

Decay $D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e$ in the context of chiral perturbation theory

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We study the decay $D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e$, using $SU(2)_L \otimes SU(2)_R$ chiral perturbation theory for heavy charmed mesons and vector mesons, in the kinematic regime where $p_M \cdot p_\pi / m_M$ (here $M = D^0$ or K^{*0}) is much smaller than the chiral symmetry-breaking scale $\Lambda_{\chi SB}$ ($\Lambda_{\chi SB} \sim 1$ GeV). We present the leading diagrams and amplitude, and calculate the rate, in the region where, to leading order in our calculations, the K^{*0} is at zero recoil in the D^0 rest frame. The rate thus calculated is given in terms of a known form factor and depends on the $DD^*\pi$ coupling constant g_D of the heavy (charmed) meson chiral perturbation theory Lagrangian. A measurement of the above decay, in the aforementioned kinematic regime, can result in the extraction of an experimental value for g_D , accurate at the level of our approximations, and gives us a measure of the validity of approaches based on chiral perturbation theory in studying similar processes. [S0556-2821(96)02123-6]

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I. INTRODUCTION

A systematic expansion for calculating a wide range of processes involving hadrons at low momenta is provided by chiral perturbation theory. Apart from its uses in describing the strong interactions of the pseudo Goldstone bosons (PGB's), that is, the octet of the lightest pseudoscalar hadrons, chiral perturbation theory has been applied in describing the interactions of PGB's with heavy matter fields, such as nucleons and heavy mesons containing a charm or a bottom quark [1,2].

Chiral perturbation theory, based on $SU(3)_L \otimes SU(3)_R$ chiral symmetry, has also been applied to the interactions of PGB's with the lowest-lying vector mesons ρ 's, K^* 's, ω , and ϕ [3]. Reference [4] makes use of this approach in studying the decay of the τ in the channels $\tau \rightarrow \rho \pi \nu_\tau$, $\tau \rightarrow K^* \pi \nu_\tau$, and $\tau \rightarrow \omega \pi \nu_\tau$. This last decay mode is used to extract the absolute value of the $K^* K^* \pi$ coupling constant $|g_2^{(K^*)}|$ from experimental data [5]. The decay rates in the above modes are calculated, using $SU(2)_L \otimes SU(2)_R$ chiral symmetry only, thus not treating the strange quark mass as small.

In the present work, we use chiral perturbation theory for heavy charmed mesons [6] and heavy vector mesons [3], in the limit of $SU(2)_L \otimes SU(2)_R$ chiral symmetry, to calculate the differential decay rate for $D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e$ in a region of phase space where the pion is "soft" in the rest frames of both the D^0 and the K^{*0} . Currently, there is an upper bound of 1.3% on the branching ratio for this decay mode. To calculate the leading order amplitude, we need the matrix element of the left-handed current $s \gamma_\mu (1 - \gamma_5) c$, between a D^* and a K^* . Since the form factors for this current are not measured, we relate them to the form factors of the same left-handed current between a D and a K^* , using heavy quark symmetry [7,8]. At the present time, one of the form factors of the $D \rightarrow K^*$ current is still unmeasured. The dependence of the decay amplitude on this form factor is eliminated when we study the decay in the region where the K^* is near zero recoil. In this restricted region, the calculated differential decay rate depends on the $DD^*\pi$ coupling constant g_D of the heavy charmed meson chiral perturbation Lagrangian

[coupling constant g of Eq. (12) in Ref. [6]]. There is an experimental bound on the value of g_D , but the value of this constant remains to be measured. There have been theoretical attempts at the determination of g_D , such as those involving radiative D^* decays [9,10]. In this paper, we present an independent theoretical approach for obtaining the value of g_D . The results of our paper can be compared to the data on $D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e$ to extract the experimental value of g_D , which can in turn provide a measure of the validity of the methods used here, by calculating other processes that depend on g_D and comparing with data.

We will introduce the chiral perturbation theory relevant to this work in the next section. In Sec. III, we present expressions for the left-handed hadronic currents for $D \rightarrow K^*$ and $D^* \rightarrow K^*$, and the leading amplitude for the decay. Section IV contains the prediction for the differential decay rate, in a restricted region of phase space that will be discussed. The concluding remarks are presented in Sec. V, followed by an appendix in which some useful formulas are presented.

II. CHIRAL PERTURBATION THEORY

In this section, we introduce the formalism necessary for the calculations discussed in this paper. In the rest of this work, the words "heavy meson" refer to a meson containing a charm or a bottom quark, unless otherwise specified. We will use notation similar to that of Refs. [6, 4]. We start with the strong interactions of pions and heavy mesons, under $SU(2)_L \otimes SU(2)_R$ chiral symmetry. The pions are incorporated into a 2×2 special unitary matrix

$$\Sigma = \exp(2i\Pi/f_\pi), \quad (1)$$

where

$$\Pi = \begin{bmatrix} \pi^0/\sqrt{2} & \pi^+ \\ \pi^- & -\pi^0/\sqrt{2} \end{bmatrix}. \quad (2)$$

Under chiral $SU(2)_L \otimes SU(2)_R$, $\Sigma \rightarrow L \Sigma R^\dagger$, where $L \in SU(2)_L$ and $R \in SU(2)_R$. At the leading order in chiral perturbation theory, f_π is given by the pion decay constant $f_\pi \approx 132$ MeV. To describe the interactions of pions with other fields, it is convenient to define

$$\xi \equiv \exp\left(\frac{i\Pi}{f_\pi}\right) = \sqrt{\Sigma}. \quad (3)$$

Under chiral $SU(2)_L \otimes SU(2)_R$,

$$\xi \rightarrow L \xi U^\dagger = U \xi R^\dagger, \quad (4)$$

where U is a complicated function of L, R , and the pion fields Π . In the special case where $L=R=V$ in the unbroken $SU(2)_V$ vector subgroup, $U=V$.

We present an effective Lagrangian for the strong interactions of low momentum pions [in the case of $SU(3)_L \otimes SU(3)_R$, these results will include kaons and the η , as well] with the ground state heavy mesons with $Q\bar{q}^a$ flavor quantum numbers, where $a=1,2$ and $q^1=u$, and $q^2=d$. The light degrees of freedom have $s_l^{\pi_l} = \frac{1}{2}^-$ spin-parity quantum numbers, in these heavy mesons. In the limit where the mass of the heavy quark $m_Q \rightarrow \infty$, the spin of the light degrees of freedom combines with the spin of the heavy quark to yield two degenerate doublets, consisting of an $SU(2)_V$ antiduallet of pseudoscalar mesons, denoted by P_a , and an $SU(2)_V$ antiduallet of vector mesons, denoted by P_a^* . We are interested in the case $Q=c$, for which the pseudoscalar mesons are D^0 and D^+ and the vector mesons are D^{*0} and D^{*+} . The above-mentioned strong interaction Lagrangian, in addition to the usual symmetries, such as parity and Lorentz invariance, must have heavy quark symmetry, at the leading order. To proceed, it is convenient to incorporate the P_a and $P_{a\mu}^*$ meson fields into a 4×4 matrix H_a [6,11]:

$$H_a = \frac{1+\not{v}}{2} (P_{a\mu}^* \gamma^\mu - P_a \gamma_5). \quad (5)$$

Note that the heavy fields P_a and $P_{a\mu}^*$ only destroy their respective mesons of four-velocity v and do not create the corresponding antiparticles. We have $v^\mu P_{a\mu}^* = 0$. Under $SU(2)_L \otimes SU(2)_R$,

$$H_a \rightarrow H_b U_{ba}^\dagger, \quad (6)$$

where the repeated index b is summed over 1 and 2, and U was introduced in Eq. (4). Under the heavy quark spin symmetry group $SU(2)_v$, we have

$$H_a \rightarrow S H_a, \quad (7)$$

where $S \in SU(2)_v$. Lorentz transformations act on H_a according to

$$H_a \rightarrow D(\Lambda) H_a D(\Lambda)^{-1}, \quad (8)$$

where $D(\Lambda)$ is an element of the 4×4 matrix representation of the Lorentz group.

We introduce

$$\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0. \quad (9)$$

Thus we get

$$\bar{H}_a = (P_{a\mu}^{*\dagger} \gamma^\mu + P_a^\dagger \gamma_5) \frac{1+\not{v}}{2}. \quad (10)$$

The transformation laws for $\bar{H}_a$, corresponding to those in Eqs. (6), (7), and (8), are

$$\bar{H}_a \rightarrow U_{ab} \bar{H}_b, \bar{H}_a \rightarrow \bar{H}_a S^{-1}, \quad \bar{H}_a \rightarrow D(\Lambda) \bar{H}_a D(\Lambda)^{-1}. \quad (11)$$

The effective Lagrangian that describes the strong interactions of pions and heavy mesons, at leading order (one derivative), is then given by

$$\begin{aligned} \mathcal{L}^{(H)} = & -i \text{Tr} \bar{H}_a v \cdot \partial H_a + i \text{Tr} \bar{H}_a H_b (v \cdot V)_{ba} \\ & - g_D \text{Tr} \bar{H}_a H_b \gamma_\lambda \gamma_5 (A^\lambda)_{ba}, \end{aligned} \quad (12)$$

where

$$V_\nu = \frac{1}{2} (\xi \partial_\nu \xi^\dagger + \xi^\dagger \partial_\nu \xi) \quad (13)$$

and

$$A_\lambda = \frac{i}{2} (\xi \partial_\lambda \xi^\dagger - \xi^\dagger \partial_\lambda \xi). \quad (14)$$

At the present time, there is only a bound on the coupling constant g_D of Eq. (12), $g_D^2 < 0.45$, coming from the experimental bound $\Gamma(D^{*+} \rightarrow D^0 \pi^+) < 0.09$ MeV [12]. The explicit chiral symmetry-breaking terms contain light quark masses:

$$\delta \mathcal{L}^{(1)} = \lambda_\xi \text{Tr} \bar{H}_b H_a (M_\xi)_{ab} + \lambda_\Sigma \text{Tr} \bar{H}_a H_a (M_\Sigma + \Sigma^\dagger M)_{bb}. \quad (15)$$

In Eq. (15),

$$M_\xi = \frac{1}{2} (\xi M \xi + \xi^\dagger M \xi^\dagger), \quad (16)$$

where $M = \text{diag}(m_u, m_d)$ is the 2×2 light quark mass matrix.

Equation (12) yields the propagators $i \delta_{ab}/2v \cdot k$ and $-i \delta_{ab}(g^{\mu\nu} - v^\mu v^\nu)/2v \cdot k$ for P_a and P_a^* , respectively, where k is a small residual momentum. The leading heavy quark spin symmetry-breaking effects at order Λ_{QCD}/m_Q , induced by the color-magnetic operator [13], are given by

$$\delta \mathcal{L}^{(2)} = \frac{\lambda_Q}{m_Q} \text{Tr} \bar{H}_a \sigma^{\mu\nu} H_a \sigma_{\mu\nu}. \quad (17)$$

In Eq. (15), λ_ξ and λ_Σ are dimensionless constants independent of the heavy quark mass, whereas λ_Q of Eq. (17), with dimension 2, has a logarithmic dependence on the heavy quark mass [13], calculable in perturbative QCD. Under $SU(3)_L \otimes SU(3)_R$, the correspondence with the notation of Ref. [6] is given by

$$g_D = g, \quad \lambda_\xi = 2\lambda_1, \quad \lambda_\Sigma = \lambda'_1, \quad \lambda_Q = \lambda_2. \quad (18)$$

The only effect of the term in Eq. (17) is to change the P_a and P_a^* propagators, and an appropriate field redefinition [6] will yield $i \delta_{ab}/2v \cdot k$ and $-i \delta_{ab}(g^{\mu\nu} - v^\mu v^\nu)/2(v \cdot k - \Delta)$ for the aforementioned propagators, respectively, where

$$\Delta = m_{P^*} - m_P = \frac{-2\lambda_Q}{m_Q}. \quad (19)$$

In this paper $Q=c$, and the mass difference $\Delta = m_{D^*} - m_D = 145$ MeV $\sim m_\pi$. Thus Δ is considered to be of order 1 derivative and has a leading order contribution in our subsequent calculations.

The K^* and K^* fields are introduced as doublets [4]:

$$K_\mu^* = \begin{bmatrix} K_\mu^{*+} \\ K_\mu^{*0} \end{bmatrix}, \quad \bar{K}_\mu^* = \begin{bmatrix} K_\mu^{*-} \\ K_\mu^{*0} \end{bmatrix}. \quad (20)$$

Under chiral $SU(2)_L \otimes SU(2)_R$,

$$K_\mu^* \rightarrow UK_\mu^*, \quad \bar{K}_\mu^* \rightarrow U^* \bar{K}_\mu^*. \quad (21)$$

The doublets K_μ^* and $\bar{K}_\mu^*$ are related by charge conjugation C which acts on the fields as follows:

$$CK_\mu^* C^{-1} = -\bar{K}_\mu^*, \quad C\xi C^{-1} = \xi^T. \quad (22)$$

The vector meson fields are treated as heavy with fixed four-velocity $\underline{v}^{\prime\mu}$, $v'^2=1$, satisfying the constraint $v' \cdot K^* = v' \cdot \bar{K}^* = 0$. Interactions of the form $V \rightarrow V' X$, where V and V' are K^* 's and X is either the vacuum or one or more soft pions, are given by a Lagrangian of the form

$$\mathcal{L} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{\text{int}} + \mathcal{L}_{\text{mass}} - \frac{i}{2} \mathcal{L}_{\text{width}}. \quad (23)$$

The interaction Lagrangian is given by

$$\mathcal{L}_{\text{int}} = ig_2^{(K^*)} \bar{K}_\mu^{*\dagger} A_\lambda^T \bar{K}_\nu^* v'_\sigma \epsilon^{\mu\nu\lambda\sigma} + ig_2^{(K^*)} K_\mu^{*\dagger} A_\lambda K_\nu^* v'_\sigma \epsilon^{\mu\nu\lambda\sigma}. \quad (24)$$

Here $g_2^{(K^*)}$ in Eq. (24) and g_2 of the Lagrangian in Eq. (11) of Ref. [3] are equal, at leading order in $SU(3)_L \otimes SU(3)_R$ chiral perturbation theory. Note that for the vector mesons $K_\mu^{*-} \neq K_\mu^{*+}$, and so on. In the heavy vector meson chiral perturbation theory, K_μ^{*+} destroys a K_μ^{*+} , but it does not create its antiparticle K_μ^{*-} . The field K_μ^{*-} creates a K_μ^{*-} .

The kinetic terms in Lagrangian (23) are

$$\begin{aligned} \mathcal{L}_{\text{kin}} = & -iK_\mu^{*\dagger} v' \cdot \partial K_\mu^* - iK_\mu^{*\dagger} v' \cdot \nabla K_\mu^* - i\bar{K}_\mu^{*\dagger} v' \cdot \partial \bar{K}_\mu^* \\ & + i\bar{K}_\mu^{*\dagger} v' \cdot \nabla \bar{K}_\mu^*. \end{aligned} \quad (25)$$

The mass terms which explicitly break chiral symmetry are given by

$$\begin{aligned} \mathcal{L}_{\text{mass}} = & \lambda_2^{(K^*)} K_\mu^{*\dagger} M_\xi K_\mu^* + \lambda_2^{(K^*)} \bar{K}_\mu^{*\dagger} M_\xi^T \bar{K}_\mu^* \\ & + \sigma_8^{(K^*)} \text{Tr}(M_\xi) K_\mu^{*\dagger} K_\mu^* + \sigma_8^{(K^*)} \text{Tr}(M_\xi) \bar{K}_\mu^{*\dagger} \bar{K}_\mu^*. \end{aligned} \quad (26)$$

At leading order in $SU(3)_L \otimes SU(3)_R$ chiral perturbation theory, the couplings in Eq. (26) are related to those of Ref. [3] by

$$\lambda_2^{(K^*)} = \lambda_2 \quad \text{and} \quad \sigma_8^{(K^*)} = \sigma_8. \quad (27)$$

The K^* 's have a width $\Gamma^{(K^*)} = 50$ MeV. Since this width is comparable to the pion mass, we treat it as being of order 1 derivative, and introduce it in our Lagrangian via the terms

$$\mathcal{L}_{\text{width}} = \Gamma^{(K^*)} K_\mu^{*\dagger} K_\mu^* + \Gamma^{(K^*)} \bar{K}_\mu^{*\dagger} \bar{K}_\mu^*. \quad (28)$$

Note that the terms in Eqs. (24), (25), and (28) are of order 1 derivative and are considered of leading order, whereas the terms in Eq. (26) are proportional to light quark masses, which means they are of order 2 derivatives and, thus, considered nonleading in our calculations. In this formalism, the K^* propagator is given by

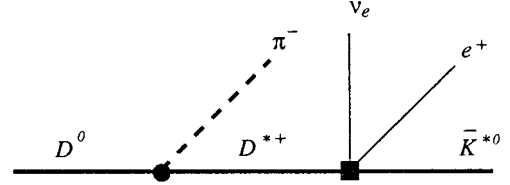


FIG. 1. The D^* -pole diagram contribution to the amplitude in Eq. (40). The solid square represents the hadronic left-handed current, and the solid circle represents the $DD^*\pi$ coupling proportional to g_D .

$$\frac{-i(g^{\mu\nu} - v'^\mu v'^\nu)}{v' \cdot k' + i\Gamma^{(K^*)}/2}, \quad (29)$$

where k' is the small residual momentum of the K^* .

III. CURRENTS AND THE AMPLITUDE

The part of the effective Hamiltonian $\mathcal{H}_W$ for weak semi-leptonic decay of D^0 that contributes to $D^0 \rightarrow K^{*0} \pi^- e^+ \nu_e$, at the quark level, is given by

$$\mathcal{H}_W = \frac{G_F}{\sqrt{2}} V_{cs} \bar{\nu}_e \gamma_\mu (1 - \gamma_5) e^+ \bar{s} \gamma^\mu (1 - \gamma_5) c, \quad (30)$$

where G_F is the Fermi constant, V_{cs} is an element of the Cabibbo-Kobayashi-Maskawa matrix, and the spinors represent the corresponding fermions. Experimentally, we have $|V_{cs}| \approx 1$. The only Feynman diagrams that contribute at leading order in $SU(2)_L \otimes SU(2)_R$ chiral perturbation theory are presented in Figs. 1 and 2. We note that, of the nonleading diagrams, there are two that are not readily seen as higher order in our chiral perturbation expansion. One is the diagram with a direct $D^0 K^{*0} \pi^-$ current coupling to the leptonic current or the nonpole diagram. The other diagram is the one in which the K^{*-} in Fig. 2 has been replaced by a K^- or the kaon-pole diagram.

To see why the nonpole diagram is subleading, we note that, under $SU(2)_L \otimes SU(2)_R$, the quark level current $\bar{s} \gamma^\mu (1 - \gamma_5) c$ is a singlet, and thus the hadronic current which represents the quark level current must be a singlet as well. To make a singlet out of the H , K_μ^* , and ξ fields, we need to have a $\partial^\mu \xi$ in the hadronic current expression. Thus, for the amplitude A_1 of the nonpole diagram, we have $A_1 \propto p_\pi$. We have $p_\pi \sim m_\pi$, for a valid perturbative expansion, and thus $A_1 \rightarrow 0$, as $m_\pi \rightarrow 0$, in the chiral limit.

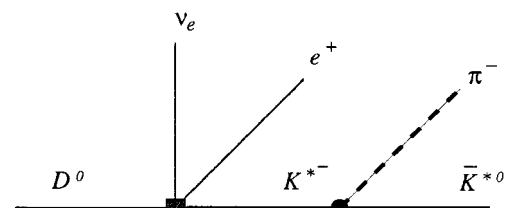


FIG. 2. The K^* -pole diagram contribution to the amplitude in Eq. (40). The solid square represents the hadronic left-handed current, and the solid circle represents the $K^* K^* \pi$ coupling proportional to $g_2^{(K^*)}$.

In the kaon-pole diagram, the $K^- \bar{K}^{*0} \pi^-$ coupling is proportional to $\partial^\mu \xi$, because only the pion has derivative coupling, being the only pseudo Goldstone boson in the vertex. Hence, for the amplitude A_2 of this diagram, we get $A_2 \propto p_\pi / (p_K^2 - m_K^2)$. Noting that the kaon is far off shell, we see that $A_2 \rightarrow 0$, as $p_\pi \rightarrow 0$, in the chiral limit. In order to have a consistently systematic expansion in $SU(2)_L \otimes SU(2)_R$ chiral perturbation theory, we must treat the nonpole and the kaon-pole diagrams as subleading in our calculations, since the amplitudes for the leading order diagrams in Figs. 1 and 2 do not vanish in the chiral limit.

We need expressions for $\langle K^* | \bar{s} \gamma^\mu (1 - \gamma_5) c | D^* \rangle$ and $\langle K^* | \bar{s} \gamma^\mu (1 - \gamma_5) c | D \rangle$, in order to write down the amplitude for the decay. We write [14]

$$\langle K^*(p', \varepsilon') | V_\mu | D(p) \rangle = i g \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{*\prime\nu} (p + p')^\lambda (p - p')^\sigma \quad (31)$$

and

$$\begin{aligned} \langle K^*(p', \varepsilon') | A_\mu | D(p) \rangle = & f \varepsilon_\mu^{*\prime} + a^{(+)} (\varepsilon^{*\prime} \cdot p) (p + p')_\mu \\ & + a^{(-)} (\varepsilon^{*\prime} \cdot p) (p - p')_\mu, \end{aligned} \quad (32)$$

where p' and ε' are the four-momentum and the polarization of the K^* , p is the four-momentum of the D , $V_\mu \equiv \bar{s} \gamma_\mu c$, $A_\mu \equiv \bar{s} \gamma_\mu \gamma_5 c$, and g , f , $a^{(+)}$, and $a^{(-)}$ are experimentally measurable form factors which are functions of $p \cdot p'$. At the present time, $a^{(-)}$, having a contribution to $D \rightarrow K^* l \nu_l$ that is proportional to the lepton mass, remains unmeasured, but g , f , and $a^{(+)}$ have been measured [12]. Let $p = m_D v$ and $p' = m_{K^*} v'$, where v and v' are the four-velocities of the D and the K^* , respectively. Then, as a function of $y \equiv v \cdot v'$, we have [15]

$$f(y) = \frac{1.8 \text{ GeV}}{1 + 0.63(y - 1)}, \quad (33)$$

$$a^{(+)}(y) = -\frac{0.17 \text{ GeV}^{-1}}{1 + 0.63(y - 1)}, \quad (34)$$

and

$$g(y) = \frac{0.51 \text{ GeV}^{-1}}{1 + 0.96(y - 1)}. \quad (35)$$

Note that the sign of g depends on the convention for the sign of Levi-Civita tensor, which we take to be $\epsilon_{\mu\nu\lambda\sigma} = -\epsilon^{\mu\nu\lambda\sigma} = 1$.

Next, we will write an expression for $\langle K^* | \bar{s} \gamma_\mu (1 - \gamma_5) c | D^* \rangle$ in terms of the form factors g , f , $a^{(+)}$, and $a^{(-)}$. Later on, we will only consider the region of phase space where K^{*0} is nearly at rest in the decaying D^0 rest frame and the dependence of the amplitude on the unknown form factor $a^{(-)}$ is negligible. We treat the charm quark as heavy and use heavy quark symmetry to write down the following expression [8] for the current $\langle K^* | V_\mu - A_\mu | M^{(H)} \rangle$, where $M^{(H)}$ is either a D or a D^* with polarization ε :

$$\begin{aligned} \langle K^* | V_\mu - A_\mu | M^{(H)} \rangle \\ = \text{Tr} [K^*(v, v', \varepsilon') \gamma_\mu (1 - \gamma_5) M^{(H)}(v)]. \end{aligned} \quad (36)$$

In Eq. (36),

$$M^{(H)}(v) = \frac{1 + \not{v}}{2} \begin{cases} -\gamma_5 & \text{for } D, \\ \not{\varepsilon} & \text{for } D^*, \end{cases} \quad (37)$$

and

$$K^*(v, v', \varepsilon') = (C_1 + C_2 \not{v}') \not{\varepsilon}^{*\prime} + (C_3 + C_4 \not{v}') (\varepsilon^{*\prime} \cdot v) \quad (38)$$

is the most general form that can be written down for the wave function of K^* consistent with heavy quark symmetry for $M^{(H)}(v)$ at leading order. C_i , $i=1,2,3,4$, are form factors which can be expressed in terms of f , $a^{(+)}$, $a^{(-)}$, and g by comparing the expression for $\langle K^* | V_\mu - A_\mu | D \rangle$ from Eqs. (31) and (32) with the one given by Eq. (36) above. With the C_i determined in this way, Eq. (36) yields the following expression for $\langle K^* | V_\mu - A_\mu | D^* \rangle$, in terms of the form factors of $\langle K^* | V_\mu - A_\mu | D \rangle$:

$$\begin{aligned} \langle K^*(v', \varepsilon') | V_\mu - A_\mu | D^*(v, \varepsilon) \rangle = & m_D m_{K^*} \left(i \epsilon_{\mu\nu\lambda\sigma} \varepsilon^{\nu\prime} \left[(a^{(+)} - a^{(-)}) v^\lambda v'^\sigma (\varepsilon^{*\prime} \cdot v) - \frac{f}{m_D m_{K^*}} \varepsilon^{*\prime\lambda} v^\sigma \right] \right. \\ & + \left[\frac{f}{m_D m_{K^*}} + \frac{m_D}{m_{K^*}} (a^{(+)} + a^{(-)}) + (a^{(+)} - a^{(-)}) (v \cdot v') \right] (\varepsilon^{*\prime} \cdot v) \varepsilon_\mu + 2g (\varepsilon \cdot v') \varepsilon_\mu^{*\prime} \\ & + \left\{ 2g [i \epsilon^{\alpha\beta\gamma\delta} \varepsilon_\alpha v_\beta \varepsilon_\gamma^{*\prime} v'_\delta + (\varepsilon \cdot \varepsilon^{*\prime}) (v \cdot v') - (\varepsilon \cdot v') (\varepsilon^{*\prime} \cdot v)] - (a^{(+)} - a^{(-)}) (\varepsilon \cdot v') \right. \\ & \left. \left. \times (\varepsilon^{*\prime} \cdot v) - \frac{f}{m_D m_{K^*}} (\varepsilon \cdot \varepsilon^{*\prime}) \right\} v_\mu - 2g (\varepsilon \cdot \varepsilon^{*\prime}) v'_\mu \right), \end{aligned} \quad (39)$$

where we have used the identity $\varepsilon_\alpha \varepsilon_\beta^{*\prime} v'_\lambda v_\sigma g^{\lambda[\sigma} \varepsilon^{\alpha\beta\gamma\mu]} = 0$ to write down the first term proportional to v_μ .

We see that the expressions for $\langle K^* | V_\mu - A_\mu | D \rangle$ and $\langle K^* | V_\mu - A_\mu | D^* \rangle$, given by Eqs. (31), (32), and (39), depend on the value of $a^{(-)}$, which, as mentioned before, is not measured. The value of this form factor could, in principle, be measured, given enough data on $D \rightarrow K^* \mu^+ \nu_\mu$, where

the antimuon is massive enough to make the measurement possible. However, for the rest of this paper, we work in the kinematic regime where K^{*0} is at rest, to leading order in our calculations, in the rest frame of decaying D^0 . Thus the pion can be soft in the rest frames of both the D^0 and the K^{*0} , as required by chiral perturbation theory. In this region, we have $v = v'$, $v \cdot v' = 1$, and $\varepsilon \cdot v' = \varepsilon^{*\prime} \cdot v = 0$. The amplitude

for the decay $D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e$, at leading order in $SU(2)_L \otimes SU(2)_R$ chiral perturbation theory, represented by Feynman diagrams of Figs. 1 and 2, is given by

$$\begin{aligned} \mathcal{M}(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e) &= \frac{G_F V_{cs} f}{\sqrt{2} f_\pi} \left\{ \frac{g_D}{v \cdot p_\pi + \Delta} [\epsilon^{\mu\nu\lambda\sigma} v_\nu p_{\pi\lambda} \epsilon_\sigma^{*'} - i(p_\pi \cdot \epsilon^{*'}) v^\mu] \right. \\ &\quad \left. + \frac{g_2^{(K^*)}}{v \cdot p_\pi + i(\Gamma^{(K^*)}/2)} \epsilon^{\mu\nu\lambda\sigma} p_{\pi\nu} \epsilon_\lambda^{*'} v_\sigma \right\} \\ &\quad \times \bar{u}_{(\nu)} \gamma_\mu (1 - \gamma_5) v_{(e)}, \end{aligned} \quad (40)$$

where $u_{(\nu)}$ and $v_{(e)}$ are the spinors for the electron neutrino and the positron, respectively. Note that in the recoilless K^{*0} limit used here, the dependence of the amplitude on $a^{(-)}$ is eliminated. We have $f(1) = 1.8$ GeV, from Eq. (33) above.

IV. DIFFERENTIAL DECAY RATE

Before presenting the differential decay rate, let us introduce the following kinematic variables [16,17]. The invariant mass of the $K^* \pi$ system $m_{K^* \pi}$, where $m_{K^* \pi} \equiv \sqrt{(p' + p_\pi)^2}$, is given by

$$m_{K^* \pi}^2 = m_{K^*}^2 + m_\pi^2 + 2m_{K^*} m_\pi x, \quad (41)$$

where

$$x \equiv \frac{v \cdot p_\pi}{m_\pi}. \quad (42)$$

The invariant mass of the lepton pair is denoted by $m_{e\nu}$, where $m_{e\nu} \equiv \sqrt{(p_e + p_\nu)^2}$, p_e is the four-momentum of the positron, and p_ν is the four-momentum of the electron neutrino. The angle formed by the three-momentum of the K^{*0} in the $K^* \pi$ center-of-mass frame and the line of flight of the $K^* \pi$ in the D^0 rest frame is denoted by θ_K , and the angle between the positron three-momentum in the $e^+ \nu$ center-of-mass frame and the line of flight of the $e^+ \nu$ in the D^0 rest frame is denoted by θ_e . The angle ϕ is formed by the normals to the $K^* \pi$ and $e^+ \nu$ planes in the D^0 rest frame. The sense of ϕ is from the normal to the $K^* \pi$ plane to that of the $e^+ \nu$ plane.

The amplitude given by Eq. (40) is obtained in a region where the K^{*0} is “near zero recoil,” and later we specify a region of phase space that corresponds to this approximation. Using the above kinematic variables, in the limit $m_\pi/m_{K^*} \rightarrow 0$, one obtains the following differential decay rate for $D^0 \rightarrow K^{*0} \pi^- e^+ \nu_e$, in a region where K^{*0} is near zero recoil:

$$\begin{aligned} \frac{d^{(5)}\Gamma(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e)}{dx dm_{e\nu}^2 d(\cos\theta_K) d(\cos\theta_e) d\phi} &= \frac{G_F^2 |V_{cs}|^2 f^2(1)}{(4\pi)^6 f_\pi^2 m_D^3} \left(\frac{m_\pi}{m_{K^*}} \right)^2 (x^2 - 1)^{3/2} \sqrt{T_0} \\ &\times \left\{ \frac{g_D^2}{(x + \delta)^2} (T_1 + T_2) \right. \\ &\quad \left. + \left[\frac{g_2^{(K^*)2}}{x^2 + \gamma^2} + \frac{2x g_D g_2^{(K^*)}}{(x + \delta)(x^2 + \gamma^2)} \right] T_2 \right\}, \end{aligned} \quad (43)$$

where

$$\gamma \equiv \frac{\Gamma^{(K^*)}}{2m_\pi}, \quad \delta \equiv \frac{\Delta}{m_\pi}, \quad (44)$$

$$T_0 \equiv (m_D^2 + m_{K^*}^2 - m_{e\nu}^2)^2 - 4m_D^2 m_{K^*}^2, \quad (45)$$

$$T_1 \equiv X^2 \sin^2 \theta_e, \quad (46)$$

$$\begin{aligned} T_2 &\equiv \sin^2 \theta_K \sin^2 \theta_e (X^2 + m_{K^*}^2 m_{e\nu}^2 \cos^2 \phi) \\ &\quad + m_{K^*}^2 m_{e\nu}^2 (1 + \cos^2 \theta_K \cos^2 \theta_e) \\ &\quad - \left(\frac{m_{K^*} m_{e\nu}}{2} \right) \sin(2\theta_K) \sin(2\theta_e) \cos \phi \sqrt{X^2 + m_{K^*}^2 m_{e\nu}^2}, \end{aligned} \quad (47)$$

and

$$X \equiv \sqrt{\frac{(m_D^2 - m_{K^*}^2 - m_{e\nu}^2)^2}{4} - m_{K^*}^2 m_{e\nu}^2}. \quad (48)$$

We can integrate the expression in Eq. (43) for the differential decay rate in order to obtain a total decay rate $\Gamma(D^0 \rightarrow K^{*0} \pi^- e^+ \nu_e)$ for a limited volume of phase space consistent with the regime of validity of chiral perturbation theory and the recoilless K^{*0} approximation. In this region of phase space, we demand that (a) $p \cdot p_\pi/m_D \ll \Lambda_{\chi\text{SB}}$ and $p' \cdot p_\pi/m_{K^*} \ll \Lambda_{\chi\text{SB}}$, where $\Lambda_{\chi\text{SB}} \sim 1$ GeV, in order to have a valid perturbative expansion, and (b) $|\vec{p}'|/m_{K^*} \ll 1$, where $\vec{p}'$ is the three-momentum of the K^{*0} , in the rest frame of the D^0 , to ensure that we stay in a region of phase space where $v = v'$, corresponding to a recoilless K^{*0} , at leading order.

The constraints in (a) and (b) above are not very exact, and an appropriate choice for the region of phase space that satisfies our requirements may, in principle, be made only after comparing the shapes of different distributions from experimental data with the predictions from the theoretical differential decay rate presented in Eq. (43). The possibility of making such comparisons between experimental data and our theory depends on the availability and resolution of the data in the restricted phase space region of our calculations.

However, we proceed to make a reasonable choice for the region of integration, given the constraints mentioned in (a) and (b) above. To satisfy $p' \cdot p_\pi/m_{K^*} \ll \Lambda_{\chi\text{SB}}$, we require $x \in [1, 2]$. The quantities $p \cdot p_\pi/m_D$ and $|\vec{p}'|$ depend on the values of the variables x , $m_{e\nu}$, and $\cos \theta_K$, as shown in the Appen-

dix. By inspection, for $x \in [1, 1.5]$, $\cos \theta_K \in [-1, 1]$, and $m_{e\nu}^2 \in [0.593, (m_D - m_{K^* \pi})^2]$, we have $p \cdot p_\pi / m_D < 200$ MeV and $|\vec{p}'| < 240$ MeV. Integrating the expression in Eq. (43) over $m_{e\nu}^2 \in [0.593, (m_D - m_{K^* \pi})^2]$, $x \in [1, 1.5]$, $\cos \theta_K \in [-1, 1]$, $\cos \theta_e \in [-1, 1]$, and $\phi \in [0, 2\pi]$ yields the partial decay width $\Gamma_1(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e)$ and

$$\begin{aligned} \Gamma_1(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e) \\ = 9.01 \times 10^{-17} [(0.016)g_D^2 \pm (0.024)g_D + 0.013] \text{ GeV}, \end{aligned} \quad (49)$$

where $\pm$ corresponds to $g_2^{(K^*)} = \pm 0.6$. With the angular limits of integration the same, if we assume that, without exceeding the realm of validity of our theory considerably, the region of integration may be expanded to $x \in [1, 2]$ and $m_{e\nu}^2 \in [0.510, (m_D - m_{K^* \pi})^2]$, where $p \cdot p_\pi / m_D < 245$ MeV and $|\vec{p}'| \leq 330$ MeV, we obtain a partial decay width $\Gamma_2(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e)$, which is nearly an order of magnitude larger,

$$\begin{aligned} \Gamma_2(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e) \\ = 9.01 \times 10^{-17} [(0.176)g_D^2 \pm (0.231)g_D + 0.117] \text{ GeV}, \end{aligned} \quad (50)$$

where, again, $\pm$ corresponds to $g_2^{(K^*)} = \pm 0.6$. We mentioned before that present data suggest $g_D^2 < 0.45$. Let us take $g_D^2 = 0.3$ in order to get some estimates of the size of the branching ratio in our region of phase space. Since the total width $\Gamma_{D^0} = 1.586 \times 10^{-12}$ GeV for the D^0 , Γ_1 gives a branching ratio B_1 ,

$$B_1(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e) = \begin{cases} 2 \times 10^{-6}, & g_D g_2^{(K^*)} > 0, \\ 3 \times 10^{-7}, & g_D g_2^{(K^*)} < 0, \end{cases} \quad (51)$$

in a region where $m_{e\nu}^2 \in [0.593, (m_D - m_{K^* \pi})^2]$ and $x \in [1, 1.5]$. For the branching ratio B_2 , corresponding to Γ_2 , we get

$$B_2(D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e) = \begin{cases} 2 \times 10^{-5}, & g_D g_2^{(K^*)} > 0, \\ 3 \times 10^{-6}, & g_D g_2^{(K^*)} < 0, \end{cases} \quad (52)$$

in our restricted region of phase space, where $m_{e\nu}^2 \in [0.510, (m_D - m_{K^* \pi})^2]$ and $x \in [1, 2]$. For the smaller value of B_1 , the corresponding region can most likely be explored at a fixed target experiment, or a τ -charm or B factory only, since the present experiments are not able to measure such small branching ratios for the decay of the D^0 . However, the values of B_2 lie reasonably close to the present measurable range and g_D can be extracted from the data if its absolute value is not too much smaller than $\sqrt{0.3}$.

V. CONCLUSIONS

In this work, we have presented a systematic calculation of the differential decay rate for $D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e$, in a restricted kinematic region, based on the $SU(2)_L \otimes SU(2)_R$ chiral perturbation theory formalism. Since this decay rate

depends on the unmeasured $DD^*\pi$ coupling constant g_D of Eq. (12), one can use our results to extract g_D from experimental data. Currently, there is an experimental bound $g_D^2 < 0.45$ on this coupling constant.

We have treated D , D^* , and K^* as heavy matter fields and applied the chiral perturbation theory formalism to describe the strong couplings of the π^- to D , D^* , and K^* , assuming that the π^- is “soft” in the rest frames of the heavy matter fields. The leading order (one derivative, here) amplitude involves $\langle K^* | \bar{s} \gamma_\mu (1 - \gamma_5) C | D \rangle$ and $\langle K^* | \bar{s} \gamma_\mu (1 - \gamma_5) C | D^* \rangle$ left-handed hadronic currents, and we have presented an expression, in Eq. (39), for the $D^* \rightarrow K^*$ current, in terms of the form factors of the $D \rightarrow K^*$ current. One of the $D \rightarrow K^*$ form factors, denoted by $a^{(-)}$, remains unmeasured. However, we have restricted our phase space to a region where the K^{*0} is near zero recoil in the D^0 rest frame, making the dependence of the amplitude on $a^{(-)}$ negligible, at leading order. Note that this restriction on the phase space is made in accordance with the requirement that the pion be soft in the rest frames of both the D^0 and the K^{*0} , in order to have a valid chiral perturbation theory expansion.

We have presented an expression for the differential decay rate, in the aforementioned region of phase space, in Eq. (43). To get the branching ratio in this region, one has to make a reasonable choice for the limits of the integration that yields the partial width in the restricted volume of phase space. For $|g_D| = \sqrt{0.3}$ as an allowed value for the magnitude of the $DD^*\pi$ coupling, we get the branching ratios B_1 and B_2 , corresponding to two reasonable choices for the integration region. The first choice is expected to be strict enough for a good leading order approximation, and it can probably be explored at a fixed target experiment, or a τ -charm or B factory only. We believe that expanding the region of integration beyond that which corresponds to the second choice can result in a considerable departure from the proper regime for our approximations. The branching ratios in this region are close to the present measurable range. For a rough experimental value of g_D , we can expect that the branching ratio for a typical region of integration will be of order 10^{-5} .

The volume of phase space in which our theory is valid may be best selected after consulting the data. However, the values of B_2 suggest that for a volume of phase space that is close to our second choice presented here and provided that $|g_D|$ is not too much smaller than the value used here, $\sqrt{0.3}$, an experimental value for g_D can be extracted using the data on $D^0 \rightarrow \bar{K}^{*0} \pi^- e^+ \nu_e$ and given the sign of $g_2^{(K^*)}$. In the heavy quark limit ($m_Q \rightarrow \infty$), g_D is equal to g in Eq. (12) of Ref. [6]. There are other processes, with fewer final state particles, that involve the constant g_D . $D^* \rightarrow D \pi$ is an example of such a process, where one, in principle, can measure the value of g_D from the knowledge of the partial width for this decay channel. This assumes knowing the full width of D^* , which is difficult to measure, since the D^* is too short lived for time-of-flight measurements and too narrow for a reliable measurement of its full width.

Our results represent the $SU(2)_L \otimes SU(2)_R$ chiral perturbation theory predictions at the level of one derivative, corresponding to $Q(v_H \cdot p_\pi / 1 \text{ GeV})$, where v_H is the four-velocity of the D^0 or K^{*0} . Thus we expect to have significant corrections to our results at the level of two derivatives, suppressed

by one power of $v_H \cdot p_\pi / 1 \text{ GeV}$, only. Hence, at next to leading order, operators containing two derivatives of the pion field, insertions of the light quark mass terms, nonpole diagrams, and additional pole diagrams, such as the one in which the K^{*-} is replaced by a K^- in Fig. 2, contribute.

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APPENDIX

The following relations [16] are useful in calculating the scalar products of the various four-vectors introduced in this paper. Let

$$\begin{aligned} R &\equiv p' + p_\pi, & Q &\equiv p' - p_\pi, \\ K &\equiv p_e + p_\nu, & L &\equiv p_e - p_\nu. \end{aligned} \quad (\text{A1})$$

We then get

$$\begin{aligned} Q \cdot R &= m_{K^*}^2 - m_\pi^2, \\ K \cdot L &= 0, \\ R \cdot K &= \frac{m_D^2 - m_{K^*}^2 - m_{e\nu}^2}{2}, \\ |\vec{K}|^2 &\equiv \frac{(m_{K^*}^2 + m_\pi^2 - m_{K^*}^2)^2}{4m_{K^*}^2} - \frac{m_{K^*}^2 m_\pi^2}{m_{K^*}^2}, \end{aligned}$$

$$\beta = \frac{2|\vec{K}|}{m_{K^*} m_\pi},$$

$$\chi = \sqrt{(R \cdot K)^2 - m_{K^*}^2 m_{e\nu}^2},$$

$$K \cdot Q = \frac{m_{K^*}^2 - m_\pi^2}{m_{K^*}^2} R \cdot K + \beta \chi \cos \theta_K,$$

$$R \cdot L = \chi \cos \theta_e,$$

$$\begin{aligned} Q \cdot L &= \left(\frac{m_{K^*}^2 - m_\pi^2}{m_{K^*}^2} \right) \chi \cos \theta_e + \beta (R \cdot K) \cos \theta_K \cos \theta_e \\ &\quad - \beta m_{K^*} m_{e\nu} \sin \theta_K \sin \theta_e \cos \phi, \end{aligned}$$

$$2\eta \equiv p_D \cdot R = \frac{m_D^2 + m_{K^*}^2 - m_{e\nu}^2}{2},$$

$$2\zeta \equiv p_D \cdot Q = (m_{K^*}^2 - m_\pi^2) \left(1 + \frac{R \cdot K}{m_{K^*}^2} \right) + \beta \chi \cos \theta_K. \quad (\text{A2})$$

In the above, p_D denotes the four-momentum of the D^0 . Also, in the limit where $(m_\pi/m_{K^*}) \rightarrow 0$, we have $\chi \rightarrow X$, where X is defined in Eq. (48).

We have the following expressions for the magnitude of the K^{*0} three-momentum and the energy of the π^- , in the rest frame of the D^0 , denoted by $|\vec{p}'|$ and E_π , respectively:

$$\begin{aligned} |\vec{p}'| &= \sqrt{\left(\frac{\eta + \zeta}{m_D} \right)^2 - m_{K^*}^2}, \\ E_\pi &= \frac{\eta - \zeta}{m_D}. \end{aligned} \quad (\text{A3})$$

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