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Analysis of Vector-Meson–Photoproduction Experiments in the ρ-ω Interference Region*

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Vector-meson photoproduction on nuclei in the ρ^0-ω interference region has been observed both by means of experiments detecting photoproduction of e^+e^- pairs and by means of experiments detecting photoproduction of $\pi^+\pi^-$ pairs. These two phenomena are closely related to each other and to the corresponding interference phenomenon in $e^+e^- \to \pi^+\pi^-$. It is shown how the data on all such phenomena can be expressed in terms of three model-independent complex parameters: ϵ, the intrinsic ρ-ω mixing parameter, r_ω, the ratio of ω-photon to ρ-photon coupling at an energy corresponding to the mass of the ω, and P_ω/P_ρ, the ratio of nuclear photoproduction amplitudes for ω and ρ. Existing data on the interference phenomena are combined with data on the ratios of ρ and ω production cross sections to determine these parameters. It is found that $|\epsilon| = 0.034 \pm 0.004$, $|r_\omega| \equiv \gamma_\rho/\gamma_\omega = 0.34 \pm 0.07$, and $|P_\rho/P_\omega| = 3.44 \pm 0.19$ is the most consistent representation of the data. Because present experiments have uncertainties at the 10% level, it turns out that only relative phases can be determined. If r_ω is assumed to be real, the most consistent representation yields $\arg\epsilon = 95° \pm 15°$ and $\arg(P_\omega/P_\rho) = 0° \pm 21°$. The former result is in agreement with the expectations from mixing theory. The latter is interpreted in terms of a simple vector-dominance model and is shown to agree with simple peripheralism, that is, equality of the ρ and ω forward scattering amplitudes. Because of disagreements in the data from different laboratories, there may be some doubt about the reality of r_ω. It appears that the phase of r_ω could be determined in a model-independent way by an electron-positron colliding-beam experiment on interference in $\pi^+\pi^-$ production at the 1% level.

I. GENERAL REMARKS

Recent experiments[1] on the coherent forward photoproduction of electron-positron pairs and of pairs of pions from light nuclei exhibit typical ρ-ω interference behavior when the pair mass falls in the region of the mass of the ω meson. The purpose of this note is to review some salient features of the interpretation of this phenomenon on the basis of general phenomenological theory and to show how it may be combined with information on other phenomena associated with electromagnetic production of ρ and ω mesons to give a model-independent parametrization of the data. The propagator method[2] is the basis for our analysis, but other methods would lead to essentially the same results. The parametrization will be carried out in such a way as to separate three different aspects of the physics: The intrinsic ρ-ω mixing, the vector-meson coupling to photons, and the mechanism for production of vector mesons from hadrons. Therefore, the tabulation of experimental results in terms of these model-independent parameters provides a convenient way to disentangle the physical processes and to compare experimental results directly with models. This latter point will be illustrated by use of a simple version of the vector-dominance model.

We shall also make use of existing experimental data to obtain values of the model-independent parameters. Because of uncertainties in the data, these physical parameters are not all determined at the present time.

The "other phenomena" to be considered here include the ρ-ω interference in the 2π mode produced by electron-positron collisions,[3] the production of ρ^0 and ω mesons in electron-positron collisions[4] determined by 2π and 3π modes, respectively, and the photoproduction of ρ^0 and ω as determined by these same modes.[5] Under the assumption that the masses and widths of the ρ^0 and ω resonances are known, the data on each of the three interference experiments may be used to determine the ratio of the amplitude of the ω resonance to the amplitude of the ρ resonance for the particular process. We shall denote these ratios (ω to ρ) of resonance amplitudes by the complex numbers $\xi(\gamma, 2e)$, $\xi(\gamma, 2\pi)$, and $\xi(2e, 2\pi)$ for e^+e^- photoproduction, $\pi^+\pi^-$ photoproduction, and $\pi^+\pi^-$ production in electron-positron collisions, respectively.

These three ratios may be expressed in terms of the three model-independent complex parameters which are:

(1) The ratio, r_ω, of the ω-photon to ρ-photon coupling for the *unmixed* states (i.e., states of definite G parity). In general, this coupling ratio, r,

may be an energy-dependent complex number and r_ω, which is involved here, denotes its value at the ω mass.

(2) The intrinsic ρ-ω parameter ϵ, which is a universal complex constant, independent of the production mechanism, measuring the mixing of ρ and ω states due to G-violating (presumably electromagnetic) interactions.

(3) The ratio P_ω/P_ρ of the amplitudes for photoproduction of the *physical* ω and ρ mesons.

In principle, the three experiments giving $\xi(\gamma, 2e)$, $\xi(\gamma, 2\pi)$, and $\xi(2e, 2\pi)$ determine the three physical parameters.

The experiments giving 2π and 3π resonance cross sections may be used to determine the ratios of ρ and ω cross sections both in photoproduction and in e^+-e^- collisions, and these ratios may also be expressed directly in terms of the physical parameters r_ω, ϵ, and P_ω/P_ρ. Therefore, the five experiments together should serve to overdetermine the three complex parameters.

The value of P_ω/P_ρ depends on the strong interaction between vector meson and nucleus. Hence, it may be used to test specific models of the photoproduction process. The only model which we shall consider here is the simplest version of vector dominance. Then P_ω/P_ρ is related to the ratio of forward scattering amplitudes of the ω and ρ mesons as well as to ϵ and r_0, where r_0 is the value of r at the photon mass (where it is expected to be a real number).

Within the context of vector dominance, it is then possible to test the assumption of peripheralism, which would lead to equal forward scattering amplitudes for ω and ρ in its simplest form, and to verify $r_\omega = r_0$, a relationship to be expected in the simplest form of vector dominance. It will be shown that most of the available data are consistent with both of these assumptions taken together, but because of the experimental uncertainties mentioned above, the phase of r_ω cannot be determined and must, therefore, be assumed to bring about agreement.

II. BASIC RELATIONSHIPS

The measured cross section in the interference region for each of the phenomena under discussion may be analyzed in terms of a resonance amplitude, A_R, and various coherent and incoherent background amplitudes. Because of its strong energy dependence, the relative resonance amplitude A_R can usually be separated from the others. It is taken to have the form

$$A_R(m^2) \sim \frac{1}{m^2 - m_\rho{}^2 + i m_\rho \Gamma_\rho} + \frac{\xi}{m^2 - m_\omega{}^2 + i m_\omega \Gamma_\omega}, \quad (1)$$

where m is the total mass of the electron-positron or pion pair, as the case may be, and ξ is the ratio of the complex amplitudes of ω and ρ. This is the general form to be expected when the mixing of ρ and ω is treated by the propagator method.[2,6] It is shown in Ref. 2 how the value of ξ depends upon the particular process under consideration.

In order to obtain the appropriate expression for ξ in terms of the underlying parameters, it is desirable to standardize the mixing notation. If $|\rho_0\rangle$ and $|\omega_0\rangle$ are the unmixed states, we write for the physical states

$$|\rho\rangle = |\rho_0\rangle - \epsilon |\omega_0\rangle, \quad (2a)$$

$$|\omega\rangle = |\omega_0\rangle + \epsilon |\rho_0\rangle. \quad (2b)$$

The way in which the coefficient ξ in Eq. (1) may be expressed in terms of the three physical parameters by use of Eq. (2) has been treated in detail by many authors, but the essential points of the physical argument are simple enough to repeat them here: We start by noting that the physical states $|\rho\rangle$ and $|\omega\rangle$ are defined as those that propagate as particles having a physical mass m_V and a total decay width Γ_V. Thus, the propagator for each physical state at four-momentum P_μ is proportional to

$$[m^2 - (m_V - \tfrac{1}{2} i \Gamma_V)^2]^{-1} \approx (m^2 - m_V{}^2 + i m_V \Gamma_V)^{-1}, \quad (3)$$

where $m^2 = -\sum P_\mu{}^2$ is the invariant square of the four-momentum.

Let us assume that a physical vector-meson state $|V\rangle$ is produced in some given process with an amplitude T_V. It will then proceed to propagate with an amplitude

$$T_V (m^2 - m_V{}^2 + i m_V \Gamma_V)^{-1}.$$

If, then, the amplitude for decay of the state $|V\rangle$ into some mode, j, is $D_V(j)$, the total amplitude for resonance production of that mode in the process under consideration is proportional to

$$A_j(m^2) = \frac{T_\rho D_\rho(j)}{m^2 - m_\rho{}^2 + i m_\rho \Gamma_\rho} + \frac{T_\omega D_\omega(j)}{m^2 - m_\omega{}^2 + i m_\omega \Gamma_\omega}.$$

Comparison with Eq. (1) leads immediately to the result

$$\xi(j) = T_\omega D_\omega(j) / T_\rho D_\rho(j). \quad (4)$$

The next step is to express $D_V(j)$ in terms of the decay amplitude of the unmixed states $|V_0\rangle$, the reason being that clear selection rules apply to the decay of the unmixed states since they are states of definite G parity. The necessary relationship between the decay amplitudes $D_V(j)$ of the physical state and the amplitude $D_V^0(j)$ of the state of definite G parity is clearly given by the linear relationship between the corresponding states, Eq. (2):

$$D_\rho(j)=D_\rho^0(j)-\epsilon D_\omega^0(j)\,, \tag{5a}$$

$$D_\omega(j)=D_\omega^0(j)+\epsilon D_\rho^0(j)\,. \tag{5b}$$

We now need only consider specific cases to obtain the desired relationships. In the case of the 2π mode, we assume that only the even-G-parity state, namely, $|\rho_0\rangle$, can decay into it. Although direct electromagnetic effects violating G-parity conservation would permit a small $|\omega_0\rangle$ decay amplitude into the 2π mode, it is expected that these effects would be an order of magnitude less[7] than ϵ, the mixing parameter, and all effects of this order will be neglected in the following. This assumption that $D_\omega^0(2\pi)=0$ may be inserted in Eq. (5) to obtain

$$D_\omega(2\pi)/D_\rho(2\pi)=\epsilon\,, \tag{6}$$

i.e., any decay of the physical state $|\omega\rangle$ into the 2π mode is due entirely to mixing.

At this point we should note that the energy-dependent factor introduced by Gounaris and Sakurai[8] drops out of our considerations. This factor, relating to the energy dependence of the ρ width, is characteristic of the coupling of the unmixed $|\rho_0\rangle$ state to the 2π mode. As long as that is the dominant 2π coupling contributing to $D_\omega(2\pi)$, the Gounaris-Sakurai factor is common to both terms in the interference phenomenon. The factor would appear in Eq. (3) in the form suggested by Augustin *et al.*[3] only if the direct $\omega_0\to 2\pi$ amplitude $D_\omega^0(2\pi)$ were much larger than the amplitude caused by mixing, namely, $\epsilon D_\rho^0(2\pi)$.

In the case of the electron-positron mode, the decay is assumed to take place via a virtual photon. Then if the coupling between *unmixed* meson and photon is f_V, Eq. (5) yields

$$D_\rho(2e)\sim f_\rho-\epsilon f_\omega\,,$$

$$D_\omega(2e)\sim f_\omega+\epsilon f_\rho\,,$$

with a common constant of proportionality depending on the $\gamma\to 2e$ amplitude. Thus,

$$D_\omega(2e)/D_\rho(2e)=(r+\epsilon)/(1-\epsilon r)\,, \tag{7}$$

where

$$r(m^2)=f_\omega(m^2)/f_\rho(m^2)\,. \tag{8}$$

Since we are primarily interested in the interference region where $m\approx m_\omega$, we shall replace $r(m^2)$ by its value $r_\omega=r(m_\omega{}^2)$ on the assumption that if there is any variation of the vector-meson–photon coupling with energy, it is a slow variation. Furthermore, we shall find that $|\epsilon r|\ll 1$ and can therefore write

$$D_\omega(2e)/D_\rho(2e)\approx r_\omega+\epsilon\,. \tag{9}$$

We are now in a position to write down the values of ξ for photoproduction. We have denoted the amplitudes T_V for photoproduction of the unmixed states by $T_V=P_V$. Thus, for photoproduction of pion pairs

$$\xi(\gamma,2\pi)=\epsilon P_\omega/P_\rho\,, \tag{10}$$

and for photoproduction of electron-positron pairs

$$\xi(\gamma,2e)=(\epsilon+r_\omega)P_\omega/P_\rho\,. \tag{11}$$

To obtain the corresponding result for 2π production in electron-positron collisions, the amplitude T_V for production of vector mesons in electron-positron collisions is needed. This may be treated as the inverse of the decay of vector mesons into electron-positron pairs, but the amplitude is not simply obtained by taking the conjugate complex of $D_V(2e)$ since the transformation described by Eq. (2) is not unitary. We must use instead the inverse transformation,

$$|\rho_0\rangle=|\rho\rangle+\epsilon\,|\omega\rangle\,, \tag{12a}$$

$$|\omega_0\rangle=|\omega\rangle-\epsilon\,|\rho\rangle\,, \tag{12b}$$

valid to order ϵ. When the electron-positron pair collides to form a virtual photon, the photon transition to the unmixed vector-meson state is given by the complex conjugate, f_V^*, of the coupling (to take account of the possibility that that coupling is not real). The state produced in this way is then of the form $f_\rho^*|\rho_0\rangle+f_\omega^*|\omega_0\rangle$. Then by substituting from Eq. (12), one may immediately obtain the amplitudes for production of physical states,

$$T_\rho(e^+e^-)\sim f_\rho^*-\epsilon f_\omega^*\,,$$

$$T_\omega(e^+e^-)\sim f_\omega^*+\epsilon f_\rho^*\,.$$

Therefore, in the approximation used above for $D_V(e^+e^-)$,

$$T_\omega(e^+e^-)/T_\rho(e^+e^-)\approx r_\omega^*+\epsilon\,, \tag{13}$$

and for the production of pion pairs in electron-positron collisions Eqs. (4) and (6) give

$$\xi(2e,2\pi)=(\epsilon+r_\omega^*)\epsilon\,. \tag{14}$$

The other relationships that are of interest concern the production cross sections at the peak of the resonance. These are determined by observing the dominant decay modes: the 2π mode for the ρ and the 3π mode for the ω resonance. The peak-resonance amplitude is proportional to

$$T_V D_V/im_V\Gamma_V\,,$$

where $|D_V|^2\approx\Gamma_V$ since it concerns the dominant decay mode. Thus, the ratio of peak cross sections is

$$\sigma_\omega(\text{peak})/\sigma_\rho(\text{peak})\approx|T_\omega/T_\rho|^2(m_\rho\Gamma_\rho/m_\omega\Gamma_\omega)\,. \tag{15}$$

TABLE I. Experimental results used in the analysis. The reference numbers refer to footnotes giving the sources of data.

Quantity measured	Parameter	Measured value(s)	Ref.
Interference amplitude, photoproduction of $\pi^+\pi^-$	$\xi(\gamma,\ 2\pi)$	$(0.0097\pm0.008)e^{i(104^\circ\pm5^\circ)}$	1(d)
		$(0.014\ \pm0.016)e^{i(84^\circ\pm6^\circ)}$	1(e)
		$(0.0106\pm0.0012)e^{i(96^\circ\pm15^\circ)}$	1(b)
		$(0.01\pm0.001)e^{i(95^\circ\pm15^\circ)}$	Note [a]
Interference amplitude, photoproduction of e^+e^-	$\xi(\gamma,\ 2e)$	$(0.106\pm0.022)e^{i(41^\circ\pm20^\circ)}$	1(a)
		$(0.142\,^{+0.039}_{-0.032})e^{i(100^\circ\,^{+38^\circ}_{-30^\circ})}$	1(c)
Integrated resonance cross section, (photoproduction of 2π)/(photoproduction of 3π)	η	11.8 ± 1.3	5
Interference amplitude, $e^+e^-\to\pi^+\pi^-$	$\xi(2e,\ 2\pi)$	$(0.016\pm0.004)e^{i(95^\circ\pm15^\circ)}$	3 [b]
Peak resonance cross section, $e^+e^-\to\pi^+\pi^-$	$\sigma_\rho(ee)$	$1.69\pm0.21\ \mu\text{b}$	4(a)
Peak resonance cross section, $e^+e^-\to\pi^+\pi^-\pi^0$	$\sigma_\omega(ee)$	$1.82\pm0.34\ \mu\text{b}$	4(b)

[a] This is an "average" value for the three experiments that is used as the basis of the analysis here.
[b] Since the energy dependence of the ρ width is made explicit in Ref. 3, the adjustment in phase described at the end of Sec. II has been included.

On the other hand, the cross sections integrated over the width of the resonances satisfy

$$\sigma_\omega(\text{int})/\sigma_\rho(\text{int})\approx|T_\omega/T_\rho|^2\,. \tag{16}$$

We shall find uses for both of these forms.

The important results of this section are given by Eqs. (10), (11), and (14). For the sake of simplicity, both the production and decay amplitudes have been assumed to be spin-independent in writing down these equations. Furthermore, the small modifications associated with the differences between propagation of spin-one and spin-zero particles have been ignored, as have some of the effects associated with the energy dependence of the ρ width. These latter effects lead to small corrections in phase in Eq. (2), and all equations derived therefrom, but these changes can easily be made[6] when they are required to match the accuracy of experimental data.

It should be noted that Eq. (1) [or Eq. (3)] with constant Γ_ρ is a correct representation of the amplitude, even when the energy dependence of the ρ width is taken into account. The effect of making the energy dependence explicit in the ρ-resonance term is compensated in the interference region by introducing a small phase change in the definition of ξ.

III. GENERAL ANALYSIS OF EXISTING DATA

The determination of the three complex parameters ϵ, r_ω, and P_ω/P_ρ from the photoproduction data can be expedited by noting that Eqs. (10) and (11) giving the two interference parameters may be combined to write

$$\xi(\gamma,2e)-\xi(\gamma,2\pi)=r_\omega(P_\omega/P_\rho)\,. \tag{17}$$

The other information concerning photoproduction that will be used here is the ratio, η, of cross sections for production of the ρ meson and the ω meson. These cross sections correspond to the integrated cross sections of Eq. (16) and since $T_V\equiv P_V$ for photoproduction,

$$\eta=\sigma_{\gamma\rho}/\sigma_{\gamma\omega}=|P_\rho/P_\omega|^2\,. \tag{18}$$

The electron-positron collision data from the storage rings give the peak cross sections $\sigma_\rho(ee)$ for resonance production of the 2π mode[4(a)] and

TABLE II. Approximate equations for physical parameters, valid to about 10% in magnitude and 10° in phase.

	Equation	Source in text
(A)	$\epsilon P_\omega/P_\rho=\xi(\gamma,\ 2\pi)$	Eq. (10)
(B)	$r_\omega P_\omega/P_\rho=\xi(\gamma,\ 2e)$	Eq. (21)
(C)	$\epsilon r_\omega^*\approx\xi(2e,\ 2\pi)$	Eq. (14)
(D)	$\lvert P_\rho/P_\omega\rvert^2=\eta$	Eq. (18)
(E)	$\lvert r_\omega\rvert^2\approx\Gamma_\omega\sigma_\omega(ee)/\Gamma_\rho\sigma_\rho(ee)$	Eq. (19)
(F)	$\arg\xi(\gamma,\ 2e)\approx\arg\xi(\gamma,\ 2\pi)-\arg\xi(2e,\ 2\pi)$	Eq. (22) and Eq. (C) (above)

$\sigma_\omega(ee)$ for resonance production of the 3π mode[4(b)] as well as[3] the interference parameter $\xi(2e, 2\pi)$. The last of these is to be applied to Eq. (14), while the first two may be combined through Eqs. (13) and (15) to give

$$|\epsilon + r_\omega^*|^2 = \frac{m_\omega \Gamma_\omega}{m_\rho \Gamma_\rho} \frac{\sigma_\omega(ee)}{\sigma_\rho(ee)}. \qquad (19)$$

The relevant data that are presently available for both photoproduction and electron-positron collisions are given in Table I. Since the data clearly have uncertainties in magnitude of the order of 10% or more and uncertainties in phase of 10° or more, terms in the equations used for this analysis, which can be shown on the basis of the data to be of this order or smaller, will be dropped.

It is immediately apparent from Table I that

$$\xi(\gamma, 2\pi)/\xi(\gamma, 2e) \simeq 0.1. \qquad (20)$$

Therefore, Eq. (17) may be simplified to

$$r_\omega(P_\omega/P_\rho) \simeq \xi(\gamma, 2e), \qquad (21)$$

and this result combined with Eq. (10) yields

$$\epsilon/r_\omega \simeq \xi(\gamma, 2\pi)/\xi(\gamma, 2e). \qquad (22)$$

Equation (22) along with Eq. (20) establishes

$$|\epsilon/r_\omega| \simeq 0.1 \qquad (23)$$

as an experimental fact. Having established this, it is clear from Eq. (22) that the approximation we are using results from the fact that the effect of ρ-ω mixing on the e^+e^- mode of the ω is negligible (at the 10% level) while the 2π mode of the ω is entirely due to mixing.

For the same reason, simplifications of Eqs. (14) and (19) are also possible. As a result of these simplifications, the basic equations to be used for the analysis at the 10% level of the five types of experiments are those that are shown in Table II. We see immediately from Table II that although, in principle, the magnitudes and phases of the three parameters ϵ, r_ω, and P_ω/P_ρ could be determined from the data, at this level of approximation only relative phases can be determined. Data at the 1% level will be required to determine the phases independently from the e^+e^- collisions by means of more precise equations such as Eqs. (14) and (19). However, when the analysis is to be made at the more precise level, additional corrections that have been omitted for the sake of simplicity, such as those described at the end of Sec. II, will be required.

Results obtained by applying the equations of Table II to the data of Table I are shown in Table III. It is evident that the magnitudes of ϵ and r_ω determined independently from photoproduction and electron-positron collisions are quite consistent. The value of $|r_\omega|$ is also consistent with the predictions of the simplest version of SU(3).[9] We note that $|r_\omega|$ is often denoted by $\gamma_\rho/\gamma_\omega$ in the literature.

Comparison of phases determined from the two types of experiments is simplified by combining the data on interference in the 2π mode from both

TABLE III. The results obtained by applying the equations of Table II to the data of Table I.

	Result	Source
	From photoproduction	
1.	$\epsilon(P_\omega/P_\rho) = (0.01 \pm 0.001)e^{i(95° \pm 15°)}$	Table II, Eq. (A)
2a.	$r_\omega(P_\omega/P_\rho) = (0.10 \pm 0.02)e^{i(41° \pm 20°)}$	Table II, Eq. (B), and Ref. 1(a)
2b.	$r_\omega(P_\omega/P_\rho) = (0.14 \pm 0.035)e^{i(100° \pm 35°)}$	Table II, Eq. (B), and Ref. 1(c)
3.	$\lvert P_\rho/P_\omega\rvert = 3.44 \pm 0.19$	Table II, Eq. (D)
4.	$\lvert\epsilon\rvert = 0.034 \pm 0.004$	Lines 1 and 3 above
5a.	$\lvert r_\omega\rvert = 0.34 \pm 0.07$	Lines 2a and 3 above
5b.	$\lvert r_\omega\rvert = 0.44 \pm 0.12$	Lines 2b and 3 above
	From e^+e^- collisions	
6.	$\epsilon r_\omega^* = (0.016 \pm 0.004)e^{i(95° \pm 15°)}$	Table II, Eq. (C)
7.	$\lvert r_\omega\rvert = 0.33 \pm 0.03$	Table II, Eq. (E)
8.	$\lvert\epsilon\rvert = 0.048 \pm 0.012$	Lines 6 and 7 above
	Combined data	
9.	$\arg\xi(\gamma, 2e) = 0° \pm 21°$	Table II, Eq. (F), and line 6 above

electron-positron and photoproduction experiments, as shown in line 9 of Table III. This independent determination of the phase of $\xi(\gamma, 2e)$ is to be compared with the two directly determined values shown in Table I. Evidently the results of Ref. 1(a) do not give a significantly different phase, but those given in Ref. 1(c) are quite inconsistent with the others.

Although its phase cannot be determined from existing data, it is reasonable to assume that the photon–vector-meson coupling is real, since the numerical value of the ratio of coupling constants, $|r_\omega| \approx \frac{1}{3}$, is consistent with the simplest notions of photon–vector-meson coupling. Then, r_ω is real and from line 6 of Table III,

$$\arg\epsilon \approx 95° \pm 15° , \tag{24}$$

or ϵ is essentially imaginary, which is in good agreement with the prediction of Gourdin, Stodolsky, and Renard[10] based on the assumption that the mixing of the vector-meson states is due entirely to electromagnetic coupling.

IV. VECTOR DOMINANCE AND THE INTERPRETATION OF P_ω/P_ρ

The assumption of vector dominance of photoproduction takes its simplest form when the resonance production amplitude is assumed to be dominated by diagrams of the form shown in Fig. 1. The double lines labeled with a "V" in these diagrams represent the vector-meson propagators. It is important to realize that the propagator on the left of the diagram is evaluated on the photon mass shell, i.e., at $m^2=0$, while that on the right is evaluated at the mass of the 2π or $2e$ system.

The choice of mass also may affect the value of meson-photon coupling, f_V, to be used. We have noted earlier that on the right-hand side of the diagram in Fig. 1(b) f_V is to be evaluated at the ω-meson mass. On the other hand, f_V is to be evaluated at $m^2=0$ on the left-hand side of both diagrams. The corresponding ratio of coupling constants is denoted by r_0, and r_0 is expected to be a real number since no absorptive processes can contribute at $m^2=0$.

The ratio P_ω/P_ρ may be evaluated in accordance with general mixing theory for the diagrams in Fig. 1. The procedure is analogous to the one used in Sec. II to obtain production amplitudes in electron-positron collisions. In the present instance, the photon produces vector-meson states of definite G parity with the (real) amplitude $f_V(m^2=0)=f_V^0$ and the state so formed is

$$f_\rho^0|\rho_0\rangle + f_\omega^0|\omega_0\rangle . \tag{25}$$

By means of Eq. (12) this state may be expressed

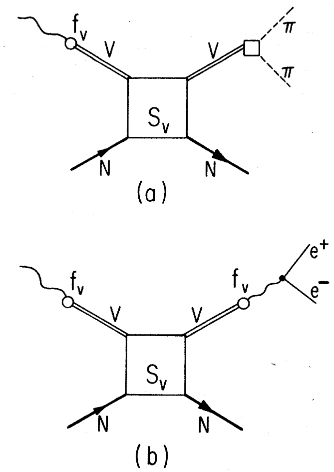

FIG. 1. Diagrams of photoproduction of vector-meson resonances in the vector-dominance model. N stands for nucleus, V for vector-meson propagator, f_V for photon–vector-meson coupling. S_V is the nuclear scattering amplitude for vector mesons. (a) Resonance photoproduction of pion pairs. (b) Resonance photoproduction of electron-positron pairs.

in terms of the physical states which propagate to the nuclear vertex (S_V in Fig. 1) in accordance with Eq. (3) with $m^2=0$. However, our consistent neglect of terms of the order Γ_V/m_V and $(m_\rho - m_\omega)/m_V$ means that the $|\rho\rangle$ and $|\omega\rangle$ propagators should be treated as equal; hence the structure of the state arriving at the vertex is approximately given by Eq. (25).

The interaction S_V conserves G parity in the simplest case of diffractive scattering and, if the forward scattering amplitudes are denoted by S_V^0, the state after scattering is simply

$$S_\rho^0 f_\rho^0|\rho_0\rangle + S_\omega^0 f_\omega^0|\omega_0\rangle . \tag{26}$$

In order to obtain the amplitudes P_ρ and P_ω of the physical states, Eq. (26) must be expressed in terms of physical states by means of Eq. (12). In this way we find

$$P_\omega/P_\rho = \epsilon + r_0 S_\omega^0/S_\rho^0 , \tag{27}$$

where a correction of order ϵr_0 is neglected.

According to Table III, $|\epsilon| \ll |P_\omega/P_\rho|$, and it is a good approximation to rewrite Eq. (27) as

$$r_0 S_\omega^0/S_\rho^0 = P_\omega/P_\rho . \tag{28}$$

The simplest model of vector dominance would suggest that r is independent of m^2 and real; hence $r_\omega = r_0$. Using this model we then find from Table III either

$$|S_\omega^0/S_\rho^0| = 0.85 \pm 0.14 \tag{29a}$$

or

$$|S_\omega^0/S_\rho^0| = 0.61 \pm 0.17 , \tag{29b}$$

depending on the choice of values of $|r_\omega|$. According to Eq. (B) in Table II, the phase of S_ω^0/S_ρ^0 obtained in this model is the same as the phase of $\xi(\gamma, 2e)$. Thus, the model gives

$$\arg(S_\omega^0/S_\rho^0) = 0° \pm 21° \tag{30}$$

if we use the result of the combined data from Table III (line 9). As we have already noted, this result is in rough agreement with the results of Ref. 1(a), but not with those of Ref. 1(c).

Equations (29a) and (30) suggest

$$S_\omega^0/S_\rho^0 = 1\,, \tag{31}$$

a result that is in accord with the simplest model of peripheral scattering, which would suggest that the ρ and ω have the same, or nearly the same, forward scattering amplitude.

Although the experimental results of Ref. 1(c) are *not* consistent with the other experimental results used in obtaining Eq. (31), it is of some interest that they *are* consistent with Eq. (31). This agreement requires that r_ω be taken to be approximately imaginary (rather than real), i.e., $r_\omega \approx ir_0$ [Eq. (28) and Eq. (B) of Table II]. It is also consistent with ϵ being approximately imaginary [Eq. (A) of Table II], but it is clearly quite inconsistent with the phase of $\xi(2e, 2\pi)$ measured in the storage-ring experiments [Eq. (C) of Table II] as well as with the other measurement of $\xi(\gamma, 2e)$, Ref. 1(a).

V. CONCLUSIONS

We have shown how the measurements on relative amplitudes for ρ and ω production in the (forward) photoprocess and in electron-positron collisions may be used to determine three physically interesting parameters in a model-independent way. These complex parameters are the ρ-ω mixing parameter, ϵ, the relative ω and ρ coupling to photons at the ω mass, r_ω, and the ratio of nuclear photoproduction amplitudes for ω and ρ.

Existing data, which are good to roughly the 10% level, have been analyzed in terms of these parameters. It is found that at this level of accuracy, the magnitudes of the physical parameters and their relative phases can be determined. They are evaluated from the data with results that are in good agreement with general expectations when the unknown phase is fixed by making the natural assumption that r_ω is real. The largest errors and greatest uncertainties in the analysis occur in regard to the determination of the interference of the amplitudes for resonance photoproduction of electron-positron pairs.

The results may be used to test particular models, and they are applied here with some success to the model based on simple diffractive forward scattering of the vector mesons.

The absolute phase of r_ω enters the equations of the analysis only in terms of the order of 10%; therefore, its determination will require that the colliding-beam experiments be carried out to the 1% level or better. However, this determination will also require that other small effects be incorporated into the equations used for the analysis. These small effects include the effect of the energy dependence of the ρ width on the mixing equations, Eqs. (2). They also introduce new parameters,[6] such as the direct 2π decay amplitude of the state $|\omega_0\rangle$, which is expected to be a 10% effect. Another parameter that should be included at this level is the ratio of the coupling constants for "mass mixing" and "current mixing." Presumably, these additional parameters could also be determined from a complete set of data at this level of precision.

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[7]The mixing parameter ϵ also depends on the G-violating electromagnetic coupling between ρ and ω because the states $|\rho_0\rangle$ and $|\omega_0\rangle$ are neither stable nor degenerate. In the case of perfect degeneracy of stable states, the mixing is of order one-to-one (that would even be true for unstable states in the unlikely event that widths were also exactly equal), but in the present case it depends on the ratio of the coupling to the difference in complex masses. Since m_ρ and m_ω are very nearly equal, the complex mass difference is essentially $i(\Gamma_\rho - \Gamma_\omega) \simeq i\Gamma_\rho$, and ϵ differs from the G-violating coupling by a factor of $m_\rho/i\Gamma_\rho \simeq -7i$.

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