s-Channel Helicity Conservation in Elastic Processes*

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The unitarity equation for elastic partial-wave amplitudes is considered. Defining helicity-total-flip elastic amplitudes as those in which both helicities change sign, it is observed that the unitarity equation strongly suggests that the imaginary part of the helicity-total-flip partial-wave amplitude is much smaller than that of the nonflip amplitude at high energies. Appealing to a plausibility argument, the ratio of helicity-total-flip to helicity-nonflip partial-wave amplitudes is estimated to be $(1/s)^{1/2}$ or smaller. The implications of this result for the full amplitude, the possibility of extending the result to other amplitudes if factorization is true, and the implications for Regge theory are discussed.

I. INTRODUCTION

Following the discovery by Ballam *et al.*[1] that s-channel helicity is conserved in ρ photoproduction up to $t=-0.4$ GeV^2, it was suggested by Gilman *et al.*[2] that s-channel helicity conservation (SCHC) might be a general property of all Pomeranchukon-exchange processes. The experimental evidence, however, indicates that although SCHC does hold in πN elastic scattering[2,3] and in ρ photoproduction[1] (which to some degree of approximation may be similar to ρp elastic scattering), it does not hold for diffractive processes[4] like $\pi p \to A_1 p$. There have been a number of theoretical papers on the implications of SCHC, but little model-independent work has been done on its origins.[5]

In this paper we will suggest that a limited form of SCHC should be expected to hold for elastic processes as a constraint of unitarity and parity conservation. We will deal with partial-wave helicity amplitudes so that the well-known kinematic factor that requires the total s-channel helicity to be conserved in the forward direction (angular momentum conservation) has already been factored out. The physical basis for our argument is that the helicity-nonflip partial-wave amplitude is truly elastic; the initial and final states are the same. Other helicity amplitudes are in some sense inelastic; the initial and final states are different and the amplitude is not simply related to a total cross section.

In Sec. II we will express this idea by means of the partial-wave unitarity equation. The imaginary part of the nonflip amplitude is the sum of the absolute squares of production amplitudes. It is essentially the partial-wave total cross section.

The other helicity amplitudes are not the sums of absolute squares. Intuitively we might expect cancellations between the terms, but since we know neither the magnitudes of the terms (compared with the corresponding terms in the nonflip sums) nor their relative phases, little can be said concretely. We will see, however, that the helicity-total-flip amplitude (in which both helicities change sign) is again built up from the absolute squares of production amplitudes, the same amplitudes that appear in the nonflip case. Now, however, the positive- and negative-parity intermediate states enter the sum with opposite sign. Since the intermediate states have a great many degrees of freedom at high energy, we will argue that there is probably a large cancellation, and therefore that the total-flip amplitude should be much smaller than the nonflip.

After mentioning the possibility of extending this result to other amplitudes if high-energy elastic scattering is dominated by the exchange of a factorizable Pomeranchukon, we conclude Sec. II with a brief discussion of the implications for the full amplitudes.

In Sec. III we make some additional dynamical assumptions but do not restrict ourselves to any single model. From these we estimate that the ratio of the helicity-total-flip to the helicity-nonflip partial-wave amplitudes is $(1/s)^{1/2}$ or smaller (s is measured in GeV^2).

Section IV is devoted to a brief discussion of the constraints placed on t-channel Regge residues by approximate SCHC (and therefore by unitarity).

II. UNITARITY FOR HELICITY PARTIAL-WAVE AMPLITUDES

The basic argument is extremely simple. The partial-wave unitarity equation for an elastic two-body process is

$$\mathrm{Im}\langle \lambda_3\lambda_4|A^J(s)|\lambda_1\lambda_2\rangle = \sum_q \rho_q \langle \lambda_3\lambda_4|A^J(s)|JMPq\rangle\langle \lambda_1\lambda_2|A^J(s)|JMPq\rangle^* . \tag{1}$$

In writing down a unitarity equation any complete orthonormal set of intermediate states can be used. We have chosen states of definite total angular momentum J, z component $M=\lambda=\lambda_1-\lambda_2$, and parity P. The label q represents all the other variables needed to describe the state completely (number and kinds of particles, internal orbital angular momenta, spins, subenergies, etc.). The parity P is actually determined by the other variables. Also, ρ_q is a positive definite kinematic factor. For two-body intermediate states $\rho_q=2k_q/(s)^{1/2}$, where k_q is the center-of-mass momentum of the state. Equation (1) can be derived by writing down the ordinary (linear momentum) unitarity equation, choosing intermediate states as described above, and expanding the initial and final two-body states in terms of angular momentum states.[6]

There are four special cases of (1).

Case (i): Helicity-nonflip. This is the case in which $\lambda_1=\lambda_3$ and $\lambda_2=\lambda_4$. Equation (1) becomes

$$\mathrm{Im}\langle \lambda_1\lambda_2|A^J(s)|\lambda_1\lambda_2\rangle = \sum_q \rho_q |\langle \lambda_1\lambda_2|A^J(s)|JMPq\rangle|^2 , \tag{2}$$

a sum of positive terms.

Case (ii): Helicity-total-flip. This means that $\lambda_1=-\lambda_3$ and $\lambda_2=-\lambda_4$. The unitarity equation is

$$\mathrm{Im}\langle -\lambda_1 -\lambda_2|A^J(s)|\lambda_1\lambda_2\rangle = \eta_1\eta_2(-1)^{J-s_1-s_2}\sum_q P\rho_q |\langle \lambda_1\lambda_2|A^J(s)|JMPq\rangle|^2 . \tag{3}$$

We have used the assumption of parity conservation and the fact that (with the phase conventions of Ref. 6)

$$P_{\mathrm{op}}|JM-\lambda_1-\lambda_2\rangle = \eta_1\eta_2(-1)^{J-s_1-s_2}|JM\lambda_1\lambda_2\rangle . \tag{4}$$

Of course, η_i and s_i are the intrinsic parity and spin of particle i, and P_{op} is the reflection operator. Comparing (2) and (3), we see that the imaginary part of the helicity-nonflip (total-flip) partial-wave amplitude is the sum (difference) of positive contributions from the positive- and negative-parity intermediate states. At high energies the intermediate states are extremely complicated and have very many degrees of freedom. Not only are we summing over the number and type of particles, but also over many internal orbital angular momenta. It is difficult to imagine how either the positive- or negative-parity states could dominate at very high energies. Hence, we believe that the helicity-total-flip amplitude should be much smaller than the nonflip for large s.[7] A specific mechanism for the cancellation will be suggested later in the paper.

Cases *(i)* and *(ii)* exhaust the possibilities for πN elastic scattering. For more complicated spin structures, such as the hypothetical $\rho p\to\rho p$ reaction, there are two more possibilities.

Case (iii): Helicity-partial-flip. This means that $\lambda_1=-\lambda_3$ or $\lambda_2=-\lambda_4$. Nothing can be said directly from unitarity about this amplitude. If we make the additional assumption that high-energy elastic scattering is dominated by the exchange of a Pomeranchukon trajectory with factorizable residues, we can argue that this amplitude should also be small. For example, by first considering $\rho\pi$ and πp scattering, we can conclude that the t-channel Pomeranchukon residues must be such as to prevent the s-channel helicities of the ρ or the p from changing sign in any diffractive reaction, including ρp elastic scattering.

Case (iv). This includes all other amplitudes. Nothing can be said rigorously from unitarity.

We will now state our definition of SCHC: For very high-energy elastic scattering, the imaginary parts of the helicity-total-flip partial-wave amplitudes are much smaller than those of the nonflip. If factorization holds, the result can be extended to partial-flip amplitudes. Of course, high-energy diffractive amplitudes are expected to be largely imaginary. Applicable reactions include πN, KN, ρp, γp, $\bar{p}p$, pn, pp, etc. For pp we mean that a final proton should maintain the helicity of the initial proton with which it has a small momentum transfer. The result should also hold for ρ photoproduction to the extent that the amplitudes, including the spin structure in the center-of-mass frame, are the same as for ρp elastic scattering. However, we have found no reason for amplitudes like

$$\langle \lambda_\rho=0\ \lambda_p=\tfrac{1}{2}|A^J(s)|\lambda_\gamma=1\ \lambda_p=\tfrac{1}{2}\rangle ,$$

included in case *(iv)*, to be small.

Our results cannot be generalized to inelastic diffractive processes because even for case *(i)* the unitarity sum would not be a sum of positive terms. From factorization, however, we expect that the proton helicity should be conserved in processes like $\pi p\to A_1 p$.

Now consider the full amplitudes. It is well known that the functions $d^J_{\lambda\mu}(\theta)$ ($\mu=\lambda_3-\lambda_4$) that appear in the partial-wave expansion carry the kinematic factor $(\sin\frac{1}{2}\theta)^{|\lambda-\mu|}$. This factor causes the flip amplitudes to vanish in the forward direction, as required by angular momentum conservation. The coefficient of this factor, however, is very large; for large J it is $J^{\mu-\lambda}/(\mu-\lambda)!$ at $\theta=0$ (for $\mu>\lambda$). For diffractive scattering, we expect all of the partial waves up to J near $\frac{1}{2}b(s)^{1/2}$, where $b\approx 5$ GeV^{-1}, to be important.[8] The reader can easily verify that these facts imply that the ratio R of helicity-flip to nonflip amplitudes is

$$R=f(t)(-t)^{|\lambda-\mu|/2}X\ll 1\,, \tag{5}$$

where t is in units of GeV^2 in the kinematic factor, X is the typical ratio of the corresponding partial-wave amplitudes, and $f(t)$ is a function that depends on the details of the amplitudes but is close to unity. For πN or KN scattering this means

$$\left|\frac{mA}{sB}\right|\approx f(t)X\ll 1\,, \tag{6}$$

where A and B are the Chew-Goldberger-Low-Nambu (CGLN) amplitudes[9] and m is the nucleon mass. This, of course, suggests that A decouples from the Pomeranchukon.

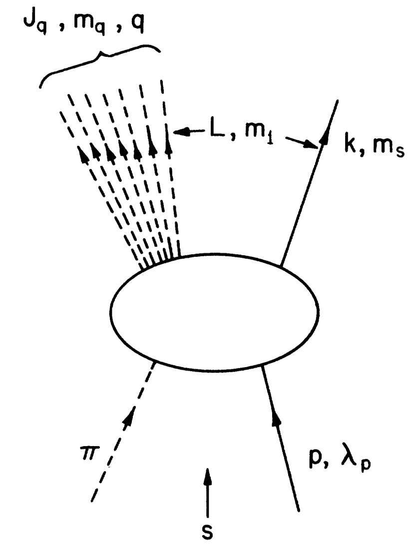

FIG. 1. Production amplitude treated formally as a two-body amplitude.

III. A POSSIBLE MECHANISM FOR CANCELLATION

Now let us discuss one possible mechanism for the cancellation between positive- and negative-parity intermediate states. This will require additional dynamical assumptions, but even if this specific calculation should turn out to be wrong, the general argument following (4) will not be affected.

For πp elastic scattering, for example, each intermediate state will contain at least one nucleon. For these production amplitudes we will assume that one of these nucleons is a fragment of the initial proton, i.e., that it has an energy distribution peaked near $\frac{1}{2}(s)^{1/2}$ and that its angular distribution is sharply peaked in the forward direction. These assumptions are compatible with most models (such as the multiperipheral) and are supported by experiments on related processes (such as $pp\to p+\text{anything}$[10]). Under these assumptions there is a forward-backward asymmetry in πp production reactions, corresponding to a superposition of positive- and negative-parity states.

To estimate this effect more carefully, consider the amplitude illustrated in Fig. 1. A π and a p in a linear momentum state interact to produce a single nucleon of three-momentum k and z component of spin m_s (our intermediate states are chosen to have definite parity), and some configuration of other particles. In a purely formal sense we can think of this as a two-body reaction. The second "particle" has a definite angular momentum J_q, z component m_q, and a definite parity in its own center of mass; q describes its other quantum numbers and subenergies. True two-body intermediate states are a special case. Finally, the two "particles" have relative orbital angular momentum L and z component m_1. This amplitude is

$$\langle kLm_1m_s;J_qm_qq|A^\dagger(s)|\theta=\phi=0\,\lambda_p\rangle = \int d^2\hat{n}f_Q(\hat{n})Y_{Lm_1}(\hat{n})\,, \tag{7}$$

where $f_Q(\hat{n})$ is the angular distribution of the final nucleon and Q represents all of the quantum numbers of the final state except L and m_1. For fixed Q and m_1 our previous assumptions guarantee that the amplitude varies smoothly with L up to L of the order of k (in GeV), where it cuts off. Both even and odd values of L enter with equal importance.

If we now label the states in (2) and (3) by $|JMP;kLJ_qJ_rq\rangle$, where J_r is the sum of J_q and the nucleon spin, we can easily show that

$$16\pi(2J+1)\langle\lambda_p|A^J(s)|JMP;kLJ_qJ_rq\rangle=\sum_{m_1m_sm_q}\langle J_rM-m_1|J_qm_q\tfrac{1}{2}m_s\rangle \times\langle JM|Lm_1J_rM-m_1\rangle\langle\theta=\phi=0\,\lambda_p|A(s)|kLm_1m_s;J_qm_qq\rangle\,. \tag{8}$$

All of the amplitudes on the right-hand side vary uniformly with L up to $L=O(k)$. This at least suggests that both even and odd values of L occur with equal importance on the left-hand side for each fixed value of J, k, J_r, J_q, and q, although this cannot really be proved without a detailed knowledge of the m_1, m_s, and m_q dependence of the amplitudes on the right-hand side. With the other variables fixed, the parity of the final state on the left-hand side changes when L is changed by one unit. Hence, when we sum over L in (2), approximately k terms of comparable magnitude add together. In (3), however, the terms cancel, and only on the order of one term survives. We must then integrate over the distribution of k values, which is peaked near $\frac{1}{2}(s)^{1/2}$. Hence, we expect the contribution of the states of a given J_r, J_q, and q to the helicity-nonflip amplitude to be larger by a factor $(s)^{1/2}$ than the contribution to the total-flip amplitude. Additional cancellations can occur as we sum over the other variables. Although complicated by symmetry problems, the argument can be extended to the case in which there are several nucleons in the intermediate state, as long as one of them is a fragment of the initial proton and the others have no preferred direction. Therefore, we expect

$$R<(-t)^{|\lambda-\mu|/2}[\epsilon f_1(t)+f_2(t)/(s)^{1/2}], \tag{9}$$

where f_1 and f_2 are functions of t, and ϵ is a very small (possibly zero) constant generated by the intermediate states that do not fulfill our dynamical assumptions.

Similar arguments can be made for $\bar{p}p$, etc. For $\rho\pi$ scattering, we must make the additional (questionable) assumption that there is always one pion that can be considered a fragment of the initial ρ and that all of the other pions are actually decay products of produced ρ's.

IV. IMPLICATIONS FOR REGGE THEORY

Let us now use our results as a constraint on Regge residues for elastic processes. Thews[11] has shown that exact SCHC [including the vanishing of case-*(iv)* amplitudes] is compatible with factorizable Pomeranchukon exchange only at infinite s. This is because the elements of the helicity crossing matrix can be written as a power series in $1/s$, the coefficients depending on t. If we require that the contributions of the constant terms to all of the s-channel helicity-changing amplitudes exactly vanish, we can uniquely determine the ratios of the t-channel flip residues to nonflip residues.[11,12] These ratios being determined, the $1/s$ and higher-order terms can be calculated and do not vanish; hence, the Pomeranchukon contribution to R in (5) should decrease as $1/s$. For the Pomeranchukon part of πN or KN scattering the ratio is predicted to be $m(-t)^{1/2}/s$. If the residues do not arrange themselves perfectly there could also be a small energy-independent contribution to R.

For more complicated spin structures we do not really expect the case-*(iv)* amplitudes to vanish; we still have certain linear relations between the residues, due to the constraints on the case-*(ii)* and -*(iii)* amplitudes, but we cannot determine all of the ratios.

The residues of secondary trajectories could also be arranged so as to give SCHC to first order, but there is no special reason to expect this.[13] Hence, we expect the leading contribution to the s-channel flip amplitude to be given by the secondary trajectories. This is smaller than the nonflip amplitude by $s^{\Delta(t)}\approx s^{-1/2}$, where $\Delta(t)$ is the difference between the secondary and Pomeranchukon trajectories. Finally, the t-channel residues must be such as to give the $(-t)^{|\lambda-\mu|/2}$ kinematic factor in the s-channel amplitude. Hence, these simple Regge arguments reproduce (9).

We can restate these conclusions if we accept the Harari-Freund hypothesis[14] that the low-energy background is associated with Pomeranchukon exchange: The flip-to-nonflip ratio for the background should vary as $1/s$. Hence, vestiges of SCHC may persist to fairly low energies for the background. SCHC should not hold for the low-energy resonance amplitude, however; rather, the helicity structure is entirely determined by the spins and parities of the resonances.

V. CONCLUSION

We have made a plausibility argument that helicity-total-flip elastic amplitudes should be much smaller than helicity-nonflip amplitudes because of unitarity. This requirement places constraints on the ratios of the t-channel Pomeranchukon residues. If factorization is true, s-channel helicity-partial-flip amplitudes should also be small at high energies. These results apply only to elastic amplitudes. Inelastic diffractive amplitudes, which are not subject to any simple unitarity constraints, are not expected to have any simple s-channel properties except those which follow from factorization.

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Spectral Representations and Their Application to Inclusive Processes*

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We give a representation for the product of two currents sandwiched between two-particle states, akin to the Deser-Gilbert-Sudarshan representation of commutators. In this case, arguments based on causality are not available for the two-particle matrix element. It is shown that in the process $pp\to\mu^+\mu^-$ +anything, the cross section cannot decrease rapidly in $q_\perp$ (the perpendicular momentum transfer) for $q_\perp{}^2 \ll q^2$ (q is the $\mu^+\mu^-$ momentum), unless it decreases extremely rapidly in q^2. An application of the free-quark model to the product of two pion currents yields Feynman scaling for the process $pp\to\pi$ +anything, when proper account is taken of the rapid decrease in $q_\perp$ of this cross section.

I. INTRODUCTION

The Deser-Gilbert-Sudarshan (DGS) representation[1,2] (a specialized version of the Jost-Lehmann-Dyson representation), based on causality and spectrum conditions, has been of great value in the investigation of deep-inelastic electroproduction and neutrino production.[3] For these semileptonic inclusive reactions, matrix elements of the type

$$\langle p|[J_\mu(x), J_\nu(0)]|p\rangle \tag{1}$$

(where $|p\rangle$ is a single-hadron state) determine the cross sections. Thus the commutator, with its causal structure, is directly accessible experimentally.

The situation is different for hadronically induced inclusive processes, such as $pp\to\mu^+\mu^-$ +anything or $pp\to\pi$ +anything. Cross sections for these processes depend on matrix elements of the type

$$\langle p_1 p_2\,\mathrm{in}|J(x)J(0)|p_1 p_2\,\mathrm{in}\rangle\,, \tag{2}$$

which are of a different character from (1) for several reasons. First, the experiments measure the ordinary (Wightman) product, not the commutator, so that conventional causality arguments are not available for constructing a spectral representation for (2). Second, the two-particle states $|p_1p_2\,\mathrm{in}\rangle$ behave very much like an unstable single-particle state would, and this leads to some nontrivial technical complications with the spectrum conditions. Third, one must distinguish between in and out states in (2), while this is irrelevant for (1). This is of no consequence for the general nature of the spectral representations we shall give for matrix elements of the type (2), except for certain reality properties.

In this paper, we give a spectral representation for matrix elements such as (2), based on a straightforward generalization of the conventional DGS representation for (1). The needed general-