Spontaneous baryogenesis with observable *CP* violation

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Using a left-right symmetric model with spontaneous *CP* violation and the hypothesis of a weakly first-order electroweak phase transition we derive a relation between the produced baryon asymmetry and the observed parameter ε, describing *CP* violation in the *K* system.

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I. INTRODUCTION

The electroweak baryogenesis models came as a response to the discovery of the existence in the early Universe of rapid baryon-number-violating anomalous processes [1], which could have erased any preexisting asymmetry. In this way the problem of preserving some postinflationary baryon asymmetry until now [2] can be replaced by the problem of generating the baryon asymmetry of the Universe (BAU) at the latest possible stage: the electroweak phase transition (EWPT) [3]. This scheme has also opened the exciting possibility of further testing the standard model (SM) and probe its numerous extensions.

The task of baryogenesis is twofold: first one has to ensure that Sakharov's conditions [4] are satisfied and then check that an efficient mechanism is realized. It is very nice and intriguing that the SM can satisfy by its own the form point: *C* and *CP* violation, *B* nonconservation, and an out-of-equilibrium stage through a first-order phase transition [3]. Whether this model is able to generate the right asymmetry is still an open question [5] and the most popular point of view is to consider the baryon excess of the Universe as a hint for something standing beyond the SM (see [6] and [7] and references therein).

In this paper we investigate the left-right (*LR*) model based on the gauge group $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$, already considered in [8] and [9]. One of the most interesting suggestions is that, if *CP* is spontaneously violated, then it should be possible to relate the baryon asymmetry of the Universe to the low energy *CP* violation phenomenology and in particular to the precisely measured ϵ parameter in the K_0-$\bar{K}_0$ system. We think this model deserves further attention since it offers a common source to two "well measured" manifestations of *CP* violation in nature.

At first sight it is cosmologically troublesome to deal with a spontaneously broken discrete symmetry due to the appearance of domain walls [10]. Also, spontaneous *CP* violation would allow for the generation of both matter and antimatter and would lead to a globally neutral universe. However, these problems may be circumvented by separating the scale of spontaneous *CP* violation from the scale where baryogenesis takes place.

In [9] it was supposed that the electroweak phase transition was strongly first order, occurring through nucleation and expansion of bubbles of true vacuum with *thin* walls. The reflection of quarks on the moving wall in a *CP*-violating way creates a charge excess which is converted into a baryon excess by the rapid *B*-violating processes occurring in the symmetric phase.[1]

Here we will follow the opposite hypothesis, i.e., that the expanding bubbles have a *thick* wall.[2] The appropriate mechanism here is the so called spontaneous baryogenesis [13] in which the BAU is created within the wall. We will get a strong upper bound for the produced baryon asymmetry in terms of the ϵ parameter and purely dynamical quantities ($\kappa, m_{\rm top}, M_2 \approx M_R$), and the signature of the quark mass matrices). If we further suppose that this *LR* model "saturates" ϵ, we get the nice result that the BAU and Reϵ must have the same sign.

II. SPONTANEOUS *CP* VIOLATION IN THE *LR* MODEL

The salient points of the *LR* model are the following [14]: the explicit *LR* symmetry is broken to the SM one at a scale $M_R \sim 1$ TeV through the vacuum expectation value (VEV) of a scalar field transforming as a triplet under $SU(2)_R$; the electroweak symmetry is broken through the VEV of a scalar bidoublet, i.e., a field in the $(\frac{1}{2},\frac{1}{2},0)$ representation of the gauge group. The Yukawa couplings of quarks read

$$\mathcal{L}_{\rm Yukawa} = -\Gamma_{ij}\bar{q}_{iL}\phi q_{jR} - \Delta_{ij}\bar{q}_{iL}\tilde{\phi} q_{jR} , \tag{2.1}$$

where ϕ is the bidoublet field, with a VEV

$$\langle \phi \rangle = \begin{bmatrix} v & 0 \\ 0 & w \end{bmatrix} \tag{2.2}$$

and $\tilde{\phi} \equiv \sigma_2 \phi^* \sigma_2$. The coupling matrices Δ and Γ are real

[1]This is the so-called charge transport mechanism proposed in [11].

[2]For a study of the left-right phase transition with a simpler scalar content, see [12].

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and symmetric due to the assumed CP invariance of the original Lagrangian.

CP violation in the quark sector occurs if v and w^* are complex relative to each other. After a $SU(2)_L$ or $SU(2)_R$ transformation it is possible to express this by one single phase α:

$$\langle \phi \rangle = e^{i\alpha/2} \begin{bmatrix} |v| & 0 \\ 0 & |w| \end{bmatrix} . \tag{2.3}$$

As noted in [15] and further analyzed in [16], the simplest model cannot give rise to a nontrivial phase without fine-tuning, but the minimal extension of adding a pseudoscalar singlet suffices to solve the problem. This will be further discussed below.

In the chosen phase convention the mass matrices of the quarks are given by

$$\begin{aligned} M^{(u)} &= v\Gamma + w^*\Delta = |v| e^{i\alpha/2}(\Gamma + re^{-i\alpha}\Delta) , \\ M^{(d)} &= w\Gamma + v^*\Delta = |v| e^{i\alpha/2}(r\Gamma + e^{-i\alpha}\Delta) , \end{aligned} \tag{2.4}$$

with $w/v^* = |w/v| e^{i\alpha} = re^{i\alpha}$. These are complex symmetric matrices which can be diagonalized by two unitary matrices

$$\begin{aligned} M^{(u)} &= e^{i\alpha/2} U D^{(u)} U^T , \\ M^{(d)} &= e^{i\alpha/2} V D^{(d)} V^T . \end{aligned} \tag{2.5}$$

At the difference of the SM the L and R quarks are not rotated independently. This has as important consequence that the mixing matrices K_L and K_R [the generalizations of the Kobayashi-Maskawa (KM) matrix] are not independent and in the basis (phase convention) we have chosen they are related by

$$K_L = U^\dagger V = K_R^* . \tag{2.6}$$

Note that the factorization of $e^{i\alpha/2}$ in (2.5) leaves (2.6) invariant. In the same way one can factorize, e.g., $|v|$ without changing the rotation matrices U and V. Note also that, for the case of N_f flavors, there are $(N_f^2 - N_f + 1)$ physical phases in the mixing matrices and so it is possible to have CP violation already with two generations. The nicety of the LR model with spontaneous CP violation is that all the phases are functions of α and that in the limit of small $r \sin\alpha$ they are analytically calculable. Furthermore, an enhancement of the $\Delta S = 2$ channel ensures a small value of ϵ'/ϵ for dynamical reasons [17].

The key point, as remarked by Chang [18], is that in the limit where $r \sin\alpha$ goes to zero in (2.4) there is no CP violation. With the hypothesis of small $y = r \sin\alpha$ (*a posteriori* verified) one may expand K_L as follows:

$$K_L = e^{-i\alpha/2}(K_0 + iyK_1) + O(y^2) , \tag{2.7}$$

where the lowest order matrix K_0 is real and experimentally known, being equal to $|K_{\mathrm{KM}}|$, up to signs. As shown in [18] K_1 is calculable as a function of the measured mixing angles and quark masses.

We will now show that it is possible to use the vacuum (at $T=0$) results to describe the features of CP violation in the quark sector during the EWPT, provided $r = |w/v|$, is constant during the phase transition, and α is small. Let us define from (2.4) the matrices

$$\begin{aligned} \tilde{M}^{(u)} &= \Gamma + re^{-i\alpha}\Delta = \Gamma + r\cos\alpha\Delta - ir\sin\alpha\Delta = U\tilde{D}^{(u)}U^T , \\ \tilde{M}^{(d)} &= r\Gamma + e^{-i\alpha}\Delta = e^{-i\alpha}(r\cos\alpha\Gamma + \Delta + ir\sin\alpha\Gamma) \\ &= V\tilde{D}^{(d)}V^T . \end{aligned} \tag{2.8}$$

As the $\tilde{D}$'s are dimensionless they can only be functions of r and α. Chang's expansion of U, V, $D^{(d)}$, and $D^{(u)}$ is given by

$$\begin{aligned} U &= U_0 - iyU_1 + O(y^2) , \\ V &= e^{-i\alpha/2}(V_0 + iyV_1) + O(y^2) , \\ D^{(u,d)} &= D_0^{(u,d)} + iyD_1^{(u,d)} + O(y^2) . \end{aligned} \tag{2.9}$$

It can be shown [18,19] that these matrices have the following properties: U_0 and V_0 are orthogonal, they diagonalize the real part of (2.8) and $K_0 = U_0^\dagger V_0$; $D_1^{(u,d)} = 0$; $U_0^\dagger U_1$ and $V_0^\dagger V_1$ are real and symmetric. As appears from (2.8) and (2.9) if r is constant during the EWPT and α is small, U_0, V_0, and $\tilde{D}_0^{(u,d)}$ are constant to order α. Moreover one has, from [19],

$$\begin{aligned} (1-w_\alpha^2)(U_0^\dagger U_1)ij &= \frac{(K_0 D^{(d)} K_0^T)_{ij}}{m_i^{(u)} + m_j^{(u)}} - \tfrac{1}{2} w_\alpha \delta_{ij} \\ &\approx \frac{(K_0 \tilde{D}_0^{(d)} K_0^T)_{ij}}{\tilde{m}_i^{(u)} + \tilde{m}_j^{(u)}} - \tfrac{1}{2} r \delta_{ij} , \\ (1-w_\alpha^2)(V_0^\dagger V_1)ij &= \frac{(K_0^T D^{(u)} K_0)_{ij}}{m_i^{(d)} + m_j^{(d)}} - \tfrac{1}{2} w_\alpha \delta_{ij} \\ &\approx \frac{(K_0^T \tilde{D}_0^{(u)} K_0)_{ij}}{\tilde{m}_i^{(d)} + \tilde{m}_j^{(d)}} - \tfrac{1}{2} r \delta_{ij} , \end{aligned} \tag{2.10}$$

where $w_\alpha = r\cos\alpha \approx r$ to the same order. One concludes that $U_0^\dagger U_1$ and $V_0^\dagger V_1$ are also almost constant during the EWPT. Hence the matrices $U_{(0,1)}$, $V_{(0,1)}$ and $\tilde{D}^{(u,d)}$ introduced above are the same as in the $T=0$ vacuum. All the effects of CP violation in the quark sector during the EWPT can be parametrized by the sole variation of the phase α or more properly of the small expansion parameter y. This property will be extensively used below but we want to show first that our hypotheses of constant r and small α may indeed be easily satisfied.

The VEV's of α and r are given by the minimization of the most general scalar potential compatible with LR and CP symmetry. It is shown in [16] that, without fine-tuning, α must be 0 or π. In order to have spontaneous CP violation a simple extension is proposed: a pseudoscalar singlet is introduced which transforms under P and CP as

$$P: \eta \to -\eta , \quad CP: \eta \to -\eta . \tag{2.11}$$

This allows us to introduce in the scalar potential $V = V_\phi + V_\Delta + V_{\Delta\phi}$ the additional terms

$$V_{\eta\phi} = iC_1 \langle \eta \rangle \eta \mathrm{Tr}(\phi_2^\dagger \phi_1 - \phi_1^\dagger \phi_2) + C_2 \eta^2 \mathrm{Tr}(\phi_2^\dagger \phi_1 + \phi_1^\dagger \phi_2) , \tag{2.12}$$

which contain a term odd in α. Then a nonzero VEV for α is obtained by minimizing the potential:

$$\tan\alpha = \frac{C_1\langle\eta\rangle\eta}{C_2\eta^2 + B_4\sigma_R^2}, \tag{2.13}$$

where B_4 is simply a combination of couplings from $V_{\Delta\phi}$. Also, with $\tan s \equiv 1/r$,

$$\tan 2s = \frac{(B_4\cos\alpha)\sigma_R^2 + (C_1\sin\alpha + C_2\cos\alpha)\eta^2}{A_3\sigma_R^2}, \tag{2.14}$$

where η and σ_R are the VEV's of the pseudoscalar singlet and the right scalar triplet, respectively. The other parameters are combinations of dimensionless couplings in the scalar potential. For $\eta \sim 100$ GeV and $\sigma_R \sim 100$ TeV,

$$\tan\alpha \approx C_1\eta^2/B_4\sigma_R^2 \tag{2.15}$$

has a well defined sign and is monotonically varying from 0 to some *small* finite value as η varies from 0 to its VEV during the EWPT. Also, as α is small,

$$\tan 2s \approx B_4/A_3 = \text{const}. \tag{2.16}$$

The temperature-dependent corrections to the above results can only be logarithmic and should consequently barely change the conclusions.

Some important remarks are in order here. It is a well known fact that the breaking of a discrete symmetry leads to the cosmologically troublesome formation of domain walls [10]. This problem may be solved in different ways. The most popular one supposes the existence of an inflationary period [20]. It is also possible that quantum gravity effects lift the degeneracy of the "vacua" and so make the walls unstable [21].[3] Ultimately the result has to be that the VEV of the field η has a definite sign throughout the observable Universe. However, this may not be enough for the purpose of electroweak baryogenesis. Clearly, the scale of spontaneous *CP* violation and the scale at which baryogenesis takes place must be different. If not, an equal amount of matter and antimatter is generated and one is left with no asymmetry.

In view of these considerations we may imagine different scenarios. First, it may be simply that *CP* is spontaneously broken in the scalar sector at a high scale and fed down to the quarks only at the electroweak scale. More explicitly, there may be another pseudoscalar field χ with couplings such as $\chi\eta^3$ and a VEV great enough for one of the above mentioned mechanisms to be realized, while keeping the η's VEV still at the EW scale. Alternatively, it may be that the VEV of the pseudoscalar field η is much greater than the electroweak scale, while our criteria on α and r are still satisfied. However, $\langle\eta\rangle$ cannot be too high, because in the decoupling limit $\langle\eta\rangle\to\infty$ there is no more *CP* violation in the couplings to the quarks [16]. Surely more elegant solutions may be found but this goes beyond the scope of this paper.

[3]This may even provide a mechanism for baryogenesis in models with a broken discrete symmetry [22].

III. SPONTANEOUS BARYOGENESIS WITH SPONTANEOUS *CP* VIOLATION

In [9] it was argued that the phase transition in the model we consider could be strongly first order because of the presence of trilinear couplings in the scalar potential. However, the astonishing complexity of the potential cannot allow us to exclude the opposite possibility, i.e., that, for some values of the parameters, the EWPT is weakly first order, occurring through nucleation and expansion of bubbles of true vacuum with a thick wall.

The mechanism in [9] used the reflection and transmission of heavy quarks on the wall in a *CP*-violating way. If the wall is thick, the reflection probability at threshold is exponentially suppressed and the mechanism breaks down. A possible alternative is to create the baryon asymmetry within the wall. For this we will suppose that all the quarks are in quasithermal equilibrium within the wall. This means that the thermalization time of the particles $\tau_{\text{th}} \approx (0.25-0.08T)^{-1}$ [6] is significantly smaller than the characteristic time of the moving wall $\tau_{\text{wall}} = \delta_{\text{wall}}/v_{\text{wall}}$, where δ_{wall} and v_{wall} are respectively the thickness and the velocity of the wall. The characteristic time for baryon number violation τ_B is estimated to be of the order $(\alpha_W^4 T)^{-1} \approx 10^6/T$ in the symmetric phase and is much longer in the broken phase. Baryon-number-violating processes are thus always out of thermal equilibrium. We further suppose that within the wall $\ddot{\alpha} \ll \dot{\alpha}$ so that the phase is quasistatic.

The hypothesis $\tau_{\text{th}} \ll \tau_{\text{wall}}$ with $\ddot{\alpha} \ll \dot{\alpha}$ is referred as the *adiabatic regime* in [6]. Thermal equilibrium is maintained through the bubble wall by fast interactions, while chemical equilibrium may not be satisfied as some processes, among which is baryon number violation, are comparatively slower. The departure from chemical equilibrium is handled by introducing effective chemical potentials due to the interaction with the slowly varying phase α.

To see how this may happen we go back to the Yukawa Lagrangian for the quarks (2.1). In the rest frame of the plasma, the time-varying phase of the bidoublet scalar field may be removed by a time-dependent rotation of the quark fields. We choose the rotation which diagonalizes the couplings with the scalar field:

$$\begin{aligned} u_L &\to u_L' = e^{-i\alpha/4}U^\dagger u_L, \\ u_R &\to u_R' = e^{i\alpha/4}U^T u_R, \\ d_L &\to d_L' = e^{-i\alpha/4}V^\dagger d_L, \\ d_R &\to d_R' = e^{i\alpha/4}V^T d_R, \end{aligned} \tag{3.1}$$

where we have used the convention of (2.5) and (2.6). The first consequence of (3.1) is that the kinetic part of the quark Lagrangian gives rise to a new term,

$$\mathcal{L}_K \to \mathcal{L}_K' + \mathcal{L}_{\text{eff}}, \tag{3.2}$$

where

$$\mathcal{L}_{\text{eff}} = -\tfrac{1}{4}\{\bar{d}_L'(\partial\!\!\!/\alpha)d_L' - \bar{d}_R'(\partial\!\!\!/\alpha)d_R' + \bar{u}_L'(\partial\!\!\!/\alpha)u_L' - \bar{u}_R'(\partial\!\!\!/\alpha)u_R'\} + i\bar{d}_L'(V^\dagger\partial\!\!\!/V)d_L' + i\bar{d}_R'(V^T\partial\!\!\!/V^*)d_R' + i\bar{u}_L'(U^\dagger\partial\!\!\!/U)u_L' + i\bar{u}_R'(U^T\partial\!\!\!/U^*)u_R' . \quad (3.3)$$

One has also to consider another contribution. Indeed our rotation, being chiral, has a gauge anomaly and the full Lagrangian is then modified as

$$\mathcal{L} \to \mathcal{L} + \mathcal{L}_{\text{eff}} + \mathcal{L}_\Theta = \mathcal{L} + \mathcal{L}_{\text{eff}} + \frac{\Theta(x)}{16\pi^2}\text{Tr}\,W_{\mu\nu}\tilde{W}^{\mu\nu} , \quad (3.4)$$

where

$$\Theta(x) = \text{Arg}\,\det M^{(u)}M^{(d)} \quad (3.5)$$

and $W_{\mu\nu} = (g/2i)\sum_a \lambda_a W^a_{\mu\nu}$, $\lambda_a/2$ being the gauge group generators in the appropriate representation.

We will first concentrate on the contribution given by (3.3). For this, we use the expansions (2.9) of U and V to get

$$\mathcal{L}_{\text{eff}} = -\tfrac{1}{4}\{-\bar{d}_L'(\partial\!\!\!/\alpha)d_L' + \bar{d}_R'(\partial\!\!\!/\alpha)d_R' + \bar{u}_L'(\partial\!\!\!/\alpha)u_L' - \bar{u}_R'(\partial\!\!\!/\alpha)u_R'\} - \bar{d}_L'(V_0^\dagger V_1\partial\!\!\!/y)d_L' + \bar{d}_R'(V_0^\dagger V_1\partial\!\!\!/y)^*d_R' + \bar{u}_L'(U_0^\dagger U_1\partial\!\!\!/y)u_L' - \bar{u}_R'(U_0^\dagger U_1\partial\!\!\!/y)^*u_R' , \quad (3.6)$$

where we have used that $U_{(0,1)}$ and $V_{(0,1)}$ are constant matrices.

As y is time dependent, this new part of the Lagrangian is of the form

$$\mathcal{L}_{\text{eff}} = -\sum_i \mu_i \bar{q}_i \gamma_0 q_i , \quad (3.7)$$

where the summation is over flavors and chiralities, and μ is proportional to $\dot{y}$: the μ's look like chemical potentials [6] though being dynamical quantities rather than Lagrange multipliers as in thermodynamics.

The quark distributions will try to minimize this effective potential, possibly through baryon-number-violating processes, which could lead to a net baryon excess at the end of the EWPT [6]. First, let us suppose that both left and right baryon-number-violating processes are effective at the same rate. Then the rate for baryon asymmetry production is proportional to the rate for B violation times the sum of the effective chemical potentials or, in the above formulation, to the trace of *all* the matrices in (3.6):

$$\dot{n}_B \propto \Gamma_B\{[\tfrac{3}{4}\dot{\alpha} - \dot{y}\,\text{Tr}(V_0^\dagger V_1)]_{d_L} + [-\tfrac{3}{4}\dot{\alpha} + \dot{y}\,\text{Tr}(V_0^\dagger V_1)^*]_{d_R} \times [-\tfrac{3}{4}\dot{\alpha} + \dot{y}\,\text{Tr}(U_0^\dagger U_1)]_{u_L} + [\tfrac{3}{4}\dot{\alpha} - \dot{y}\,\text{Tr}(U_0^\dagger U_1)^*]_{u_R}\} . \quad (3.8)$$

As $U_0^\dagger U_1$ and $V_0^\dagger V_1$ are real matrices, the different contributions cancel to zero. Now, from (3.6) and (3.8) it is clear that the effective potential can also be minimized through chirality-changing processes and this without producing any baryon asymmetry. So, in the two limiting cases, equal B_L and B_R violation rates or dominant chirality changes, no baryon asymmetry is produced. This reflects the fact that C and P violation are necessary ingredients of baryogenesis.

In reality, chirality-changing processes are dominant only for the heavy quarks and in this case no B asymmetry can be produced. For the light quarks the rate for sphaleron-like transitions is higher or of the same order as the rate for chirality-changing processes. Moreover, R-sphaleron transitions are completely negligible due to the high LR symmetry breaking scale. Figure 1 gives a naive illustration of the above considerations.

One could deduce that in the light quark sector, B-violating processes act differently on the L and the R quarks and a net B asymmetry can be produced. However, we have verified that this contribution to the asymmetry is at most of the same order of the anomalous contribution (3.4) that we will discuss below. Thus in the following we will neglect the baryon asymmetry contribution due to (3.6) as it can modify our final prediction for n_B at most by a factor of ~ 1 [23].

We now turn to the discussion of the other contribution in (3.4):

$$\mathcal{L}_\Theta = \frac{\Theta(x)}{16\pi^2}\text{Tr}\,W_{\mu\nu}\tilde{W}^{\mu\nu} . \quad (3.9)$$

A priori such a term arises for each gauge group of the model, but the only ones relevant for baryogenesis are $SU(2)_L$ and $SU(2)_R$ because they are chiral and have a nontrivial topological structure. Moreover, as the $SU(2)_R$ symmetry is broken, there is a high energy barrier (a R sphaleron [9]) between the different R vacua, and topological fluctuations are exponentially suppressed in this sector of the model at the electroweak temperature. So, as in the SM, "rapid" baryon number violation will occur through configurations of $SU(2)_L$.

Note that (3.4) also gives a contribution in the strong sector. Because parity is spontaneously broken, the LR model we consider belongs to a class of models in which Θ_{strong} is *a priori* calculable, but the final value of Θ we will obtain largely exceeds the bound from the electric dipole moment (EDM) of the neutron [24]. Some fine-tuning, i.e., the introduction of a nonzero Θ_{strong} bare, is

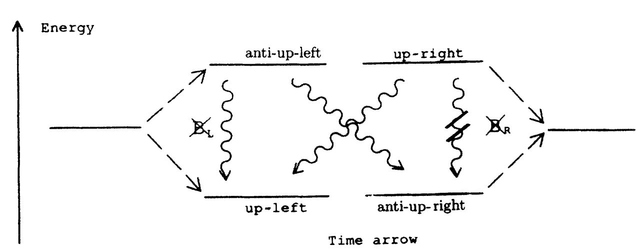

FIG. 1. Effective potential for the left-handed and right-handed quark density due to the time-dependent phase α. Both L-baryon-number-violating and chirality-changing transitions may take place. R-baryon-number-violating processes are negligible due to LR symmetry breaking.

thus necessary in order to cancel the $\Theta_{\rm strong}$. As our mechanism of baryogenesis is only sensitive to the *variation* of Θ, this will not affect our final result.

From the anomaly equation

$$\partial_\mu j_5^\mu = -\frac{1}{8\pi^2}\mathrm{Tr}\, W_{\mu\nu}^L \tilde{W}_L^{\mu\nu} , \tag{3.10}$$

we deduce the divergence of the baryonic current

$$\partial_\mu j_B^\mu = -\frac{N_f}{2}\partial_\mu j_5^\mu = \frac{N_f}{16\pi^2}\mathrm{Tr}\, W_{\mu\nu}^L \tilde{W}_L^{\mu\nu} , \tag{3.11}$$

where N_f is the number of flavors.[4] Using the anomaly equation with (3.4) we then get an effective potential for the baryon number density

$$\mathcal{L}_\Theta = \frac{\Theta}{N_f}\partial_\mu j_B^\nu = -\frac{\dot{\Theta}}{N_f} j_B^0 . \tag{3.13}$$

The fermions will try to minimize the potential (3.13) through baryon-number-violating processes. This, as pointed out in [6], is equivalent to the introduction of chemical potentials constraining the processes in a system slightly out of chemical equilibrium.

The master equation for the baryon number density is then given by [25]

$$\frac{dn_B}{dt} = -3N_f \frac{\Gamma_B}{T}\frac{\partial F}{\partial B} , \tag{3.14}$$

where F is the free energy of the system. If Θ is quasistatic during the EWPT, then $\partial F/\partial B \approx \partial V/\partial j_B^0 = \dot{\Theta}/N_f$.

As we do not know the details of the phase transition, and hence the exact time dependence of Θ, and as little is known on the exact rate for B violation in the intermediate regime of the phase transition, we will make the following simplifications: the rate per unit volume will be taken to be constant and the same as in the symmetric phase, $\Gamma_B = \kappa\alpha_W^4 T^4$ were κ parametrizes our ignorance of the exact rate but has been estimated from numerical simulations to be ~ 1 [26]; the variation of Θ during the EWPT takes its maximal value. These assumptions clearly give a very conservative upper bound for the baryon number produced during the phase transition.[5] Using the expression (2.5) for the mass matrices and the expansion (2.9) for the rotation matrices, we have the following expression for $\Theta(x)$:

$$\begin{aligned}\Theta(x) &= \mathrm{Arg}\, \det M^{(u)}M^{(d)}\\ &= \mathrm{Im}\,\mathrm{Tr}\,\ln M^{(u)}M^{(d)}\\ &= \mathrm{Im}\,\mathrm{Tr}\,\ln\{1-2iy(x)(U_0^\dagger U_1 - V_0^\dagger V_1)\} + O(y^2)\\ &= 2y(x)\mathrm{Tr}(V_0^\dagger V_1 - U_0^\dagger U_1) ,\end{aligned} \tag{3.15}$$

where we have used the small y approximation and the properties of the matrices given in the previous section. Note also that $U_0^\dagger U_1$ and $V_0^\dagger V_1$ are explicitly given by (2.10) and are *a priori* experimentally accessible quantities.

With the above assumptions we get the following upper bound on the baryon-to-entropy ratio:

$$\begin{aligned}\frac{n_B}{s} &\le \left.\frac{n_B}{s}\right|_{\rm max}\\ &= -3\frac{\kappa\alpha_W^4 T^3}{s}(\Theta_f - \Theta_i)\\ &\le -\kappa\frac{135}{g_{*S}\pi^2}\alpha_W^4 r\sin\alpha\,\mathrm{Tr}(V_0^\dagger V_1 - U_0^\dagger U_1) ,\end{aligned} \tag{3.16}$$

where we have used the fact that $\sin\alpha$ starts from zero [see (2.15)] and $s = 2\pi^2/45 g_{*S} T^3$ is the entropy density. Note that (3.16) does not depend on the bare value of Θ.

IV. PHENOMENOLOGICAL IMPLICATIONS: ϵ AND THE BAU

The ϵ parameter as predicted in the *LR* model with spontaneous *CP* violation is given by [19,29]

$$\epsilon = \epsilon_{\rm SM} + \epsilon_{LR} , \tag{4.1}$$

where

$$\epsilon_{\rm SM} = e^{i(\pi/4)} 1.34 s_2 s_3 \times \sin\delta\left\{1 + 860 S\left[\frac{m_t^2}{M_1^2}\right] s_2 \mathrm{Re} V_{ts}\right\} \tag{4.2}$$

and

[4]Note that if we take into account the anomalous contribution of the right gauge bosons, then (3.11) becomes

$$\partial_\mu j_B^\mu = \frac{N_f}{16\pi^2}\{\mathrm{Tr}\, W_{\mu\nu}^L \tilde{W}_L^{\mu\nu} - \mathrm{Tr}\, W_{\mu\nu}^R \tilde{W}_R^{\mu\nu}\} . \tag{3.12}$$

As the Θ term acts the same for the L and R sector their contributions cancel in mean, if $g_L = g_R$ as we assumed. The breaking of the *LR* symmetry at a higher scale is crucial in order to satisfy Sakharov's conditions.

[5]As pointed out in [27] and [28], the existence of anomalous chirality transitions due to *strong* sphalerons may further reduce the asymmetry. Their relevance depends on the value of $\tau_{\rm strong}/\tau_{\rm Higgs}$, i.e., the ratio of the time scale for strong sphaleron processes and the characteristic time of the phase transition. More has to be known about the dynamics of the phase transition in order to decide whether their effects have to be taken into account.

$$\epsilon_{LR}=-e^{i(\pi/4)}0.36\sin(\delta_2-\delta_1)\left[\frac{M_2}{1.4\ \text{TeV}}\right]^2 \times\left\{1+0.05\ln\left[\frac{M_2}{1.4\ \text{TeV}}\right]\right\}. \tag{4.3}$$

Some remarks are in order here.

The expression for $\epsilon_{\rm SM}$ is the same as the one calculated in the standard model from the KM matrix with three generations. M_1 is essentially the W_L boson of the SM. The product $s_2s_3\sin\delta$ is calculable in the LR model and is linear in $y=r\sin\alpha$ [19].

In the LR model, there is CP violation in the K_0-$\bar{K}_0$ system already with two generations. In that case, there are three CP-violating phases (γ,δ_1, and δ_2) in the mixing matrices which are also calculable functions of r and α. In the case of three generations, the LR contribution of the third generation is usually negligible in comparison to the one of the first two families. Moreover, the LR contribution with the first two generations usually dominates the SM one for not too heavy $W_R\approx W_2$ [29].

There is a subtlety in the fact that in a LR model the sign of the quark masses are observable. One can remove them from the mass matrices by doing a rotation on the right-handed quarks to the cost of effects in the K_R mixing matrix. So there is actually a discrete set of 2^{2N_f-1} different models. This clearly weakens the predictability of the model [29]: for example, for some signatures there are cancellations in ϵ_{LR} and $\epsilon_{\rm SM}$ so that the third generation dominates in (4.1) [29].

The interesting point for us is that ϵ is linear in $y=r\sin\alpha$, just as in our upper bound for the BAU. For the reasons given above, a complete analysis of the consequences for the LR model requires some care. For the sake of simplicity we will suppose that the LR model with two generations "saturates" ϵ in the sense that ϵ_{LR} dominates $\epsilon_{\rm SM}$ (see above). We then have the following expression for $(\delta_2-\delta_1)$ in (4.3):

$$\delta_2-\delta_1=\frac{r\sin\alpha}{1-w_\alpha^2}\left[\frac{m_uc^2+m_cs^2}{m_d}-\frac{m_us^2+m_cc^2}{m_s}+2\frac{m_d-m_s}{m_u+m_c}+2\frac{m_u-m_c}{m_d+m_s}(s^2-c^2)\right]$$
$$\approx\frac{r\sin\alpha}{1-w_\alpha^2}\left[7\,\text{sgn}\left[\frac{m_c}{m_s}\right]+6\,\text{sgn}\left[\frac{m_c}{m_d}\right]\right]. \tag{4.4}$$

Also we will limit ourselves to the first two generations of quarks in the estimate of the produced baryon asymmetry. We think this hypothesis makes the link between B and ϵ more transparent as only two generations are necessary to have CP violation. In a more realistic situation one should include the contribution of heavy quarks, especially in their mixing with the second generation. We have verified that this does not modify the order of magnitude of n_B [23]. Thus, in the two-generation case, from (2.10) we get

$$r\sin\alpha\,\text{Tr}(V_0^\dagger V_1-U_0^\dagger U_1)=\frac{1}{2}\frac{r\sin\alpha}{1-w_\alpha^2}\left[c^2\left[\frac{m_u}{m_d}-\frac{m_d}{m_u}\right]+c^2\left[\frac{m_c}{m_s}-\frac{m_s}{m_c}\right]+s^2\left[\frac{m_c}{m_d}-\frac{m_d}{m_c}\right]+s^2\left[\frac{m_u}{m_s}-\frac{m_s}{m_u}\right]\right]$$
$$\approx\frac{1}{2}\frac{r\sin\alpha}{1-w_\alpha^2}\left[7\,\text{sgn}\left[\frac{m_c}{m_s}\right]+6\,\text{sgn}\left[\frac{m_c}{m_d}\right]\right], \tag{4.5}$$

where $s\equiv\sin\theta_C$ is the Cabibbo mixing angle.

We finally have, from (3.16) and (4.3),

$$\frac{n_B}{s}\le 135\frac{\sqrt{2}}{0.72}\frac{\kappa\alpha_W^4}{\pi^2 g_{*S}}\text{Re}\epsilon\left[\frac{1.4\ \text{TeV}}{M_2}\right]^2\times\left[1-0.05\ln\left[\frac{M_2}{1.4\ \text{TeV}}\right]\right]. \tag{4.6}$$

There are only two unknowns in the upper bound, i.e., κ, describing the amount of B violation, and M_2, the mass of the charged right-handed W: these are dynamical parameters independent of CP violation. Reϵ is observable and the expression (4.6) predicts that it must have the same sign as the BAU. From the experimental value of Reϵ we get also an upper bound on the mass M_2 [20]:

$$4\times10^{-11}\le\frac{n_B}{s}<0.7\kappa\left[\frac{1.4\ \text{TeV}}{M_2}\right]^2 10^{-9} \tag{4.7}$$

or

$$M_2<6\sqrt{\kappa}\ \text{TeV}, \tag{4.8}$$

where κ is usually estimated to be of the order of 1. This upper bound is consistent with the one needed to satisfy $\epsilon\approx\epsilon_{LR}$ which is $M_2\le 19$ TeV [29].

V. CONCLUSIONS

The LR symmetry coupled to spontaneous CP violation offers the interesting opportunity to describe in a unified framework the generation of the baryon asymmetry and the LR phenomenology of the K_0-$\bar{K}_0$ system. The existence of one unique source of CP violation naturally permits one to relate n_B/s and ϵ and this is in possible agreement with the observations.

The analytical upper bound on the baryon asymmetry we have obtained rests on some crucial assumptions. The most important ones are that spontaneous CP violation and baryogenesis take place at different scales and that the phase transition is weakly first order. Although the complexity of the potential hardly allows one to draw definitive conclusions, we think it would be interesting to see to what extent these conditions can be met in a natural way.

Moreover, we have only considered the case where the *LR* model with two generations "explains" *CP* violation. The agreement in sign we have found between n_B/s and $\mathrm{Re}\,\epsilon$ could be not satisfied in some cases (inclusion of the third family, other mass signatures). Clearly, this should be further investigated and could be complemented by an analysis of the consequences on ϵ' and the electric dipole moment of the neutron.

On the other hand, the nonadiabatic charge transport mechanism is known to be more efficient if the bubble wall is thin. Even if working it out is more involved than the spontaneous baryogenesis mechanism considered here, it might prove worthwhile doing so.

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