Model of Weak- and Electromagnetic-Interaction Symmetry*

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A model that unites the intermediate boson of the weak interaction and the photon of the electromagnetic interaction into a multiplet of gauge fields is considered. Weak leptonic currents and hadronic Cabibbo currents are handled in a parallel manner and generate an SU(2) symmetry group. It is suggested how the K-spin-triplet hypothesis emerges from such an approach. The symmetry between the weak and electromagnetic interaction is spontaneously broken resulting in a massive electron and an intermediate boson.

I. INTRODUCTION

The similarity between the electromagnetic interaction and the weak interaction mediated by the photon and the intermediate boson, respectively, has attracted recent attention.[1-3] In order to overcome the obvious difference in masses and couplings of the photon A_μ and the intermediate boson W_μ, Weinberg[1] regarded the symmetry relating the weak and electromagnetic interactions as an exact symmetry of the Lagrangian but broken by the vacuum. In order to avoid unwanted massless Goldstone bosons the photon and vector bosons were introduced as gauge fields. This model of leptons was generalized to include also the hadrons by Schechter and Ueda[2] (S-U). Independently, Lee[3] has shown that the algebra generated by the product of the respective coupling constant and space integrals of the fourth components of the electromagnetic current and the weak-interaction current is that of an $\mathrm{SU}(2)\otimes\mathrm{U}(1)$ group[4] provided that the vector-boson mass M is 37.3 GeV, and introduced the K-triplet-current hypothesis which is valid in the absence of strong-interaction currents.

Our purpose is to utilize the various models of the aforementioned authors and obtain an alternative model of a unified interaction scheme that gives the usual experimental results.

The leptons are represented by an SU(2) doublet of a left-handed neutrino and electron and an SU(2) singlet of a right-handed electron considered as forming a triplet in a Euclidean space,

$$\psi_L=\begin{pmatrix}\frac{1}{2}(1+\gamma_5)\nu\\ \frac{1}{2}(1+\gamma_5)e\\ \frac{1}{2}(1-\gamma_5)e\end{pmatrix}=\begin{pmatrix}\nu_L\\ e_L\\ e_R\end{pmatrix}.$$

The left- and right-spinning electrons are regarded as separate particles and the first two components are left-handed so as to generate the lepton part of the $V-A$ weak-interaction currents. The symmetry groups are restricted so that observed electron-type leptons connect only with each other. The corresponding terms of the muon-type leptons are also present but are suppressed.

The hadrons can be regarded as made out of a triplet of quarks q_1, q_2, and q_2 having electrical charge $Q_c=\frac{2}{3}$, $-\frac{1}{3}$, and $-\frac{1}{3}$. For describing weak interactions, it is more suitable to use the quarks that are rotated through the Cabibbo angle θ, and Q_L is defined similarly to ψ_L:

$$Q=\begin{pmatrix}Q_1\\ Q_2\\ Q_3\end{pmatrix}=e^{i\theta\lambda_7}\begin{pmatrix}q_1\\ q_2\\ q_3\end{pmatrix}$$

$$=\begin{pmatrix}q_1\\ q_2\cos\theta+q_3\sin\theta\\ -q_2\sin\theta+q_3\cos\theta\end{pmatrix},$$

$$Q_L=\begin{pmatrix}\frac{1}{2}(1+\gamma_5)Q_1\\ \frac{1}{2}(1+\gamma_5)Q_2\\ \frac{1}{2}(1-\gamma_5)Q_3\end{pmatrix}=\begin{pmatrix}Q_{1L}\\ Q_{2L}\\ Q_{3R}\end{pmatrix}.$$

We define similarly $Q_R=(Q_{1R}, Q_{2R}, Q_{3L})$. The present model can be formulated in an equivalent manner when all three quarks in the triplet have the same handedness.

In Sec. II, the results of S-U with modifications are reexpressed in a form emphasizing the symmetric nature of the leptonic and hadronic currents. Spontaneous breakdown is discussed in Sec. III, and a suitable assumption is introduced to obtain Lee's K-triplet-current hypothesis, and the usual weak and electromagnetic interactions. The conclusions are given in Sec. IV.

II. FORMALISM

We first define the leptonic and hadronic currents that interact with the vector gauge fields

$A_{i\mu}$ and B_μ to generate the weak and electromagnetic interactions. The weak current is given in terms of ψ_L and Q_L by

$$j_{i\mu}=i(\bar\psi_L\gamma_\mu\lambda_i\psi_L+\bar Q_L\gamma_\mu\lambda_i Q_L)\,,\quad i=1,2,3 \tag{1}$$

where λ_i are the SU(3) matrices. Because the third quark in ψ_L and Q_L has the opposite handedness, $j_{i\mu}$ satisfies

$$j_{i\mu}=0\,,\quad i=4,5,6,7\,. \tag{2}$$

The additional terms corresponding to the muon are suppressed in the following and $\hbar=c=1$. The subscript i varies from 1 to 3, but the 4 and 5 components are incorporated in the hadronic currents because the quark Q is the rotated quark.

The electromagnetic current j_μ^γ is given in terms of ψ_L and Q by

$$\begin{aligned}j_\mu^\gamma&=i\bar\psi_L\gamma_\mu(\tfrac12\lambda_3+\tfrac12 Y^l)\psi_L+i\bar Q\gamma_\mu(\tfrac12\lambda_3+\tfrac12 Y)Q\\&=-i\bar e\gamma_\mu e+\tfrac13 i(2\bar q_1\gamma_\mu q_1-\bar q_2\gamma_\mu q_2-\bar q_3\gamma_\mu q_3)\,,\end{aligned} \tag{3}$$

where the matrices for the leptonic and hadronic hypercharge are defined as[5]

$$\begin{aligned}Y^l&=\frac{\lambda_8}{\sqrt3}-\frac43 I=\begin{pmatrix}-1&0&0\\0&-1&0\\0&0&-2\end{pmatrix},\\ Y&=\frac{\lambda_8}{\sqrt3}=\frac13\begin{pmatrix}1&0&0\\0&1&0\\0&0&-2\end{pmatrix}.\end{aligned} \tag{4}$$

In Eq. (3), as in the following equations, the identity

$$\bar Q\gamma_\mu OQ=\bar Q_L\gamma_\mu OQ_L+\bar Q_R\gamma_\mu OQ_R \tag{5}$$

is used, where O is an operator that can depend on the subscript μ.

The space integrals of the fourth components of (1) and (3) are

$$\begin{aligned}K_i&\equiv\tfrac12 j_i=-\tfrac12 i\int d^3x\, j_{i4}(x)\\&=\tfrac12\int d^3x\,(\bar\psi_L\gamma_4\lambda_i\psi_L+\bar Q_L\gamma_4\lambda_i Q_L)\,,\\&\qquad i=1,2,3\end{aligned} \tag{6}$$

$$Q_c=-i\int d^3x\, j_4^\gamma(x)\,. \tag{7}$$

The K_i, $i=1,2,3$, are the generators of an SU(2) group whose commutation relations are[6]

$$[K_i,K_j]=if_{ijk}K_k=i\epsilon_{ijk}K_k\,, \tag{8}$$

where ϵ_{ijk} is the completely antisymmetric third-rank tensor. The hadronic part of (6) satisfies the equal-time commutation relation of Gell-Mann.

When a Hermitian operator

$$S=Q_c-K_3 \tag{9}$$

is defined with the aid of (6) and (7), one finds

$$[S,K_i]=0\,,\quad i=1,2,3\,. \tag{10}$$

Therefore, S is the generator of a U(1) group and together with K_i, they form an SU(2)⊗U(1) group.[3,4]

The charge Q_c is also expressible as a linear combination of the third component of isotopic spin I_3 and an isoscalar component $\frac12 Y$, where Y is the hypercharge, and $Q_c=I_3+\frac12 Y$. It is of interest to obtain the relationship between these alternative decompositions. We note from (3), (6), and (7) that

$$K_3=\frac12\int d^3x\,(\bar\psi_L\gamma_4\lambda_3\psi_L+\bar Q_L\gamma_4\lambda_3 Q_L)\,, \tag{11}$$

$$\begin{aligned}Q_c&=\int d^3x\,[(\bar\psi_L\gamma_4\tfrac12\lambda_3\psi_L+\bar Q\gamma_4\tfrac12\lambda_3 Q)\\&\qquad+(\bar\psi_L\gamma_4\tfrac12 Y^l\psi_L+\bar Q\gamma_4\tfrac12 YQ)]\\&=I_3+\tfrac12 Y\,.\end{aligned} \tag{12}$$

Comparison of Eqs. (11) and (12) yields

$$K_3=I_3-\tfrac12 I_3^{hR}\,, \tag{13}$$

where

$$\tfrac12 I_3^{hR}=\int d^3x\,\bar Q_R\gamma_4\tfrac12\lambda_3 Q_R\,. \tag{14}$$

Then, from (9) and (13)

$$\begin{aligned}S&=Q_c-K_3\\&=\tfrac12[Y+I_3^{hR}]\,.\end{aligned} \tag{15}$$

In other words K_3 is purely isovector whereas S contains isoscalar and isovector components. Let $A_{i\mu}$ be the gauge field coupled to isospin $\vec I$ and to the universal left-handed SU(2) group[7] and B_μ be a singlet vector field coupled to hypercharge Y, and corresponding to a U(1) gauge group. A linear combination of leptonic currents coming from the left- and right-handed components of ψ_L forms a pure vector current to reproduce the electromagnetic vector current.

The Yang-Mills-type Lagrangian density $\mathfrak{L}_g$ invariant under isospin $\vec I$ and Y gauge transformations is[2]

$$\begin{aligned}\mathfrak{L}_g=&-\bar\psi_L\gamma_\mu(\partial_\mu+ig'\tfrac12 Y^l B_\mu-ig\tfrac12\lambda_i A_{i\mu})\psi_L\\&-\bar Q_L\gamma_\mu(\partial_\mu+ig'\tfrac12 YB_\mu-ig\tfrac12\lambda_i A_{i\mu})Q_L\\&-\bar Q_R\gamma_\mu[\partial_\mu+ig'(\tfrac12 Y+\tfrac12\lambda_3)B_\mu]Q_R\\&-\tfrac14(\partial_\mu A_{i\nu}-\partial_\nu A_{i\mu}+g\epsilon_{ijk}A_{j\mu}A_{k\nu})^2\\&-\tfrac14(\partial_\mu B_\nu-\partial_\nu B_\mu)^2\,,\end{aligned} \tag{16}$$

where the matrices for the leptonic hypercharge Y^l and hadronic hypercharge Y are defined in Eq. (4).

There is no leptonic term corresponding to the third term on the right-hand side of Eq. (16), essentially because the electron appears twice in ψ_L. This term is required to obtain the isovector part of the electromagnetic field when the photon field A_μ is identified with the particular mixture of $A_{3\mu}$ and B_μ that comes from the spontaneous breakdown mechanism, which is next discussed in Sec. III. It follows from Eq. (16) that one must assign isotopic spin $I=0$, and hypercharge $2Q$ to the Q_R fields.

III. SPONTANEOUS BREAKDOWN

In order to generate mass terms a complex doublet of a spin-zero field ϕ is introduced.[8] The invariant Lagrangian density of ϕ is

$$\begin{aligned}\mathcal{L}_\phi = &-\tfrac{1}{2}\operatorname{Tr}|\partial_\mu\phi - \tfrac{1}{2}igA_{i\mu}\lambda_i\phi + i\tfrac{1}{2}g'B_\mu\phi|^2\\ &-G_e\bar{\psi}_L(\phi+\phi^\dagger)\psi_L - V(\phi)\\ &-G_1(\bar{Q}_L\phi\lambda_6 Q_R + \bar{Q}_R\lambda_6\phi^\dagger Q_L)\\ &-G_2(\bar{Q}_{3L}Q_{3R} + \bar{Q}_{3R}Q_{3L}),\end{aligned} \tag{17}$$

where the ϕ is arranged in the matrix form

$$\phi = \begin{pmatrix} 0 & 0 & \varphi^{(+)} \\ 0 & 0 & \varphi^{(0)} \\ 0 & 0 & \end{pmatrix}, \quad \phi^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \varphi^{(-)} & \varphi^{(0)\dagger} & 0\end{pmatrix}. \tag{18}$$

The ϕ and $\phi^\dagger$ have the quantum numbers of K and $\bar{K}^+$ mesons. The last two terms on the right-hand side of (17) are invariant contributions to the quark mass.

The $V(\phi)$ is an invariant function of ϕ whose minimum is chosen to occur at ϕ given in (19), in order to have spontaneous breakdown of symmetry.[9] There is no leptonic term corresponding to the last term on the right-hand side of (17) because the neutrino mass is zero.

The Lagrangian after spontaneous symmetry breakdown is obtained by replacing ϕ in L_ϕ by[1,2]

$$\phi = \lambda\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0\end{pmatrix}, \tag{19}$$

where λ is the vacuum expectation value of $\varphi^{(0)}$, i.e., $\lambda = \langle\varphi^{(0)}\rangle$. The Lagrangian $\mathcal{L}_g$ remains unchanged and $\mathcal{L}_\phi$ becomes[1]

$$\begin{aligned}\mathcal{L}_\phi = &-\tfrac{1}{8}\lambda^2 g^2[(A_{1\mu})^2 + (A_{2\mu})^2]\\ &-\tfrac{1}{8}\lambda^2(gA_{3\mu} + g'B_\mu)^2 - \lambda G_e\bar{e}e\\ &-\lambda G_1\bar{Q}_2Q_2 - G_2\bar{Q}_3Q_3.\end{aligned} \tag{20}$$

The neutral spin-1 fields of definite mass are from (20) and its orthogonal mixture

$$\begin{aligned}A_\mu &= N(-g'A_{3\mu} + gB_\mu),\\ Z_\mu &= N(gA_{3\mu} + g'B_\mu),\end{aligned} \tag{21}$$

where

$$N = [2(g^2 + g'^2)]^{-1/2}.$$

The requirement that the quark masses are nonnegative leads to $\lambda G_1 > 0$ and $G_2 > 0$.

The interaction between the spin-1 bosons and ψ_L and Q triplets is, from (16) and (21),

$$\frac{g}{2\sqrt{2}}(j_\mu^w W_\mu^- + j_\mu^{w\dagger} W_\mu^+) - \frac{\sqrt{2}gg'}{(g^2+g'^2)^{1/2}} j_\mu^\gamma A_\mu - \frac{\sqrt{2}}{(g^2+g'^2)^{1/2}}(g'^2 S_\mu - g^2 K_{3\mu})Z_\mu, \tag{22}$$

where

$$j_\mu^w = j_{1\mu} - ij_{2\mu}, \tag{23}$$

$$W_\mu^\pm = (A_{1\mu} \mp iA_{2\mu})/\sqrt{2}, \tag{24}$$

$$j_\mu^\gamma = K_{3\mu} + S_\mu. \tag{25}$$

Equation (22) is equivalent to previous results obtained in Refs. 1 and 2, in which isotopic-spin and hypercharge are conserved.

It is of interest to see how Lee's K-spin triplet hypothesis emerges from this approach. We let the third component of the gauge field $A_{i\mu}$, i.e., $A_{3\mu}$, be identified with $-g'B_\mu/g$, thus violating the conservation of $K_{3\mu}$. When this choice is substituted into (21) and (22), one obtains

$$A_\mu = -\left(\frac{g^2+g'^2}{2}\right)^{1/2}\frac{A_{3\mu}}{g'}, \tag{26}$$

$$\frac{g}{2\sqrt{2}}(j_\mu^w W_\mu^- + j_\mu^{w\dagger} W_\mu^+) - \frac{\sqrt{2}gg'}{(g^2+g'^2)^{1/2}} j_\mu^\gamma A_\mu. \tag{27}$$

Equation (27) involves a strong assumption because of $Z_\mu = 0$, but the alternative[2] would be to introduce another U(1) gauge field to cancel off unwanted semileptonic decays resulting from Z_μ. The electric charge from (27) is

$$e = \sqrt{2}gg'/(g^2+g'^2)^{1/2}, \tag{28}$$

which on substitution in (26) yields

$$A_\mu = -\frac{g}{e}A_{3\mu}. \tag{29}$$

The identification of $A_{3\mu}$ with A_μ, i.e., $A_{3\mu} = A_\mu$, gives

$$g = -e. \tag{30}$$

It follows that $B_\mu = -gA_\mu/g'$ from (21), (29), and (30).

When A_μ is substituted for $A_{3\mu}$ and $-g'B_\mu/g$ in Eq. (16), the first three terms become

$$-\bar{\psi}_L\gamma_\mu(\partial_\mu - igQ_c A_\mu)\psi_L - \bar{Q}\gamma_\mu(\partial_\mu - ig\, Q_c A_\mu)Q + ig\bar{\psi}_L\gamma_\mu \tfrac{1}{2}\lambda_i A_{i\mu}\psi_L + ig\bar{Q}_L\gamma_\mu \tfrac{1}{2}\lambda_i A_{i\mu} Q_L\,, \quad i=1,2\,. \qquad (31)$$

Putting together (22)–(29), one obtains for the weak and electromagnetic interactions from (27) or (31)

$$-\frac{e}{2\sqrt{2}}(j_\mu^w W_\mu^- + j_\mu^{w\dagger} W_\mu^+) + ej_\mu^\gamma A_\mu\,, \qquad (32)$$

which satisfies the K-triplet-current hypotheses of Lee.[3]

We see from Eqs. (20) and (24) that the masses of the electron, intermediate boson, photon, Q_1, Q_2, and Q_3 are λGe, $\frac{1}{2}\lambda g$, 0, 0, λG_1, and G_2, respectively. Only two quarks have a mass in the present model.[2] The condition $Z_\mu = 0$ results from the fact that $K_{3\mu}$ and S_μ are not separately conserved.[3]

IV. REMARKS

The present model that is based on previous work, is an attempt to unify the weak and electromagnetic interactions in a simple manner assuming that these interactions are generated by leptonic and hadronic currents interacting with vector gauge fields representing the photon and intermediate vector bosons. For the purpose of generating mass terms of the electron, intermediate bosons, and hadrons a quartet of spin-zero mesons is introduced which are coupled to the gauge fields, leptons, and hadrons. Then, the symmetry between the weak and electromagnetic interactions is spontaneously broken.

The Lagrangian density $\mathcal{L}_g$ invariant under $\vec{I}$ and Y gauge transformations that couples the gauge fields $A_{i\mu}$ and B_μ with leptons and hadrons Q is essentially unique when we require the K-triplet-current hypothesis to be satisfied, whereas, the invariant Lagrangian density $\mathcal{L}_\phi$ is not free from arbitrariness as one would desire. The electron, intermediate boson, and the Q_2 quark have acquired a mass after spontaneous breakdown.

Whether or not the present model has any contact with nature is difficult to assess since both the intermediate boson W_μ and the quarks remain unobserved. It is perhaps premature to consider such questions as the introduction of CP violation in weak interactions. Search for observable consequences of the K-spin triplet hypothesis would be of interest, as well as the study of models starting from Eq. (22).

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[2]J. Schechter and Y. Ueda, Phys. Rev. D 2, 736 (1970).

[3]T. D. Lee, Phys. Rev. Letters 26, 801 (1971).

[4]It is noted in M. Gell-Mann, Physics 1, 63 (1964), that the weak and electric charge operators generate the algebra of $SU(2)\otimes U(1)$. In Ref. 3, this algebra is proposed for the direct physical observable (current $\times$ coupling constant).

[5]The matrices for the charge operator Q_c of the triplets is given by $Q_c = \frac{1}{2}\lambda_3 + \frac{1}{2}Y = I_3 + \frac{1}{2}Y$ for hadrons and $Q_c = I_3 + \frac{1}{2}Y^l$ for leptons. The Y is rotated hypercharge where Q_{1L}, Q_{2L}, and Q_{3R} have $Y = \frac{1}{3}$, $\frac{1}{3}$, and $-\frac{2}{3}$, respectively. This approach appears preferable over the alternative one of using the original quarks q and then defining the weak current as $(j_{1\mu} \pm ij_{2\mu})\cos\theta + (j_{4\mu} \pm ij_{5\mu})\sin\theta$. The same symbol is used for the generators and the matrices for Q_c, I_3, and Y.

[6]The K^+, K^-, and K^0 defined by $K^+ = K_1 + iK_2$, $K^- = K_1 - iK_2$, and $K^0 = K_3$ satisfy the commutation relations $[K^+, K^-] = 2K^0$, $[K^\pm, K^0] = \mp K^\pm$.

[7]The $A_{i\mu}$ ($i = 1, 2, 3$) does not couple to the third component of ψ_L, the singlet right-handed lepton.

[8]One can also introduce the symmetry breaking directly as in S. L. Glashow, Nucl. Phys. 22, 579 (1961).

[9]Articles on relevant literature on spontaneous symmetry breakdown can be found in Refs. 1 and 2.