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Production and Propagation of Particles with $A > 81$ in the Galaxy*

T. B. Kaiser, J. R. Wayland,† and G. Gloeckler‡
Center for Theoretical Physics of the Department of Physics and Astronomy, University of Maryland, College Park, Maryland 20742
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A detailed mathematical model is developed to describe the transformation of the charge composition of superheavy ($A > 81$) cosmic rays by spallation on interstellar hydrogen in the galaxy. If one takes a single source which suddenly injects particles with an r-process charge spectrum t_0 years in the past, the calculated relative abundance ratios in four atomic mass groups are consistent with measured values for $t_0 \approx 2.5\times 10^5$ years. We further conclude that for any initial source distribution the upper limit on t_0 is 10^6 years.

I. INTRODUCTION

One of the least understood aspects of cosmic rays is the presence of particles with charges greater than that of iron. The observation of the superheavy[1,2] and even the transuranium component[3] is particularly difficult to explain within the context of simple propagation theories for cosmic-ray particles. Yet a reasonable description of the propagation of particles from cosmic-ray sources to observers which adequately explains the presence of the superheavy component would not only help in our understanding of the physical processes within our galaxy, but would also allow us to estimate the composition of particles at the sources, and thus help in limiting the possible candidates for the origin of cosmic rays. In fact, much has already been done along these lines, particularly for iron and lighter nuclei.[4]

We investigate certain aspects of the propagation of high-energy nuclei ($\gtrsim$1 GeV/nucleon) with atomic mass numbers larger than 81. Although a steady-state, homogeneous source distribution can reasonably account for the measured low-energy cosmic-ray charge ratios,[5] a random and discrete source distribution is probably required to explain the presence of the very heavy nuclei observed.[6] Depending upon the strength and space-time characteristics of the source, its contribution may at one extreme dominate a subregion of the total confinement volume or simply add to the general background. It is very reasonable to expect, and, of course, consistent with observations,[7] that much of this background consists of protons and light elements. These not only have rather long lifetimes against nuclear interactions,[7] but also are products of the fragmentations of heavier particles.[7] Superimposed upon this background are local disturbances resulting from specific sources which will provide, for a finite interval in space and time, cosmic rays with a component in the trans-iron region. In view of this picture, it is likely for the superheavy component to exhibit a measurable transient behavior.[8]

After developing a general model that includes, as possibilities, all of the above considerations, we use the recently measured superheavy charge spectra to establish age limits on cosmic-ray sources. In this paper we consider only the simplest source distribution, i.e., a single source that injects particles with a given charge spectrum at some time t_0 in the past. While the occurrence of only one source in the local confinement volume (that region of the galactic disk within a few hundred parsecs of the sun[9,10]) during the past t_0 years is not totally improbable, more than one source is more likely. We use the single-source hypothesis because of its mathematical simplicity and, as will be seen, because it allows one to avoid a choice of geometry and boundary conditions and commitment to a specific propagation model. The effects of a random distribution of discrete sources will be examined in future work.

II. MATHEMATICAL FORMULATION

We begin by considering the transport and fragmentation of high-energy cosmic rays in the galaxy. We choose to describe the propagation of cosmic particles in the turbulent magnetic field by diffusion with appropriate boundary conditions. Because only particles with kinetic energies higher than 1 GeV/nucleon are considered, we are justified in neglecting energy losses by ionization of the interstellar gas.[7] Furthermore, we neglect continuous statistical acceleration and consider nuclear interactions only with hydrogen, the dominant constituent of the interstellar gas. Then the continuity equation satisfied by the directionally averaged intensity $j(A, Z, \epsilon, \vec{r}, t)$ of nuclei with atomic mass A, charge Z, and kinetic energy per nucleon ϵ at position $\vec{r}$, and time t is[7]

$$\frac{\partial j}{\partial t} - \vec{\nabla}\cdot(D\vec{\nabla}j) = \frac{c\beta}{4\pi}Q - n_{\mathrm{H}}c\beta\sigma_I j + n_{\mathrm{H}}c\beta\sum_{A',Z'}\int_0^\infty d\epsilon'\sigma_{A'Z',AZ}(\epsilon',\epsilon)j'. \tag{1}$$

Here D is the diffusion coefficient which can, in general, depend on A, Z, ϵ, $\vec{r}$, and t. $Q(A, Z, \epsilon, \vec{r}, t)$ represents the energy spectra of cosmic rays at their sources of origin and the distribution of these sources. σ_I is the total inelastic cross section for the interaction of nuclei with interstellar hydrogen, n_{H} is the number density of hydrogen, and $c\beta$ the speed of the nuclei. The last term describes the production of nuclei from spallation on interstellar hydrogen. Thus $\sigma_{A'Z',AZ}(\epsilon', \epsilon)$ is the spallation cross section for the production of a nucleus characterized by (A, Z, ϵ) from the spallation of a nucleus with (A', Z', ϵ'). Making the usual assumption[7] that ϵ is the same for the spallation products as for the incident nucleus,

$$\int_0^\infty d\epsilon'\sigma_{A'Z',AZ}(\epsilon',\epsilon)j(A',Z',\epsilon',\vec{r},t) = \sigma_{A'Z',AZ}\,j(A',Z',\epsilon,\vec{r},t). \tag{2}$$

To proceed further we assume that the diffusion coefficient is independent of A, Z, ϵ, $\vec{r}$, t, and n_{H} is constant throughout the region of interest. Furthermore, we restrict σ_I to depend only[11] on A and take note of the fact[12] that the spallation cross section is independent of Z'. When Eq. (1) is summed over Z, holding the mass number A constant, we obtain for each value of A

$$\frac{\partial j_A}{\partial t} - \vec{\nabla}\cdot(D\vec{\nabla}j_A) = \frac{c\beta}{4\pi}Q_A - n_{\mathrm{H}}c\beta\sigma_I(A)j_A + n_{\mathrm{H}}c\beta\sum_{A'}\sigma_{A'A}j_{A'}, \tag{3}$$

where the subscript A indicates summation over Z. For later comparison with results of nucleosynthesis studies and their relation to possible cosmic-ray sources it is convenient to partition the isobaric families described by Eq. (3) into five groups as given in Table I. The first group contains mass numbers 180 to 205, while the fifth group includes A values 82 to 97. The continuity equation for each of the five groups is obtained by summing Eq. (3) over A values within the respective groups. The resulting equation for the ith group reads

$$\frac{\partial J_i}{\partial t} - \vec{\nabla}\cdot(D\vec{\nabla}J_i) = \frac{c\beta}{4\pi}Q_i - n_{\mathrm{H}}c\beta\sigma_i J_i + n_{\mathrm{H}}c\beta\sum_{k<i}\sigma_{ki}J_k. \tag{4}$$

J_i and Q_i are, respectively, $\sum j_A$ and $\sum Q_A$, while

TABLE I. Mass groups and corresponding charge groups.

Group No.	$A_{\min}$	$A_{\max}$	$Z_{\min}$	$Z_{\max}$
1	180	205	74	81
2	140	179	58	73
3	115	139	49	57
4	98	114	43	48
5	82	97	36	42

σ_{ki} is $\sigma_{A'A}$ first averaged over A' and summed over A. The average absorption cross section for group i is $\sigma_i = \sigma_{Ii} - \sigma_{ii}$ where σ_{Ii} is the average value of $\sigma_I(A)$.

Ginzburg and Syrovatskii[7] have given general solutions to equations similar to Eq. (4). They find that fragmentation and propagation effects may be treated separately, i.e.,

$$J_i(\epsilon,\vec{r},t) = \int_0^\infty d\tau\, J_i^{(p)}(\tau)G(\epsilon,\vec{r},t,\tau), \tag{5}$$

where $G(\epsilon, \vec{r}, t, \tau)$ describes the propagation of particles and $J_i^{(p)}(\tau)$ the effect of fragmentation. The functions $J_i^{(p)}$ satisfy

$$\frac{\partial J_i^{(p)}}{\partial \tau} + n_{\mathrm{H}}c\beta\sigma_i J_i^{(p)} = n_{\mathrm{H}}c\beta\sum_{k<i}\sigma_{ki}J_k^{(p)}, \tag{6}$$

with initial conditions $J_i^{(p)}(0) = q_i$. q_i is the source strength of the ith group. Here we made the assumption that shapes of the energy spectra and distribution of sources are identical for all of the groups considered, that is, $Q_i(\epsilon, \vec{r}, t) = q_i\chi(\epsilon, \vec{r}, t)$. The solution to the system of fragmentation equations (6) is[7]

$$J_i^{(p)}(\tau) = \sum_{k=1}^{i}\sum_{l=1}^{k}a_{ikl}q_l e^{-\tau/\tau_k}, \tag{7}$$

where the absorption time $\tau_k = (n_{\mathrm{H}}c\beta\sigma_k)^{-1}$ and the coefficients a_{ikl} depend only on the appropriate absorption and spallation cross sections. [A precise definition of a_{ikl} is given in Eq. (14.20) of Ref. 7.]

The propagation function $G(\epsilon, \vec{r}, t, \tau)$ can be evaluated for various geometries and for sources arbitrarily distributed in space and time. For our purposes it is sufficient that it can be put in the form[7,13]

$$G(\epsilon,\vec{r},t,\tau) = \int d^3r'\,\Psi(\vec{r},\vec{r}',\tau)\chi(\epsilon,\vec{r}',t-\tau). \tag{8}$$

The solution of Eq. (4) can then be expressed as

$$J_i(\epsilon,\vec{r},t) = \sum_{l=1} q_l\int_0^\infty d\tau\, b_{il}(\tau)\int d^3r'\,\Psi(\vec{r},\vec{r}',\tau)\chi(\epsilon,\vec{r}',t-\tau), \tag{9}$$

where

$$b_{il}(\tau)=\sum_{k=l}^{i}a_{ikl}e^{-\tau/\tau_k}. \tag{10}$$

Equation (9) is valid for sources arbitrarily distributed in space and time. In the case of an instantaneous source, $\chi(\epsilon,\vec{r},t)=\chi(\epsilon,\vec{r})\delta(t-t_0)$, however, the solution simplifies to

$$J_i(\epsilon,\vec{r},t)=G_0(\epsilon,\vec{r},t-t_0)\sum_{l=1}^{i}b_{il}(t-t_0)q_l, \tag{11}$$

$$\frac{J_i(\epsilon,\vec{r},t)}{J_1(\epsilon,\vec{r},t)}=\sum_{l=1}^{i}\frac{b_{il}(t-t_0)}{b_{11}(t-t_0)}\frac{q_l}{q_1}, \tag{12}$$

where

$$G_0(\epsilon,\vec{r},t-t_0)=\int d^3r'\Psi(\vec{r},\vec{r}',t-t_0)\chi(\epsilon,\vec{r}').$$

The inverse of Eqs. (11) and (12), which have the simple form[7]

$$q_i=G_0^{-1}\sum_{l=1}^{i}b_{il}(t_0-t)J_l(\epsilon,\vec{r},t), \tag{13}$$

$$\frac{q_i}{q_1}=\sum_{l=1}^{i}\frac{b_{il}(t_0-t)}{b_{11}(t_0-t)}\frac{J_l(\epsilon,\vec{r},t)}{J_1(\epsilon,\vec{r},t)}, \tag{14}$$

enables the computation of source strengths in terms of observed intensities.

III. COMPUTATIONAL CONSIDERATIONS

In order to compute the average cross sections σ_i and σ_{ik}, we need, of course, the explicit dependence for $\sigma_I(A)$ and $\sigma_{A'Z',AZ}$. To fill in gaps in the experimental cross-section data, we will use semiempirical relationships. The expression adopted for the total inelastic cross section is[11]

$$\sigma_I(A)=50A^{0.769}\ \text{mb}. \tag{15}$$

For the spallation cross section we use a slightly modified version of a semiempirical formula developed by Rudstam.[12] Above 2.1 GeV/nucleon he finds

$$\sigma_{A'Z',AZ}=\frac{\sigma_I(A')f_1(A')Pd^{2/3}A'^{-2e'/3}}{1.79[(e^{PA'}-1)(1-2e'/3PA')+2e'/3PA']}$$
$$\times\exp[PA-R(A)|Z-Z_P(A)|^{3/2}], \tag{16}$$

with

$$f_1(A')=e^{-g+hA'},$$
$$R(A)=dA^{-e'}, \tag{17}$$
$$Z_P(A)=SA-TA^2,$$

and $P=0.056$, $d=11.8$, $e'=0.45$, $S=0.486$, $T=0.00038$, $g=0.25$, and $h=0.0074$.

To obtain $\sigma_{A'A}$, Eq. (16) must be summed over Z. It is convenient to replace the summation with an integration. Since $\sigma_{A'Z',AZ}$ falls off sharply for large values of $|Z-Z_P|$, the range of integration may be taken to be $0\leq Z\leq\infty$,

$$\sigma_{A'A}=\int_0^\infty dZ\ \sigma_{A'Z',AZ}$$
$$=\frac{\sigma_I(A')f_1(A')PA'^{-2e'/3}}{(e^{PA'}-1)(1-2e'/3PA')+2e'/3PA'}(e^{PA}A^{2e'/3}). \tag{18}$$

For our purpose the expression (18) is deficient in two respects. First of all, it does not apply[12] when $A=A'-1$. In this case we follow Waddington's[14] suggestion based on experimental results obtained by Winzeler[15] that one take

$$\sigma_{A',A'-1}=0.08\sigma_I(A'). \tag{19}$$

Secondly, if the cross section as given by Eqs. (18) and (19) is summed over A, one finds that the ratio of the total spallation cross section to the total inelastic cross section is given approximately by

$$\frac{\sum_{A=1}^{A'-1}\sigma_{A'A}}{\sigma_I(A')}\approx 0.08+e^{-0.369+0.00757A'}.$$

The expression on the right exceeds unity for $A'\geq 40$, which, of course, is not allowed on physical grounds. Comparison of this expression with the first of Eqs. (17) suggests that the simplest way to avoid this difficulty is to reassign the parameters g and h, so as to force the ratio to equal unity. (In justification of this procedure, it might be pointed out that Rudstam's values were obtained by least-squares fits to experimental cross-section data in which target mass numbers less than 75 predominated.) The new values determined in this way are

$$g=-0.0353,\quad h=-0.000165. \tag{20}$$

The average cross sections σ_{ik} computed from Eqs. (15) and (17)–(20) are given in Table II.

IV. DATA

A. Charge-Spectrum Measurements

Data, in the form of charge spectra, from four balloon experiments[3,16-19] were used to compile a

TABLE II. Average cross sections, σ_{ik} (mb).

Product group \ Target group	1	2	3	4	5
1	1390	1390	110	21	7.4
2		1550	770	136	48
3			1010	729	242
4				681	741
5					579

TABLE III. Experimental data used to derive composite mass spectrum at the top of the atmosphere.

Experiment	Reference	Minimum Z, A	Total number of events used	Collecting power (m^2 day sr)	Vertical depth (g/cm^2)	Average depth (g/cm^2)
Texas I	3, 16	38, 88	11	5.79	4.5	5.4
Texas II	16, 17	35, 78	52	30.4	3.5	4.4
Barndoor III	18	39, 89	17	19.3	3.5	4.9
Monster I and II	17, 19	36, 82	78	124, 107, 89, 20, 20 [a]	3.0	4.6

[a] The collecting power varied for different mass groups in this experiment; entries in the table are in decreasing order of mass.

composite mass spectrum at the top of the atmosphere. The Z-to-A correspondence was assumed to be given by the β stability line.

The minimum detectable charge varied from one experiment to another (see Table III) making the choice of the lower boundary of the lightest mass group somewhat ambiguous. It was chosen so as to include as much of the experimental data as possible without the necessity of undue extrapolation of data for those experiments with higher charge thresholds. 95% of all events with $Z \leq 42$ ($A \leq 97$) were retained, and approximately 12% of the total intensity in group 5 was due to extrapolation.

To derive the composite intensities at the top of the atmosphere, the data from each experiment were employed in the following way. The total number of events and intensity at the detector in each mass group were determined from the reported or deduced collecting power (see Table III). In the case of the Barndoor III data, the average values of the charge as measured in plastic and emulsion were used. The uncertainties in intensities were taken to be

$$(\Delta J_i)^2 = \left(\frac{\sqrt{n_i}}{n_i} J_i\right)^2, \qquad (21)$$

where n_i is the number of events in the ith mass group.

The intensities were then propagated to the top of the atmosphere using Eq. (13) with $G_0 = 1$ (no diffusion). Spallation cross sections for interaction with the atmosphere were assumed to be given by Rudstam's formula, with

$$\sigma_I(A) \to \sigma_I(A, A') = 50(A^{0.3845} + A'^{0.3845} - 1)^2 \text{ mb}, \qquad (22)$$

and $A' = 14$, a simple generalization[7] of Eq. (15). The effective mean free paths deduced from Rudstam's formula with Eq. (22) are given in Table IV. The average path length traversed in air was taken to be the vertical depth divided by the average cosine of the zenith angle (see Table III). Some idea of the importance of spallation in the atmosphere can be obtained from Table V, which presents the effect in the case of the Monster I and II data.

The intensities at the top of the atmosphere in each mass group were then weighted by the appropriate collecting powers and added to obtain the composite intensity ratios given in Table VI. The uncertainties given there are the result of propagating uncertainties at the detectors, given by Eq. (21). It should be pointed out that these are to be considered lower bounds on the uncertainties, since other contributing sources of error (charge resolution, assumed Z-to-A conversion, Rudstam's approximation, average-depth approximation) have been ignored. Furthermore, a systematic underestimate of all intensities is due to neglect of attenuation in the detectors.

B. Nucleosynthesis

The only nucleosynthesis studies that have implied the existence of appreciable amounts of elements with $A > 81$ are those based on the r process. The fundamental paper is by Seeger, Fowler, and Clayton (SFC),[20] who compute source strengths normalized by subtraction of solar-system s-process abundances. Wagoner[21] has obtained abundances from the r process under conditions of high-temperature explosion for elements up to $A = 27$, and these would be the natural choice for the present application were they extended to high A. Lacking such data, we will rely on the SFC results.

If the mass spectrum is subdivided into groups in conformity with peaks in the abundance distribution obtained by SFC, the mass groups and the corresponding source strength ratios are as given in Tables I and VI. The ambiguity in the fifth mass

TABLE IV. Effective mean free path for interaction in the atmosphere.

Mass group	Mean free path (g/cm^2)
1	10.5
2	16.1
3	13.5
4	12.4
5	13.5

TABLE V. Effect of spallation in atmosphere in the case of Monster I and II.

Mass group	Intensity at top of atmosphere $(m^2 \text{ day sr})^{-1}$	Intensity at detector $(m^2 \text{ day sr})^{-1}$	Percentage of intensity at detector due to spallation products from heavier mass groups
1	0.124	0.0806	0
2	0.138	0.140	26
3	0.318	0.258	12
4	0.851	0.650	9
5	0.951	0.850	16

group reflects uncertainty about the r-process contribution to the abundance peak[20] at $Z=36$.

V. RESULTS

Equation (12) was used to compute the intensities attributable to a single r-process source distribution. Intensity ratios rather than the intensities were considered, in order to eliminate the peculiarities of particular diffusion models. The results are plotted in Fig. 1 as functions of elapsed time since source turn-on, and compared with the observed ratios shown as shaded bands (1-standard-deviation limits). The two curves for the J_5/J_1 ratio reflect uncertainties about the r-process contribution to the $Z=36$ abundance peak. Note that although comparison of theory with observation leads to ranges of allowed propagation times only in groups 2, 3, and 4, and overlapping times in only the first two of these, the discrepancies involved are not large. If the uncertainties in the observed intensity ratios are doubled, for example, consistency is achieved for a propagation time of about 2.5×10^5 years. Such an adjustment is not unreasonable, in view of the fact that several sources of error have been neglected.

If, on the other hand, the lack of agreement is accepted as real, at least two possible explanations present themselves. The most obvious is that the assumption of a single source is unrealistic. Generalization to a random distribution of discrete sources necessitates a choice of geometry and boundary conditions, and will be investigated in a future paper. Another possibility is that the SFC r process is not adequate as a source of superheavy nuclei. This raises the question of what a recalculation of r-process abundances based on more accurate cross-section data might reveal.

To obtain further information on the applicability of the source model as well as to derive constraints on the age of a single source of arbitrary abundance distribution, the observed intensity ratios were propagated backward in time by means of Eq. (14). The results of this computation are given in Fig. 2. The most significant feature is the fact that the ratio q_2/q_1 is negative for $t-t_0 \gtrsim 10^6$

TABLE VI. Source strength ratios according to SFC (Ref. 20) and observed intensity ratios (Refs. 3, 16–19).

Group No.	$(q_i/q_1)_{SFC}$	$(J_i/J_1)_{obs}$
1	1.00	1.00
2	0.78	1.46 ± 0.54
3	3.22	2.71 ± 0.95
4	1.07	5.78 ± 1.95
5	7.28, 11.7	4.86 ± 1.77

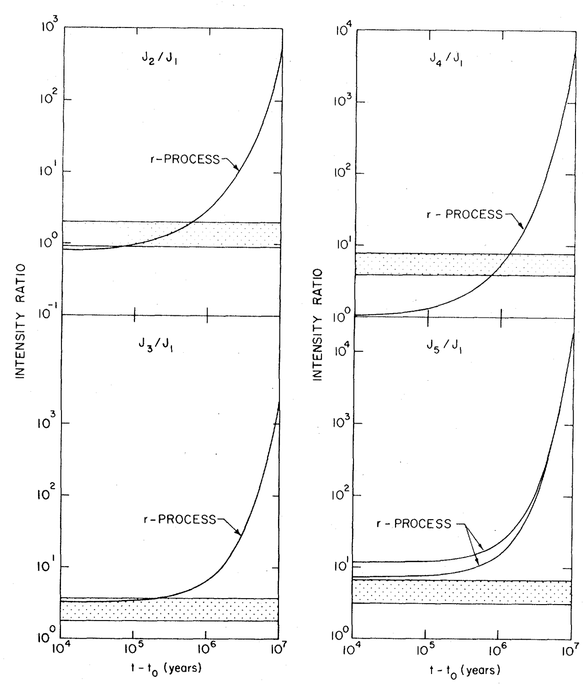

FIG. 1. Time dependence of intensity ratios computed from Eq. (12) and r-process source distribution (Ref. 20) for mass groups of Table I. Shaded areas represent observed ratios (Refs. 3, 16–19) (±1 standard deviation). The two curves given for J_5/J_1 are explained in the text.

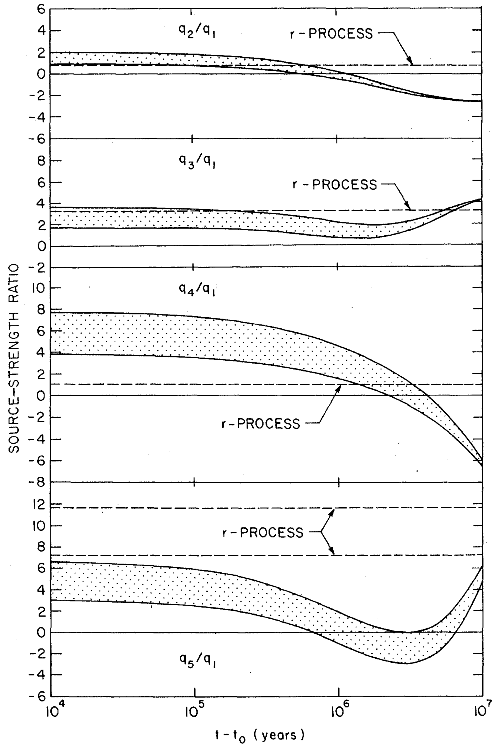

FIG. 2. Dependence of source-strength ratios (±1 standard deviation) on elapsed time between source turn-on and observation, as computed from Eq. (14) using observed intensity ratios (Refs. 3, 16–19) for mass groups of Table I. Dashed lines give r-process source-strength ratios (Ref. 20). The two possibilities for q_5/q_1 are explained in the text.

years. Since q_2 feeds particles into the lighter mass groups, the plots of $q_{3,4,5}/q_1$ are not physically meaningful for propagation times greater than about 10^6 years. We deduce that no single source, r process or otherwise, occurring more than 10^6 years ago could be responsible for the intensities of superheavy nuclei observed.

In comparing the computed source-strength ratios with predictions of the SFC r process, times identical to those from the forward-propagation calculation were obtained.

VI. CONCLUSIONS

We have investigated the propagation of high-energy (>1 GeV/nucleon) nuclei with atomic mass greater than 81. Because of rather short lifetimes against nuclear interactions, a random and discrete source distribution was considered. Only the simplest case, i.e., a single source, was investigated in this paper. By taking the ratio of intensities we have avoided commitment to any specific propagation model.

The strongest conclusion we can reach comes from the physical requirement that the source strength connot be negative. For a single source we note that the oldest age is approximately 10^6 years, independent of any assumptions about source composition.

If a single source following a conventional r-process abundance distribution is assumed, the most likely value for $t - t_0$ is about 2.5×10^5 years. A relatively young source of the superheavy component is thus implied if present observations and cross-section formulas are assumed correct.

A random distribution of discrete r-process sources provides a more realistic mechanism for the production of the superheavy cosmic rays, and such a model will be investigated in future work. The present need is clearly for more experimental data on superheavy intensities and spallation cross sections. A recalculation of r-process abundances based on more accurate cross-section information would also be of much value.

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†Present address: Department of Physics, Texas A&M University, College Station, Texas 77843.

‡ Alfred P. Sloan Foundation Fellow.

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Magnetic Spin Monopole from the Yang-Mills Field

A. Joseph
Department of Physics and Astronomy, Tel-Aviv University, Ramat-Aviv, Israel
(Received 8 September 1970)

A generalization of the magnetic monopole has been derived in the context of the Yang-Mills field. This is achieved through the observation that the quantization of the ordinary magnetic monopole derives from closure of the angular momentum and magnetic field components on the Lie algebra of the Euclidean group E(3) and the fact that the Yang-Mills field is the generalization of the Maxwell field obtained by replacing the ordinary derivative by a covariant derivative. This leads to defining equations for the generalized monopole which are explicitly solved for the case when SU(2) is the gauge group of the field. The resulting system, referred to as the spin monopole, is investigated in some detail.

I. INTRODUCTION

A conviction in the existence of a symmetry between the electric and magnetic fields leads to the adoption of the magnetic monopole as the fundamental unit of magnetic matter. As first shown by Dirac,[1] the existence of such an entity requires the quantization of the product of the electric and magnetic pole strengths. It has long been supposed that this quantization is somehow responsible for the existence of the unit of charge, though the search for magnetic monopoles has as yet proved entirely unfruitful.[2] Recently Schiff[3] has revived this belief in an attempt to use the monopole quantization to explain the failure of quarks to appear as free individuals, though his method has not gone without criticism.[4] In a different spirit, but with a similar objective, Schwinger[5] has further suggested that the particle which carries the magnetic monopole is at the same time a manifestation of the elusive quark itself.

It is our present objective to develop the notion of a fundamental unit of matter which is capable of producing a charge quantization in the spirit of Dirac's monopole and at the same time carries the SU(3) internal quantum numbers of the hadrons in the spirit of Schwinger's quark. For this purpose we seek a theory which embraces this internal symmetry and yet retains the structure of the Maxwell equations as expressed through the properties of the exterior derivative and Poincaré invariance.

A particularly pleasing example of such a theory is provided by the Yang-Mills field.[6] From a physical standpoint it may be regarded – and indeed was first developed in this fashion[6] – as the natural generalization of the Maxwell field accommodating a noncommutative gauge group. We may choose this gauge group to be SU(3) and in so doing effect an interplay between the internal hadron symmetry and the field equations. Mathematically, the Yang-Mills field represents the generalization of the Maxwell field obtained by replacing the ordinary derivative by a covariant derivative. This structural resemblance to the Maxwell field proves a useful asset in its analysis.

In the following we shall derive a form of magnetic monopole from the Yang-Mills field by exploiting its relationship with the Maxwell field and then investigate the properties of this new system. More specifically, we first develop the quantization of the magnetic monopole through certain algebraic identities which lead to the Lie algebra of the Euclidean group E(3) and the requirement that the