Deep inelastic scattering of leptons from nuclear targets and the BFKL Pomeron

Andrzej Bialas and Wieslaw Czyz
Institute of Physics of the Jagellonian University, Reymonta 4, 30-059 Kraków, Poland

Wojciech Florkowski
H. Niewodniczański Institute of Nuclear Physics, Radzikowskiego 152, 31-342 Kraków, Poland

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We calculate shadowing in the process of deep inelastic interactions of leptons with nuclei in the perturbative regime of QCD. We find appreciable shadowing for heavy nuclei (e.g., Pb) in the region of a small Bjorken scaling variable $10^{-5} \leqslant x \leqslant 10^{-3}$. This shadowing depends weakly on Q^2, but it may be strongly influenced, especially at $x \geqslant 10^{-3}$, by the existence of real parts of the forward scattering amplitudes. [S0556-2821(97)03709-0]

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I. INTRODUCTION

There is a considerable progress in calculations of various cross sections for deep inelastic lepton scattering in the Lipatov limit ($\nu \to \infty$ and Q^2 fixed at a large value [1]). In particular a picture has been proposed [2,3] which reproduces the basic features of Balitskii-Fadin-Kuraev-Lipatov (BFKL) Pomeron, e.g., its Regge behavior which was earlier [1] obtained from a direct resummation of the Feynman diagrams of quantum chromodynamics. Subsequently, the model of [2,3] was applied to calculate the total cross section, the diffractive dissociation cross section, and the corresponding structure functions for the proton target [4–8].

In view of the successful experimental program for the so-called ''small-x physics'', which is being pursued at the DESY ep collider HERA for proton targets and will be, possibly, extended to nuclear targets, it makes sense to adapt the results of [4–8] also to nuclear targets. So, in this paper, we apply the results of [4–8] to calculate the shadowing in deep inelastic lepton-nucleus interactions.

The problem of nuclear shadowing in deep inelastic lepton-nucleus interactions has been recently discussed in many papers. References [9–14] give an up-to-date picture of this field.

For a description and discussion of the basic physical ideas of the model used in the present calculations we refer the reader to the article [8]. Here, a brief summary will suffice. The model was initially designed for ''quarkonium''-''quarkonium'' scattering, i.e., for scattering of two bound $q\bar{q}$ states with a small transverse separation. At high energy, in perturbative QCD, such a ''quarkonium'' state evolves (cascades) into a bunch of colorless dipoles whose number increases as a power of the incident energy. The interaction occurs *via* two gluon exchange between the pairs of dipoles (one from each ''quarkonium''). Navelet, Peschanski, Royon, and Wallon [4,5] applied this model to deep inelastic lepton-proton scattering. They assumed, naturally, that the virtual photon at high Q^2 can be described by a ''quarkonium.'' For the target proton, which certainly does not resemble a ''quarkonium,'' they made a rather bold assumption that it can nevertheless be approximated by a collection of ''quarkonia'' with an average ''quarkonium'' radius to be determined from the data. It was thus rather surprising that such a seemingly crude model described very reasonably the existing data on F_2 and on gluon structure function in a fairly large range of Q^2 and of Bjorken x. To obtain good fits to the experimental data the authors of Refs. [4,5] had only to normalize F_2. We show below that this normalization corresponds to $n_{\rm eff} \approx 6.7$ ''quarkonia'' in a nucleon.

In the present paper we apply these ideas to the calculations of shadowing in deep inelastic scattering from nuclear targets. We hope that this can provide an independent check of the hypothesis of Ref. [4], and give more information about the nucleon structure at very small Bjorken x.

The total virtual transverse photon-''quarkonium'' cross section is one of the basic inputs in computations of nuclear shadowing and we will use the following integral representation of $\sigma_{\rm tot}^T = (\Sigma_n P_n \sigma_n)^T$ (n is the index of the Good and Walker eigenmodes of absorption, see below)

$$(\Sigma_n P_n \sigma_n)^T = \sigma_{\rm tot}^T(x,Q^2) = \frac{4N_c\alpha^2 n_{\rm eff}\alpha_{\rm em}e_f^2}{\pi} r_0^2 \int_{c-i\infty}^{c+i\infty} \frac{d\gamma}{2\pi i} x^{(-\alpha N_c/\pi)\chi(\gamma)} \times 2\pi \left(\frac{Qr_0}{2}\right)^{\gamma-2} h^T(\gamma). \tag{1}$$

This integral representation is our version of $\sigma_{\rm tot}^T$ discussed in [4]. We have checked through numerical evaluation of Eq. (1) that it agrees well with the results of Refs. [4,5] provided $n_{\rm eff} \approx 6.7$. In Eq. (1), N_c is the number of colors involved in the process, and $n_{\rm eff}$ is an effective number of ''quarkonia'' in the nucleon. $\alpha_{\rm em} = \frac{1}{137}$, e_f^2 is the sum of the squares of quark charges, and $r_0 \approx 0.8$ fm is approximately the ''quarkonium'' diameter:

$$h^T(\gamma) = \frac{4}{\gamma(2-\gamma)} \times \frac{\Gamma\left(3-\frac{1}{2}\gamma\right)\Gamma^3\left(2-\frac{1}{2}\gamma\right)\Gamma\left(2+\frac{1}{2}\gamma\right)\Gamma\left(1+\frac{1}{2}\gamma\right)}{\Gamma(4-\gamma)\Gamma(2+\gamma)}, \tag{2}$$

and $\chi(\gamma)$ is the eigenvalue of the BFKL kernel defined in terms of $\psi(\gamma)=(d/d\gamma)[\ln\Gamma(\gamma)]$ as

$$\chi(\gamma)=2\psi(1)-\psi\left(1-\frac{1}{2}\gamma\right)-\psi\left(\frac{1}{2}\gamma\right). \tag{3}$$

$\chi(1)$ determines the slope of the BFKL Pomeron

$$\Delta_p=\alpha_p-1=\frac{\alpha N_c}{\pi}\chi(1), \tag{4}$$

where α is a strong coupling constant.

Since we are interested in the structure function $F_2\sim(\sigma_{\rm transv}+\sigma_{\rm longitud})$ one should add to Eq. (1), which gives the transverse virtual photon-''quarkonium'' total cross section, the contribution of the longitudinal virtual photon in order to have a complete virtual photon-''quarkonium'' total cross section. As it was shown in Ref. [5], one gets the longitudinal photon contribution by replacing the function $h^T(\gamma)$ of Eq. (2) by $h^L(\gamma)$:

$$h^L(\gamma)=h^T(\gamma)\frac{(2-\gamma)\gamma}{2\left(2-\frac{1}{2}\gamma\right)\left(1+\frac{1}{2}\gamma\right)}. \tag{5}$$

Since the integrand in Eq. (1) is strongly peaked at $\gamma=1$, we can approximate the above replacement by $h^L(\gamma)\approx\frac{2}{9}h^T(\gamma)$. Hence, the total cross section is

$$\sigma_{\rm tot}=\sigma_{\rm tot}^T+\sigma_{\rm tot}^L\approx\frac{11}{9}\sigma_{\rm tot}^T. \tag{6}$$

The success of Eq. (1) (compare [4,5]) encourages us to apply the results of [4–8] to the evaluation of nuclear shadowing.

In the next section we present the general formula for the shadow in the deep inelastic virtual photon-nucleus interactions. In Sec. III we apply it to calculate the ''triple Pomeron'' [6] contribution to shadowing. In Sec. IV we do the same for the ''quasielastic'' contributions [7]. Section V presents results and conclusions. Two appendices give details of derivations of some formulas.

II. BASIC FORMULAS FOR VIRTUAL PHOTON-NUCLEUS INTERACTIONS

Many theoretical papers on this and related subjects have been published in the past and our task is merely to adapt the existing material for our purposes. Our main formula, which gives the deviation from unity of the ratio R_A of the total virtual photon-nucleus cross section to the incoherent sum of the cross sections on individual nucleons, is a slight generalization of the formula proposed more than 25 years ago by Gottfried and Yennie [15] and then developed by Yennie and Bauer *et al.* [16]. It goes as follows:

$$\begin{aligned}\Delta_A(x,Q^2)&=R_A(x,Q^2)-1\\&=-\frac{2(A-1)}{\sigma_{\rm tot}}{\rm Re}\sum_n P_n\Bigg\{\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{z_1}dz_2\\&\times\left[\int d^2b' f_n(\mathbf{b}')\right]^2\rho(\mathbf{b},z_1)\rho(\mathbf{b},z_2)e^{i(z_1-z_2)mx_p}\\&\times\left[1-\int_{z_1}^{z_2}dz_3\rho(\mathbf{b},z_3)\int d^2b' f_n(\mathbf{b}')\right]^{A-2}\Bigg\}.\end{aligned} \tag{7}$$

Negative Δ_A means there is shadowing, positive Δ_A antishadowing. Here, $x=Q^2/2m\nu$ is the Bjorken variable with $Q^2=-q^2$ (the negative of the four-momentum squared of the virtual photon), m the nucleon mass, and ν the energy loss of the incident lepton (in the rest frame of the target nucleus).

$$x_p=\frac{x}{\beta},\quad \beta=\frac{Q^2}{Q^2+M^2}, \tag{8}$$

where M is the mass of the virtual photon excitation. In this formula we neglected the correlations between the nucleons inside the target. The index n stands for the Good and Walker eigenmodes of absorption [17], which propagate through nuclear matter without diffractive excitations. $\mathbf{b}$ is the impact parameter of the virtual photon, and P_n is the probability of realization of the nth eigenmode. $\Sigma_n P_n\sigma_n=\sigma_{\rm tot}$ is the total transverse virtual photon-nucleon cross section. f_n is the forward scattering amplitude of the nth mode.

In [6–8], only second order interactions in dipole-dipole cross section of the diffractively excited systems were considered. Then, only two classes of processes seem to be dominant [8]: the ''triple Pomeron'' contribution where the ''quarkonium'' scatters inelastically, and the ''quasielastic'' contribution where the ''quarkonium'' scatters quasielastically from the target proton.

Note that the results of Refs. [6–8] we are going to use to calculate Eq. (7) do not give us P_n and $f_n(\mathbf{b})$ but only $\Sigma_n P_n f_n(\mathbf{b})f_n(\mathbf{b})$. However, $\Sigma_n P_n f_n(\mathbf{b})f_n(\mathbf{b})f_n(\mathbf{b})$ and higher order expressions have, so far, not been calculated (e.g., in the case of an exchange of three Pomerons one would have to calculate a three dipole distribution function), hence the last factor in Eq. (7), $[\ \ldots\ ]^{A-2}$, can, at present, be only approximated (see below).

Note also that when the transverse spatial extension of f_n is much smaller than the nuclear radius [hence the transverse extension of $\rho(\mathbf{b},z)$], we can approximate

$$f_n(\mathbf{b})=\delta^{(2)}(\mathbf{b})\int d^2b' f_n(\mathbf{b}'). \tag{9}$$

This approximation was employed in Eq. (7). Physically, this is the question of the relative transverse sizes of the cascade of dipoles [6–8] and of the nucleus. We have checked accuracy of the assumption (9) for $\rho(\mathbf{b},z)$ suitable for large nuclei (see Sec. V) and found it acceptable; $\rho(\mathbf{b},z)$'s are taken in form of Saxon-Woods distributions.

Now, we will discuss the other input, $\Sigma_n P_n f_n f_n$, from [6–8] and to make it concise we shall proceed through numerous references to many formulas of the papers of Refs. [6–8].

III. ''TRIPLE POMERON'' CONTRIBUTION TO THE SHADOWING

We assume $A \gg 1$, hence a large nucleus where Eq. (9) is applicable. We first work out only the double scattering factor in Eq. (7). We modify the formula (21) of [6] introducing two impact parameters $\tilde{\mathbf{b}},\tilde{\mathbf{b}}'$ (to describe collisions with two different nucleons) and taking the inverse Mellin transformation of both sides of Eq. (21) of [6]:

$$\int d^2\tilde{b}d^2\tilde{b}'[\Sigma_n' P_n f_n(\tilde{\mathbf{b}}) f_n(\tilde{\mathbf{b}}')]$$
$$= 2\pi\alpha^5 n_{\rm eff}^2 N_c x_p^{-2\Delta_p}\left(\frac{2a_p}{\pi}\right)^3 \int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\rho^{2-\gamma}\beta^{-\alpha N_c\chi(\gamma)/\pi}$$
$$\times\int\frac{d^2\tilde{b}d^2\tilde{b}'}{\tilde{b}^2\tilde{b}'^2}\int dx_{12}dx_{02}W(x_{12},x_{02})$$
$$\times D(\tilde{\mathbf{b}},x_{02})D(\tilde{\mathbf{b}}',x_{12}), \tag{10}$$

where

$$a_p=[7\alpha N_c\zeta(3)\ln(1/x_p)/\pi]^{-1}=a(x_p), \tag{11}$$

and Σ_n' denotes a partial summation over the parameters specifying diffractive eigenstates (for details see [6]). The prime is added because the summation over the masses M of the diffractively excited states is left until the very end. $W(x_{12},x_{02})$ is a symmetric function of x_{12} and x_{02}, and, e.g., for $x_{02}<x_{12}$

$$W(x_{12},x_{02})=x_{12}^{\gamma-2}F\left[1-\frac{1}{2}\gamma,1-\frac{1}{2}\gamma;1;\left(\frac{x_{02}}{x_{12}}\right)^2\right], \tag{12}$$

F being the hypergeometric function, and

$$D(\tilde{b},x)=\int d^2rdz\Phi(r,z)r\ln\left(\frac{\tilde{b}^2}{rx}\right)\exp\left[-\frac{a_p}{2}\ln^2\left(\frac{\tilde{b}^2}{rx}\right)\right], \tag{13}$$

where $\Phi(r,z)$ is the square of the proton wave function. For small a_p, one can evaluate Eq. (10) analytically (the relevant integrals are given in Appendix A) and obtain for the transverse virtual photon

$$\int d^2\tilde{b}d^2\tilde{b}'[\Sigma_n' P_n f_n(\tilde{\mathbf{b}}) f_n(\tilde{\mathbf{b}}')]$$
$$\equiv\langle f_n^2\rangle(\rho,\beta,x_p)$$
$$= 32\pi\alpha^5 n_{\rm eff}^2 N_c x_p^{-2\Delta_p}\left(\frac{a_p}{\pi}\right)^{1/2}$$
$$\times\int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\rho^{2-\gamma}\beta^{-\alpha N_c\chi(\gamma)/\pi}V(\gamma)r_0^{2+\gamma}e^{\gamma^2/4a_p}, \tag{14}$$

where

$$V(\gamma)=\int_0^1 F\left(1-\frac{1}{2}\gamma,1-\frac{1}{2}\gamma;1;y^2\right)dy, \tag{15}$$

and $r_0=\int d^2rdz\Phi(r,z)r$ is a nonperturbative parameter determined from the data. A fit to the proton structure function gives $r_0\approx 0.8$ fm [4,5].

At this point we introduce the nuclear algorithm with A collisions inside the nucleus. We write

$$t_A=\langle f_n^2\rangle\left\{1-f_{\rm eff}\int_{z_1}^{z_2}dz_3\rho(\mathbf{b},z_3)\right\}^{A-2}, \tag{16}$$

and

$$T_A=A(A-1)\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{z_1}dz_2 e^{i(z_1-z_2)mx_p}$$
$$\times\rho(\mathbf{b},z_1)\rho(\mathbf{b},z_2)t_A. \tag{17}$$

The question now is what to take for $f_{\rm eff}$? As we have already remarked, the results given in [6–8] do not go beyond the double scattering. Also, [6–8] do not provide us with amplitudes for the eigenstates of absorption but with amplitudes averaged over many eigenstates. In order to have such amplitudes, one would have to do new and, possibly, rather complicated calculations. Therefore, in order to *estimate* the multiple scattering corrections, we shall use the full amplitude for dipole-proton scattering at fixed ρ (this is the diameter of the ''quarkonium'' representing the virtual photon):

$$f_{\rm eff}(r_0,\rho,x_p)=\int d^2\tilde{b}T(r_0,\rho,\tilde{b},x_p), \tag{18}$$

where [7]

$$T(r_0,\rho,\tilde{b},x_p)=\pi\alpha^2 n_{\rm eff}\frac{r_0\rho}{\tilde{b}^2}\ln\left(\frac{\tilde{b}^2}{r_0\rho}\right)x_p^{-\Delta_p}\left(\frac{2a_p}{\pi}\right)^{3/2}$$
$$\times\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{\tilde{b}^2}{r_0\rho}\right)\right]. \tag{19}$$

The integration over $d^2\tilde{b}$ can be done (see Appendix A) and we obtain

$$f_{\rm eff}(r_0,\rho,x_p)=2^{3/2}\pi\alpha^2 n_{\rm eff}r_0\rho x_p^{-\Delta_p}\left(\frac{a_p}{\pi}\right)^{1/2}$$
$$\times\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{r_0}{\rho}\right)\right]. \tag{20}$$

A comment is in order here. To estimate the multiple scattering corrections one can use something other than $f_{\rm eff}$ effective amplitude, e.g., the one proposed in Ref. [14], Eq. (5). In our notation it amounts to taking the ratio $\langle f_n^2\rangle/f_{\rm eff}$ instead of $f_{\rm eff}$. As expected, $\langle f_n^2\rangle/f_{\rm eff}>f_{\rm eff}$ but both are of the same order of magnitude. Since the complete expressions for multiple scattering corrections need direct calculations of the contributions of interactions with three and more nucleons which are not done, we choose $f_{\rm eff}$ as a somewhat simpler expression.

The last point is to average T_A over the *probability* $\tilde{\Phi}$ to find a dipole in the virtual transverse or longitudinal photon [2]:

$$\tilde{\Phi}^T(\hat{z},\rho,Q)=2\frac{N_c\alpha_{\rm em}}{(2\pi)^2}e_f^2[\hat{z}^2+(1-\hat{z})^2]\hat{Q}^2K_1^2(\hat{Q}\rho), \quad (21a)$$

$$\tilde{\Phi}^L(\hat{z},\rho,Q)=8\frac{N_c\alpha_{\rm em}}{(2\pi)^2}e_f^2\hat{z}(1-\hat{z})\hat{Q}^2K_0^2(\hat{Q}\rho). \quad (21b)$$

Here,

$$\hat{Q}=\sqrt{\hat{z}(1-\hat{z})}Q. \quad (22)$$

Thus, the ''triple Pomeron''(TP), transverse (T), and longitudinal (L) contributions to the shadow, within the approximation (9), are

$$\Delta_A^{\rm TP(T,L)}(x,Q^2)=-\frac{2(A-1)}{\sigma_{\rm tot}}\int_{10x}^1 d\beta{\rm Re}\Bigg\{\int_0^1 d\hat{z}\int_0^{r_0}d^2\rho\tilde{\Phi}^{T,L}(\hat{z},\rho)\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{z_1}dz_2e^{i(z_1-z_2)mx_p}\rho(\mathbf{b},z_1)\rho(\mathbf{b},z_2)$$
$$\times\langle f_n^2\rangle(\rho,\beta,x_p)\Bigg[1-f_{\rm eff}(r_0,\rho,x_p)\int_{z_1}^{z_2}dz_3\rho(\mathbf{b},z_3)\Bigg]^{A-2}\Bigg\}. \quad (23)$$

For comments on the limits of the integration over β and ρ see Sec. V.

It is convenient to extract from Eq. (23) a correction factor $C_{ms}(x,Q^2)$ for multiple interactions higher than double. This is the factor which multiplies the double scattering contribution to give $\Delta_A^{\rm TP}(x,Q^2)$. Thus, we have

$$\Delta_A^{\rm TP(T,L)}(x,Q^2)=-C_{ms}(x,Q^2)\frac{2(A-1)}{\sigma_{\rm tot}}\int_{10x}^1 d\beta$$
$$\times{\rm Re}\Bigg\{\int_0^1 d\hat{z}\int_0^{r_0}d^2\rho\tilde{\Phi}^{T,L}(\hat{z},\rho)\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{z_1}dz_2e^{i(z_1-z_2)mx_p}\rho(\mathbf{b},z_1)\rho(\mathbf{b},z_2)\langle f_n^2\rangle(\rho,\beta,x_p)\Bigg\}. \quad (24)$$

$C_{ms}(x,Q^2)$ will be used in the next section to compute the shadow of the whole multiple scattering process from the shadow of the double scattering contribution. We believe that the level of accuracy of our calculations and the size of $C_{ms}(x,Q^2)$ justify this approximation.

IV. THE QUASIELASTIC CONTRIBUTIONS TO THE SHADOWING

This time the Ref. [7] provides us with the basic formulas and we will assume $A\gg1$; hence, again, we accept the approximation (9). The method used in [7] to obtain the double scattering contribution is different than in the case of ''triple Pomeron'' [6] and we now have at our disposal the probability *amplitude* for the intermediate state which, in the present case, is the one dipole state. Therefore, the procedure of integration over $\tilde{b}$ is done at the level of the amplitude which, subsequently, is contracted with the probability amplitude for finding a dipole in a photon of a given polarization, and the result squared.

So, starting from the formulas (19) and (20) of Ref. [7], we first replace $d\sigma_{T,L}/dM^2$ by $d\sigma_{T,L}/d\beta=(Q^2/\beta^2)(d\sigma_{T,L}/dM^2)$, do the integration of the amplitude T over $\tilde{b}$, and obtain the following double scattering terms in the approximation (9):

$$\frac{d\tilde{\sigma}_T}{d\beta}=\frac{N_c\alpha_{\rm em}e_f^2}{2\pi}\frac{Q^4}{\beta^2}\int_0^1 d\hat{z}\hat{z}^2(1-\hat{z})^2[\hat{z}^2+(1-\hat{z})^2]$$
$$\times\Bigg[\int_0^{r_0}d\rho\rho f_{\rm eff}(r_0,\rho,x_p)K_1(\hat{Q}\rho)J_1(\hat{M}\rho)\Bigg]^2 \quad (25a)$$

and

$$\frac{d\tilde{\sigma}_L}{d\beta}=4\frac{N_c\alpha_{\rm em}e_f^2}{2\pi}\frac{Q^4}{\beta^2}\int_0^1 d\hat{z}\hat{z}^3(1-\hat{z})^3$$
$$\times\Bigg[\int_0^{r_0}d\rho\rho f_{\rm eff}(r_0,\rho,x_p)K_0(\hat{Q}\rho)J_0(\hat{M}\rho)\Bigg]^2, \quad (25b)$$

where $f_{\rm eff}(r_0,\rho,x_p)$ is given by Eq. (18), and $\hat{M}=\sqrt{\hat{z}(1-\hat{z})}M$, with M being the mass of the virtual photon excitation.

In the double scattering cross sections (25) $f_{\rm eff}$ appears quadratically. Therefore, since the basic ingredient of $C_{ms}(x,Q^2)$ defined in Sec. III is the factor $[1-f_{\rm eff}\int_{z_1}^{z_2}dz_3\rho(\mathbf{b},z_3)]^{A-2}$ [defined through the same amplitude $f_{\rm eff}$ as in Eq. (25)], we accept that by multiplying $\Delta_A^{\rm QE}$ for just the double interaction inside the target nucleus by C_{ms} we obtain a reasonable approximation for $\Delta_A^{\rm QE}$ with all

possible (i.e., A) interactions included. The error made in this approximate procedure is not significant because C_{ms} is close to unity throughout the relevant region of x and Q^2 (see Fig. 3 below).

Following steps similar to those which led us to the formula (23), we obtain for the quasielastic contributions to the shadow of the transverse and longitudinal virtual photons the expressions

$$\Delta_A^{\mathrm{QE}(T)}(x,Q^2)=-\frac{2(A-1)}{\sigma_{\mathrm{tot}}}C_{ms}(x,Q^2)\int_{10x}^{1}d\beta\,\mathrm{Re}\Bigg\{\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{z_1}dz_2 e^{i(z_1-z_2)mx_p}\rho(\mathbf{b},z_1)\rho(\mathbf{b},z_2)\frac{N_c\alpha_{\mathrm{em}}e_f^2}{2\pi}\frac{Q^4}{\beta^2}$$
$$\times\int_0^1 d\hat{z}\hat{z}^2(1-\hat{z})^2[\hat{z}^2+(1-\hat{z})^2]\Bigg[\int_0^{r_0}d\rho\rho f_{\mathrm{eff}}(r_0,\rho,x_p)K_1(\hat{Q}\rho)J_1(\hat{M}\rho)\Bigg]^2\Bigg\} \tag{26a}$$

and

$$\Delta_A^{\mathrm{QE}(L)}(x,Q^2)=-\frac{2(A-1)}{\sigma_{\mathrm{tot}}}C_{ms}(x,Q^2)\int_{10x}^{1}d\beta\,\mathrm{Re}\Bigg\{\int d^2b\int_{-\infty}^{+\infty}dz_1\int_{-\infty}^{z_1}dz_2$$
$$\times e^{i(z_1-z_2)mx_p}\rho(\mathbf{b},z_1)\rho(\mathbf{b},z_2)\frac{4N_c\alpha_{\mathrm{em}}e_f^2}{2\pi}\frac{Q^4}{\beta^2}\int_0^1 d\hat{z}\hat{z}^3(1-\hat{z})^3\Bigg[\int_0^{r_0}d\rho\rho f_{\mathrm{eff}}(r_0,\rho,x_p)K_0(\hat{Q}\rho)J_0(\hat{M}\rho)\Bigg]^2\Bigg\}, \tag{26b}$$

where $f_{\mathrm{eff}}(r_0,\rho,x_p)$ is given by Eq. (20).

Clearly, setting $C_{ms}=1$ reduces both Eqs. (24) and (26) to the correct expressions for the double scattering contributions to the shadow. In fact, as we have already stressed, the double scattering term is more reliable than the multiple scattering correction because its input from [6–8] comes exclusively from the ''first principles.'' That is why it is of interest to discuss deuterium target. This will be discussed in a forthcoming paper.

V. DISCUSSION OF THE RESULTS AND THE CONCLUSIONS

Before presenting any results, a comment on the real parts of the amplitudes f is in order. First, the dependence of σ_{tot} on x, hence on $2m\nu\approx s$, implies that $\mathrm{Re}[f]$ must be different from zero. This follows from the derivative analyticity relations for the forward elastic hadronic amplitudes worked out in, e.g., Ref. [18]. We can introduce, approximately, this effect into all our expressions derived so far performing the replacements

$$\langle f_n^2\rangle\to\langle f_n^2\rangle(1-i\hat{\alpha})^2,\quad f_{\mathrm{eff}}\to f_{\mathrm{eff}}\,(1-i\hat{\alpha}), \tag{27}$$

where $\hat{\alpha}$ is the ratio of the real to imaginary parts of the forward amplitude. We use the leading term of the expansion of the forward amplitudes f given in [18]:

$$\hat{\alpha}=\frac{\mathrm{Re}f}{\sigma_{\mathrm{tot}}}\approx\tan\left(\frac{1}{2}\pi\Delta_p\right). \tag{28}$$

We accept $\Delta_p\approx 0.3$, hence $\hat{\alpha}\approx 0.5$.

Now the results. We have calculated all contributions to the shadow Δ_A from the triple Pomeron and the quasielastic diffractions for two target nuclei: lead (Pb, $A=208$) and calcium (Ca, $A=40$). All integrations in Eqs. (23) and (26) were done numerically with an extensive help of MATHEMATICA (more details are given in Appendix B).

The single nucleon densities $\rho(\mathbf{s},z)$ were taken in the standard form

$$\rho(\mathbf{s},z)=\frac{1}{V(1+e^{(r-r_A)/a})}, \tag{29}$$

with $r=\sqrt{\mathbf{s}^2+z^2}$, $r_A=1.2$ fm $A^{1/3}$, and $a=0.54$ fm,

$$V=\int d^2s dz(1+e^{(r-r_A)/a})^{-1}, \tag{30}$$

so that $\int d^2s dz\rho(\mathbf{s},z)=1$.

We assumed $\Delta_p=0.3$ which, through relation (4) and using $N_c=3$, gives the strong coupling constant $\alpha=0.11$. This is an acceptable set of parameters characterizing the BFKL Pomeron. We set $n_{\mathrm{eff}}=6.7$ to recover the fit to the experimental data of the F_2 of the nucleon [6–8].

In all integrations we restricted the values of x_p to

$$x_p=\frac{x}{\beta}\leqslant 0.1, \tag{31a}$$

which, as seen from Eq. (7), amounts to excluding distances $(z_1-z_2)\leqslant 2$ fm from the double and higher multiplicity interactions; physically, a reasonable condition to impose. This was done because in construction of the triple Pomeron contribution to Δ_A, Eq. (23), see also Appendix A, we explicitly used the approximation of small a_p, hence small x_p. We have checked that changing the upper limit for x_p, Eq. (31a), by a factor 1.5 or 2 does not change significantly Δ_A^{TP}, at the values of x for which we have done the calculations, namely, $x\leqslant 0.01$. This gives the lower limit for integration over β: $\beta>10x$.

The upper limit for β, on the other hand, should be taken close to unity because small masses give important contributions to shadowing. So, the integration range for β is taken as

$$10x<\beta<1\,. \tag{31b}$$

All this makes almost all *asymptotic* formulas for the ''triple Pomeron'' given in Refs. [6–8] unacceptable for our purposes because they lead to infinities for integrations over β's close to unity.

We wish to make it clear: by extending our calculation to β close to 1, hence to very small masses M, we go beyond the region of β's for which the original formulation of Refs. [4–8] was devised. We make this extrapolation because *the complete integral representations of various* $\Sigma_n P_n f_n f_n$ *given also in* [6–8] *lead to well-defined finite results, even for* $\beta\to 1$ and only they are employed in our calculations. The price to pay is loss of simple analytic expressions and necessity of numerical integrations. Although the asymptotic form of $\sigma_{\rm tot}=\Sigma_n P_n\sigma_n$ does not lead to any such problems, to be consistent with our treatment of $\Sigma_n P_n f_n f_n$, we also used its integral representation (1).

We set the cutoff parameter r^* of the integration over ρ, following the discussion of Ref. [7], at $r^*=r_0$. Unfortunately, in the case of nuclear targets the saturation of the values of the integrals over ρ does not look as good as that in the case of the nucleon target (see Fig. 2 of Ref. [7]), Therefore, we should treat r^* as a parameter with the lower bound equal to r_0. This implies that taking $r^*=r_0$ we tend to underestimate the shadow.

One more general comment: we accept that the contribution to Δ_A of the triple Pomeron, of the quasielastic transverse, and of the quasielastic longitudinal processes, are additive. In other words, we do not introduce any interferences between them.

In Figs. 1(a), 1(b) the total shadow $\Delta_A=\Delta_A^{\rm TP(T)}+\Delta_A^{\rm TP(L)}+\Delta_A^{\rm QE(T)}+\Delta_A^{\rm QE(L)}$ is presented for Pb and Ca targets. As we can see, the pattern of all curves for these two targets is very similar, only the size of the shadow for Pb is a factor of 2 larger than that for Ca. We have also checked through explicit calculations that all components of Δ_A of Pb are by the same factor ($\approx$ 2) larger than that of Ca. Therefore, we will, henceforth, show only the results for Pb.

In Figs. 2(a), 2(b) we show $\Delta_A^{\rm TP}$ with (a) and without (b) presence of the real part of the forward scattering amplitudes. Comparing this figure with Fig. 1 we see that $\Delta_A^{\rm TP}$ dominates Δ_A, and that the presence of $\hat{\alpha}\neq 0$ makes shadowing smaller. In fact a different from zero $\hat{\alpha}$ makes $|\Delta_A|$ smaller: by 16% at very small x and by 60% (!) at $x=0.01$. (The role of the real parts of the forward amplitudes at various x has been recently discussed in [14,19].)

In Figs. 3(a), 3(b) we show the correction factor for multiple scattering higher than double, $C_{ms}(x,Q^2)$, with (a) and without (b) presence of $\hat{\alpha}$. We can see that in either case, C_{ms} is close enough to unity to justify our procedures in construction of Δ_A.

The following conclusions result from inspection of the Figs. 1–3.

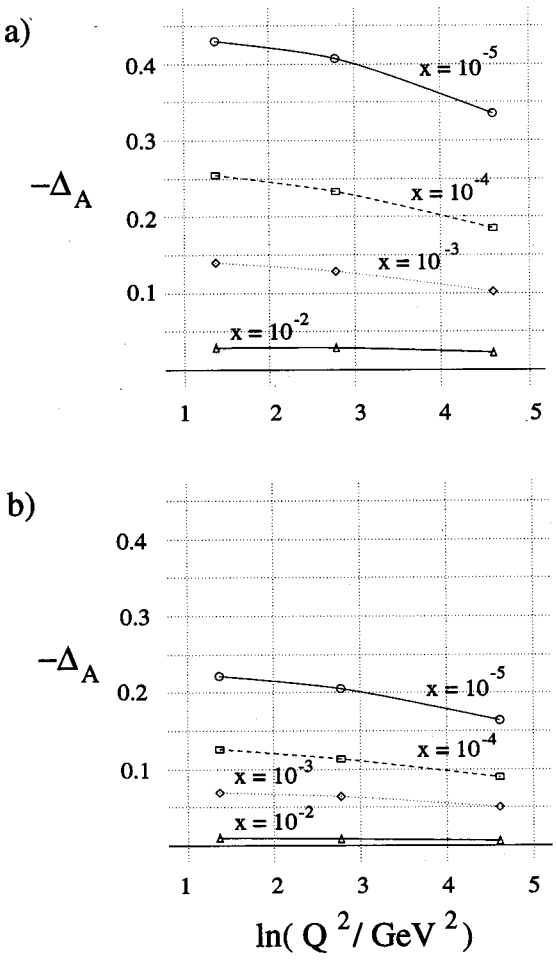

FIG. 1. The total shadow Δ_A for (a) Pb and (b) Ca nuclei. $-\Delta_A$ is plotted vs $\ln(Q^2/\mathrm{GeV}^2)$ for four values of the Bjorken scaling variable x. The ratio of the real to imaginary parts of the forward amplitudes is set at $\hat{\alpha}=\tan(\frac{1}{2}\pi\Delta_p)\approx 0.5$.

(i) In the region $x<10^{-3}$, our calculation shows an appreciable shadowing, increasing with decreasing x. The x dependence of Δ_A follows approximately the power law

$$\Delta_A\sim x^{-\lambda}, \tag{32}$$

with $\lambda\approx 0.25$, somewhat smaller than the assumed value of $\Delta_p=0.3$. An indication of saturation is seen at $Q^2=4$ GeV^2.

(ii) Δ_A decreases slowly with increasing Q^2. The effect is somewhat stronger at small x and large Q^2.

(iii) Multiple scattering corrections account for less than 20% of the value of Δ_A.

(iv) The existence of the real parts of the forward amplitudes makes shadowing appreciably smaller, especially at x close to $x=10^{-2}$.

Several comments are in order.

(a) The rather weak Q^2 dependence of Δ_A seems puzzling if one takes into account that in the dipole model the total (virtual) photon-hadron cross section decreases as Q^{-1} at large Q [3,4,8]. One would thus naively expect $\Delta_A\sim\langle f^2\rangle/\langle f\rangle$ to decrease also as Q^{-1} which is another version of the Bjorken-Gribov [20,21] paradox. In the dipole

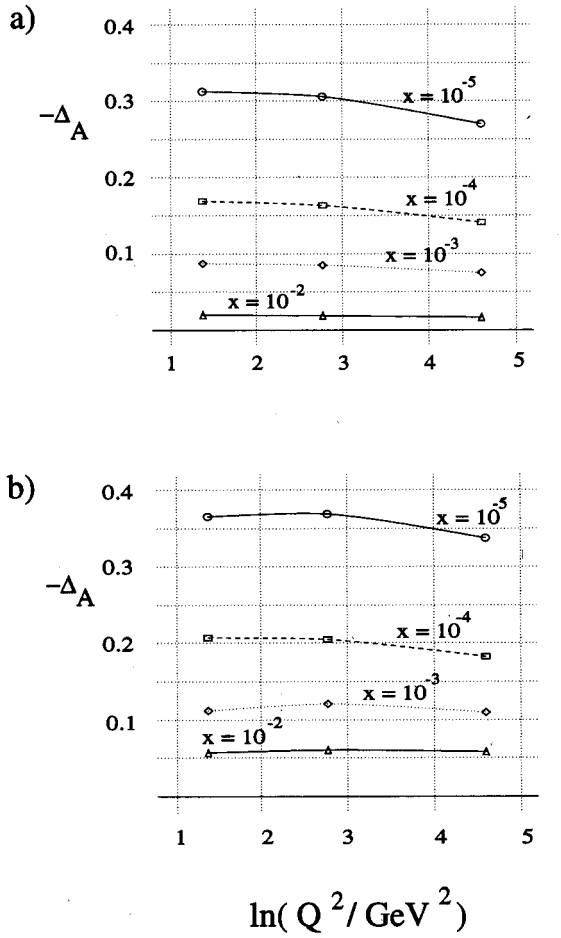

FIG. 2. The ''triple Pomeron'' contribution to Δ_A with $\hat{\alpha}\neq 0$ (a), and with $\hat{\alpha}=0$ (b), for Pb target.

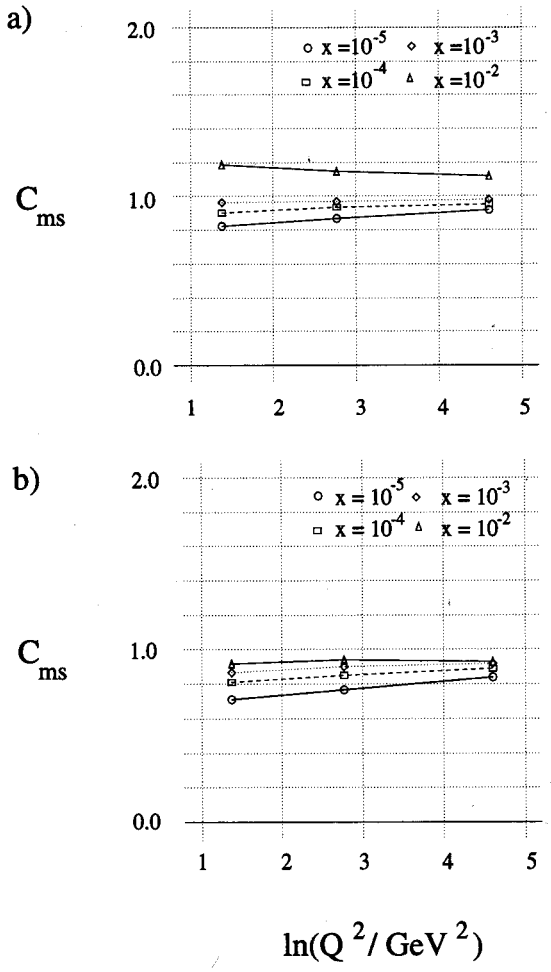

FIG. 3. The correction factor $C_{ms}(x,Q^2)$ for higher than double scatterings with $\hat{\alpha}\neq 0$ (a), and with $\hat{\alpha}=0$ (b), for Pb target.

model the paradox is resolved (and approximate scaling of Δ_A recovered) because of the cascade nature of the intermediate state which interacts inside the nucleus. This cascade nature implies very strong correlations between the dipoles in the quarkonium representing the virtual photon so that the double dipole density does not resemble at all the product of single densities and both $\langle f^2\rangle$ and $\langle f\rangle$ behave as Q^{-1} (apart from logarithmic corrections). The approximate scaling of Δ_A follows. It is interesting to observe that this explanation is radically different from that proposed in the parton model (first paper of [2], [21,22]).

(b) It should be pointed out that, although our calculation is based on perturbative QCD (as is the whole dipole approach), it, nevertheless, requires two ''nonperturbative'' parameters (r_0 and $n_{\rm eff}$) describing structure of the nucleon. These parameters were estimated from the data on the total (virtual) photon-proton cross section as they cannot be calculated in the dipole model. Particularly important for the absolute value of the shadowing turns out to be $n_{\rm eff}$, the average number of ''quarkonia'' forming the proton. This confirms the point of view that the perturbative QCD alone cannot describe fully deep inelastic scattering even at very small x [23]. It is rather remarkable, however, that the required ''nonperturbative'' input is so restricted.

(c) As was already pointed out in the main text, our calculation relies heavily on the calculation of the deep inelastic diffraction reported in [6,7]. This is a consequence of close relation between the nuclear shadowing and diffraction (see, e.g., [13]). Nevertheless, it is worth mentioning, because of the power-law tail in the impact parameter dependence of the quasielastic and triple Pomeron amplitudes, nuclear shadowing is much more sensitive to the details of the effective cutoff in the photon-nucleon impact parameter plane than is the total diffractive cross section. At this point the two calculations are substantially different.

(d) Our treatment of multiple scattering is only approximate. To improve on this point, it is necessary to calculate directly at least the contribution of interaction with three nucleons which would give an estimate of the accuracy of our approximation. This seems a feasible, although probably a complicated, calculation.

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APPENDIX A

Here, we give more details of some intermediate steps of calculations of Sec. III. First, the integration over $\tilde{\mathbf{b}}$ and $\tilde{\mathbf{b}}'$ in Eq. (10). It can be done using the formula

$$\int_{\max(r,x)}^{\infty} d^2\tilde{b}\,\tilde{b}^{-2}\ln\left(\frac{\tilde{b}^2}{rx}\right)\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{b^2}{rx}\right)\right]$$
$$=\pi\int_0^{\infty} du(u+h)\exp\left[-\frac{1}{2}a_p(u+h)^2\right]$$
$$=\frac{\pi}{a_p}e^{-(1/2)a_p h^2}, \tag{A1}$$

where $h=\ln(r/x)$.

The integration over d^2rdz is done with the help of the (asymptotic, $a_p\to 0$) formula

$$\int \Phi(r)r\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{r}{x}\right)\right]d^2rdz$$
$$=r_0\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{r_0}{x}\right)\right], \tag{A2}$$

where $r_0=\int\Phi(r)rd^2rdz$. The argument justifying Eq. (A2) goes as follows. We start with the identity

$$\left(\frac{r}{x}\right)\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{r}{x}\right)\right]=2\sqrt{\frac{\pi}{2a_p}}x_p^{\Delta_p}\int\frac{d\gamma}{2\pi i}\left(\frac{r}{x}\right)^{\gamma}$$
$$\times\exp\left[-\frac{\alpha N_c}{\pi}\chi(\gamma)\ln(x_p)\right], \tag{A3}$$

and write

$$\int \Phi(r)r\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{r}{x}\right)\right]d^2rdz$$
$$=2\sqrt{\frac{\pi}{2a_p}}x_p^{\Delta_p}x\int\frac{d\gamma}{2\pi i}\exp\left[-\frac{\alpha N_c}{\pi}\chi(\gamma)\ln(x_p)\right]$$
$$\times\int\Phi(r)\left(\frac{r}{x}\right)^{\gamma}d^2rdz. \tag{A4}$$

Let us denote

$$\int\Phi(r)r^{\gamma}d^2rdz\equiv\tilde{r}_0^{\gamma}. \tag{A5}$$

Of course, $\tilde{r}_0$ depends on γ. But, in the limit $a_p\to 0$, the main contribution to the integral (A4) comes from the region $\gamma\approx 1$ and we approximate $\tilde{r}_0$ by $r_0=\tilde{r}_0$ ($\gamma=1$). Using this we obtain from Eq. (A4)

$$\int\Phi(r)r\exp\left[-\frac{1}{2}a_p\ln^2\left(\frac{r}{x}\right)\right]d^2rdz$$
$$=2\sqrt{\frac{\pi}{2a_p}}x^{\Delta_p}x\int\frac{d\gamma}{2\pi i}\exp\left[-\frac{\alpha N_c}{\pi}\chi(\gamma)\ln(x_p)\right]\left(\frac{r_0}{x}\right)^{\gamma}. \tag{A6}$$

We use again Eq. (A3) and obtain Eq. (A2).

Now, the formula (14): Introducing Eqs. (A1) and (A2) into Eq. (10), we obtain

$$\int d^2\tilde{b}d^2\tilde{b}'[\Sigma_n' P_n f_n(\tilde{\mathbf{b}})f_n(\tilde{\mathbf{b}}')]$$
$$=\frac{16\pi\alpha_{\rm eff}^4\alpha N_c r_0^2}{x_p^{2\Delta_p}}\left(\frac{a_p}{\pi}\right)\int_{c-i\infty}^{c+i\infty}\frac{d\gamma}{2\pi i}\rho^{2-\gamma}\beta^{-\alpha N_c\chi(\gamma)/\pi}$$
$$\times\int dx_{12}dx_{02}W(x_{12},x_{02})\exp\left[-\frac{1}{2}a_p(h_0^2+h_1^2)\right], \tag{A7}$$

where $h_0=\ln(r_0/x_{02})$, $h_1=\ln(r_0/x_{12})$.

Next point is the integration over $dx_{02}dx_{12}$. This we do as in [6] (although the accuracy of this procedure may not be very good). Let us repeat the steps. Since the integrand is symmetric in x_{02},x_{12}, we have

$$I\equiv\int_0^{\infty}dx_{12}dx_{02}W(x_{12},x_{02})\exp\left[-\frac{1}{2}a_p(h_0^2+h_1^2)\right]$$
$$=2\int_0^{\infty}dx_{12}e^{-(1/2)a_p h_1^2}x_{12}^{\gamma-1}\int_0^{x_{12}}\frac{dx_{02}}{x_{12}}e^{-(1/2)a_p h_0^2}F\left(\frac{x_{02}}{x_{12}}\right). \tag{A8}$$

The second integral can be approximated (in the limit $a_p\approx 0$) by $e^{-(1/2)a_p h_1^2}V(\gamma)$, where $V(\gamma)$ is given below Eq. (14). Substituting this into Eq. (A8), we obtain

$$I=2V(\gamma)\int_0^{\infty}\frac{dx_{12}}{x_{12}}x_{12}^{\gamma}\exp\left[-a_p\ln^2\left(\frac{r_0}{x_{12}}\right)\right]. \tag{A9}$$

This is an easy integral [Gaussian, after substituting $u=\ln(r_0/x_{12})$] and we finally obtain

$$I=2V(\gamma)r_0^{\gamma}\sqrt{\frac{\pi}{a_p}}e^{\gamma^2/4a_p}. \tag{A10}$$

This completes the integrations in Eq. (A7) and we obtain Eq. (14).

APPENDIX B

We comment here on our numerical calculations. The contour integrals in Eqs. (1) and (14) and the integral of the hypergeometric function defining $V(\gamma)$ were calculated with MATHEMATICA.

For a fixed value of Bjorken's x, the contour integral in Eq. (14) becomes a function of two variables: ρ and β. Such a two-dimensional function was tabulated (for $x=10^{-5}$,

$x=10^{-4}$, $x=10^{-3}$, and $x=10^{-2}$) and used as an input in the FORTRAN program calculating the triple Pomeron contribution (23). Similarly, the integral over $\hat{z}$ of the probability distribution $\tilde{\Phi}(\hat{z},\rho,Q)$ was first tabulated with the help of MATHEMATICA, and then used as an input in the FORTRAN program calculating Eq. (23).

MATHEMATICA was used once again to calculate and tabulate the integrals over ρ of the Bessel functions in Eq. (26).

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