the total excitation energy is greater than the uncertainty energy, which in four dimensions is G_4/r_e^3 , below which scale the statistical assumptions break down. We can express the physical charges Q_i in terms of moduli and integral charges n_i as

$$\begin{aligned} Q_1 &= \frac{n_1}{L_6 L_7} \left(\frac{\kappa_{11}}{4\pi} \right)^{2/3}, \quad Q_2 = \frac{n_2}{L_4 L_5} \left(\frac{\kappa_{11}}{4\pi} \right)^{2/3}, \\ Q_3 &= \frac{n_3}{L_2 L_3} \left(\frac{\kappa_{11}}{4\pi} \right)^{2/3}, \quad Q_4 = 8\pi G_4 \left(\frac{n_4}{L_1} \right), \end{aligned} \quad (\text{A7})$$

where L_i is the length of the i th internal circle, and $\kappa_{11}^2 = 8\pi G_4 \Pi_i L_i$. Such integral charges arise from the toroidal compactification of an 11-dimensional solution, and can be interpreted in terms of intersecting brane representations in M theory [7,8]. Q_4 is the Kaluza-Klein charge, deriving from the quantized momentum in a circle direction.

The entropy of the BPS state takes the modular-independent form

$$S = 2\pi \prod_i \sqrt{n_i}, \quad (\text{A8})$$

and so we would expect charged emission to be suppressed as the entropy loss in emitting one unit of Kaluza-Klein charge is

$$\Delta S = \pi \sqrt{\frac{n_1 n_2 n_3}{n_4}}, \quad (\text{A9})$$

which is generally large. Exponential suppression by precisely this factor is implied in Eq. (A6), since the BPS masses of particles carrying the Kaluza-Klein charge are quantized in units of $2\pi/L_1$ and

$$\sqrt{\frac{r_1 r_2 r_3}{r_4}} = \sqrt{\frac{n_1 n_2 n_3}{n_4}} \left(\frac{L_1}{4\pi} \right). \quad (\text{A10})$$

We find that the rate of energy loss by neutral emission is

$$\left(\frac{d\Delta E}{dt} \right)_{\text{neut}} \approx \frac{\pi^2}{30} A_h T_H^4, \quad (\text{A11})$$

which has a different temperature dependence to the five-dimensional expression. The rate of energy loss by Kaluza-Klein charged emission is

$$\left(\frac{d\Delta E}{dt} \right)_4 \approx \frac{\pi^3}{15} A_h T_H^4 \frac{1}{\mu_4} \sqrt{\frac{n_1 n_2 n_3}{n_4}} \exp - \pi \sqrt{\frac{n_1 n_2 n_3}{n_4}}, \quad (\text{A12})$$

where μ_4 is the deviation from one of the Kaluza-Klein potential on the horizon. Note the higher exponential suppression than in five dimensions, deriving from the expression for the entropy. If n_4 is much greater than the product of the other three charges, we must allow for emission of not only minimally charged particles and

$$\left(\frac{d\Delta E}{dt} \right)_4 \approx \frac{\pi^3}{90} A_h T_H^4 \frac{1}{\mu_4} \sqrt{\frac{n_1 n_2 n_3}{n_4}}. \quad (\text{A13})$$

Then the modular independent ratios of energy loss rates are

$$\frac{(d\Delta E/dt)_4}{(d\Delta E/dt)_{\text{neut}}} \approx \frac{2\pi}{\mu_4} \sqrt{\frac{n_1 n_2 n_3}{n_4}} \exp - \pi \sqrt{\frac{n_1 n_2 n_3}{n_4}}, \quad (\text{A14})$$

except for $n_4 \gg n_1 n_2 n_3$ when

$$\frac{(d\Delta E/dt)_4}{(d\Delta E/dt)_{\text{neut}}} \approx \frac{\pi}{3\mu_4} \sqrt{\frac{n_4}{n_1 n_2 n_3}}, \quad (\text{A15})$$

which is very large close to extremality. Our modular independent ratios are in agreement with those derived from the emission rates in [13] and [14]. So, unless one integral charge is much greater than the product of the other three, charged emission does not play a role in the decay and measurements of the microstate by repeated scattering processes appear to be feasible.

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