Multiperipheral Theory of Deep-Inelastic Neutrino-Nucleon Scattering

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The recently proposed multiperipheral theory of deep-inelastic electron-nucleon scattering is applied to deep-inelastic neutrino-nucleon scattering. The behavior of the structure functions is derived in the limit of large energy ν ($\nu \equiv p \cdot q/m$), and large momentum transfer squared q^2, with $\omega \equiv 2m\nu/(-q^2)$ fixed but large. In particular, scaling is derived. The model predicts that an energetic nucleon-antinucleon pair should be produced in this limit. The model also predicts that

$$\frac{d\sigma^{\nu p}}{d(1/\omega)} + \frac{d\sigma^{\nu n}}{d(1/\omega)} > \frac{d\sigma^{\bar{\nu} p}}{d(1/\omega)} + \frac{d\sigma^{\bar{\nu} n}}{d(1/\omega)},$$

and the inequality approaches at a known rate an equality as $\omega \to \infty$. The natural scaling variable that arises in the model is $\bar{\omega} \equiv (p+q)^2/(-q^2)$. The generality of our derivation, in the sense of requiring only Regge behavior and a particular asymptotic off-shell dependence, is discussed. The multiperipheral theory is compared with theories based on other models.

I. INTRODUCTION

Recently we proposed a multiperipheral theory for deep-inelastic electron-nucleon scattering.[1] We showed how a simple modification of the Amati-Bertocchi-Fubini-Stanghellini-Tonin (ABFST) multiperipheral model[2] (MPM) can explain many general features of deep-inelastic e-N scattering. We also showed the generality of our derivation. When an infinitesimal damping factor is introduced for the momentum transfer of the exchanged nucleon, our derivation requires only Regge behavior and the particular asymptotic off-shell dependence given by the ABFST MPM. If instead of introducing an infinitesimal damping factor a cutoff is introduced, then our derivation requires only Regge behavior. The justification for introducing this extra damping factor or cutoff is because the nucleon propagator does not sufficiently damp out large transverse momenta corresponding to large momentum transfer. It is also possible that this extra damping factor may be generated within our model (see Sec. IV). Thus, our derivation can be valid for more general multiperipheral models or even general Regge models.

The idea that Regge poles may dominate the asymptotic behavior of the structure functions in the Bjorken limit was first suggested by Abarbanel, Goldberger, and Treiman[3] and independently by Harari.[4] Furthermore, the former authors used a ϕ^3 theory coupling to a spinless photon to study the behavior of the Regge residue function in the large-q^2 limit. They found that this behavior nearly gives Bjorken scaling and attributed the nonscaling behavior as due to their negligence of the photon spin. Altarelli and Rubinstein[5] took into account the spin of the photon and found that νW_2 scales, but $W_1 \to 0$ in the Bjorken limit (i.e., trivial scaling). Drell, Levy, and Yan[6] (DLY) derived nontrivial scaling for both W_1 and νW_2 by using a pseudoscalar-meson–nucleon field theory. They found that in the leading-logarithm approximation, the dominant diagram is the nucleon-exchange diagram. However, this nucleon-exchange dominance is an important undesirable feature of the DLY model, because hadronic multiparticle production processes are dominated by meson exchange.

In Ref. 1 we showed that to derive nontrivial scaling, what is essential is that the photon should couple to two spin-$\frac{1}{2}$ particles, but the rest of the multiperipheral chain can be identical to the hadronic multiparticle production chain of meson exchange. Thus, multiperipheral models used to describe high-energy hadronic production processes also essentially describe virtual Compton scattering. The only modification is an additional link in the multiperipheral chain where the virtual photon is coupled to two spin-$\frac{1}{2}$ particles. One way to interpret our model is that in the Bjorken limit with ω large, the virtual photon behaves like a spin-$\frac{1}{2}$ particle-antiparticle pair, and the virtual particle in this pair interacts multiperipherally with the target nucleon. A novel prediction of our model is that an energetic N-$\bar{N}$ pair should be produced in deep-inelastic e-N scattering in the Bjorken limit with ω large.

In this paper, we apply this model to study deep-inelastic ν-N scattering. In Sec. II, we describe the kinematics and briefly review our model. In Sec. III, we derive our results. Section IV contains a discussion of our results and a comparison of our model with other models.

II. KINEMATICS AND REVIEW OF MODEL

If only the final lepton is observed in unpolarized ν-N scattering, then, to lowest order in the weak coupling constant G, all information is contained in $W_{\mu\nu}$ or the three structure functions W_1, W_2, and W_3. They are defined by

$$W_{\mu\nu}^{\nu,\bar{\nu}} \equiv 4\pi^2 \frac{E_p}{m} \sum_n \langle p | J_\mu^{\mp}(0) | n \rangle \langle n | J_\nu^{\pm}(0) | p \rangle (2\pi)^4 \delta^4(q+p-p_n)$$
$$= -g_{\mu\nu} W_1^{\nu,\bar{\nu}}(q^2,\nu) + \frac{p_\mu p_\nu}{m^2} W_2^{\nu,\bar{\nu}}(q^2,\nu) + i\frac{\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta}{2m^2} W_3^{\nu,\bar{\nu}}(q^2,\nu) + \cdots, \tag{2.1}$$

where $|p\rangle$ is a one-nucleon state with four-momentum p_μ and mass m, $J_\mu(x)$ is the total hadronic weak current operator, q_μ and $\nu \equiv p\cdot q/m$ are respectively the four-momentum carried by the hadronic weak current and its energy in the laboratory system. The $\cdots$ in (2.1) represent the three terms proportional to q_μ or q_ν. Since $W_{\mu\nu}$ is contracted with the lepton vertex, these three terms are negligible in the limit of neglecting the lepton masses. The upper sign and lower sign in (2.1) refer respectively to ν and $\bar{\nu}$ scattering. The nucleon spin has been averaged in the definition of $W_{\mu\nu}^{\nu,\bar{\nu}}$. The terms W_1 and W_2 are due to the parity-conserving contributions, i.e., the vector-vector and axial-vector–axial-vector contributions. The term W_3 is due to the parity-violating contribution, i.e., the vector–axial-vector interference contribution. The differential cross section in the laboratory system is given by

$$\frac{d^2\sigma^{\nu,\bar{\nu}}}{dE'd\cos\theta} = \frac{G^2}{\pi} E'^2 \left[W_2^{\nu,\bar{\nu}}(q^2,\nu)\cos^2(\tfrac{1}{2}\theta) + 2W_1^{\nu,\bar{\nu}}(q^2,\nu)\sin^2(\tfrac{1}{2}\theta) \pm W_3^{\nu,\bar{\nu}}(q^2,\nu)\left(\frac{E+E'}{m}\right)\sin^2(\tfrac{1}{2}\theta)\right], \tag{2.2}$$

where E' and θ are the final energy and the scattering angle of the lepton, and E is the energy of the incident ν (or $\bar{\nu}$), all measured in the laboratory system. The upper and lower signs in (2.2) again refer respectively to ν and $\bar{\nu}$ scattering.

Equation (2.1) can be rewritten as

$$W_{\mu\nu}^{\nu,\bar{\nu}} = 4\pi^2 \frac{E_p}{m} \int d^4x\, e^{-iq\cdot x} \langle p | [J_\mu^{\mp}(x), J_\nu^{\pm}(0)] | p \rangle . \tag{2.3}$$

From (2.3) we see that if we set the Cabibbo angle, θ_C, equal to zero, then the neutrino and antineutrino structure functions are related by

$$W_i^{\bar{\nu}p} = W_i^{\nu n}, \qquad W_i^{\bar{\nu}n} = W_i^{\nu p}, \tag{2.4}$$

where the superscripts p and n refer respectively to a proton target and a neutron target. Since experimentally θ_C is small, in the rest of the paper we work in the $\theta_C = 0$ approximation. Thus, we only have to consider neutrino scattering.

To construct a model for deep-inelastic ν-N scattering, it is sufficient to construct a model for the absorptive part of the forward elastic amplitude for the hadronic weak current scattering off a nucleon. Our model for the latter is a simple modification of the ABFST MPM. We assume that with the exception of the last link that couples directly to the current, the high-energy current-nucleon scattering amplitude is dominated by t-channel spinless-meson poles. For the last link we assume a combination of spin-0 and spin-$\frac{1}{2}$ t-channel poles. If the experimental ratio of the longitudinal and transverse photoabsorption cross sections, σ_S/σ_T, stays near its present small value[7] of 0.18 when ω gets large, then we have shown in Ref. 1 the contribution from spin-$\frac{1}{2}$ coupling (to the current) dominates over the contribution from spin-0 coupling. As we will show in Sec. III, the neutrino structure functions W_1 and W_2 are related to the corresponding electron structure function[8] by multiplicative factors; it is reasonable to assume this dominance of spin-$\frac{1}{2}$ coupling is also

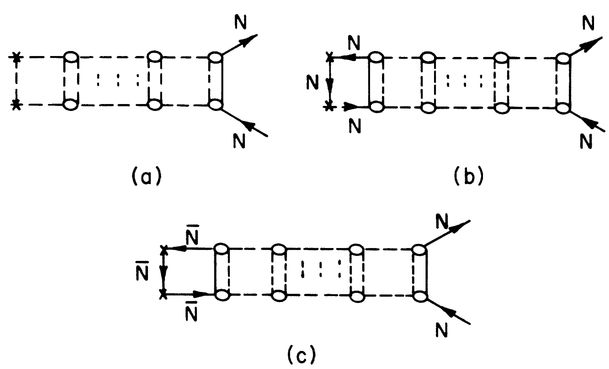

FIG. 1. Three contributions to one term in the unitarity sum. Dashed and solid lines are respectively pions and nucleons (or antinucleons). The crosses represent the hadronic weak currents.

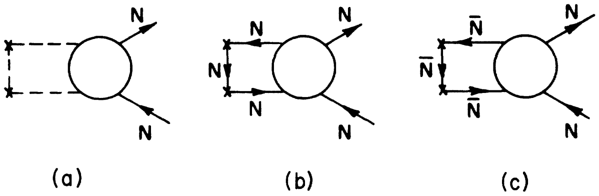

FIG. 2. Three contributions to the absorptive part of the forward elastic amplitude for the hadronic weak current scattering off a nucleon.

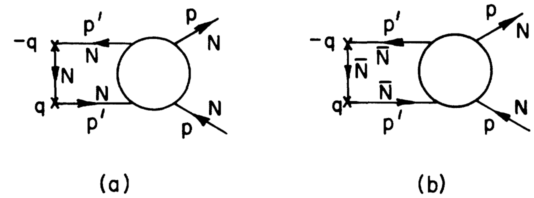

FIG. 3. Definitions of momentum variables: p, p', and q are respectively the momenta of the target nucleon, the exchanged nucleon (or antinucleon), and the hadronic weak current.

true in ν-N scattering. Since we are only interested in constructing a simple model which can explain the general features of experimental data, we do not concern ourselves here with the question of what happens when higher spins are included at the current vertex. However, since all particles which are stable under strong interaction (except for the Ω^-) have either spin 0 or spin $\frac{1}{2}$, it may be that higher-spin contributions in some average sense are already included in our model.

Therefore, for a fixed number of particles in the unitarity sum, there are three contributions, corresponding to Figs. 1(a), 1(b), and 1(c). For brevity in writing, we specifically assume that the particles coupling directly to the current are pions and nucleons (or antinucleons). In actuality, they may be members of the corresponding SU(3) octets. The absorptive part of the forward elastic amplitude for the hadronic weak current scattering off a nucleon is the sum of diagrams of Fig. 1. This sum is represented by Fig. 2. The result for the contribution from Fig. 2(a) can be directly carried over from Ref. 1. In the Bjorken limit with $\omega \equiv 2m\nu/(-q^2)$ large, the results are

$$\begin{aligned} &W_1^\pi(q^2,\nu) \approx \bar{c}\,\omega^\alpha \frac{\ln(-q^2)}{(-q^2)} \approx 0\,, \\ &\nu W_2^\pi(q^2,\nu) \approx 2m\bar{c}\,\omega^{\alpha-1}\,, \qquad (2.5) \\ &W_3^\pi(q^2,\nu) = 0\,, \end{aligned}$$

where $\bar{c}$ is a constant and the superscript π refers to the contribution from spin-0 coupling. The contributions from Figs. 2(b) and 2(c) are calculated in Sec. III.

III. DERIVATION OF RESULTS

We first calculate the contribution from Fig. 2(b). The momentum variables are defined in Fig. 3(a). The method follows closely that of Ref. 1. After averaging over the spin of the target nucleon, we can describe the forward absorptive part of the N-N amplitude by

$$[-A(p',p)+\not{p}B_1(p',p) + \not{p}'B_2(p',p)+(\not{p}'\not{p}-\not{p}\not{p}')C(p',p)]^{NN}\,.$$

The reason for the presence of the B_2 and C terms[9] is because the exchanged nucleon is off its mass shell. We then attach the additional link to the hadronic weak current by using conventional Feynman rules. The tensor $W^N_{\mu\nu}$ [superscript N refers to the contribution from Fig. 2(b) where the exchanged particle is a nucleon] is then given by

$$\begin{aligned} W^N_{\mu\nu}(q,p) &= \frac{g^2}{8\pi^3}\int d^4p' \frac{\delta((p'-q)^2-m^2)}{(p'^2-m^2)^2}\mathrm{Tr}\{[-A(p',p)+\not{p}B_1(p',p)+\not{p}'B_2(p',p)+(\not{p}'\not{p}-\not{p}\not{p}')C(p',p)]^{NN} \\ &\qquad \times(\not{p}'+m)\gamma_\nu(1-\gamma_5)(\not{p}'-\not{q}+m)\gamma_\mu(1-\gamma_5)(\not{p}'+m)\} \\ &= \frac{g^2}{8\pi^3}\int d^4p' \frac{\delta((p'-q)^2-m^2)}{(p'^2-m^2)^2}\mathrm{Tr}\{2[-A(p',p)+\not{p}B_1(p',p)+\not{p}'B_2(p',p)+(\not{p}'\not{p}-\not{p}\not{p}')C(p',p)]^{NN} \\ &\qquad \times(\not{p}'+m)\gamma_\nu(\not{p}'-\not{q})\gamma_\mu(1-\gamma_5)(\not{p}'+m)\}\,, \qquad (3.1) \end{aligned}$$

where g is the current-N-$\bar{N}$ vertex function, which has been assumed to be a constant. This is not so unreasonable an assumption, because as we will see the dominant contribution in our model comes from the region where the exchanged nucleon is very spacelike. An interesting problem is to actually construct a realistic model for the vertex function which has the property of becoming approximately a constant when the current and another leg are very spacelike.[10] The case of a constant vertex function is not the only case in which our derivation of scaling is valid. We could have described the vertex by a function $g(q,p')$ and kept g inside the integrals. At the end we could then decide for what possible forms of $g(u,u')$

($u \equiv -q^2$ and $u' \equiv -p'^2$) is our scaling result valid. In this paper, we consider only the case where g is a constant.

From Eq. (3.1) it is obvious that the $C(p',p)$ term gives zero contribution to the trace. Therefore, we can rewrite (3.1) as

$$W^N_{\mu\nu}(q,p)=\frac{g^2}{8\pi^3}\int d^4p'\,\frac{\delta((p'-q)^2-m^2)}{(p'^2-m^2)^2}\mathrm{Tr}\{2[-A(p',p)+\not p B_1(p',p)+\not p' B_2(p',p)]^{NN}$$
$$\times(\not p'+m)\gamma_\nu(\not p'-\not q)\gamma_\mu(1-\gamma_5)(\not p'+m)\}. \tag{3.2}$$

Comparing (3.2) with the analogous equation for the e-N case,[11] we can conclude that

$$W^\nu_i\;/W^e_i=2g^2/f^2\,,\quad \text{for } i=1,2 \tag{3.3}$$

where g and f are the constant vertex functions for the ν case and the e case, respectively.

Let us denote the trace in (3.2) by $T^N_{\mu\nu}(q,p',p)$. Taking the trace, we find

$$\begin{aligned}T^N_{\mu\nu}(q,p',p)=&-16mA(p',p)[-p'\cdot(p'-q)g_{\mu\nu}+2p'_\mu p'_\nu-(p'_\mu q_\nu+p'_\nu q_\mu)+i\epsilon_{\mu\nu\alpha\beta}p'^\alpha q^\beta]\\
&+8B_1(p',p)\{[(p'^2-m^2)p\cdot(p'-q)-2p'\cdot p\,p'\cdot(p'-q)]g_{\mu\nu}+2p'\cdot p[2p'_\mu p'_\nu-(p'_\mu q_\nu+p'_\nu q_\mu)]\\
&\qquad-(p'^2-m^2)[p_\mu(p'-q)_\nu+p_\nu(p'-q)_\mu]\\
&\qquad-i\epsilon_{\mu\nu\alpha\beta}[m^2p^\alpha(p'-q)^\beta+p'\cdot(p'-q)p'^\alpha p^\beta-p'\cdot p\,p'^\alpha q^\beta]\\
&\qquad+i\epsilon_{\nu\gamma\alpha\beta}p^\alpha q^\beta p'^\gamma p'_\mu-i\epsilon_{\mu\gamma\alpha\beta}p^\alpha q^\beta p'^\gamma p'_\nu\}\\
&+8B_2(p',p)\{-(p'^2+m^2)p'\cdot(p'-q)g_{\mu\nu}+(p'^2+m^2)[2p'_\mu p'_\nu-(p'_\mu q_\nu+p'_\nu q_\mu)]+i\epsilon_{\mu\nu\alpha\beta}p'^\alpha q^\beta(p'^2+m^2)\}.\end{aligned} \tag{3.4}$$

Equation (3.4) is exact. The reason for writing it out in totality is to facilitate checking of our derivation. After dropping terms which are obviously proportional to q_μ or q_ν, we get

$$\begin{aligned}T^N_{\mu\nu}(q,p',p)\equiv&-A(p',p)T^A_{\mu\nu}(q,p',p)+B_1(p',p)T^{B_1}_{\mu\nu}(q,p',p)+B_2(p',p)T^{B_2}_{\mu\nu}(q,p',p)\\
\approx&-16mA(p',p)[-p'\cdot(p'-q)g_{\mu\nu}+2p'_\mu p'_\nu+i\epsilon_{\mu\nu\alpha\beta}p'^\alpha q^\beta]\\
&+8B_1(p',p)\{[(p'^2-m^2)p\cdot(p'-q)-2p'\cdot p\,p'\cdot(p'-q)]g_{\mu\nu}+4p'\cdot p\,p'_\mu p'_\nu-(p'^2-m^2)(p_\mu p'_\nu+p_\nu p'_\mu)\\
&\qquad-i\epsilon_{\mu\nu\alpha\beta}[m^2p^\alpha(p'-q)^\beta+p'\cdot(p'-q)\,p'^\alpha p^\beta-p'\cdot p\,p'^\alpha q^\beta]\\
&\qquad+i\epsilon_{\nu\gamma\alpha\beta}p^\alpha q^\beta p'^\gamma p'_\mu-i\epsilon_{\mu\gamma\alpha\beta}p^\alpha q^\beta p'^\gamma p'_\nu\}\\
&+8B_2(p',p)\{-(p'^2+m^2)p'\cdot(p'-q)g_{\mu\nu}+2(p'^2+m^2)p'_\mu p'_\nu+i\epsilon_{\mu\nu\alpha\beta}p'^\alpha q^\beta(p'^2+m^2)\}.\end{aligned} \tag{3.5}$$

We now change to invariant variables by defining $W^N_{\mu\nu}(s,u)$ by

$$W^N_{\mu\nu}(q,p)=\iint ds\,du\,W^N_{\mu\nu}(s,u)\delta((p+q)^2-s)\delta(u+q^2)\,. \tag{3.6}$$

Similar definitions are defined for the A and B terms. We then get

$$W^N_{\mu\nu}(s,u)=\frac{g^2}{8\pi^3}\iint\frac{ds'du'}{(u'+m^2)^2}[-A(s',u')Q^A_{\mu\nu}+B_1(s',u')Q^{B_1}_{\mu\nu}+B_2(s',u')Q^{B_2}_{\mu\nu}]\,, \tag{3.7}$$

where

$$Q^{A,B_1,B_2}_{\mu\nu}(s,u;s',u';m^2)\equiv\int d^4p'\,T^{A,B_1,B_2}_{\mu\nu}(q,p',p)\delta((p'-q)^2-m^2)\delta((p+p')^2-s')\delta(u'+p'^2)\,. \tag{3.8}$$

The next step is to calculate $Q^{A,B_1,B_2}_{\mu\nu}$. We note that

$$\begin{aligned}&\int d^4p'\,p'_\mu p'_\nu\,\delta((p'-q)^2-m^2)\delta((p+p')^2-s')\delta(u'+p'^2)=e_1g_{\mu\nu}+e_2p_\mu p_\nu+\cdots\,,\\
&\int d^4p'\,p'_\mu\,\delta((p'-q)^2-m^2)\delta((p+p')^2-s')\delta(u'+p'^2)=e'_1p_\mu+e'_2q_\mu\,,\end{aligned} \tag{3.9}$$

where $\cdots$ represent terms proportional to q_μ or q_ν. To leading order in the limit

$$s \to \infty\,, \quad u \to \infty\,, \quad \text{with } \overline{\omega} \equiv s/u \text{ fixed but large}\,, \tag{3.10}$$

we find

$$\begin{aligned} e_1 &\approx \tfrac{1}{2}[u(s'/s) - u']Q_{\text{ABFST}}\,, \\ e_2 &\approx (u^2/s^2)Q_{\text{ABFST}}\,, \\ e_1' &\approx -(u/s)Q_{\text{ABFST}}\,, \\ e_2' &\approx (s'/s)Q_{\text{ABFST}}\,, \end{aligned} \tag{3.11}$$

where Q_{ABFST} is the ABFST boundary function[2] defined by

$$Q_{\text{ABFST}}(s,u;s',u';m^2) = \int d^4p'\, \delta((p'-q)^2 - m^2)\delta((p+p')^2 - s')\delta(u' + p'^2)\,. \tag{3.12}$$

Substituting (3.5) into (3.8) and the result into (3.7), and using (3.9) and (3.11), we find to leading order in the limit (3.10)

$$\begin{aligned} W^N_{\mu\nu}(s,u) \approx \frac{g^2}{8\pi^3}\iint \frac{ds'\,du'}{(u'+m^2)^2} Q_{\text{ABFST}}(s,u;s',u';m^2) \\ \times\Bigg[-g_{\mu\nu}[-8muA(s',u') + 4(su'-us')B_1(s',u') + 4uu'B_2(s',u')] \\ + \frac{p_\mu p_\nu}{m^2}\left(-32m^3\frac{u^2}{s^2}A(s',u') + 16\frac{m^2u}{s^2}(su'-us')B_1(s',u') + 16m^2\frac{u^2u'}{s^2}B_2(s',u')\right) \\ + \frac{i\epsilon_{\mu\nu\alpha\beta}p^\alpha q^\beta}{2m^2}\left(32\frac{m^3u}{s}A(s',u') - \frac{16m^2}{s}(su'-us')B_1(s',u') - 16m^2\frac{uu'}{s}B_2(s',u')\right)\Bigg]\,. \end{aligned} \tag{3.13}$$

We now use the information from the ABFST MPM. This is regarding the asymptotic behavior of $A(s',u')$, $B_1(s',u')$, and $B_2(s',u')$. From Ref. 1, we know that for s' large

$$\begin{aligned} A(s',u') &\approx s'^{\alpha}\phi^A_\alpha(u')\,, \\ B_1(s',u') &\approx s'^{\alpha-1}\phi^{B_1}_\alpha(u')\,, \\ B_2(s',u') &\approx s'^{\alpha}\phi^{B_2}_\alpha(u')\,, \end{aligned} \tag{3.14}$$

where for u' large

$$\begin{aligned} \phi^A_\alpha(u') &\approx c_A/(u')^{\alpha+1}\,, \\ \phi^{B_1}_\alpha(u') &\approx c_{B_1}/(u')^{\alpha}\,, \\ \phi^{B_2}_\alpha(u') &\approx c_{B_2}/(u')^{\alpha+1}\,. \end{aligned} \tag{3.15}$$

Using (3.14) and (3.15) to do the s' and u' integrals of (3.13), we can obtain the three structure functions by comparing (3.13) with (2.1). Here we explicitly work out the integration only for the B_1 term. Since (3.13) shows that the three structure functions are related to each other by multiplicative factors, it suffices to calculate $W_1^{B_1}(s,u)$. Thus

$$W_1^{B_1}(s,u) = \frac{4g^2}{8\pi^3}\int\int \frac{ds'\,du'}{(u'+m^2)^2} Q_{\text{ABFST}}(s,u;s',u';m^2)(su'-us')B_1(s',u'). \tag{3.16}$$

Substituting (3.14) into (3.16), we find in the limit (3.10)

$$W_1^{B_1}(s,u) \approx \frac{4g^2s}{8\pi^3}\int_0^s \frac{ds'}{s}\int_{\frac{s'}{s}\left(u+\frac{m^2}{1-s'/s}\right)}^{\infty} du'\,\frac{(u'-us'/s)s'^{\alpha-1}}{(u'+m^2)^2}\phi^{B_1}_\alpha(u')\,. \tag{3.17}$$

Following the method of Ref. 1, we let $x \equiv s'/s$ and interchange variables of integration to get

$$W_1^{B_1}(s,u)=\frac{4g^2}{8\pi^3}s^\alpha\int_0^\infty du'\frac{\phi_\alpha^{B_1}(u')}{(u'+m^2)^2}$$
$$\times\left(\frac{u'\{(u+u'+m^2)-[(u+u'+m^2)^2-4uu']^{1/2}\}^\alpha}{\alpha(2u)^\alpha}-\frac{u\{(u+u'+m^2)-[(u+u'+m^2)^2-4uu']^{1/2}\}^{\alpha+1}}{(\alpha+1)(2u)^{\alpha+1}}\right). \tag{3.18}$$

Substituting (3.15) into (3.18) and choosing b such that for $u'>b$ Eq. (3.15) is valid, we find

$$W_1^{B_1}(s,u)\approx\frac{4g^2}{8\pi^3}\left(\frac{s}{u}\right)^\alpha\left[\left(\frac{1}{\alpha}-\frac{1}{\alpha+1}\right)\left(\int_0^b du'\frac{u'^{\alpha+1}\phi_\alpha^{B_1}(u')}{(u'+m^2)^2}+c_{B_1}\int_b^u\frac{du'}{u'}\right)+c_{B_1}\left(\frac{u^\alpha}{\alpha}\int_u^\infty\frac{du'}{u'^{\alpha+1}}-\frac{u^{\alpha+1}}{\alpha+1}\int_u^\infty\frac{du'}{u'^{\alpha+2}}\right)\right]. \tag{3.19}$$

Equation (3.19) implies that

$$W_1^{B_1}(s,u)\sim\overline{\omega}^\alpha\ln u\,. \tag{3.20}$$

This means that the asymptotic off-shell behavior of the ABFST MPM as given by (3.15) requires only an infinitesimal damping factor to give rise to scaling. Note that we did not have to know how (3.14) and (3.15) are derived. Once (3.14) and (3.15) are true, our derivation follows. Also, if a cutoff for u' is introduced, then our derivation only requires (3.14). This substantiates the statements made in the first part of the Introduction about the generality of our derivation.

Similar results are obtained for the A and B_2 terms, except that no damping factor is necessary for the A term. Therefore, in the limit (3.10) we can write our result as

$$\begin{aligned}W_1^N(s,u)&=c_1^N\overline{\omega}^{\alpha_1}\,,\\ sW_2^N(s,u)&=4m^2c_2^N\overline{\omega}^{\alpha_2-1}\,,\\ sW_3^N(s,u)&=-4m^2c_3^N\overline{\omega}^{\alpha_3}\,,\end{aligned} \tag{3.21}$$

where the subscripts in c^N and α denote the leading trajectory which can be exchanged. As mentioned earlier, W_3 is the vector–axial-vector current interference term. Since the vector current resembles the ρ meson in its internal quantum numbers and the axial-vector current resembles the A_1, the leading trajectory contributing to W_3 is the ω trajectory.[12] The leading trajectory contributing to W_1 and W_2 is the Pomeranchuk trajectory. Equation (3.21) thus becomes

$$\begin{aligned}W_1^N(s,u)&=c_P^N\overline{\omega}^{\alpha_P}\,,\\ sW_2^N(s,u)&=4m^2c_P^N\overline{\omega}^{\alpha_P-1}\,,\\ sW_3^N(s,u)&=4m^2c_\omega^N\overline{\omega}^{\alpha_\omega}\,.\end{aligned} \tag{3.22}$$

The sign change for W_3 from (3.21) to (3.22) results from the fact that the ω contribution to pp or pn scattering is negative[13] and we have adopted the convention that c_ω^N in (3.22) is positive.

We now can quickly calculate the contribution from Fig. 2(c). The momentum variables are defined in Fig. 3(b). The equation analogous to (3.2) is

$$W_{\mu\nu}^{\overline{N}}(q,p)=\frac{g^2}{8\pi^3}\int d^4p'\frac{\delta((p'-q)^2-m^2)}{(p'^2-m^2)^2}$$
$$\times\mathrm{Tr}\{2[A(p',p)+\not{p}B_1(p',p)+\not{p}'B_2(p',p)]^{\overline{N}N}$$
$$\times(\not{p}'-m)\gamma_\mu(\not{p}'-\not{q})\gamma_\nu(1-\gamma_5)(\not{p}'-m)\}. \tag{3.23}$$

Note that in going from N-N scattering to $\overline{N}$-N scattering, there is a sign change for the A term in the amplitude that is sandwiched between spinors. This sign change just compensates for the sign change of the m terms due to the change from using u spinors to v spinors (note that the A contribution comes from terms linear in m). The switch in places of the μ and ν indices mean that there should be a sign change for W_3, but not for W_1 and W_2. There is however an additional change in going from Fig. 2(b) to Fig. 2(c), as illustrated in Figs. 4(a) and 4(b). In the t channel, one obtains the quantum numbers of Fig. 4(b) from Fig. 4(a) by rotation in isospin space followed by charge conjugation, i.e., trajectories which have odd $C(-1)^I$ (such as the ω-trajectory) change sign. This means W_3 changes sign, but not W_1 and W_2. Combining the two effects, we see that we get the same relative signs[14] as in Eq. (3.22). Of course, the magnitude of the constants may be different due to the difference between the magnitudes of the residues of $\overline{N}$-N scattering and N-N scattering.

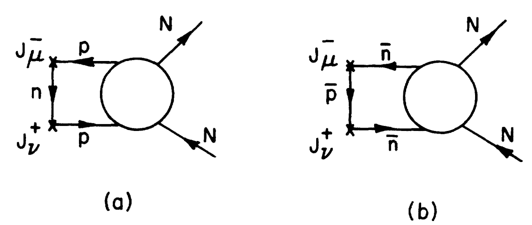

FIG. 4. The allowed quantum numbers corresponding to Figs. 2(b) and 2(c).

Remembering that $\bar{\omega} \approx \omega$ for large ω, we can write our final result for the structure functions in the limit of large ν and q^2 with ω fixed but large:

$$\begin{aligned} W_1(\nu, q^2) &= c_P \omega^{\alpha_P}, \\ \nu W_2(\nu, q^2) &= 2m c_P \omega^{\alpha_P - 1}, \qquad (3.24) \\ \nu W_3(\nu, q^2) &= 2m c_\omega \omega^{\alpha_\omega - 1}. \end{aligned}$$

We now derive a relationship between the neutrino and antineutrino differential cross sections. Let us define $F_2(\omega) \equiv \nu W_2$. Then $mW_1 = \frac{1}{2}\omega F_2(\omega)$ and $\nu W_3 = a\omega^{1+\Delta\alpha} F_2(\omega)$, where $a \equiv c_\omega / c_P$ and $\Delta\alpha \equiv \alpha_\omega - \alpha_P \approx -0.6$. Substituting these into (2.2), we get

$$\frac{d^2\sigma^{\nu,\bar\nu}}{dE' d\cos\theta} = \frac{G^2 E'^2}{\pi\nu} F_2^{\nu,\bar\nu}(\omega) \times \left\{ \cos^2(\tfrac{1}{2}\theta) + \frac{\omega\nu}{m}\left[1 \pm a\omega^{\Delta\alpha}\left(\frac{2E-\nu}{\nu}\right)\right]\sin^2(\tfrac{1}{2}\theta) \right\}. \qquad (3.25)$$

Changing variables to $y \equiv \nu/E$ and $1/\omega$, and integrating over y, we get

$$\frac{d\sigma^{\nu,\bar\nu}}{d(1/\omega)} \approx \frac{G^2}{\pi}(mE) F_2^{\nu,\bar\nu}(\omega)(\tfrac{2}{3} \pm \tfrac{1}{3} a\omega^{\Delta\alpha}). \qquad (3.26)$$

As first pointed out by Bjorken,[15] this linear rise with energy of the ν (or $\bar\nu$) cross section is a consequence of scaling of the structure functions. When we make use of Eq. (2.4), this leads to the prediction that

$$\left[\frac{d\sigma^{\nu p}}{d(1/\omega)} + \frac{d\sigma^{\nu n}}{d(1/\omega)}\right] - \left[\frac{d\sigma^{\bar\nu p}}{d(1/\omega)} + \frac{d\sigma^{\bar\nu n}}{d(1/\omega)}\right] = \frac{G^2 mE}{\pi}\left[F_2^{\nu p}(\omega) + F_2^{\nu n}(\omega)\right](\tfrac{2}{3} a\omega^{\Delta\alpha}). \qquad (3.27)$$

Therefore, we predict

$$\frac{d\sigma^{\nu p}}{d(1/\omega)} + \frac{d\sigma^{\nu n}}{d(1/\omega)} > \frac{d\sigma^{\bar\nu p}}{d(1/\omega)} + \frac{d\sigma^{\bar\nu n}}{d(1/\omega)} \qquad (3.28\text{a})$$

and

$$\frac{d\sigma^{\nu p}}{d(1/\omega)} + \frac{d\sigma^{\nu n}}{d(1/\omega)} \underset{\omega\to\infty}{\sim} \frac{d\sigma^{\bar\nu p}}{d(1/\omega)} + \frac{d\sigma^{\bar\nu n}}{d(1/\omega)}. \qquad (3.28\text{b})$$

Our model also predicts that

$$\sigma^{\nu n} > \sigma^{\nu p}. \qquad (3.29)$$

This follows when we consider the relative sign of various Regge contributions for the two cases of a proton target and a neutron target. Symbolically, when the target is a proton,

$$\begin{aligned} W_1 &= P - \rho, \\ \nu W_2 &= P - \rho, \\ \nu W_3 &= \omega - A_2. \end{aligned}$$

When the target is a neutron

$$\begin{aligned} W_1 &= P + \rho, \\ \nu W_2 &= P + \rho, \\ \nu W_3 &= \omega + A_2. \end{aligned}$$

As in the case of e-N scattering,[1] our model predicts that the average multiplicity of final state hadrons is $\langle n\rangle \sim \ln\omega$.[16] This follows from (3.24). This logarithmic growth in ω of the multiplicity is consistent with the fact that our derivation requires ω to be large. Large ω means a long multiperipheral chain and so justifies the use of the integral equation approach.

IV. DISCUSSION AND COMPARISON WITH OTHER MODELS

In comparing our theoretical predictions with present experimental data, we should keep in mind that present experimental data are too preliminary and too meager to enable us to differentiate various theoretical models. Furthermore, present data have not reached the large-ω region. Nevertheless, present data[17] are consistent with our predictions. The data show that in the incident energy range 2–14 GeV, the neutrino total cross section rises linearly with E, giving support to Eq. (3.26) and the scaling result of (3.24). The data are compatible with $2mW_1 = \omega\nu W_2$, in agreement with (3.24). The data suggest

$$\sigma^{\nu p} + \sigma^{\nu n} > \sigma^{\bar\nu p} + \sigma^{\bar\nu n},$$

consistent with (3.28a). The data also require $\sigma^{\nu n} > \sigma^{\nu p}$, in agreement with (3.29).

There are two distinctive predictions of our model which future experiments at large values of ω will either verify or reject. One is that the differential cross section as a function of $1/\omega$ for antineutrinos should approach that for neutrinos, and the rate of approach is given by Eq. (3.27). The other is that an energetic N-$\bar N$ pair[18, 19] should be produced near the direction of the spatial part of the momentum carried by the current. If such an energetic N-$\bar N$ pair is not observed, then either our model is not valid or σ_S/σ_T increases from its present small value so that spin-$\frac{1}{2}$ coupling does not dominate over spin-0 coupling.

The extra damping factor that is required in our derivation of scaling is not necessarily an *ad hoc* input. The necessary damping may come from the vertex function's deviation from a constant.

Another possibility for generating the necessary damping is to modify the original ABFST formulation by including off-shell dependences for the low-energy kernel.

In Secs. I and III we have already made several comparisons of our model and the DLY model[6]. We now mention the other differences. Because within their model the dominant diagram is the nucleon-exchange ladder, they were allowed not to identify their output trajectories with ordinary hadronic trajectories. That is why the same c and α appear in all three equations of (3.24). Thus, they predict

$$\frac{d\sigma^{\nu p}}{d(1/\omega)}+\frac{d\sigma^{\nu n}}{d(1/\omega)}=3\left[\frac{d\sigma^{\bar{\nu} p}}{d(1/\omega)}+\frac{d\sigma^{\bar{\nu} n}}{d(1/\omega)}\right],$$

corresponding to $a=1$ and $\Delta\alpha=0$ in our Eq. (3.26). In DLY's field-theory derivation, to each order only the leading term is kept in the leading-logarithm approximation; they had to assume that the sum of these leading terms is the dominant contribution. On the other hand, in the multiperipheral integral equation approach, one is certain that the neglected terms do not contribute to the leading contribution. In this respect, the multiperipheral derivation is more rigorous. Note that once a damping factor is introduced, the dominant contribution comes from a region of finite u'. This allows all other momentum transfers to be small, consistent with the notion of peripheralism and the dominance of meson exchange.

In one respect, our model is similar to the parton model.[20] Both models assume that the interaction of the hadronic weak current (or hadronic electromagnetic current in the e-N case) is pointlike. However, the way scaling comes about is different. In the parton model, scaling comes from energy-momentum conservation after assuming that the partons are on the mass shell both before and after the interaction with the current. In our model, scaling comes from Regge behavior and asymptotic off-shell dependence. In the parton model, either there should be partons in the final state (partons generally are assumed not to be ordinary particles; they may be quarks or gluons) or there is final-state interaction (which then destroys the simplicity of the parton model). On the other hand, our model only involves ordinary hadrons.

Besides using somewhat different methods of derivation, our multiperipheral model has several differences from the duality-parton model of Landshoff and Polkinghorne.[21] The multiperipheral model gives a definite off-shell behavior as shown in Eq. (3.15), whereas Landshoff and Polkinghorne just assumed that their quark-hadron amplitude goes to zero rapidly as the virtual quark mass becomes large. The multiperipheral model treats all trajectories on the same footing. The duality-parton model assumes that there are two separate parts: the contribution from resonances (which they calculate explicitly using a Veneziano-like model) and the contribution from the exotic states (corresponding to the Pomeranchuk contribution which they cannot calculate explicitly). Our currents couple to ordinary mesons and nucleons. Their currents couple to quarks, so their model either produces quarks or needs final-state interaction.[22] Furthermore, Figs. 2(b) and 2(c) give rise to comparable contributions in our model. In their model, Fig. 2(b) with the exchanged nucleon replaced by a quark is exotic and corresponds to the diffractive contribution; Fig. 2(c) with $\bar{N}$ replaced by an antiquark is nonexotic and corresponds to the nondiffractive contribution. The two contributions are not comparable in their model.

We conclude with the following remark. The natural scaling variable that arises from our model is the variable $\bar{\omega}\equiv(p+q)^2/(-q^2)$ [see, for example, Eq. (3.22)]. This variable is simply related to the variable ω' introduced by Bloom and Gilman[23] to illustrate more explicitly the concept of duality in lepton-nucleon scattering. The relation is $\bar{\omega}=\omega'-1$. Strictly speaking, our model is valid only in the large $\bar{\omega}$ region. However, because of duality it may be possible that our model is also applicable in some average sense in a region of smaller $\bar{\omega}$.

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[8]Strictly speaking, W_1^ν and W_2^ν are related to the isovector parts of W_1^e and W_2^e.

[9]We thank Professor M. Goldberger for pointing out the presence of the C term. However, as is shown in the text, this C term gives zero contribution to $W_{\mu\nu}$. It can also be shown that in the case of e-N scattering, this C term in the Bjorken limit does not contribute to the leading contribution.

[10]In Ref. 1 we presented a plausibility argument for this assumption when the current is electromagnetic; we have not yet been able to extend this plausibility argument to the case of a hadronic weak current.

[11]See Eq. (3.12) in the second paper of Ref. 1. Note that the m term in the factor $(\not p' - \not q + m)$ in Eq. (3.12) does not contribute to the leading contribution. There was an arithmetic error in writing down Eq. (3.12): The constant factor before the integral should be $f^2/8\pi^3$, instead of $f^2/16m\pi^5$.

[12]Of course, to generate the ω trajectory with the ABFST MPM for the hadronic absorptive part, one must include the exchanges of the pions' SU(3) partners.

[13]See, e.g., P. D. B. Collins and E. J. Squires, *Regge Poles in Particle Physics* (Springer, Berlin, 1968), p. 207.

[14]In the Drell-Levy-Yan model, the relative sign between W_1 and W_3 is negative for Fig. 2(b) and positive for Fig. 2(c). Since within their model of pseudoscalar-meson–nucleon coupling the latter contribution dominates, the relative sign for the sum is still positive. The reason their relative sign for Fig. 2(b) differs from ours is because they did not associate their output trajectories with ordinary hadronic trajectories. Their result corresponds to our Eq. (3.21), instead of Eq. (3.22).

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