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# Light-Cone Commutators and a Sum Rule for High-Energy Neutrino Scattering*

Dennis Corrigan
*Department of Physics, University of Calgary, Calgary 44, Alberta, Canada*
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Using canonical expressions for certain current commutators on the light cone, we derive a sum rule which relates the structure function $W_3$ (arising from vector–axial-vector interference) measured in high-energy neutrino experiments to the structure functions measured in electroproduction.

Recently Kogut, Soper, and Bjorken[1] have shown how one may discuss quantum electrodynamics in the infinite-momentum frame by specifying canonical commutators (and anticommutators) on the light cone rather than at equal times. Using this formalism Cornwall and Jackiw[2] discussed current commutators on the light cone and derived a number of interesting sum rules. The fact that certain of these light-cone commutators could be expressed in an interaction-independent way enabled Dicus, Jackiw, and Teplitz[3] to use them to rederive several standard current-algebra sum rules. They found modifications in some of these and were also able to derive new sum rules not obtainable by the usual current-algebra methods.[4]

In this brief note we will use certain of the commutators of Ref. 2 and the techniques of Ref. 3 to derive an interesting sum rule involving one of the structure functions measured in inelastic neutrino scattering from nucleons. Also, using simple isotopic-spin arguments, an integral over this structure function will be related to an integral over the structure functions measured in inelastic electron scattering from nucleons.

Let us define

$$\frac{1}{2\pi}\int d^4x\, e^{iq\cdot x}\langle p|[J_\mu^\pm(x), J_\nu^\mp(0)]|p\rangle = -g_{\mu\nu}W_1^{(\nu,\bar\nu)} + \frac{p_\mu p_\nu}{m^2}W_2^{(\nu,\bar\nu)} - \frac{i}{2m^2}\epsilon_{\mu\nu\lambda\rho}p^\lambda q^\rho W_3^{(\nu,\bar\nu)} + \cdots, \tag{1}$$

where $J_\mu^+$ is the Cabibbo current, $J_\mu^-$ its adjoint, $m$ the nucleon mass, and an average over the nucleon spin is implied. Then the cross section for the process $\nu(\bar\nu)+$ proton $\rightarrow l(\bar l)+$ hadrons may be written[5]

$$\frac{d^2\sigma^{(\nu,\bar\nu)}}{dE'\Omega} = \frac{G^2}{2\pi}\frac{E'}{E}\left[W_2^{(\nu,\bar\nu)}\cos^2(\tfrac{1}{2}\theta) + 2W_1^{(\nu,\bar\nu)}\sin^2(\tfrac{1}{2}\theta) \mp \left(\frac{E+E'}{m}\right)\sin^2(\tfrac{1}{2}\theta)W_3^{(\nu,\bar\nu)}\right]. \tag{2}$$

Here $E$ is the incident neutrino energy and $E'$ and $\theta$ are, respectively, the energy and scattering angle of the final lepton. The invariant functions $W_1$, $W_2$, and $W_3$ depend on $q^2$ and $\nu = p\cdot q$ only.

The sum rule we wish to present involves the structure function $W_3(q^2,\nu)$ which is present due to the parity-violating nature of the weak interactions. Using the notation of Ref. 3 we find the fixed-$q^2$ sum rule ($\theta_C$ = Cabibbo angle),

$$\frac{1}{2m}\int_0^\infty d\nu[W_3^{(\bar\nu)}(q^2,\nu) - W_3^{(\nu)}(q^2,\nu)] = m\int_0^\infty d\alpha[(2\cos^2\theta_C + \sin^2\theta_C)\bar V_3^1(0,\alpha) + \sqrt{3}\,\sin^2\theta_C \bar V_8^1(0,\alpha)]. \tag{3}$$

This shows that the integral on the left-hand side should be independent of $q^2$. Before indicating how this result is derived, we note the sum rule of Dicus *et al.* obtained for the structure function $W_2$ measured in electroproduction,[6] viz.,

$$\frac{2}{m}\int_0^\infty d\nu\left(\frac{-2\nu}{q^2}\right)W_2^{ep}(q^2,\nu) = -\frac{m}{3\sqrt3}\int_0^\infty d\alpha[2\sqrt3\bar V_3^1(0,\alpha) + 4\sqrt2\,\bar V_0^1(0,\alpha) + 2\bar V_8^1(0,\alpha)]. \tag{4}$$

Combining Eqs. (3) and (4) in an obvious way we arrive at the following interesting relation:

$$\int_0^\infty d\nu(W_3^{\bar\nu p} - W_3^{\nu p} - W_3^{\bar\nu n} + W_3^{\nu n}) = -6(2\cos^2\theta_C + \sin^2\theta_C)\int_0^\infty d\nu\left(\frac{-2\nu}{q^2}\right)(W_2^{ep} - W_2^{en}). \tag{5}$$

Finally, if the functions $W_2$ and $W_3$ obey the scaling properties suggested by Bjorken,[7] viz., as $\nu\rightarrow\infty$ with fixed $\omega = -q^2/2\nu$,

$$(\nu/m)W_a(q^2,\nu)\rightarrow F_a(\omega), \quad a=2,3 \tag{6}$$

and if we make a small error in our sum rule by setting $\theta_C = 0$, we derive a relation between the scale functions $F_2$ and $F_3$, i.e.,

$$\int_0^1 \frac{d\omega}{\omega}(F_2^{\nu p} - F_2^{\bar{\nu} p}) = 6\int_0^1 \frac{d\omega}{\omega^2}(F_2^{ep} - F_2^{np}). \qquad (7)$$

The most interesting form of the sum rule is probably Eq. (7) which relates the difference of neutrino and antineutrino scattering from protons to the difference of electron scattering from protons and neutrons. At present, sufficient data do not exist to test this sum rule – especially as it depends sensitively on the small-$\omega$ behavior of the scale functions – but presumably it should be possible to check the sum rule as the statistics of the neutrino and electron scattering experiments improve.

As was pointed out in Ref. 3, the fact that the integral on the right-hand side of Eq. (5) should be independent of $q^2$ was derived earlier by Cornwall, Norton, and the present author.[8] The $q^2=0$ value for this integral can be estimated from the work of Damashek and Gilman[9] who found its value to be about +1.0 – consistent with the Born term alone. If the neutron contribution is small – again consistent with the vanishing Born term – then the right-hand side of Eq. (5) would be rather large – the order of +6.0. It will be interesting to see if such a large difference between neutrino and antineutrino scattering is seen.

It should be noted that since the model of Refs. 2 and 3 requires $F_L = F_1 - F_2/2\omega = 0$, the sum rule of Eq. (7) can be restated as follows:

$$\int_0^1 \frac{d\omega}{\omega}(F_3^{\nu p} - F_3^{\bar{\nu} p}) = 12\int_0^1 \frac{d\omega}{\omega}(F_1^{ep} - F_1^{en}), \qquad (8)$$

which is saturated by the parton-quark-model relation $12(F_1^{ep} - F_1^{en}) = F_3^{\nu p} - F_3^{\bar{\nu} p}$ derived by Llewellyn Smith.[10]

The derivation of Eq. (3) is a straightforward exercise in applying the techniques of Ref. 3. Define

$$\frac{i}{2m}\epsilon^{\mu\nu\lambda\rho}p_\lambda q_\rho D_3^{ab}(q^2,\nu) = \int d^4x\, e^{iq\cdot x}\langle p|[V_a^\mu(x), A_b^\nu(0)] + [A_a^\mu(x), V_b^\nu(0)]|p\rangle. \qquad (9)$$

Choosing $\mu=+$, $\nu=-$, we set $q^+=0$, multiply by $1/2\pi$, and integrate over $q^-$.[11] Then, as in Ref. 3,

$$\text{Left-hand side} = \frac{\epsilon^{ij}p_i q_j}{p^+}\frac{1}{2\pi}\int d\nu \frac{i}{2m}D_3^{ab}(q^2,\nu),$$

$$\text{Right-hand side} = \int dx^- \int d^2x_\perp\, e^{-i\vec{q}_\perp\cdot\vec{x}_\perp}\langle p|[V_a^+(x), A_b^-(0)]_{\rm LC} + [A_a^+(x), V_b^-(0)]_{\rm LC}|p\rangle,$$

where LC denotes the light-cone commutator.

Now according to Cornwall and Jackiw[2]

$$\begin{aligned}[V_a^+(x), A_b^-(0)] - if_{abc}A_c^-(0)\delta(x^-)\delta^2(\vec{x}_\perp) &= -\tfrac{1}{4}(if_{abc}+d_{abc})(\partial_-[\epsilon(x^-)\delta^2(\vec{x}_\perp)A_c^-(x|0)] + \tfrac{1}{2}\partial_j\{\epsilon(x^-)\delta^2(\vec{x}_\perp)[A_c^j(x|0) - i\epsilon^{jk}V_{kc}(x|0)]\}) \\ &\quad + \tfrac{1}{8}i\epsilon(x^-)\delta^2(\vec{x}_\perp)\bar{\psi}(x)P_-\gamma_5\Lambda_{ab}^-\psi(0) - \text{H.c.}\end{aligned}$$

When this equation is inserted between the nucleon states and the spin average is taken, the only term which survives on the right-hand side is the one involving $V_{kc}(x|0)$. The terms involving the (local or bilocal) axial-vector currents obviously vanish. The last term does not contribute because it is easily seen to be a linear combination of a pseudoscalar (no pseudoscalar can be formed from $x$ and $p$) and a pseudotensor of rank 2 [the only such pseudotensor which can be formed from $x$ and $p$ vanishes because the term is multiplied by $\delta^2(\vec{x}_\perp)$]. Thus, the right-hand side reduces to

$$\text{Right-hand side} = \tfrac{1}{2}\epsilon^{jk}q_j\int dx^-\epsilon(x^-)\langle p|f_{abc}\bar{\mathcal{V}}_{kc}(x|0) - d_{abc}\mathcal{V}_{kc}(x|0)|p\rangle.$$

This can then be further simplified to yield

$$\text{Right-hand side} = \epsilon^{jk}q_j p_k\int_0^\infty (d\alpha/p^+)f_{abc}\bar{V}_c^1(0,\alpha).$$

Hence, we have a sum rule for the $D_3^{ab}$,

$$-\frac{1}{2m}\int_0^\infty d\nu\, D_3^{[ab]}(q^2,\nu) = \pi f_{abc}\int_0^\infty d\alpha\,\bar{V}_c^1(0,\alpha), \qquad (10)$$

where, as in Ref. 3, the $[ab]$ refers to the antisymmetric combination $(D_3^{ab} - D_3^{ba})/2i$. Using this result, our starting point, Eq. (3), is easily established.

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[2]J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971).

[3]D. A. Dicus, R. Jackiw, and V. L. Teplitz, Phys. Rev. D 4, 1733 (1971). Throughout we use the notations of this reference except where explicitly stated.

[4]Related work on light-cone algebras has been done by H. Fritzsch and M. Gell-Mann, in Proceedings of the Eighth Coral Gables Conference on Symmetry Principles at High Energies, University of Miami, 1971 (to be published), and D. J. Gross and S. B. Treiman, Phys. Rev. D 4, 1059 (1971).

[5]See, e.g., J. D. Bjorken, in *Selected Topics in Particle Physics, International School of Physics "Enrico Fermi," Course XLI,* edited by J. Steinberger (Academic, New York, 1967). Terms omitted in Eq. (1) give rise to contributions in the cross section proportional to the lepton mass and are ignored here.

[6]We here correct a sign error of Ref. 3 and use the conventional $W_2$ in terms of which the electroproduction cross section is written as in Ref. 5.

[7]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[8]J. M. Cornwall, D. Corrigan, and R. E. Norton, Phys. Rev. D 3, 536 (1971).

[9]M. Damashek and F. J. Gilman, Phys. Rev. D 1, 1319 (1970).

[10]C. H. Llewellyn Smith, Nucl. Phys. B17, 277 (1970).

[11]Recall that the $\pm$ components of the 4-vector $V^m$ are given by $V^\pm = (V^0 \pm V^3)/\sqrt{2}$.

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# Addendum to "Recalculation of the Crossed-Graph Contribution to the Fourth-Order Lamb Shift"

J. A. Fox

*Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850*
*and Department of Physics, University of Utah, Salt Lake City, Utah 84112*

(Received 30 September 1971)

The discrepancy among the various calculations of the crossed-graph contribution to the fourth-order Lamb shift is discussed and resolved.

Recently, the results of two additional calculations of the slope of the Dirac form factor of the free-electron vertex at zero momentum transfer have appeared.[1,2] These results agreed with each other, disagreed with the results of Ref. 3 and 4, and yielded the following:

$$F_1'(0) = \frac{\alpha^2}{m^2\pi^2}\left[-\tfrac{13}{36}\ln\lambda^{-2} + \tfrac{1}{72}\pi^2 + \tfrac{7}{12}\pi^2\ln 2 - \tfrac{7}{8}\zeta(3) - \tfrac{1181}{1728}\right]. \tag{A1}$$

The methods used to compute $F_1'(0)$ were vastly different: the one by Petermann using standard Feynman theory, the other by Remiddi *et al.* using dispersion techniques. Since both approaches gave the same answer and one differed from the result of Ref. 4, the present author decided to recheck his results completely.

Reanalysis showed that through an organizational error the coefficients of two terms, $I(3,4,3,2,5)$ and $I(2,2,3,2,5)$, were assigned the incorrect values, 6 and $-\frac{11}{2}$, respectively; the correct values are +1 and $+\frac{3}{4}$. In addition, Petermann[1] uncovered a genuine error in one of the integrals, namely, $I(3,1,1,3,4)$, for which Soto[3] listed the value

$$I(3,1,1,3,4) = \tfrac{1}{32}\pi^2 - \tfrac{1}{12}. \tag{A2}$$

The correct value is

$$I(3,1,1,3,4) = \tfrac{1}{72}\pi^2 + \tfrac{1}{6}. \tag{A3}$$

When the correct coefficients for $I(3,4,3,2,5)$ and $I(2,2,3,2,5)$ and the result of Eq. (A3) are employed in the present author's result [Eq. (3a), Ref. 4], it agrees with Eq. (A1).

It is both interesting and appropriate to note that Petermann lists a total of five integrals which he claims are incorrect. They are listed in Table I. Entry 3 of this table has already been discussed. The other terms reveal an important fact: Table II of Ref. 3 contains four misprints. For when the present author recomputed these integrals, he found them to be in agreement with the values he had used originally. Moreover, they were exactly the same as they appeared in preprint form.

These results led to an investigation of the table of coefficients for these integrals (Ref. 3, Table I). Petermann claimed to have found agreement with only about 75% of the published values for these coefficients. The present author, on the other hand, found generally excellent agreement between