

Nonperturbative treatment of heavy quarks and mesons

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A formalism for studying heavy quarks in terms of model Dyson-Schwinger equations is developed. The formalism is the natural extension of a technique which has proved successful in a number of studies of light hadron physics. The dressed heavy quark propagator, calculated to leading order in the inverse quark mass, is incorporated in a treatment of mesons consisting of a heavy quark and light antiquark via the ladder approximation Bethe-Salpeter equation. In the limit of infinite heavy quark mass the model is found to respect the spectrum degeneracies present in heavy quark effective theory. An exploratory numerical analysis of a simple form of the model is carried out to assess its viability for studying D and B mesons. [S0556-2821(97)02501-0]

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I. INTRODUCTION

There has been a great deal of recent theoretical interest in heavy quark mesons, that is, mesons containing charm quarks or bottom quarks such as B and D mesons. The basis of this interest is heavy quark effective theory (HQET) [1], which hinges on the realization that in the limit of infinite quark mass extra symmetries, and hence degeneracies, occur beyond those normally associated with QCD [2]. It is a property of many existing HQET analyses, particularly of meson spectrum calculations, that dynamical self-dressing of the quarks is either not included at all or rarely included in a nonperturbative fashion. Examples of recent spectrum calculations in which heavy quarks are modeled essentially by free propagators are given in Ref. [3]. However, one could argue that a theory which does not prevent quarks from being on mass shell is not in keeping with the spirit of a confinement and may lead, in certain instances, to incorrect results. In Bethe-Salpeter equation calculations, for instance, there is in principle nothing to prevent the occurrence of spurious production thresholds.

At the other end of the meson spectrum, namely the physics of light quarks, the Dyson-Schwinger equation (DSE) technique [4] has enjoyed considerable success in modeling a number of light meson phenomena. Basic to the DSE technique is the quark propagator. The spacelike structure of this propagator has been well modeled within the DSE technique for light quarks. At a phenomenological level, model quark propagators based on numerical studies of approximate Dyson-Schwinger equations have proved efficacious in calculating light meson spectra [5–7], electromagnetic form factors of the pion and kaon [8], π - π scattering lengths [9], quark loop contributions to ρ - ω mixing [10], and the anomalous $\gamma\pi\rightarrow\gamma$ and $\gamma\pi\rightarrow\pi\pi$ form factors [11].

The aim of this paper is to formulate an extension of the DSE technique to the realm of heavy quarks. In the applications of the the DSE technique to light quarks listed above, it has not been necessary to determine the behavior of quark propagators well away from the real spacelike p^2 axis. How-

ever, for the purpose of accurately modeling confined heavy quarks, because the self-energy is small compared with the current quark mass m_Q , it is particularly important to know the analytic structure of the heavy quark propagator near the (timelike) bare fermion mass pole $p_{\text{Minkowski}}^2 = m_Q^2$. The method frequently used within the DSE technique of fitting analytic functions to the numerically determined spacelike quark propagator is inadequate for this task. It is for this reason that new ideas must be developed within the DSE technique if it is to be adapted to heavy quarks.

It is the philosophy of the DSE technique that confinement in QCD should be signaled by the absence of timelike poles in the quark propagator. This analytic structure is the result of nonperturbative quark self-dressing, which in principle could be demonstrated by a full treatment of the quark Dyson-Schwinger equation. Many numerical calculations of approximate quark Dyson-Schwinger equations in confining field theories have demonstrated propagator solutions which are free of timelike poles [4], though not necessarily free of complex conjugate poles away from the real p^2 axis [12]. Herein we shall examine in detail the analytic structure of the heavy quark propagator in the limit of infinite quark mass. We shall also consider the Bethe-Salpeter equation (BSE) for bound states of heavy quarks and light antiquarks. Our eventual aim is to make contact with the observed spectrum of D and B mesons. However, at the present level of approximation we are unable to find solutions to our model BSE because of spurious poles in the model quark propagators. We suggest ways in which the model can be improved to overcome this problem.

The layout of the paper is as follows. In Sec. II we develop the basic formalism for the heavy quark propagator. In Sec. III we consider the Bethe-Salpeter equation for mesons consisting of one heavy and one light antiquark. Sections IV and V deal with scalar and vector mesons, respectively, and demonstrate that the symmetries of HQET are preserved within the heavy quark limit of the DSE technique. Some numerical results are given in Sec. VI and conclusions drawn in Sec. VII.

II. HEAVY QUARK PROPAGATOR

Before understanding the procedures which must be employed to ensure the extension of the DSE technique to heavy quarks, it is important to appreciate that a naive translation of the light quark sector DSE technique, by setting the quark mass to large values in the existing formalism and computer codes, simply will not work. This is because the interesting physical phenomena are driven by nonperturbative contributions to the heavy quark propagator in the vicinity of the bare fermion mass pole. These contributions are of order $1/m_Q$ compared with the bare fermion propagator. Earlier studies of the heavy fermion limit of another confining theory, three-dimensional QED (QED3) [13], demonstrate categorically that for fermions whose mass is greater than the typical scale of the interaction of the theory, this information cannot be accurately obtained by an extrapolation of the spacelike solution of the fermion propagator produced by the usual DSE technique, and a new formalism must be developed.

We begin with the general form for the inverse of the dressed quark propagator consistent with Lorentz and *CPT* covariance:

$$S^{-1}(p) = i\gamma \cdot p A(p^2) + B(p^2), \quad (2.1)$$

where A and B are scalar functions. We use a Euclidean metric in which timelike four-vectors satisfy $p^2 < 0$. Within the DSE technique the full quark Dyson-Schwinger equation is typically truncated to a manageable approximation. For the purposes of illustrating the extension to the heavy quark sector, we consider the simplest viable approximation to the quark DSE,

$$S^{-1}(p) - i\gamma \cdot p - m_Q = \frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu S(q) \gamma_\mu \Delta(p - q), \quad (2.2)$$

in which the quark-gluon vertex has been replaced by a bare vertex, and the dressed gluon propagator is modeled by a representation $g^2 D_{\mu\nu}(k) = \delta_{\mu\nu} \Delta(k^2)$ in a Feynman-like gauge. Our formalism for calculating model heavy quark propagators can be extended without trouble to include more realistic vertex *Ansätze* (to allow for gauge covariance [4]) and to more realistic model gluon propagators (see, for instance, [14]).

In the limit of heavy quark masses, $m_Q \rightarrow \infty$, the dressed quark propagator is dominated by the bare form $S_{\text{bare}}^{-1} = i\gamma \cdot p + m_Q$. However, it is important to isolate from the full propagator order $1/m_Q$ corrections to the bare propagator which drive confining and remnant chiral symmetry-breaking effects. To this end we set

$$A(p^2) = 1 + \frac{\Sigma_A(k)}{m_Q}, \quad B(p^2) = m_Q \left(1 + \frac{\Sigma_B(k)}{m_Q} \right), \quad (2.3)$$

where, in the spirit of HQET, we have introduced a new momentum variable

$$p_\mu = im_Q v_\mu + k_\mu, \quad (2.4)$$

with v_μ a constant unit four-vector.

The functions Σ_A and Σ_B can be expanded as a series in $1/m_Q$ and the DSE solved order by order as a set of coupled integral equations. In the Appendix we derive the set of four coupled equations required to solve the heavy quark propagator to $O(1/m_Q)$. To leading order, we find that Σ_A and Σ_B can be written as functions of the single variable $k \cdot v$, and the quark propagator becomes

$$S(p) = \frac{1 + \not{v}}{2} \frac{1}{ik \cdot v + \Sigma_B(k \cdot v) - \Sigma_A(k \cdot v)} + O(m_Q^{-1}). \quad (2.5)$$

Substituting into Eq. (2.2), changing the variable of integration to k' defined by $q_\mu = im_Q v_\mu + k'_\mu$, and projecting out coefficients of $\not{v}$, $\not{k}$, and I gives a set of integral equations which can be summarized in the equation [see Eqs. (A8) and (A9)]

$$\Sigma(k \cdot v) = \frac{4}{3} \int \frac{d^4 k'}{(2\pi)^4} \Delta(k - k') \frac{1}{ik' \cdot v + \Sigma(k' \cdot v)}, \quad (2.6)$$

where

$$\Sigma(k \cdot v) = \Sigma_B(k \cdot v) - \Sigma_A(k \cdot v). \quad (2.7)$$

Note that to leading order in $1/m_Q$ the heavy quark self-energy is specified by a single, complex-valued, scalar function. This equation is numerically tractable once the function Δ is specified, and provides the required information regarding fermion self-dressing in the vicinity of the bare fermion mass pole, $k_\mu \approx 0$, in the limit $m_Q \rightarrow \infty$.

Note also that the change of integration variable from q to k' involves an assumption that the propagator, and hence the functions $A(q^2)/[q^2 A(q^2)^2 + B(q^2)^2]$ and $B(q^2)/[q^2 A(q^2)^2 + B(q^2)^2]$, be analytic over the region

$$\text{Re}(q^2) > -m_Q^2 + \frac{[\text{Im}(q^2)]^2}{4m_Q^2}. \quad (2.8)$$

In the limit $m_Q \rightarrow \infty$ this translates to the requirement that $1/[ik \cdot v + \Sigma(k \cdot v)]$ be analytic for $\text{Im}(k \cdot v) < 0$.

III. BETHE-SALPETER EQUATION FOR HEAVY-LIGHT MESONS

The combination of a bare vertex DSE and ladder approximation Bethe-Salpeter equation produces well the spectrum of light quark mesons [5,6]. Here we extend the formalism to mesons containing one heavy quark and one light antiquark in the limit $m_Q \rightarrow \infty$. We note that our work differs in a fundamental way from that of Villate *et al.* [15] who use an instantaneous approximation to study heavy quarkonium states via the combination of a DSE and BSE. Because the light quark present in our case must be treated in a fully relativistic fashion, the instantaneous approximation is not applicable.

The homogeneous ladder approximation to the meson Bethe-Salpeter equation in a Feynman-like gauge is

$$\Gamma(p, P) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \gamma_\mu S_q(q-\xi P) \Gamma(q, P) \\ \times S_Q(q+(1-\xi)P) \gamma_\mu. \quad (3.1)$$

Here $S_q(p)$ is the full light quark propagator, $S_Q(p)$ the full heavy quark propagator, and $\delta_{\mu\nu}\Delta(p-q)$ the same effective gluon propagator as that employed in the DSE. The Bethe-Salpeter amplitude $\Gamma(q, P)$ is defined so that external outgoing and incoming quark lines carry momenta $q+(1-\xi)P$ and $q-\xi P$, respectively.

For the light quark, we write the propagator as

$$S_q(p) = -i\not{p}\sigma_V(p^2) + \sigma_S(p^2) = \frac{1}{i\not{p}A_q(p^2) + B_q(p^2)}. \quad (3.2)$$

The vector and scalar parts σ_V and σ_S are related to the functions A_q and B_q simply by dividing by the quantity $p^2 A_q^2(p^2) + B_q^2(p^2)$. Using Eq. (2.2), with the quark mass replaced by the light current quark mass m_q , A_q and B_q are given by the integral equations

$$p^2[A_q(p^2) - 1] = \frac{8}{3} \int \frac{d^4 q}{(2\pi)^4} p \cdot q \frac{A_q(q^2)}{q^2 A_q^2(q^2) + B_q^2(q^2)} \\ \times \Delta(p-q), \quad (3.3)$$

$$B_q(p^2) - m_q = \frac{16}{3} \int \frac{d^4 q}{(2\pi)^4} \frac{B_q(q^2)}{q^2 A_q^2(q^2) + B_q^2(q^2)} \Delta(p-q). \quad (3.4)$$

For the heavy quark we use the leading order heavy quark propagator, Eq. (2.5),

$$S_Q(im_Q v_\mu + k_\mu) = \frac{1 + \not{v}}{2} \frac{1}{ik \cdot v + \Sigma(k \cdot v)}, \quad (3.5)$$

where $\Sigma(k \cdot v)$ is obtained from Eq. (2.6).

We choose to work in the rest frame of the meson and set

$$P_\mu = (\mathbf{0}, i(m_Q + \delta)), \quad (3.6)$$

where δ is the contribution to the meson mass from the ‘‘brown muck’’ [1] of light quarks and glue surrounding the heavy quark. Our aim is to obtain the BSE in the limit $m_Q \rightarrow \infty$ in a form containing δ , but independent of m_Q . It is convenient to reparametrize the momentum partitioning in Eq. (3.1) via

$$\xi = \frac{\eta \delta}{m_Q + \delta}. \quad (3.7)$$

Since the light quarks Eqs. (3.3) and (3.4), are solved in the first instance for real, spacelike momenta, we see that the choice $\eta=0$ has the advantage that no numerical extrapolation of S_q to complex momenta is required. On the other hand, using the freedom inherent in v_μ to set $v_\mu = (\mathbf{0}, 1)$, we see the choice $\eta=1$ has the advantage that no extrapolation of the numerical solution of the heavy quark self-energy $\Sigma(k_4)$ is required away from the real k_4 axis. In practice, one

expects that the optimum choice of η requiring minimal numerical extrapolation of either propagator lies somewhere between these values.

Before deriving integral equations for scalar and vector heavy-light mesons, it is convenient make some definitions. Define the projection operators

$$\Lambda_\pm = (1 \pm \gamma_4)/2. \quad (3.8)$$

The light quark propagator can then be written as

$$S_q(q - \xi P) = S_+ \Lambda_+ + S_- \Lambda_- + S_\perp \hat{\mathbf{q}}, \quad (3.9)$$

where

$$\hat{\mathbf{q}} = (q_1 \gamma_1 + q_2 \gamma_2 + q_3 \gamma_3) / |\mathbf{q}|, \quad (3.10)$$

with $|\mathbf{q}| = \sqrt{q_1^2 + q_2^2 + q_3^2}$ and

$$S_+(q_4, |\mathbf{q}|) = \sigma_S(s) - (\eta \delta + i q_4) \sigma_V(s), \quad (3.11)$$

$$S_-(q_4, |\mathbf{q}|) = \sigma_S(s) + (\eta \delta + i q_4) \sigma_V(s), \quad (3.12)$$

$$S_\perp(q_4, |\mathbf{q}|) = -i |\mathbf{q}| \sigma_V(s), \quad (3.13)$$

where

$$s = |\mathbf{q}|^2 + q_4^2 - \eta^2 \delta^2 - 2i \eta \delta q_4. \quad (3.14)$$

With $v_\mu = (\mathbf{0}, 1)$, the heavy quark propagator is, from Eq. (3.5),

$$S_Q[q + (1-\xi)P] = \frac{\Lambda_+}{i q_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}. \quad (3.15)$$

We see that, by judicious choice of momentum partitioning, the quark propagators, and hence the BSE, are independent of m_Q in the $m_Q \rightarrow \infty$ limit.

IV. SCALAR AND PSEUDOSCALAR MESONS

The general forms of the pseudoscalar and scalar Bethe-Salpeter amplitudes are given by [16]

$$\Gamma^{\text{pseud}}(q, P) = (f_1 + f_2 \not{P} + f_3 \hat{\mathbf{q}} + f_4 [\hat{\mathbf{q}}, \not{P}]) \gamma_5, \quad (4.1)$$

$$\Gamma^{\text{scalar}}(q, P) = f_1 + f_2 \not{P} + f_3 \hat{\mathbf{q}} + f_4 [\hat{\mathbf{q}}, \not{P}], \quad (4.2)$$

where $f_1 - f_4$ are functions of q^2 , P^2 and $q \cdot P$. With the P_μ chosen as Eq. (3.6), these can equally well be written in terms of the projection operators $\Lambda_\pm$ as

$$\Gamma^{\text{pseud}}(q) = (f_+ \Lambda_+ + f_- \Lambda_- + g_+ \hat{\mathbf{q}} \Lambda_+ + g_- \hat{\mathbf{q}} \Lambda_-) \gamma_5, \quad (4.3)$$

$$\Gamma^{\text{scalar}}(q) = f_+ \Lambda_+ + f_- \Lambda_- - g_+ \hat{\mathbf{q}} \Lambda_+ - g_- \hat{\mathbf{q}} \Lambda_-, \quad (4.4)$$

where $f_\pm$ and $g_\pm$ are functions of q_4 and $|\mathbf{q}|$.

By substituting Eqs. (3.9) and (3.15) for the quark propagators into Eq. (3.1) and making use of the identities

$$\gamma_\mu \Lambda_\pm \gamma_\mu = \Lambda_\pm + 3 \Lambda_\mp, \quad \gamma_\mu \hat{\mathbf{q}} \Lambda_\pm \gamma_\mu = -\hat{\mathbf{q}}, \quad (4.5)$$

a set of coupled integral equations can be projected out. For the pseudoscalar meson we obtain

$$f_-(p) = \frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \times \frac{S_- f_-(q) + S_\perp g_-(q)}{i q_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \quad (4.6)$$

$$f_+(p) = 3f_-(p), \quad (4.7)$$

$$g_-(p) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \hat{\mathbf{p}} \cdot \hat{\mathbf{q}} \Delta(p-q) \times \frac{S_+ g_-(q) + S_\perp f_-(q)}{i q_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \quad (4.8)$$

$$g_+(p) = g_-(p). \quad (4.9)$$

The scalar equations can be obtained from these by making the replacements $\sigma_S \rightarrow -\sigma_S$ in Eqs. (3.11) and (3.12).

V. VECTOR AND AXIAL VECTOR MESONS.

Llewellyn-Smith [16] gives the general form of the $J^P = 1^-$ BS amplitude as

$$\Gamma_\mu(q, P) = T_\mu(A + q \cdot PB\hat{\mathbf{q}} + C\hat{\mathbf{P}} + D[\hat{\mathbf{q}}, \hat{\mathbf{P}}]) - Q_\mu(q \cdot PB + 2\hat{\mathbf{P}}D) + Q_\mu(E + F\hat{\mathbf{q}} + q \cdot PG\hat{\mathbf{P}} + H[\hat{\mathbf{q}}, \hat{\mathbf{P}}]), \quad (5.1)$$

where

$$T_\mu = \left(\delta_{\mu\nu} - \frac{P_\mu P_\nu}{P^2} \right) \gamma_\nu, \quad Q_\mu = \left(\delta_{\mu\nu} - \frac{P_\mu P_\nu}{P^2} \right) q_\nu, \quad (5.2)$$

and $A-H$ are functions of q^2 , P^2 , and $q \cdot P$. The second line of Eq. (5.1) serves the purpose of ensuring that $A-H$ are even functions of $q \cdot P$ if the fermion is odd under charge conjugation. Since we are interested in $q-Q$ mesons, charge conjugation is not important, and this line can be ignored.

Working in the vector meson rest frame, Eq. (3.6), the general form of the amplitude can be recast in the form $\Gamma_\mu = (\vec{\Gamma}, 0)$ with

$$\vec{\Gamma}(q) = \vec{\Gamma}^+(q)\Lambda^+ + \vec{\Gamma}^-(q)\Lambda^-, \quad (5.3)$$

where

$$\vec{\Gamma}^\pm(q) = \vec{\gamma}(f_\pm + \hat{\mathbf{q}}g_\pm) + \hat{\mathbf{q}}(h_\pm + \hat{\mathbf{q}}k_\pm), \quad (5.4)$$

with $f_\pm$, $g_\pm$, $h_\pm$, and $k_\pm$ functions of q_4 and $|\mathbf{q}|$, $\vec{\gamma} = (\gamma_1, \gamma_2, \gamma_3)$, and $\hat{\mathbf{q}} = (q_1, q_2, q_3)/|\mathbf{q}|$. Using the identity $\vec{\gamma}\hat{\mathbf{q}} = \hat{\mathbf{q}} + \frac{1}{2}[\vec{\gamma}, \hat{\mathbf{q}}]$, it is convenient to absorb part of the $g_\pm$ terms into a redefined $h_\pm$ and write instead

$$\Gamma_i^\pm(q) = (\delta_{ij}f_\pm + \hat{q}_i\hat{q}_j k_\pm)\gamma_j + \frac{1}{2}g_\pm[\gamma_i, \hat{\mathbf{q}}] + h_\pm\hat{q}_i. \quad (5.5)$$

The following relations are then useful for extracting the scalar coefficient functions from $\Gamma_i^\pm$:

$$f_\pm(p) = \frac{1}{8}(\delta_{ij} - \hat{p}_i\hat{p}_j)\text{tr}[\gamma_j\Gamma_i^\pm(p)], \quad (5.6)$$

$$g_\pm(p) = -\frac{1}{16}\text{tr}\{\gamma_i, \hat{\mathbf{p}}\}\Gamma_i^\pm(p), \quad (5.7)$$

$$h_\pm(p) = \frac{1}{4}\text{tr}[\hat{p}_i\Gamma_i^\pm(p)], \quad (5.8)$$

$$k_\pm(p) = \frac{1}{8}(3\hat{p}_i\hat{p}_j - \delta_{ij})\text{tr}[\gamma_j\Gamma_i^\pm(p)]. \quad (5.9)$$

Substituting Eq. (5.5) into the BSE (3.1) together with the propagators Eqs. (3.9) and (3.15), and making use of the identities

$$\gamma_\mu\Lambda_\pm\gamma_\mu = \Lambda_\pm + 3\Lambda_\mp, \quad \gamma_\mu\gamma_i\Lambda_\pm\gamma_\mu = -\gamma_i(\Lambda_\pm + \Lambda_\mp), \quad (5.10)$$

$$\gamma_\mu[\gamma_i, \hat{\mathbf{q}}]\Lambda_\pm\gamma_\mu = [\gamma_i, \hat{\mathbf{q}}](\Lambda_\pm - \Lambda_\mp), \quad (5.11)$$

we obtain the following set of coupled integral equations:

$$f_+(p) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \{ [-S_- f_+(q) + S_\perp g_+(q)] - \frac{1}{2}[S_- k_+ + S_\perp(g_+ + h_+)] [1 - (\hat{\mathbf{p}} \cdot \hat{\mathbf{q}})^2] \} \times \frac{1}{i q_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \quad (5.12)$$

$$f_-(p) = f_+(p), \quad (5.13)$$

$$g_+(p) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \times [S_+ g_+(q) - S_\perp f_+(q)] \hat{\mathbf{p}} \cdot \hat{\mathbf{q}} \times \frac{1}{i q_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \quad (5.14)$$

$$g_-(p) = -g_+(p), \quad (5.15)$$

$$h_+(p) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \times \{ S_+ h_+(q) + S_\perp [f_+(q) + k_+(q)] \} \hat{\mathbf{p}} \cdot \hat{\mathbf{q}} \times \frac{1}{i q_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \quad (5.16)$$

$$h_-(p) = 3h_+(p), \quad (5.17)$$

$$k_+(p) = -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \frac{1}{2} \{ S_- k_+(q) + S_\perp [g_+(q) + h_+(q)] \} [1 - 3(\hat{\mathbf{p}} \cdot \hat{\mathbf{q}})^2] \times \frac{1}{i q_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \quad (5.18)$$

$$k_-(p) = k_+(p). \quad (5.19)$$

These equations can be simplified greatly by setting

$$\tilde{f} = 3f_+ + k_+, \quad (5.20)$$

$$\tilde{g} = h_+ - 2g_+, \quad (5.21)$$

$$\tilde{h} = g_+ + h_+, \quad (5.22)$$

$$\tilde{k} = k_+. \quad (5.23)$$

This gives

$$\begin{aligned} \tilde{f}(p) = & \frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \Delta(p-q) \\ & \times \frac{S_-(q)\tilde{f}(q) + S_+(q)\tilde{g}(q)}{iq_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \end{aligned} \quad (5.24)$$

$$\begin{aligned} \tilde{g}(p) = & -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \hat{\mathbf{p}} \cdot \hat{\mathbf{q}} \Delta(p-q) \\ & \times \frac{S_+(q)\tilde{g}(q) + S_-(q)\tilde{f}(q)}{iq_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \end{aligned} \quad (5.25)$$

$$\begin{aligned} \tilde{h}(p) = & -\frac{4}{3} \int \frac{d^4 q}{(2\pi)^4} \hat{\mathbf{p}} \cdot \hat{\mathbf{q}} \Delta(p-q) \\ & \times \frac{S_+(q)\tilde{h}(q) + S_-(q)\tilde{k}(q)}{iq_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}, \end{aligned} \quad (5.26)$$

$$\begin{aligned} \tilde{k}(p) = & -\frac{2}{3} \int \frac{d^4 q}{(2\pi)^4} (1 - 3(\hat{\mathbf{p}} \cdot \hat{\mathbf{q}})^2) \Delta(p-q) \\ & \times \frac{S_-(q)\tilde{k}(q) + S_+(q)\tilde{h}(q)}{iq_4 - (1-\eta)\delta + \Sigma[q_4 + i(1-\eta)\delta]}. \end{aligned} \quad (5.27)$$

Equations (5.24) and (5.25) are equivalent to the scalar equations (4.6) and (4.8). They represent the 1^- meson whose light quark spin content is $j = \frac{1}{2}$, which, as predicted by HQET, is degenerate with the scalar 0^- meson. Equations (5.26) and (5.27) represent the 1^- meson whose light quark spin content is $j = \frac{3}{2}$.

The 1^+ meson Bethe-Salpeter amplitude is obtained by multiplying the general form Eq. (5.3) through by γ_5 . The corresponding set of integral equations can be obtained from those above by making the replacement $\sigma_S \rightarrow -\sigma_S$ in Eqs. (3.11) and (3.12).

VI. NUMERICAL CALCULATIONS

In order to explore the potential of our formalism we consider here the simple Gaussian gluon propagator ansatz

$$\Delta(p-q) = \frac{3}{16} (2\pi)^4 \frac{\mu^2}{\alpha^2 \pi^2} e^{-(p-q)^2/\alpha}. \quad (6.1)$$

This model propagator is simple enough to render the angular integrals in both the SDE and BSE analytically tractable, but has the disadvantage that it underestimates the gluon

interaction strength in the ultraviolet. However, the evidence from light meson applications of the SDE technique is that it is the infrared behavior of the gluon propagator which most influences the low energy physics. Furthermore, the large mass expansion makes sense for nonperturbative interactions, but not for hard gluons. By using a Gaussian model gluon propagator we provide an ultraviolet cutoff within a treatment which does not admit hard gluon contributions. We therefore regard this model propagator as an acceptable starting point. For small α , Eq. (6.1) reduces to the infrared-dominant model

$$\lim_{\alpha \rightarrow 0} \Delta(p-q) = \frac{3}{16} (2\pi)^4 \mu^2 \delta^4(p-q), \quad (6.2)$$

which has frequently been employed in exploratory applications of the DSE technique to light quarks [17,18].

Substituting Eq. (6.1) into the heavy quark DSE (2.6) with $v_\mu = (0,1)$ gives

$$\Sigma(k_4) = \frac{\mu^2}{4(\alpha\pi)^{1/2}} \int_0^\infty dk'_4 \left[\frac{e^{-(k_4-k'_4)^2/\alpha}}{ik'_4 + \Sigma(k'_4)} + \frac{e^{-(k_4+k'_4)^2/\alpha}}{-ik'_4 + \Sigma(k'_4)^*} \right]. \quad (6.3)$$

Here we have assumed $\Sigma(-k_4) = \Sigma(k_4)^*$, this being a property of the obvious extension to negative k_4 of the solution

$$\lim_{\alpha \rightarrow 0} \Sigma(k_4) = \begin{cases} \frac{1}{2} [-ik_4 + \sqrt{\mu^2 - k_4^2}] & \text{if } 0 \leq k_4 < \mu, \\ \frac{i}{2} [-k_4 + \sqrt{k_4^2 - \mu^2}] & \text{if } k_4 \geq \mu, \end{cases} \quad (6.4)$$

to the algebraic equation arising from the infrared dominant propagator, Eq. (6.2). Substituting the Gaussian propagator, Eq. (6.1), into the light quark DSE's (3.3) and (3.4) gives

$$\begin{aligned} A_q(p^2) - 1 = & \frac{\mu^2}{\alpha p^2} \int_0^\infty q^3 dq \frac{A_q(q^2)}{q^2 A_q(q^2)^2 + B_q(q^2)^2} \\ & \times e^{-(p^2+q^2)/\alpha} I_2\left(\frac{2pq}{\alpha}\right), \end{aligned} \quad (6.5)$$

$$\begin{aligned} B_q(p^2) - m_q = & \frac{2\mu^2}{\alpha p} \int_0^\infty q^2 dq \frac{B_q(q^2)}{q^2 A_q(q^2)^2 + B_q(q^2)^2} \\ & \times e^{-(p^2+q^2)/\alpha} I_1\left(\frac{2pq}{\alpha}\right), \end{aligned} \quad (6.6)$$

where I_1 and I_2 are modified Bessel functions.

Here we shall carry out an initial study of the viability of the model Gaussian gluon propagator in the light quark chiral limit $m_q \rightarrow 0$ and heavy quark limit $m_Q \rightarrow \infty$. In this limit the parameter α in Eq. (6.1) can be scaled to unity in all equations by making the replacements

$$\mu = \sqrt{\alpha} \hat{\mu}, \quad \delta = \sqrt{\alpha} \hat{\delta}, \quad (6.7)$$

$$\Sigma(k_4) = \sqrt{\alpha} \hat{\Sigma}(\hat{k}_4), \quad A(p^2) = \hat{A}(\hat{p}^2), \quad B(p^2) = \sqrt{\alpha} \hat{B}(\hat{p}^2), \quad (6.8)$$

where

TABLE I. Position of the poles s^{pole} in the light quark propagator and k_4^{pole} in the heavy quark propagator, and the maximum accessible values of δ_{max} and corresponding momentum partitioning parameter η_{opt} . Dimensionful quantities can be extracted from the dimensionless quantities listed via Eqs. (6.7)–(6.9).

$\hat{\mu}$	$\hat{s}^{\text{pole}}$	$\hat{k}_4^{\text{pole}}$	$\hat{\delta}_{\text{max}}$	η_{opt}
2.5	$-0.126 + 0.000i$	$1.802 + 0.957i$	1.307	0.268
3.0	$-0.540 + 0.000i$	$2.285 + 1.034i$	1.769	0.415
3.5	$-1.829 + 0.919i$	$2.769 + 1.102i$	2.494	0.558
4.0	$-1.646 + 2.234i$	$3.255 + 1.162i$	2.649	0.561
5.0	$-1.246 + 4.546i$	$4.229 + 1.264i$	2.991	0.577
6.0	$-0.327 + 7.216i$	$5.207 + 1.351i$	3.294	0.589
7.5	$2.132 + 11.711i$	$4.680 + 2.649i$	4.859	0.455

$$\hat{k}_4 = k_4 / \sqrt{\alpha}, \quad \hat{p} = p / \sqrt{\alpha}. \quad (6.9)$$

It is then sufficient to explore the parameter space of the single parameter $\hat{\mu}$.

We have solved Eqs. (6.5) and (6.6) by numerical iteration for real, spacelike $\hat{p}^2$ for a range of values of $\hat{\mu}$. To extrapolate the light quark propagator from the spacelike axis into the complex p^2 plane, the right-hand sides of Eqs. (3.3) and (3.4) were integrated along the real spacelike q^2 axis using the converged solutions together with complex values of $\hat{p}^2$. We find that for $\hat{\mu} < 3$, the light quark propagator has a timelike pole, and that for larger values of $\hat{\mu}$ the pole moves away from the real axis into the complex plane. Table I lists the position $\hat{p}^2 = \hat{s}^{\text{pole}}$ of the pole as a function of $\hat{\mu}$.

To obtain a numerical solution to Eq. (6.3) for real $\hat{k}_4$ we iterated from Eq. (6.4). The numerical solution for $\hat{\mu} = 3.5$ is shown in Fig. 1. The function $\hat{\Sigma}$ is extended to complex values of $\hat{k}_4$ by integrating the right-hand side of Eq. (6.3) along the real k'_4 axis. We find numerically that the requirement stated at the end of Sec. III, viz., that the heavy quark propagator be free of singularities for $\text{Im}k_4 < 0$, is satisfied over the range of $\hat{\mu}$ listed in Table I. However, we also find

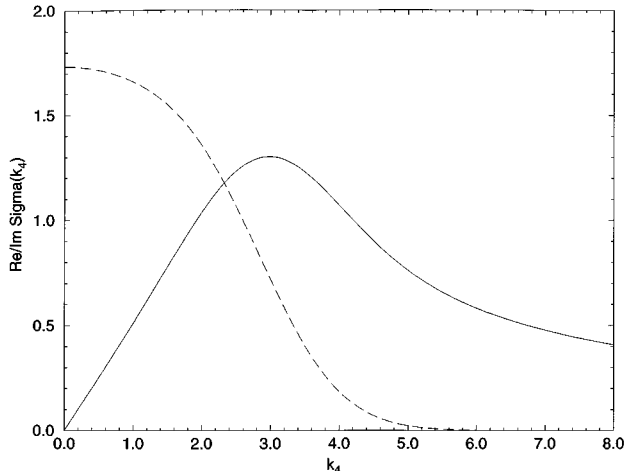


FIG. 1. The heavy quark self-energy obtained by solving Eq. (6.3): $\text{Re}\hat{\Sigma}(\hat{k}_4)$ (dashed curve) and $-\text{Im}\hat{\Sigma}(\hat{k}_4)$ (solid curve) for $\hat{\mu} = \mu / \sqrt{\alpha} = 3.5$.

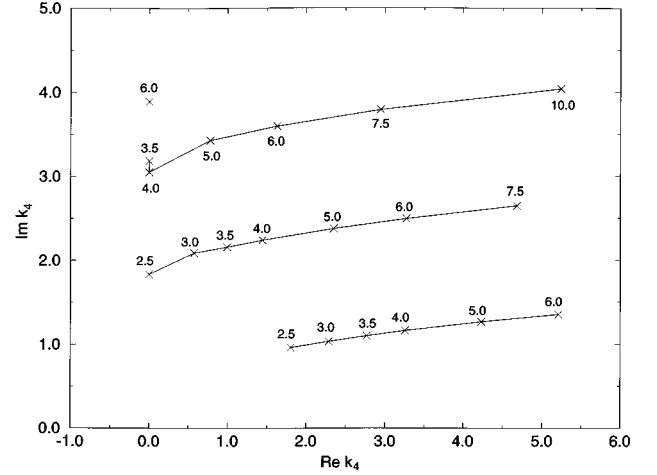


FIG. 2. The position $\hat{k}_4^{\text{pole}}$ of the poles in the heavy quark propagator for the values of $\hat{\mu}$ indicated. Only poles for which $\text{Re}k_4 > 0$ are shown, though the poles come in pairs $\hat{k}_4^{\text{pole}}$ and $-\hat{k}_4^{\text{pole}*}$.

a succession of poles which migrate inwards from $i\infty$ along the positive imaginary k_4 axis before veering away from the axis as $\hat{\mu}$ is increased. The position of these poles is plotted in Fig. 2, and the position of the pole closest to the real k_4 axis is listed in Table I. We note also that, as $\hat{\mu} \rightarrow \infty$ corresponding to the infrared-dominant limit, Eq. (6.2), the poles move farther from the origin. This is consistent with the limiting solution, Eq. (6.4), which is free from poles except at $k_4 = \infty$.

If the quarks are to be confined particles, we are restricted to values of μ for which the light quark propagator $S_q(p)$ has no pole on the real timelike p^2 axis, and for which the heavy quark propagator $1/[ik_4 + \hat{\Sigma}(k_4)]$ has no poles on the imaginary k_4 axis. While this requirement can be fulfilled by our light quark propagator solutions, unfortunately we find the heavy quark propagator is not totally free of timelike poles for any value of $\hat{\mu}$ over the range considered. It is reasonable to suggest that this fault could be rectified by improving the approximations employed by the DSE (bare vertex and Gaussian gluon propagator). In fact, in Ref. [19], it is shown that if the infrared-dominant gluon propagator, Eq. (6.2), is used in conjunction with a vertex *Ansatz* satisfying the Ward-Takahashi identity, one obtains a quark propagator which is an entire function of p^2 irrespective of the quark mass. It is therefore reasonable to assume that the timelike poles in the heavy quark propagator are an artifact of the model.

Poles which occur away from the timelike axis are, in principle, no impediment to confinement. However, solution of the BSE requires knowledge of the quark propagators over a region of the complex plane. From Eqs. (3.9)–(3.15), we see that the light quark propagator $S_q(p)$ is sampled by the BSE over the region

$$\text{Res} > \frac{(\text{Im}s)^2}{4\eta^2\delta^2} - \eta^2\delta^2, \quad (6.10)$$

where $s = p^2$, and the heavy quark propagator is sampled along the line

$$\text{Im}k_4 = (1 - \eta) \delta, \quad (6.11)$$

where δ is the “brown muck” contribution to the meson mass, and η a momentum partitioning parameter. Assuming there are no compensating zeros in the BS amplitudes, the integral in the BSE is divergent if poles are encountered in either of these two regions. Writing $\hat{s}^{\text{pole}} = X + iY$ and $\text{Im}k_4^{\text{pole}} = K$, it is straightforward to show that the model then fails if

$$\hat{\delta} > \hat{\delta}_{\text{max}} = K + \left[\frac{\sqrt{X^2 + Y^2} - X}{2} \right]^{1/2}. \quad (6.12)$$

To approach the maximum allowed value of $\hat{\delta}$ in numerical calculations, it is necessary to choose the momentum partitioning η to take its optimum value

$$\eta_{\text{opt}} = \frac{[\sqrt{X^2 + Y^2} - X]^{1/2}}{\sqrt{2K} + [\sqrt{X^2 + Y^2} - X]^{1/2}}. \quad (6.13)$$

Values of $\hat{\delta}_{\text{max}}$ and η_{opt} are plotted in Table I.

Solution of the BS equations involves the iteration of the sets of coupled linear integral equations obtained in Secs. V and VI. For the $J^P = 0^-$ meson, for instance, from Eqs. (4.6) and (4.8) and the definitions (3.11)–(3.13) we can assume without loss of generality that

$$\begin{aligned} f_-(|\mathbf{q}|, -q_4^*) &= f_-(|\mathbf{q}|, q_4)^*, \\ g_-(|\mathbf{q}|, -q_4^*) &= -g_-(|\mathbf{q}|, q_4)^*. \end{aligned} \quad (6.14)$$

Executing the spatial angular integrations analytically then gives a set of equations of the form

$$\vec{f}(|\mathbf{p}|, p_4) = \int_0^\infty dq_4 \int_0^\infty d|\mathbf{q}| K(|\mathbf{p}|, p_4; |\mathbf{q}|, q_4; \delta) \vec{f}(|\mathbf{q}|, q_4), \quad (6.15)$$

where $\vec{f} = (\text{Ref}_-, \text{Im}f_-, \text{Reg}_-, \text{Im}g_-)^T$ and the function K is a 4×4 matrix. One solves this as an eigenvalue equation of the form

$$\int d\mathbf{q} K(p, q; \delta) \vec{f}(q) = \Lambda(\delta) \vec{f}(p), \quad (6.16)$$

for a range of test values of δ until an eigenvalue $\Lambda(\delta) = 1$ is obtained.

The numerical solution to Eq. (6.16) and its analogues for the negative parity states considered in Secs. V and VI was carried out by linearly interpolating the functions contained in $\vec{f}$ on a 10×10 grid and then linearly interpolating the integrand on a 50×50 grid. A cutoff of $|\hat{\mathbf{q}}|, \hat{q}_4 < 10$, in the dimensionless units defined by Eq. (6.9) was found to be adequate.

The calculations were done for the dimensionless parameter values $\hat{\mu} = 3.5, 5.0$, and 6.0 . These values are chosen to give a broad range of $\hat{\mu}$ while ensuring that the light propagator is confining (i.e., free from poles on the real timelike axis). Poles on the imaginary k_4 axis rendering the heavy quark propagator nonconfining were sufficiently far away from the region of integration of the BSE to be considered

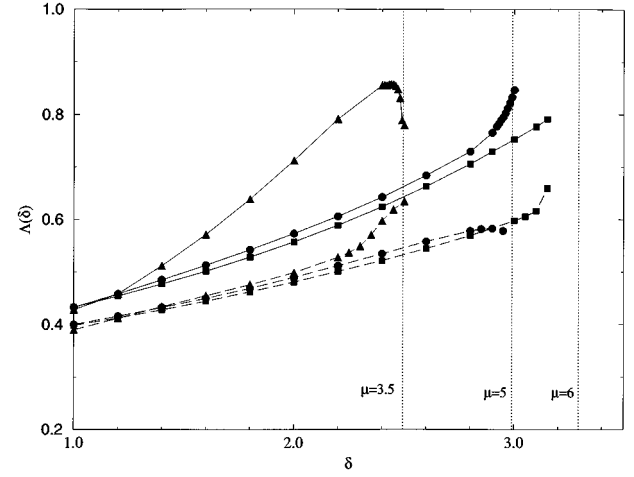


FIG. 3. Eigenvalues $\Lambda(\hat{\delta})$ from Eq. (6.16) for the degenerate $0^-/1^-$ states (solid curves) and the orbitally excited 1^- state (dashed curve) calculated with gluon propagator parameter $\hat{\mu} = 3.5$ (triangles), 5.5 (circles), and 6.0 (squares). Also shown as vertical dotted lines are the numerical limits $\hat{\delta}_{\text{max}}$ for each value of $\hat{\mu}$.

irrelevant. However, one must also consider poles elsewhere in the complex plane of both the light and heavy quark propagators, namely, those listed in Table I, and the consequent restriction, Eq. (6.12), on numerically accessible values of $\hat{\delta}$. To allow for as large a range of accessible $\hat{\delta}$ as possible, the momentum partitioning η_{opt} from Table I was used.

In Fig. 3 we plot $\Lambda(\hat{\delta})$ for the degenerate $0^-/1^-$ mesons described by Eqs. (4.6) and (4.8) [or equivalently Eqs. (5.24) and (5.25)], and the orbitally excited 1^- meson described by Eqs. (5.26) and (5.27). Also indicated in Fig. 3 are values of $\hat{\delta}_{\text{max}}$ for each value of $\hat{\mu}$. As $\hat{\delta}$ approaches $\hat{\delta}_{\text{max}}$ it is clear that the numerical calculation breaks down as the integration routine is unable to cope with the singularity in the integrand. It is also clear that any reasonable extrapolation of the smooth part of the curves would not give a solution to $\Lambda(\delta) = 1$ for any of the curves in the range $\delta < \delta_{\text{max}}$. It also appears that carrying through the calculations for higher values of $\hat{\mu}$ would be unlikely to improve the situation. We therefore conclude that, if the model is taken in its simplest form, namely, the combination of bare quark-gluon vertex and Gaussian gluon propagator in a Feynman-like gauge, spurious poles encountered in solutions to the DSE prevent the existence of solutions to the meson BSE. If the model is to be applied to the physics of D and B mesons, less crude approximations must be employed.

VII. CONCLUSIONS

We have developed a nonperturbative treatment of heavy quarks which leads in a natural way to a dynamically generated self-energy contribution to the heavy quark propagator. The treatment is based on a description of hadronic phenomena in terms of approximate Dyson-Schwinger equations which has proved highly successful within the light meson sector, namely the DSE technique [4].

The extension of the DSE technique to heavy quarks is nontrivial. The heavy quark self-energy must be calculated

near the bare current quark mass pole $p^2 = -m_Q^2$ where p is the Euclidean momentum. This is accomplished by a change of momentum integration variable in the Dyson-Schwinger equation and an expansion in $1/m_Q$. The dressed heavy quark propagator is then used in conjunction with light quark propagators obtained from the conventional DSE technique to obtain a ladder approximation Bethe-Salpeter equation for D and B mesons to leading order in $1/m_Q$. The BSE gives the meson masses in the form $m_Q + \delta$, where m_Q is essentially infinite. At this level of approximation, the predictive power of the model lies not in calculating absolute meson masses, but in calculating mass differences between different spin states or different light quark content. We recover the degeneracy between pseudoscalar and vector mesons predicted by heavy quark effective theory [1] in the limit $m_Q \rightarrow \infty$.

We have carried out a numerical analysis of the model in the limit of infinite mass heavy quark, $m_Q \rightarrow \infty$, and massless light quark, $m_q = 0$. Our calculation is not intended as a serious attempt to fit the D and B meson spectrum. Instead it is a feasibility study to determine to what extent the method is computationally viable. Solution of the BSE is numerically demanding, and the choice of model gluon propagator employed herein is primarily designed to reduce the numerical integration from three to two dimensions. We therefore use as input a particularly simple model gluon propagator, namely, a Gaussian propagator in a Feynman-like gauge. Because we are working in the chiral limit of the light quark, all dimensionful parameters can be scaled by the width of the Gaussian function, and the model is defined by a single adjustable parameter.

We find the analytic structure of the heavy quark meson is consistent with that required for the change of integration variable used in the DSE. However, we cannot unambiguously say whether the DSE leads to a confined heavy quark. Our bare vertex approximation to the DSE yields a succession of masslike poles which move off the timelike axis one by one as the parameter in the model gluon propagator is varied. As we point out below, we believe that these poles are a spurious artifact of the approximations employed.

The combination of bare vertex and Gaussian gluon propagator leads to poles in both the light and heavy quark propagators in the timelike half of the complex momentum plane over the broad range of input parameters studied. While poles off the real timelike momentum axis do not in principle spoil quark confinement, they can prevent successful solution of the BSE, which samples the quark propagators throughout a region of the complex plane. For the simple model set out above this is indeed the case. We have demonstrated numerically that the poles generated by the bare vertex approximation restrict the meson mass $m_Q + \delta$ to values of δ which do not include any bound state solutions.

In order to proceed further it will be necessary to develop an improved model which is not plagued by spurious singularities. In Ref. [19] it is demonstrated that if the bare quark gluon vertex is replaced by a vertex *Ansatz* satisfying the Ward-Takashi identity, such as the Ball-Chiu vertex [20], the quark propagator becomes an entire function. This result holds independently of the current quark mass, and so must also hold in the heavy quark limit.

Ideally one would like to incorporate quark propagator

which is an entire function into our heavy quark formalism in a way consistent with successful DSE technique studies of light quark mesons. Of particular importance to the light meson sector is the requirement that the model should produce a Goldstone pion in the chiral limit. This is achieved by ensuring that the choice of Bethe-Salpeter kernel is compatible with the choice of quark-gluon vertex in the DSE [18]. While there is no practical way at present to achieve such a compatibility with the Ball-Chiu vertex, good results can be achieved for the light pseudoscalar and vector meson octets by inverting a bare vertex approximation DSE from an entire light quark propagator to construct a separable approximation to the Bethe-Salpeter kernel [5,7]. Extension of the separable approximation in a similar manner to the heavy quark sector, beginning with the light quark propagators employed in Ref. [5], and a heavy quark propagator obtained from the Ball-Chiu vertex is perhaps the most promising way to proceed at this point. One potentially then has a single nonperturbative model of meson dynamics applicable to the entire range of meson masses.

Finally we note that, if the DSE technique is taken as a serious description of hadronic phenomena, we are led unavoidably to a treatment generically of the kind set out in this paper. The above analysis illustrates for the first time the importance of the analytic structure of heavy quark propagators and its significance in determining the heavy meson spectrum. Our analysis leads to a heavy quark self-energy which will contribute as significantly to the dynamics of the D and B mesons as that of the light quarks, and therefore cannot be ignored. Furthermore, we are unaware of any existing treatment of the heavy quark self-energy which is genuinely nonperturbative and has the potential to provide a qualitative explanation of heavy quark confinement. For these reasons we believe it is important to persevere with the application of the DSE technique to the heavy quark sector.

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APPENDIX

We derive here a set of coupled integral equations from the DSE for the heavy quark propagator to order $1/m_Q$. From Eqs. (2.1), (2.3), and (2.4) we obtain

$$S(p) = \frac{1 + \not{p}}{2} \frac{1}{ik \cdot v + \Sigma_B - \Sigma_A} + \frac{1}{2m_Q} \left[\frac{\Sigma_B + \not{p} \Sigma_A - ik}{ik \cdot v + \Sigma_B - \Sigma_A} - \frac{1 + \not{p}}{2} \frac{4ik \cdot v \Sigma_A + \Sigma_B^2 - \Sigma_A^2 + k^2}{(ik \cdot v + \Sigma_B - \Sigma_A)^2} \right] + O(m_Q^{-2}), \quad (A1)$$

where, on choosing $v_\mu = (1, \mathbf{0})$,

$$\Sigma_A = \Sigma_A(k_4, |\mathbf{k}|), \quad \Sigma_B = \Sigma_B(k_4, |\mathbf{k}|). \quad (\text{A2})$$

Clearly Σ_A and Σ_B should depend only on one variable: namely,

$$p^2 = -m_Q^2 + 2im_Q k_4 + k_4^2 + |\mathbf{k}|^2.$$

It follows that

$$\begin{aligned} \Sigma_A(k_4, |\mathbf{k}|) &= \Sigma_A\left(k_4 - \frac{i}{2m_Q} |\mathbf{k}|^2 + O(m_Q^{-2}), 0\right) \\ &= \Sigma_A(k_4, 0) - \frac{i}{2m_Q} |\mathbf{k}|^2 \frac{\partial \Sigma_A(k_4, 0)}{\partial k_4} \\ &\quad + O(m_Q^{-2}). \end{aligned} \quad (\text{A3})$$

We now make the expansion

$$\Sigma_A(k_4, 0) = \Sigma_A^0(k_4) + \frac{1}{m_Q} \Sigma_A^1(k_4) + \dots \quad (\text{A4})$$

Then

$$\begin{aligned} \Sigma_A(k_4, |\mathbf{k}|) &= \Sigma_A^0(k_4) + \frac{1}{m_Q} \left(\Sigma_A^1(k_4) - \frac{i}{2} |\mathbf{k}|^2 \Sigma_A^{0'} \right) + O(m_Q^{-2}) \\ &= \Sigma_A^0(k_4) + \frac{1}{m_Q} \tilde{\Sigma}_A^1(k_4, |\mathbf{k}|) + O(m_Q^{-2}), \end{aligned} \quad (\text{A5})$$

and similarly for Σ_B . ($\tilde{\Sigma}_A^1$ is only defined for convenience here. Eventually we shall be solving for Σ_A^0 , Σ_B^0 , Σ_A^1 and Σ_B^1 which only depend on k_4 .) For the propagator Eq. (A1) we then have

$$S(p) = \frac{1 + \not{v}}{2} \frac{1}{ik_4 + \Sigma_B^0 - \Sigma_A^0} + \frac{1}{2m_Q} \left[\frac{\Sigma_B^0 + \not{v} \Sigma_A^0 - i \not{k}}{ik_4 + \Sigma_B^0 - \Sigma_A^0} - \frac{1 + \not{v}}{2} \frac{4ik_4 \Sigma_A^0 + (\Sigma_B^0)^2 - (\Sigma_A^0)^2 + k_4^2 + |\mathbf{k}|^2 + 2(\tilde{\Sigma}_B^1 - \tilde{\Sigma}_A^1)}{(ik_4 + \Sigma_B^0 - \Sigma_A^0)^2} \right] + O(m_Q^{-2}). \quad (\text{A6})$$

The left-hand side of the DSE (2.2) then becomes

$$\frac{i \not{p}}{m_Q} \Sigma_A + \Sigma_B = -\not{v} \Sigma_A^0(k_4) + \Sigma_B^0(k_4) + \frac{1}{m_Q} [i \not{k} \Sigma_A^0(k_4) - \not{v} \tilde{\Sigma}_A^1(k_4, |\mathbf{k}|) + \tilde{\Sigma}_B^1(k_4, |\mathbf{k}|)]. \quad (\text{A7})$$

On the right-hand side, we substitute in Eq. (A6) and change the variable of integration to k' defined by $q_\mu = im_Q v_\mu + k'_\mu$. Then setting $k_\mu = (k_4, \mathbf{0})$, and projecting out coefficients of $\not{v}$, $\not{k}$, and I gives the following set of integral equations:

$$\Sigma_A^0(k_4) = \frac{4}{3} \int \frac{d^4 k'}{(2\pi)^4} \Delta(k - k') \frac{1}{ik'_4 + \Sigma_B^0(k'_4) - \Sigma_A^0(k'_4)}, \quad (\text{A8})$$

$$\Sigma_B^0(k_4) = 2 \Sigma_A^0(k_4), \quad (\text{A9})$$

$$\Sigma_A^1(k_4) = \frac{4}{3} \int \frac{d^4 k'}{(2\pi)^4} \Delta(k - k') \left[\frac{\Sigma_A^0}{ik'_4 + \Sigma_A^0} - \frac{4ik'_4 \Sigma_A^0 + 3(\Sigma_A^0)^2 + (k'_4)^2 + |\mathbf{k}'|^2 + 2\left(\Sigma_B^1 - \Sigma_A^1 - \frac{i}{2} |\mathbf{k}'|^2 \Sigma_A^{0'}\right)}{(ik'_4 + \Sigma_A^0)^2} \right] \Bigg|_{k'}, \quad (\text{A10})$$

$$\Sigma_B^1(k_4) = \frac{8}{3} \int \frac{d^4 k'}{(2\pi)^4} \Delta(k - k') \left[\frac{2 \Sigma_A^0}{ik'_4 + \Sigma_A^0} - \frac{4ik'_4 \Sigma_A^0 + 3(\Sigma_A^0)^2 + (k'_4)^2 + |\mathbf{k}'|^2 + 2\left(\Sigma_B^1 - \Sigma_A^1 - \frac{i}{2} |\mathbf{k}'|^2 \Sigma_A^{0'}\right)}{(ik'_4 + \Sigma_A^0)^2} \right] \Bigg|_{k'}. \quad (\text{A11})$$

Once a model quark propagator $\Delta(k - k')$ is specified, these equations can, in principle, be solved numerically to give the heavy quark propagator to first order in $1/m_Q$.

- [1] M. Neubert, Phys. Rep. **245**, 259 (1994).
- [2] E. V. Shuryak, Phys. Lett. **93B**, 134 (1980).
- [3] J. Zeng, *et al.*, Phys. Rev. D **52**, 5229 (1995); H.-Y. Jin *et al.*, Z. Phys. C **56**, 707 (1992); S. N. Gupta and J. M. Johnson, Phys. Rev. D **51**, 168 (1995); D. Ebert *et al.*, Nucl. Phys. **B434**, 619 (1995); Y.-B. Dai *et al.*, Z. Phys. C **60**, 527 (1993); **65**, 87 (1995).
- [4] C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33**, 475 (1994).
- [5] C. J. Burden *et al.*, “Ground-state spectrum of light quark mesons” Report No. nucl-th/9605027 (unpublished); in *Lattice ’95*, Proceedings of the International Symposium, Melbourne, Australia, edited by T. D. Kieu *et al.* [Nucl. Phys. B (Proc. Suppl.) **47**, 362 (1996)].
- [6] P. Jain and H. J. Munczek, Phys. Rev. D **48**, 5403 (1993); and references therein.
- [7] R. T. Cahill and S. M. Gunner, Phys. Lett. B **359**, 281 (1995)
- [8] C. D. Roberts, in *Chiral Dynamics: Theory and Experiment*, edited by A. M. Bernstein and B. R. Holstein, Lecture Notes in Physics Vol. 452 (Springer, Berlin 1995); C. D. Roberts, Nucl. Phys. **A605**, 475 (1996); C. J. Burden *et al.*, Phys. Lett. B **371**, 163 (1996).
- [9] C. D. Roberts, *et al.*, Phys. Rev. D **49**, 125 (1994).
- [10] K. L. Mitchell *et al.*, Phys. Lett. B **335**, 282 (1994).
- [11] M. R. Frank *et al.*, Phys. Lett. B **359**, 17 (1995); R. Alkofer and C. D. Roberts, *ibid.* **369**, 101 (1996).
- [12] P. Maris, Phys. Rev. D **52**, 6087 (1995).
- [13] T. W. Allen and C. J. Burden, Phys. Rev. D **53**, 5842 (1996).
- [14] M. R. Frank and C. D. Roberts, Phys. Rev. C **53**, 390 (1996).
- [15] J. E. Villate *et al.*, Phys. Rev. D **47**, 1145 (1993).
- [16] C. H. Llewellyn Smith, Ann. Phys. (N.Y.) **53**, 521 (1969).
- [17] H. J. Munczek, Phys. Rev. D **28**, 181 (1983); R. T. Cahill and C. D. Roberts, *ibid.* **32**, 2419 (1985).
- [18] A. Bender *et al.*, Phys. Lett. B **380**, 7 (1996).
- [19] C. J. Burden, C. D. Roberts, and A. G. Williams, Phys. Lett. B **285**, 347 (1992).
- [20] J. S. Ball and T.-W. Chiu, Phys. Rev. D **22**, 2542 (1980).