Thermal background corrections to the neutrino electromagnetic vertex in models with charged scalar bosons

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We calculate the correction to the neutrino electromagnetic vertex due to the background of electrons in a large class of models, as the supersymmetric model with explicit breaking of R parity, where charged scalar bosons couple to leptons and which are able to provide an astrophysically interesting value for the neutrino magnetic (electric) moment: $\mu_\nu \sim 10^{-12}\,\mu_B$. We show that the medium contribution to the chirality-flipping magnetic (electric) dipole moment is not significant; however, a new chirality-flipping, but helicity-conserving, term arises which does not appear in the standard model with the exchange of W gauge bosons. It signals the presence of CP and CPT asymmetries in the medium and can contribute to the emission or absorption of longitudinal photons. We estimate the contribution of this new term to the rate of the plasmon decay process $\gamma_{\rm pl} \to \nu\nu$ in the core of degenerate stars, showing that it can be comparable to the contribution coming from the vacuum magnetic (dipole) moment. We also calculate the correction to the effective potential of a propagating neutrino in the presence of a magnetic field due to a chirality-preserving contribution to the diagonal magnetic moment from the medium. This contribution is identical for particles and antiparticles and so it need not vanish for Majorana neutrinos.

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I. INTRODUCTION

It is well known that in a medium composed of a gas of electrons the dispersion relations for transverse and longitudinal photons are quite different from those in the vacuum [1]. Both modes, generally called plasmons, acquire an effective plasma mass which allows them to couple to a pair of neutrinos. Indeed, the process $\gamma_{\rm pl} \to \nu\nu$ is already predicted by the standard model (SM) of weak interactions and represents the primary source for the energy loss of degenerate plasmas, such as red giants and white dwarfs [1], and has recently received considerable attention [2]. Furthermore, if neutrinos couple to the photons through a magnetic (or electric) moment μ_ν in the vacuum, the rate of the energy loss of stellar plasmas due to μ_ν is comparable to the SM contribution for $\mu_\nu \simeq 10^{-12}\mu_B$, where μ_B indicates the Bohr magneton, and no larger values of μ_ν can be tolerated.

The great attention on the electromagnetic properties of neutrinos has also been motivated by the suggestion that the observed neutrino deficit from the Sun [3] can be explained by the interactions of solar neutrinos with the magnetic field of the outer layers of the Sun. This requires large diagonal [4] and/or transition [5] magnetic moments for the electron neutrino, of order of $\left(10^{-11} - 10^{-10}\right)\mu_B$. Moreover, a value of $\mu_\nu \sim 10^{-12}\mu_B$ is intriguing for Dirac neutrinos from the cosmological point of view [6].

Unfortunately in the SM the magnetic moment is generated at the one-loop level and is extremely small because the only scales of the problems are the mass of the neutrino m_{ν_e} and the Fermi constant G_F. Indeed [7],

$$\mu_{\nu_e} = \frac{3\,e\,G_F}{8\,\pi^2\,\sqrt{2}}\,m_{\nu_e} \simeq 3\times 10^{-19}\,\left(\frac{m_{\nu_e}}{1\,\mathrm{eV}}\right)\,\mu_B. \qquad (1)$$

It is clear that to get the interesting value of order of $10^{-12}\,\mu_B$ for the magnetic moment one has to invoke some new physics beyond the SM. Indeed, a large class of models [8, 9] which are able to provide large magnetic moments to neutrinos have the common feature of possessing new charged scalar bosons whose mass can be arbitrary [8] or fixed by the supersymmetric scale [9].

The aim of this paper is to calculate the contribution to the electromagnetic vertex of neutrinos in a background of electrons arising when these new charged scalar bosons couple to leptons.

For the sake of concreteness we have decided to perform all the calculations in a well-defined framework, namely, the supersymmetric model with explicit breaking of R parity [9,10], where neutrinos are Majorana particles. The generalization to other models for both Majorana and Dirac neutrinos is straightforward [11].

We find that the magnetic (electric) dipole moment does not receive from the medium any significant enhancement. However, a new chirality-flipping, but helicity-conserving, term appears. This new term makes possible the emission or absorption of external longitudinal photons, which in a thermal bath must be treated as new real degrees of freedom, whereas in the vacuum only transverse photons can be emitted and longitudinal photons are only virtual degrees of freedom which can propagate only in internal loops.

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We estimate the contribution coming from this new term to the plasmon decay process $\gamma_{pl} \to \nu\nu$ [1], which is the primary source of the rapid cooling of degenerate stars, and show that it can be comparable to the contribution due to the vacuum magnetic moment.

We also show that, as in the case of SM [12], one-loop thermal corrections bring in an effective charge for Majorana neutrinos in a medium as well as a magnetic (electric) diagonal dipole moment which would not be allowed in the vacuum. Moreover, the effective potential receives a correction in the presence of an external magnetic field.

The paper is organized as follows. In Sec. II we present the model we have adopted to illustrate our calculations. In Sec. III the calculations are described and general formulas for the form factors are given in terms of integrals over the electron-positron energy distribution. Some details of the calculations and the results in different limits are given in the Appendix. Then in Sec. IV we estimate the plasmon decay rate contribution from the new terms arising in the medium. Section V presents our conclusions.

II. THE MODEL

The minimal supersymmetric standard model [13] with explicit R-parity breaking [10] via L violation is described by the superpotential which, in addition to the standard Yukawa couplings, involves the $\Delta L \neq 0$ couplings

$$f^{\Delta L\neq 0} = \tfrac{1}{2}\lambda_{ijk}\,[L_i, L_j]\, e_k^c + \lambda'_{ijk}\, L_i Q_j d_k^c, \tag{2}$$

where i,j,k are generation indices, L, Q are the lepton and the quark left-handed doublets, and e^c, d^c are (the charge conjugate of) the right-handed lepton and charge $-1/3$ quark singlets, respectively. The first term in Eq. (2) gives rise to the Lagrangian

$$\mathcal{L}^{\Delta L\neq 0} = \lambda_{ijk}\left[\tilde{l}_L^j \bar{l}_R^k \nu_L^i + \left(\tilde{l}_R^k\right)^* \left(\bar{\nu}_L^i\right)^c l_L^j + \tilde{\nu}_L^i \bar{l}_R^k l_L^j - (i \leftrightarrow j)\right] + \text{H.c.}, \tag{3}$$

where $l^c = C\bar{l}^T$ means the charge conjugated of the fermion l, C being the charge conjugation matrix, and we have disregarded the second term in Eq. (2) since we are interested in a medium consisting of electrons and positrons.

In the vacuum the couplings of Eq. (3) give rise to neutrino masses and magnetic moments (after the insertion of a photon vertex in any charged internal line) through two different one-loop diagrams, see Fig. 1. In all the diagrams of Fig. 1 a helicity flip on the internal fermion line is necessary. As explicitly indicated, this also requires a mixing of the scalar leptons associated with the different chiralities. Any of the diagrams of Fig. 1 contribute to $m_{\nu_i\nu_j}$ and $\mu_{\nu_i\nu_j}$ as

$$m_{\nu_i\nu_j} \simeq \frac{\lambda_a\lambda_b}{16\pi^2}\, m\, \frac{\sin 2\theta_k}{2}\left(\frac{1}{m_{1k}^2} - \frac{1}{m_{2k}^2}\right), \tag{4}$$

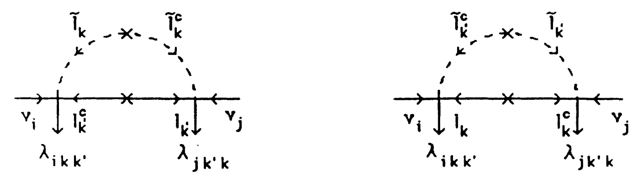

FIG. 1. Feynman diagrams contributing to $\mu_{\nu_i\nu_j}$ after a photon insertion line in any charged internal line.

$$\mu_{\nu_i\nu_j} = \mu_B \frac{\lambda_a\lambda_b}{8\pi^2}\, m_e\, m \sin 2\theta_k \left\{\frac{1}{m_{1k}^2}\left[\ln\left(\frac{m_{1k}^2}{m^2}\right) - 1\right] - \frac{1}{m_{2k}^2}\left[\ln\left(\frac{m_{2k}^2}{m^2}\right) - 1\right]\right\}, \tag{5}$$

where m is the mass of the internal charged lepton, λ_a and λ_b are the appropriate couplings, and m_{1k}, m_{2k} and θ_k are the two mass eigenvalues and the mixing angle of the $\tilde{l}_L^k \tilde{l}_R^k$ mixing matrix, respectively.

If, for instance, we examine the $\nu_e\nu_\mu\gamma$ vertex, since the contribution to $\mu_{\nu_e\nu_\mu}$ turns out to be proportional to $m_{\nu_e\nu_\mu}$ and we require $m_{\nu_e\nu_\mu} \lesssim 10$ eV, a strong bound on $\mu_{\nu_e\nu_\mu}$ is obtained, roughly $\mu_{\nu_e\nu_\mu} \lesssim 10^{-14}\mu_B$. To enhance the vacuum magnetic moment $\mu_{\nu_e\nu_\mu}$ to the astrophysically interesting value of $10^{-12}\mu_B$, one can follow Ref. [9] and impose that the lepton number $L_e - L_\mu$ remains unbroken and that the Lagrangian, in the limit of vanishing Yukawa couplings, is symmetric under an $\mathrm{SU}(2)_H$ horizontal symmetry acting on the first and the second generations. Under this assumption, the only terms which survive in Eq. (2) are

$$f^{\Delta L\neq 0} = \lambda_{123} L_e L_\mu \tau^c \;+\; \lambda_{131}\left(L_e L_\tau e^c + L_\mu L_\tau \mu^c\right). \tag{6}$$

The corresponding graphs giving rise to $m_{\nu_e\nu_\mu}$ and $\mu_{\nu_e\nu_\mu}$ are given in Fig. 2. Since under the horizontal symmetry $\mathrm{SU}(2)_H$ the mass term $m_{\nu_e\nu_\mu}$ is odd, whereas $\mu_{\nu_e\nu_\mu}$ is even, diagrams 2(a) and 2(b) and 2(c) and 2(d) tend to cancel out and to sum up for $m_{\nu_e\nu_\mu}$ and $\mu_{\nu_e\nu_\mu}$, respectively. As a consequence, $\mu_{\nu_e\nu_\mu}$ is now no longer proportional to $m_{\nu_e\nu_\mu}$ and the value $\mu_{\nu_e\nu_\mu} \simeq 10^{-12}\mu_B$ can be achieved [9].

If we now consider a medium filled up with a gas of electrons, the Lagrangian which gives rise to the finite-temperature effective vertices involving ν_e, ν_μ, and γ is

$$\mathcal{L} = \lambda_{131}\tilde{\tau}_L \bar{e}_R \nu_{eL} \;-\; \lambda_{123}\tilde{\tau}_R^*\left(\bar{\nu}_{\mu L}\right)^c e_L \;+ \text{H.c.} \tag{7}$$

This will be our starting Lagrangian in the next section.[1]

[1]Even if we are focusing on a particular vertex, $\nu_e\nu_\mu\gamma$, in a particular model, we want to stress again that the structure of the form factors derived in the next section are model independent.

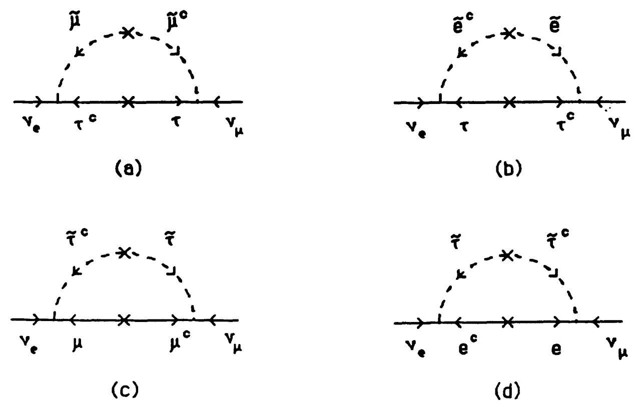

FIG. 2. Feynman diagrams contributing to $\mu_{\nu_e\nu_\mu}$ (after a photon insertion line in any charged internal line) when $L_e - L_\mu$ conservation is imposed.

III. CALCULATION OF THE VERTEX FUNCTIONS IN THE MEDIUM

A. Chirality-flipping terms

In this subsection we calculate the contribution to the chirality-flipping term $(\nu_{\mu L})^c\,\nu_{eL}\gamma$ in the medium assuming that the temperature is such that there are no charged scalar particles $\tilde{\tau}_L$ and $\tilde{\tau}_R$ in the background.

Therefore, only the electron propagator has a background-dependent part and is given by

$$S_F(k) = (\not{k} + m_e)\left[\frac{1}{k^2 - m_e^2} + 2\pi i\delta\left(k^2 - m_e^2\right)\eta\left(k\cdot u\right)\right], \tag{8}$$

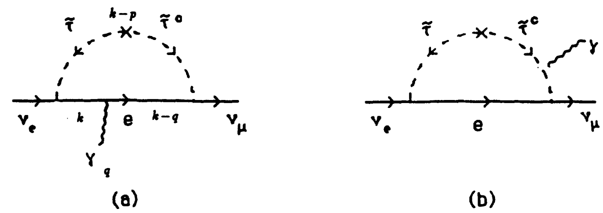

FIG. 3. Relevant Feynman diagram for the $\nu_e\nu_\mu\gamma$ vertex in a background of electrons.

where

$$\eta(x) = \frac{\theta(x)}{e^{\beta(x-\mu)} + 1} + \frac{\theta(-x)}{e^{-\beta(x-\mu)} + 1}. \tag{9}$$

Here $\theta(x)$ is the unit step function, $1/\beta$ is the temperature, μ is the electron chemical potential, and u^μ is the four-velocity of the center of mass of the medium.

The off-shell electromagnetic vertex function $\Gamma_\mu^{LR}(p_1, p_2, u)$ is defined in such a way that

$$\langle\nu_\mu(p_1)\left|J_\mu^{\rm EM}(0)\right|\nu_e(p_2)\rangle \equiv \bar{u}_\mu(p_1)\, i\Gamma_\mu^{LR}(p_1, p_2, u)\, u_e(p)\,. \tag{10}$$

Note that in the vacuum the dependence on u^μ vanishes. The diagrams which enter the calculation of Γ_μ^{LR} are shown in Fig. 3.

Since the integrals involved in the calculations of Γ_μ^{LR} are cut off by the electron-positron distribution, diagram 3(b) gives a contribution to Γ_μ^{LR} suppressed by an extra power of $1/\tilde{m}^2$, $\tilde{m}$ being the typical supersymmetric mass, relative to diagram 3(a). Therefore, we neglect it.

With these preliminaries we have to calculate the quantity

$$-i\Gamma_\mu^{LR} = e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3 \int\frac{d^4k}{(2\pi)^4}\, L\, iS_F(k-q)\,\gamma_\mu iS_F(k)\, L\left[\frac{1}{(k-p)^2 - m_{23}^2} - \frac{1}{(k-p)^2 - m_{13}^2}\right], \tag{11}$$

where $L = (1-\gamma_5)/2$ is the left-handed chirality operator.

When the electron propagator from Eq. (8) is plugged in Eq. (11), several terms are produced beyond the standard vacuum term. The terms with two factors $\eta(k\cdot u)$ contribute only to the absorptive part of the amplitude (see, for instance, D'Olivo *et al.* in Ref. [12] for further comments on this point). In this paper we will calculate only the dispersive part of the form factors. We also make the local approximation, i.e., neglect the momentum dependence of the heavy charged scalar bosons; hence $\mathcal{G}_\mu^{LR}$ reduces to

$$\Gamma_\mu^{LR} = e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right]\int\frac{d^4k}{(2\pi)^3} L\,(\not{k} - \not{q} + m_e)\,\gamma_\mu\,(\not{k} + m_e)$$
$$\times\left\{\frac{\delta\left[(k-q)^2 - m_e^2\right]}{k^2 - m_e^2}\eta\left[(k-q)\cdot u\right] + \frac{\delta\left(k^2 - m_e^2\right)}{(k-q)^2 - m_e^2}\eta\left(k\cdot u\right)\right\}L. \tag{12}$$

Making the change of variable $k \to k+q$ in the first integral of Eq. (12) we obtain

$$\Gamma_\mu^{LR} = e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right] m_e \times\left(\mathcal{I}_\mu^1 + \mathcal{I}_\mu^2 + \mathcal{I}_\mu^3\right)L, \tag{13}$$

where we have defined

$$\mathcal{I}_\mu^1 = \int\frac{d^3k}{(2\pi)^3}\frac{(f_- - f_+)}{2E}\frac{4k^\mu q^2}{(q^2 + 2k\cdot q)(q^2 - 2k\cdot q)}, \tag{14}$$

$$\mathcal{I}_\mu^2 = \int\frac{d^3k}{(2\pi)^3}\frac{(f_- + f_+)}{2E}\frac{-2i\sigma_{\mu\nu}q^\nu\, q^2}{(q^2 + 2k\cdot q)(q^2 - 2k\cdot q)}, \tag{15}$$

$$\mathcal{I}^3_\mu = \int \frac{d^3k}{(2\pi)^3} \frac{(f_- - f_+)}{2E} \frac{-4\,(k\cdot q)\; q_\mu}{(q^2 + 2k\cdot q)\,(q^2 - 2k\cdot q)}, \quad (16)$$

and $\sigma_{\mu\nu} = (i/2)\,[\gamma_\mu, \gamma_\nu]$, $k^\mu = (E, \mathbf{k})$, $E = \sqrt{|\mathbf{k}| + m_e^2}$, and we have introduced the electron and positron distributions

$$f_\mp\,(k) = \frac{1}{e^{\beta(k\cdot u \mp \mu)} + 1}. \quad (17)$$

Note that $q^\mu \Gamma^{LR}_\mu = 0$ due to the electromagnetic gauge invariance.

If the initial Lagrangian (in the vacuum) of our model respects CP (so that we take all the λ's real), we can define the four-component self-conjugate states $\chi_a = \nu_a + \eta_a\,(\nu_a)^c$, where $\eta_a = \pm 1$ are the intrinsic CP parities of χ_a's. It is well known that if χ_e and χ_μ have opposite (equal) CP parities, then their off-diagonal dipole (magnetic) moment vanishes [14]. Moreover, the correct expression for $\Gamma^{LR,\mathrm{Majorana}}_\mu$ can be derived from expressions (13)–(16) once one remembers that χ_e and χ_μ are Majorana neutrinos, so that for each Feynman diagram there exists a second diagram in which all the internal particles are replaced by their charge conjugates [14]. A practical rule to derive $\Gamma^{LR,\mathrm{Majorana}}_\mu$ is to treat neutrinos as Dirac particles and then add to $\Gamma^{LR,\mathrm{Dirac}}_\mu$ its charge conjugate part

$$\Gamma^{LR,\mathrm{Majorana}}_\mu\,(p_1, p_2, u) = \Gamma^{LR,\mathrm{Dirac}}_\mu\,(p_1, p_2, u) + \eta_e\eta_\mu\,\gamma^0\left[C\,\Gamma^{LR,\mathrm{Dirac}}_\mu\,(-p_1, -p_2, u)\;C^{-1}\right]^*\gamma^0. \quad (18)$$

If we now introduce the tensors

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}, \quad (19)$$

$$\tilde{u}_\mu = \tilde{g}_{\mu\nu} u^\nu, \quad (20)$$

and remind that, as a consequence of having taken the limit $\tilde{m} \to \infty$, the form factors depend only on q and not on p_1 and p_2 separately, the complete off-shell electromagnetic vertex function reads

$$\Gamma^{LR,\mathrm{Majorana}}_\mu = \left[F_1 \tilde{u}_\mu\,(L + \eta_e\eta_\mu R) + i\frac{F_2}{2}\,(1 - \eta_e\eta_\mu)\,\sigma_{\mu\nu}q^\nu + i\frac{F_2}{2}\sigma_{\mu\nu}\gamma_5\,(1 + \eta_e\eta_\mu)\,q^\nu\right], \quad (21)$$

where $R = (1 + \gamma_5)\,/2$ is the right-handed chirality operator and

$$F_1 = e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right]\; m_e\,2q^2 \int \frac{d^3k}{(2\pi)^2}\frac{(f_- - f_+)}{2E}\frac{(u\cdot k)}{(q^2 + 2k\cdot q)\,(q^2 - 2k\cdot q)}, \quad (22)$$

$$F_2 = -e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right]\; m_e 2q^2 \int \frac{d^3k}{(2\pi)^2}\frac{(f_- + f_+)}{2E}\frac{1}{(q^2 + 2k\cdot q)\,(q^2 - 2k\cdot q)}. \quad (23)$$

The convenience of presenting the form factors F_1 and F_2 in this form relies on the fact that F_1 and F_2 are scalars so that the integrations can be performed in the rest frame of the medium defined by setting $u^\mu = (1, \mathbf{0})$. In the rest frame of the medium we will denote the components of the four vector q^μ by

$$q^\mu = (\Omega, \boldsymbol{\mathcal{Q}}), \quad (24)$$

where $\Omega \equiv q\cdot u$ and $|\boldsymbol{\mathcal{Q}}| \equiv \sqrt{\Omega^2 - q^2}$ are manifestly scalar functions.

From expression (21) it is clear that the form factor F_2 can be regarded as an additional contribution to the magnetic (or electric) dipole moment. We note that, since the contribution to the magnetic (electric) dipole moment in the medium must be coherent with the neutrino propagation, it is necessary to take the limit $q^\mu \to 0$ in Eq. (23). When $q^\mu = 0$, the internal electron lines with four-momenta k and $k - q$ are on the mass shell, i.e., in the limit $q^\mu \to 0$, diagram 3(a) describes the process $\nu_e e \to \nu_\mu e$ with a modification of the external electron lines by the electromagnetic field. Therefore one can expect no enhancement in the medium for the magnetic (electric) dipole moment as suggested by Giunti *et al.* in Ref. [12]. The flipping term proportional to F_1 does contribute to the emission or absorption of longitudinal photons from the internal electron without changing its helicity. Let us recall that in the vacuum the magnetic (electric) moment flips both chirality and helicity[2] (at the leading order), since the photon is purely transverse, while the vertex $F_1\tilde{u}^\mu$ cannot change the helicity of the incoming neutrino since the exchanged photon is longitudinal and does have vanishing helicity. Moreover, if the chemical potential of the electron background is zero, then $f_+ = f_-$ and, therefore, $F_1 = 0$ for both Dirac and Majorana neutrinos. Indeed, if we start from a CP invariant Lagrangian in the vacuum and, for $\mu = 0$, the background is symmetric, F_1 must satisfy the relation

$$F_1\,(-p_1, -p_2, u) = -F_1\,(p_1, p_2, u), \quad (25)$$

[2]We remember that for massive neutrinos, chirality and helicity do not coincide and that in the present model the couplings which give rise to the new term also give origin to the mass of the neutrinos.

independently of whether the neutrino is of the Dirac type or Majorana type. Since in our case F_1 is only a function of $p_1 - p_2$, Eq. (25) implies that it is zero. However, we have CP as well as CPT asymmetries in the medium due to the presence of a nonvanishing electron chemical potential and therefore F_1 is not zero in general.[3] Note also that, in the particular model we have adopted to perform our calculations, the F_1 term does not respect the horizontal $\mathrm{SU}(2)_H$ symmetry of the starting Lagrangian (7) and it can be nonvanishing only because the medium, filled up with electrons and not with muons, breaks this symmetry.

The integrals (22) and (23) can be worked out analytically in different regimes either for a soft photon, namely, when the exchanged momentum q is much smaller than the momenta of the thermalized electrons (of order of T or μ), or for a hard, but almost light-cone photon, $q^2 \to 0$. The calculations for the different regimes, like the ultrarelativistic and the classical ones, can be found in the Appendix. We report here only the result for a degenerate electron gas ($T \ll \mu - m_e$) since they are of interest for the calculations of the plasmon decay rate which will be performed in the next section. We obtain

$$F_1 = e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2}-\frac{1}{m_{13}^2}\right] m_e\,\frac{1}{16\pi^2}\frac{q^2}{m_e^2\Omega^2}\frac{k_F^3}{3}, \tag{26}$$

$$F_2 = e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2}-\frac{1}{m_{13}^2}\right] m_e\,\frac{1}{4\pi^2}\frac{q^2}{m_e^2\Omega^2} \times\left[\frac{k_F E_F}{2}+m_e^2\ln\left(\frac{k_F+E_F}{m_e}\right)\right], \tag{27}$$

where $E_F = \sqrt{m_e^2+k_F^2} = \mu$ is the Fermi energy.

B. Chirality-conserving terms

In this subsection we calculate the contribution from the medium to the chirality-conserving term $\nu_e\nu_e\gamma$ (an analogous calculation can be performed for other vertices).

Repeating the same considerations of Sec. III A, we have to calculate the quantity (see Fig. 4)

$$\Gamma_\mu^{LL} = e\,|\lambda_{131}|^2\left[\frac{\cos^2\theta_3}{m_{23}^2}+\frac{\sin^2\theta_3}{m_{13}^2}\right]\int\frac{d^4k}{(2\pi)^3}\delta\left(k^2-m_e^2\right)\eta\left(k\cdot u\right)\times\left\{\frac{4q^2\,\not{k}k_\mu+q^2\left[-2i\sigma_{\mu\nu}q^\nu-2k_\mu\,\not{q}+2\left(k\cdot q\right)\gamma_\mu\right]}{\left(q^2+2k\cdot q\right)\left(q^2-2k\cdot q\right)}\right. \left.+\frac{-2\left(k\cdot q\right)\left[2\,\not{k}q_\mu-2k_\mu\,\not{q}+2\left(k\cdot q\right)\gamma_\mu\right]}{\left(q^2+2k\cdot q\right)\left(q^2-2k\cdot q\right)}\right\}L. \tag{28}$$

Again $q^\mu\Gamma_\mu^{LL}=0$ for the electromagnetic gauge invariance.

It is then easy to show that the complete electromagnetic vertex function Γ_μ^{LL} can be written under the form

$$\Gamma_\mu^{LL} = R[\tilde{F}_1\tilde{u}_\mu\,\not{u}+i\tilde{F}_2\sigma_{\mu\nu}q^\nu\,\not{u}+i\tilde{F}_3\left(\gamma_\mu u_\nu-\gamma_\nu u_\mu\right)q^\nu+\tilde{F}_4\tilde{g}_{\mu\nu}\gamma^\nu]L, \tag{29}$$

where

$$\tilde{F}_1 = -\frac{4}{3}e\,|\lambda_{131}|^2\left[\frac{\cos^2\theta_3}{m_{23}^2}+\frac{\sin^2\theta_3}{m_{13}^2}\right]q^2\int\frac{d^3k}{(2\pi)^3}\frac{(f_-+f_+)}{2E}\frac{\left[4\left(k\cdot u\right)^2-m_e^2\right]}{\left(q^2+2k\cdot q\right)\left(q^2-2k\cdot q\right)}, \tag{30}$$

$$\tilde{F}_2 = F_1, \tag{31}$$

$$\tilde{F}_3 = i\,\left(q^2\right)^{-1}\left(q\cdot u\right)\tilde{F}_1, \tag{32}$$

$$\tilde{F}_4 = \frac{8}{3}e\,|\lambda_{131}|^2\left[\frac{\cos^2\theta_3}{m_{23}^2}+\frac{\sin^2\theta_3}{m_{13}^2}\right]q^2\int\frac{d^3k}{(2\pi)^3}\frac{(f_-+f_+)}{2E}\frac{\left[\left(k\cdot u\right)^2-m_e^2\right]}{\left(q^2+2k\cdot q\right)\left(q^2-2k\cdot q\right)}. \tag{33}$$

The expression (29) holds if neutrinos are of the Dirac type. If they are of the Majorana type, then, applying the relation (18), $\Gamma_\mu^{LL,\mathrm{Majorana}}$ can be easily obtained by adding to Γ_μ^{LL} its self-conjugate term. We then obtain

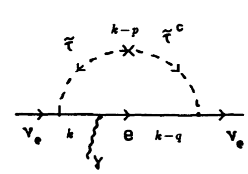

FIG. 4. Relevant Feynman diagram for the $\nu_e\nu_e\gamma$ vertex in a background of electrons.

[3]For a detailed discussion on the electromagnetic properties of neutrinos in a medium and the role played by discrete symmetries, see the first reference in [12].

$$\Gamma_{\mu}^{LL,\text{Majorana}} = -\,[\tilde{F}_1\tilde{u}_\mu \not{q} - i\tilde{F}_2\sigma_{\mu\nu}q^\nu \not{q}\gamma_5 + i\tilde{F}_3(\gamma_\mu u_\nu - \gamma_\nu u_\mu)q^\nu + \tilde{F}_4\tilde{g}_{\mu\nu}\gamma^\nu]\gamma_5, \tag{34}$$

where we have make use of the property $\gamma_5^2 = \mathbf{1}$.

The expressions for the form factors in the different regimes can be found in the Appendix.

If we now assume that neutrinos are propagating in an external field of the form $A^\mu = (\phi, \mathbf{0})$ in the rest frame of the medium, we see that $\tilde{F}_4$ yields an additional contribution to the charge radius; moreover $\tilde{F}_3$ can be regarded as an additional contribution to the electric dipole moment and $\tilde{F}_2$ to the magnetic one. It is well known that Majorana neutrinos can have neither a charge radius nor diagonal electric or magnetic dipole moments in the vacuum [14] since, for instance, $\bar{\nu}\sigma_{\mu\nu}\nu = 0$ for Majorana neutrinos. Nevertheless, already in the SM they can have electric or magnetic dipole moments in a medium of electrons which introduces CP as well CPT asymmetries [12]. We have found that additional contributions can be given by some new physics beyond the SM. These new contributions are again nonvanishing only if the medium is CP and CPT asymmetric.

To have the feeling on the importance of these new contributions, we can consider the scattering of an electron neutrino with an external static and uniform magnetic field $\mathbf{B}$. The Dirac equation for a massless neutrino spinor ψ_ν in the medium can be written as[4]

$$\mathcal{V}\psi_\nu = [(1 - a_L) \not{k} + (b_L + c_L) \not{u}]\psi_\nu = 0, \tag{35}$$

where $\mathcal{V}$ is called effective potential. The coefficients a_L and b_L have been calculated in [1] whereas

$$c_L = \boldsymbol{\mu}\cdot\mathbf{B}, \boldsymbol{\mu} = i\tilde{F}_2\left(\Omega = 0, |\mathbf{Q}| \to 0\right)\boldsymbol{\sigma}. \tag{36}$$

The value of $\tilde{F}_2$ in the limit $\Omega = 0$, $|\mathbf{Q}| \to 0$ has been found, in the nonrelativistic limit, by D'Olivo *et al.* in Ref. [12] and reads

$$\tilde{F}_2\left(\Omega = 0, |\mathbf{Q}| \to 0\right) = \frac{\beta n_-}{4m_e}, \tag{37}$$

where $n_- = e^{\beta(\mu - m_e)}\left(2m_e/\beta\right)^{3/2}\left[\Gamma\left(3/2\right)/2\pi^2\right]$ is the number density of electrons.

The meaning of Eq. (35) is that, in the presence of a magnetic field, the effective potential of a neutrino propagating through a medium gets a new contribution proportional to $|\mathbf{B}|$. The relative importance of the matter density effects thus depend on the magnitude of $\mathbf{B}$. For instance, in the Sun $|\mathbf{B}|$ is a few tenths of Tesla, the temperature is of order of 1 keV, so that c_L/b_L is very small, $c_L/b_L \simeq |\lambda_{131}|^2\,10^{-11}$ for $m_{23} \simeq m_{13} \simeq 100$ GeV.

Nevertheless the new terms for the vertex $\gamma\gamma\nu$ arising in a thermal bath can play a crucial role in different physical contexts and in the next section we want to show it explicitly by studying the plasmon decay in dense objects like red giants and white dwarfs.

IV. PLASMON DECAY

In this section we want to estimate the contribution to the energy loss of degenerate plasmas, such as red giants and white dwarfs [1] through the plasmon decay induced by the chirality-flipping term proportional to F_1.

The differential decay rate of the process $\gamma_{pl}(q) \to \nu_e(p_1)\,\nu_\mu(p_2)$ due to the F_1 term in the rest frame of the medium (there is no interference contribution with the magnetic moment term) is

$$d\Gamma_{F_1} = \frac{1}{2\Omega}\left(2\pi\right)^4\delta^{(4)}\left(q - p_1 - p_2\right)\left|\bar{\mathcal{M}}_{F_1}\right|^2\frac{d^3p_1}{\left(2\pi\right)^3 2E_1}\frac{d^3p_2}{\left(2\pi\right)^3 2E_2}, \tag{38}$$

where

$$\left|\bar{\mathcal{M}}_{F_1}\right|^2 = 4\left(e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2} - \frac{1}{m_{13}^2}\right]m_e\frac{1}{2\pi^2}\frac{q^2}{m_e^2\Omega^2}\frac{k_F^3}{3}\right)^2\frac{\left(u\cdot\eta_3\right)^2}{\left(\Omega\partial\varepsilon_L/\partial\Omega\right)}\left(p_1\cdot p_2\right). \tag{39}$$

We have neglected neutrino masses and made use of the longitudinal photon vector

$$\eta_3 = \frac{1}{\left(q^2\right)^{1/2}}\left(|\mathbf{Q}|, 0, 0, \Omega\right), \tag{40}$$

which satisfies the relation $\eta_3 \cdot q = 0$.

In expression (39) ε_L represents the dielectric constant of the longitudinal plasmons [1] and, since we are considering the case of degenerate stars, we are using expression (26). Using the Lenard's formula

$$\int\frac{d^3p_1}{2E_1}\frac{d^3p_2}{2E_2}\delta^{(4)}\left(q - p_1 - p_2\right)p_1^\mu p_2^\nu = \frac{\pi}{24}\left(2p_1^\mu p_2^\nu + g^{\mu\nu}q^2\right), \tag{41}$$

[4]Even in the SM the effective potential receives a correction in the presence of a static and uniform magnetic field, see D'Olivo *et al.* in [12], but not proportional to $\not{u}$ as in the class of models considered here.

we find that

$$\Gamma_{F_1} = \frac{1}{8\pi}\frac{1}{\Omega^2}\frac{|\mathbf{Q}|^2}{\partial\varepsilon_L/\partial\Omega}\left(e\frac{\lambda_{123}\lambda_{131}}{2}\sin 2\theta_3\left[\frac{1}{m_{23}^2}-\frac{1}{m_{13}^2}\right]m_e\frac{1}{2\pi^2}\frac{q^2}{m_e^2\Omega^2}\frac{k_F^3}{3}\right)^2 . \tag{42}$$

The energy loss rate associated to the F_1 term is then given by

$$Q_{F_1} = \int \frac{d^3\mathbf{Q}}{(2\pi)^3}\frac{\Omega\,\Gamma_{F_1}}{e^{\Omega/T}-1}(\varepsilon_L - 1)^2 . \tag{43}$$

In the case of white dwarfs (red giants before the helium flush) the electron gas is degenerate with a temperature of order of (0.01–0.1) keV ($\sim$ 8.6 keV) and the Fermi momentum k_F of order of 495 (400) keV. Therefore, rigorously speaking, the electron plasma is neither in the nonrelativistic regime nor in the ultrarelativistic one [2]. In such a case the expressions for ε_L and the dispersion relation for longitudinal photons are quite complicated and Q_{F_1} can only be found numerically. Nevertheless, to have an idea of the order of magnitude of Q_{F_1}, we can approximate ε_L and the dispersion relation as

$$\varepsilon_L = 1 - \frac{\Omega_0^2}{\Omega^2}\left[1 + \frac{3}{5}v_F^2\frac{|\mathbf{Q}|^2}{\Omega^2}\right], \tag{44}$$

$$q^2 = \Omega_0^2 + |\mathcal{Q}|^2\left[\frac{3}{5}v_F^2\frac{\Omega_0^2}{\Omega^2} - 1\right], \tag{45}$$

where $\Omega_0 = \left(4\alpha k_F^2 v_F/3\pi\right)^{1/2}$ is the plasma frequency of order 10 keV for both white dwarfs and red giants and $v_F \sim 0.7$ is the Fermi velocity. Note that, since $\Omega_0 \ll \mu$, the expression (26) of F_1 valid for both soft and hard, but almost light-cone photons, is a good approximation for the physical context under consideration.

With such approximations, Q_{F_1} can be expressed analytically and we find

$$Q_{F_1} \simeq \frac{3}{64\pi^3}\frac{\Omega_0^5}{v_F^5}\left(\frac{5}{3}\right)^{5/2}\frac{\sqrt{2\pi}}{\gamma^{5/2}}e^{-\gamma}\,\mathcal{K}^2, \tag{46}$$

where $\gamma = \Omega_0/T$ and we have expressed the coupling constants in terms of the magnetic dipole moment $\mu_{\nu_e\nu_\mu}$

$$\mathcal{K} = 4\left(\frac{\sin 2\theta_3}{\sin 2\theta_2}\right)\left(\frac{1}{m_{13}^2}-\frac{1}{m_{23}^2}\right)\frac{k_F^3\mu_{\nu_e\nu_\mu}}{6m_e m_\tau}\left\{\frac{1}{m_{12}^2}\left[\ln\left(\frac{m_{12}^2}{m^2}\right)-1\right]-\frac{1}{m_{22}^2}\left[\ln\left(\frac{m_{22}^2}{m^2}\right)-1\right]\right\}^{-1} . \tag{47}$$

In the last expression we have used the fact that the major contribution to $\mu_{\nu_e\nu_\mu}$ comes from diagrams 2(a) and 2(b). In a similar way one can find the energy loss rate due to the decay $\gamma_{pl} \to \nu_e\nu_\mu$ through the magnetic dipole moment. In the range of interest of temperatures and densities the longitudinal and transverse contributions are comparable and, for instance,

$$Q_{\rm long} \simeq \frac{1}{12}\frac{\mu_{\nu_e\nu_\mu}^2}{(2\pi)^3}\left(\frac{5}{3}\right)^{3/2}\frac{\Omega_0^7}{v_F^3}\sqrt{2\pi}\gamma^{-3/2}e^{-\gamma}. \tag{48}$$

The ratio between Q_{F_1} and $Q_{\rm long}$ is then

$$\begin{aligned}\frac{Q_{F_1}}{Q_{\rm long}} &\simeq \frac{15}{2}\frac{1}{\mu_{\nu_e\nu_\mu}^2}\frac{\mathcal{K}^2}{v_F^2\,\Omega_0^2\,\gamma}\\ &\simeq 10^{-5}\left(\frac{0.7}{v_F}\right)^2\left(\frac{10\,\text{keV}}{\Omega_0}\right)^2\left(\frac{10}{\gamma}\right)\left(\frac{k_F}{400\,\text{keV}}\right)^6\left(\frac{\sin 2\theta_3}{\sin 2\theta_2}\frac{m_{13}^2-m_{23}^2}{m_{12}^2-m_{22}^2}\right)^2\left(\frac{m_2}{m_3}\right)^8 ,\end{aligned} \tag{49}$$

where we have indicated with m_k the averaged eigenvalue of the mixing matrix $\tilde{l}_L^k\tilde{l}_R^k$. Since in supersymmetric models the factor $(\sin 2\theta_3/\sin 2\theta_2)\left(m_{13}^2 - m_{23}^2/m_{12}^2 - m_{22}^2\right)^2$ is of order of (m_τ/m_μ) [13], one can obtain, $Q_{F_1} \simeq Q_{\rm long}$ for $m_2 \simeq 2m_3$. Even if the above estimation is approximate and holds in the particular framework we have chosen, the general message one can read from it is that, going beyond the SM, one has to take into account all the possible terms which arise at finite temperature and density for the $\nu\nu\gamma$ vertex because the new terms can give non-negligible contributions to relevant processes as the plasmon decay in degenerate stars.

V. CONCLUSIONS

In this paper we have carried out an explicit calculation of the neutrino electromagnetic vertex in a background of electrons in a large class of models where charged scalar bosons couple to leptons. We have been motivated by the fact that such models are able to provide a magnetic moment as large as $\mu_\nu \sim 10^{-12}\,\mu_B$, which can play a relevant role in different astrophysical phenomena.

We have shown that the contribution from the medium to the magnetic (electric) dipole moment is not significant, but a new chirality-flipping, but helicity-conserving term, arises. This new term makes possible the emission or the absorption of longitudinal photons and can be nonvanishing only because the medium does not respect CPT. We have also estimated the contribution of this new term to the plasmon decay rate showing that it can be comparable with the contribution coming from the vacuum magnetic moment. Therefore it must be taken into account in different applications of the vertex $\nu\nu\gamma$ in a medium.

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APPENDIX

In this appendix we give some details and the complete results for the form factors introduced in the text for different regimes in the limit of soft photons, namely, when the exchanged momentum q is much smaller than the momenta of the thermalized electrons, or for a hard, but almost lightlike photon, $q^2 \to 0$.

1. Chirality-flipping terms

In the ultrarelativistic regime $(T, \mu \gg m_e)$,

$$F_1 = e\frac{\lambda_{123}\lambda_{131}}{2}\sin2\theta_3\left[\frac{1}{m_{13}^2}-\frac{1}{m_{23}^2}\right]\times\frac{1}{8\pi^2\beta}\left[a\left(m_e\beta,-\mu\right)-a\left(m_e\beta,+\mu\right)\right], \tag{A1}$$

where

$$a\left(m_e\beta,\pm\mu\right)=\ln\left[1+\mathrm{e}^{-(m_e\pm\mu)\beta}\right], \quad T>\mu \tag{A2}$$

and

$$a\left(m_e\beta,-\mu\right)\simeq\mu-m_e, \quad T<\mu, \tag{A3}$$

$$F_2 = -e\frac{\lambda_{123}\lambda_{131}}{2}\sin2\theta_3\left[\frac{1}{m_{13}^2}-\frac{1}{m_{23}^2}\right]\times\frac{1}{4\pi^2}\left[b\left(m_e\beta,-\mu\right)+b\left(m_e\beta,+\mu\right)\right], \tag{A4}$$

where

$$b\left(m_e\beta,\pm\mu\right)=\sum_{n=1}^{\infty}(-1)^n\mathrm{e}^{\mp n\beta\mu}\mathrm{Ei}\left(-n\beta m_e\right), \quad T>\mu \tag{A5}$$

and

$$b\left(m_e\beta,-\mu\right)=\ln\left(\mu/m_e\right), \quad T<\mu, \tag{A6}$$

where $\mathrm{Ei}(x)$ is the exponential-integral function.

In the classical limit $[T \ll m_e$ and $(m_e-\mu)\gg T]$,

$$F_1 = -\frac{1}{8}e\frac{\lambda_{123}\lambda_{131}}{2}\sin2\theta_3\left[\frac{1}{m_{13}^2}-\frac{1}{m_{23}^2}\right]\times q^2\frac{\sqrt{\pi}}{\Gamma(3/2)}\frac{n_-}{m_e^2\Omega^2}, \tag{A7}$$

$$F_2 = \frac{1}{4}e\frac{\lambda_{123}\lambda_{131}}{2}\sin2\theta_3\left[\frac{1}{m_{13}^2}-\frac{1}{m_{23}^2}\right]\times q^2\frac{\sqrt{\pi}}{\Gamma(3/2)}\frac{\beta n_-}{m_e^2\Omega^2}, \tag{A8}$$

where $n_- = e^{\beta(\mu-m_e)}\left(2m_e/\beta\right)^{3/2}\left[\Gamma\left(3/2\right)/2\pi^2\right]$.

2. Chirality-conserving terms

To calculate the chirality-conserving form factors we must calculate the integral

$$I_{\lambda\nu}=\int\frac{d^4k}{(2\pi)^3}\frac{\delta\left(k^2-m_e^2\right)\eta\left(k\cdot u\right)}{\left(q^2+2q\cdot k\right)\left(q^2-2q\cdot k\right)}k_\lambda k_\nu. \tag{A9}$$

Since $I_{\lambda\nu}=I_{\nu\lambda}$ and $I_{\lambda\nu}(q)=I_{\nu\lambda}(-q)$, $I_{\lambda\nu}$ must be of the form

$$I_{\lambda\nu}=Au_\lambda u_\nu+Bg_{\lambda\nu}+Cq_\lambda q_\nu. \tag{A10}$$

If we then contract $I_{\lambda\nu}$ with u^λ the result must be proportional to u_ν, from which we read that $C=0$. From Eq. (A9) we have

$$\begin{aligned}I_1&=u_\lambda u_\nu I^{\lambda\nu}\\&=A+B\\&=\int\frac{d^4k}{(2\pi)^3}\frac{\delta\left(k^2-m_e^2\right)\eta\left(k\cdot u\right)\left(k\cdot u\right)^2}{\left(q^2+2q\cdot k\right)\left(q^2-2q\cdot k\right)},\end{aligned} \tag{A11}$$

$$\begin{aligned}I_2&=g_{\lambda\nu}I^{\lambda\nu}\\&=A+4B\\&=\int\frac{d^4k}{(2\pi)^3}\frac{\delta\left(k^2-m_e^2\right)\eta\left(k\cdot u\right)m_e^2}{\left(q^2+2q\cdot k\right)\left(q^2-2q\cdot k\right)}.\end{aligned} \tag{A12}$$

The chirality-conserving form factors are then functions of I_1 and I_2:

$$\begin{aligned}\tilde{F}_1&=\tfrac{1}{3}q^2\left(4I_1-I_2\right),\\ \tilde{F}_2&=F_1,\\ \tilde{F}_3&=i\left(q^2\right)^{-1}\tilde{F}_1,\\ \tilde{F}_4&=\tfrac{1}{3}q^2\left(I_2-I_1\right).\end{aligned} \tag{A13}$$

In the ultrarelativistic regime $(T, \mu \gg m_e)$,

$$I_1=\frac{-1}{16\pi^2}\frac{1}{q^2\beta^2}\left[c\left(m_e\beta,+\mu\right)+c\left(m_e\beta,-\mu\right)\right], \tag{A14}$$

$$I_2 = \frac{-m_e^2}{16\pi^2 q^2}\left[b\left(m_e\beta, +\mu\right) + b\left(m_e\beta, -\mu\right)\right], \qquad \text{(A15)}$$

where

$$c\left(m_e\beta, \pm\mu\right) = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} e^{-n\beta(m\pm\mu)}, \quad T > \mu \qquad \text{(A16)}$$

and

$$c\left(m_e\beta, -\mu\right) = \frac{\mu^2}{2}, \quad T < \mu. \qquad \text{(A17)}$$

In the degenerate limit [$T \ll m_e$ and $(\mu - m_e) \gg T$],

$$I_1 = -\frac{1}{16\pi^2}\frac{1}{\Omega^2 m_e^2}\left[\frac{E_F^3 k_F}{4} - \frac{E_F m_e k_F}{8} - \frac{m_e^3}{8}\ln\left(\frac{E_F + k_F}{m_e}\right)\right], \qquad \text{(A18)}$$

$$I_2 = -\frac{1}{16\pi^2}\frac{1}{\Omega^2}\left[\frac{E_F k_F}{2} - \frac{m_e^2}{2}\ln\left(\frac{E_F + k_F}{m_e}\right)\right]. \qquad \text{(A19)}$$

In the classical limit [$T \ll m_e$ and $(m_e - \mu) \gg T$],

$$I_1 \simeq I_2 = -\frac{1}{16}\frac{1}{m_e^2\Omega^2}\frac{n_-\sqrt{\pi}}{\Gamma(3/2)}. \qquad \text{(A20)}$$

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