Implications of Tachyon-Like Matter for Superdense Stars

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A new equation of state of superdense matter is proposed. It is derivable by treating superdense matter as a perfect, degenerate tachyon gas. Yet the velocity of sound in the medium, β, which this equation leads to, satisfies the inequality $0 \le \beta^2 < \frac{1}{2}$. Model calculations for superdense stars based on this equation of state are presented. By appropriately choosing a certain parameter, dynamical stability can be achieved for arbitrarily large central densities. Also, a somewhat larger than usual value for the maximum mass is obtained.

The role of the equation of state of matter at ultrahigh densities (i.e., $\gtrsim$ nuclear density) in determining the over-all properties, such as the mass-energy (M) and the radius (R), of superdense stars has been investigated extensively.[1] It has been found that for all the equations of state studied so far, the general characteristics of the mass-energy versus the central-energy-density (ρ_c) curve remain essentially the same. The numerical values of the macroscopic parameters M and R vary by much less than an order of magnitude, and more importantly, by application of Chandrasekhar's variational-principle approach,[2] it has been established that the dynamical instability with respect to small radial perturbations sets in precisely at the central density corresponding to the first ("neutron-star") mass peak for all the equations of state tried.[3] In this sense, none of these equations seems to lead to results which could be considered very different from those following from the simple equation of state corresponding to the free degenerate neutron gas.

The physical restriction on the equation of state can be formulated in terms of the quantity β, which gives the velocity of sound in the medium (in units of the velocity of light c). One has

$$\beta^2 = \left(\frac{v_s}{c}\right)^2 = \frac{\partial P}{\partial \rho},$$

where P and ρ stand, respectively, for the pressure and the energy density. The physically acceptable equations of state are those equations for which β satisfies the following inequality:

$$0 \le \beta^2 \le 1\,. \tag{1}$$

The upper bound here comes from considerations of causality and special relativity while the lower bound simply means that we cannot allow "negative pressure."

It may be noted that for all the equations of state proposed hitherto for the high-density regime, β^2 is a monotonically increasing function of the energy density ρ, i.e., $\partial\beta^2/\partial\rho > 0$.

The object of this note is to propose a new equation of state, for the high-density region, which (i) is consistent with the physical requirement, Eq. (1), (ii) is different from all other equations of state in that $\partial\beta^2/\partial\rho < 0$ for high densities, (iii) when fed into the Tolman-Oppenheimer-Volkoff (TOV) relativistic equation of hydrostatic equilibrium, yields superdense stars which have significantly larger peak values for mass (M) and radius (R) than those of all other models, and finally (iv) leads to the possibility *that a stable superdense star may exist for arbitrarily high central density.*

Relegating the discussion of the motivation for this new equation of state to the last part of this note, we give here the result. Our equation of state can be exhibited in a parametric form as follows:

$$\rho = kG(p, q)\,, \tag{2a}$$

$$P = kF(p, q)\,, \tag{2b}$$

$$n = n_0 \nu(p, q)\,, \tag{2c}$$

where

$$G(p, q) = (2p^3 - p)(p^2 - 1)^{1/2} - 2(q^3 - q)(q^2 - 1)^{1/2} - \ln\{[p + (p^2 - 1)^{1/2}]/[q + (q^2 - 1)^{1/2}]\}, \tag{2d}$$

$$F(p, q) = (\tfrac{2}{3}p^3 + p - \tfrac{8}{3}q^3)(p^2-1)^{1/2} + (2q^3 - q)(q^2-1)^{1/2} + \ln\{[p + (p^2-1)^{1/2}]/[q + (q^2-1)^{1/2}]\}, \quad (2e)$$

$$\nu(q, p) = p^3 - q^3, \quad (2f)$$

and

$$k = (g m_0^4 c^5 / 16\pi^2\hbar^3), \quad n_0 = (g m_0^3 c^3/6\pi^2\hbar^3), \quad g = 2s+1. \quad (2g)$$

Here n is the number density of the particles constituting the matter, and s their spin. The quantity m_0 is a mass parameter corresponding to the particles constituting the matter.[4] The quantities ρ, P, and n are all functions of p and q. Here q is restricted to be > 1 but is otherwise free; different values of q will give us different equations of state. Once q is specified, for any n, p can be determined from Eq. (2c); in particular, $n=0$ gives $p=q$, and for any other value of n, $p>q$. Thus, for any given value of q, we have P and ρ as functions of n and consequently P as a function of ρ. Equations (2) constitute our equation of state.

By direct calculation, we find that

$$\beta^2 = (p^3 - q^3)/3p(p^2-1), \quad (3)$$

$$\partial\beta^2/\partial\rho = (-2p^3 + 3q^3p^2 - q^3)/24kp^4(p^2-1)^3. \quad (4)$$

An examination of the preceding equations reveals that

$$0 \leq \beta^2 < \tfrac{1}{2} \quad (5)$$

and that β^2 is not a monotonic function of ρ but shows a maximum at a certain energy density ρ_m corresponding to $p = p_m$, where p_m is a solution of the cubic equation

$$2p_m^3 - 3p_m^2 q^3 + q^3 = 0 \quad (6)$$

such that $p_m > q$.

In other words

$$\partial\beta^2/\partial\rho \leq 0 \text{ for } \rho \geq \rho_m. \quad (7)$$

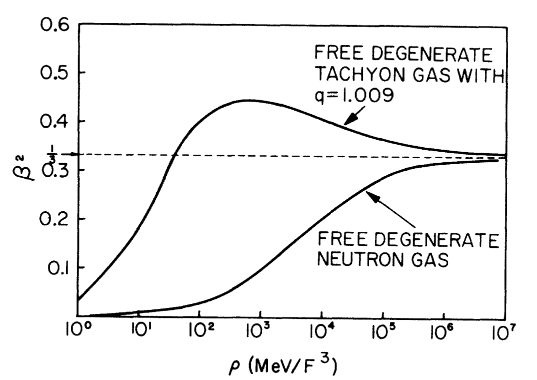

FIG. 1. A plot of β^2 vs ρ for a free degenerate Fermi gas of tachyons with $q=1.009$ and for a free degenerate neutron gas.

This behavior of β^2 as a function of ρ for $q=1.009$ is depicted in Fig. 1 and is one of the distinguishing features of our equation of state. The corresponding β^2 versus ρ curve for the free degenerate neutron gas is also shown in Fig. 1.

Having established Eq. (2) as a physically valid equation of state, we return to its application to superdense stars.

Working within the general-relativistic framework,[5] we have for the mass of the star

$$M = m(R), \quad m(r) = 4\pi \int_0^r \rho(r')r'^2 dr', \quad (8)$$

where R is the radius. One may calculate M for any given central density ρ_c, with the help of the TOV equation

$$-\frac{dP}{dr} = \frac{G(\rho + r)}{cr^4}[4\pi Pr^2 + m(r)] \bigg/ \left[1 - \frac{2Gm(r)}{c^4 r}\right] \quad (9)$$

together with our equation of state (2). The calculation of M and R from Eqs. (2) and (9) is a standard numerical exercise. We give in this note only our final results (see Fig. 2).

The dynamical stability of our model of the superdense star can be investigated by the use of Chandrasekhar's variational-principle approach.[2] This enables us to calculate the eigenfrequency ω for the lowest mode of oscillation (corresponding to small radial perturbations). A positive value of ω^2 corresponds to dynamical stability; if ω^2 is negative one has dynamical instability. The expression for ω^2 is standard[2,5] and we do not reproduce it here [it simplifies a little for the equation of state (2)].

The M versus ρ_c curve which follows from the present model is shown in Fig. 2 for two different values of the parameter q. It can be seen immediately that the shape of this curve is essentially

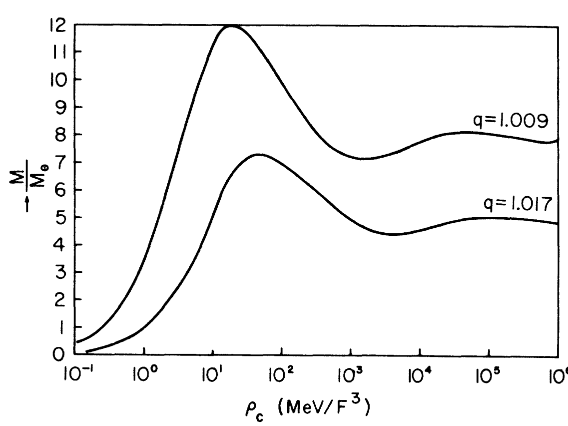

FIG. 2. The mass of superdense stars as a function of the central density for the cases $q=1.009$ and $q=1.017$.

the same as that resulting from the other known equations of state. (For $q = 1.009$, the peak value of M is $11.9M_{\odot}$ and the corresponding value of R is 99.9 km; the central density corresponding to the mass peak is 2.8×10^{13} g/cm^3.) Unlike all other equations of state, however, for our case the quantity ω^2 remains positive all along the M vs ρ_c curve (i.e., $\omega^2 > 0$ for any ρ_c) provided q is restricted[6] by the inequality $1 < q < q_{\max}$ where $q_{\max} = 1.019$. This implies that for all the equations of state (2) for which $1 < q < q_{\max}$, we obtain superdense stars which are stable with respect to radial perturbations for any central density.[7] This is indeed a new result which does not follow from any other equation of state known to us.

We conclude this note with a discussion of the motivation for our equation of state. As noted earlier, the changes which the various known equations of state produce in the gross characteristics of the superdense stars are essentially insignificant. Thus, it seems that to obtain any significantly new result, one might have to attribute to high-density matter properties fundamentally different from those underlying the conventional equations of state and yet such that they are consistent with the physical restriction given by Eq. (1). The physics underlying our equation of state is simply to consider the ground state of an assembly of free fermions for which the single-particle energy-momentum relation is that of the tachyons,[8] namely, $E(\vec{p}) = (\vec{p}^{\,2} - m_0^{\,2})^{1/2}$ with $|\vec{p}| > m_0$, rather than that of the tardyons, $E(\vec{p}) = (\vec{p}^{\,2} + m_0^{\,2})^{1/2}$ with $|\vec{p}| > 0$. It is of course quite intriguing that in a medium which is composed of tachyons, the velocity of sound is still restricted by bounds which are consistent with those for normal matter. The behavior of β^2 as a function of energy density (see Fig. 1) is a feature which distinguishes our equation of state from all earlier ones and it is this feature which seems responsible for our new result, namely, stability with respect to radial perturbations for superdense stars with ultrahigh central densities.

A derivation of Eq. (2) and some further work along the present lines will be presented elsewhere.

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[1]See, for example, J. A. Wheeler, in *Annual Review of Astronomy and Astrophysics*, edited by B. L. Goldberg (Annual Reviews, Palo Alto, Calif., 1966), Vol. 4.

[2]S. Chandrasekhar, Phys. Rev. Letters 12, 114 (1964); 15, 438 (1964).

[3]C. W. Misner and H. S. Zapolsky, Phys. Rev. Letters 12, 635 (1964).

[4]For the case of a neutron gas, for example, m_0 is simply the neutron rest mass. In our case, as will be clear from the discussion at the end of this note, it is actually the absolute value of the imaginary mass parameter of the tachyon (see Ref. 8). For numerical purposes we take it to have the same value as the neutron rest mass. The numerical value of m_0 only fixes the scale of the final results. Note that in our calculations the spin s is taken to be $\frac{1}{2}$.

[5]B. K. Harrison, K. S. Thorne, M. Wakano, and J. A. Wheeler, *Gravitational Theory and Gravitational Collapse* (Univ. of Chicago Press, Chicago, Ill., 1964), p. 50. We have chosen the trial function $\xi(r)$ to be equal to r. The Lagrangian displacement is given by $\xi(r)e^{i\omega t}$. Other choices for $\xi(r)$ do not change the results significantly.

[6]The origin and implications of this restriction on q and the consequences of our equation of state for $q \geq q_{\max}$ shall be discussed in a subsequent communication.

[7]The case of "infinite" central density is to be handled as usual by asymptotic expansions. See B. K. Harrison, Phys. Rev. 137, B1644 (1965).

[8]E. C. G. Sudarshan, Arkiv Fysik 39, 585 (1969); O. M. P. Bilaniuk, V. K. Deshpande, and E. C. G. Sudarshan, Am. J. Phys. 30, 718 (1962).