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# Dual Absorptive Model and Structure in Inelastic Reactions*

Min-Shih Chen and Frank E. Paige
*Brookhaven National Laboratory, Upton, New York 11973*
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The implications of a model for two-body scattering amplitudes combining duality and strong absorption are investigated for inelastic reactions. At $t=t_c$, where the imaginary part of the helicity-nonflip amplitude vanishes in the model, predictions about the real part can be made via fixed-$t$ dispersion relations. Strong results can be thus obtained for amplitudes which are antisymmetric under $s \leftrightarrow u$ crossing. As examples, definite predictions on $\pi^- p \to \pi^0 n$ polarization and $K_L^0 p \to K_S^0 p$ differential cross section at $t=t_c$ are made. The general case without the $s \leftrightarrow u$ antisymmetry is also discussed.

A model for two-body scattering amplitudes combining duality with strong absorption has recently been proposed.[1-3] Exclusive of the Pomeranchukon contribution, the $s$-channel helicity amplitudes[4] in this model satisfy

$$\begin{aligned} &\operatorname{Im} f^s_{\lambda_3\lambda_4;\lambda_1\lambda_2}(s,t) \propto J_{|\Delta\lambda|}(R(-t)^{1/2}),\\ &\Delta\lambda = (\lambda_3-\lambda_4)-(\lambda_1-\lambda_2), \end{aligned} \tag{1}$$

where $J_n$ is a Bessel function of order $n$ and $R$ is a characteristic radius. The proportionality sign is used in (1) to indicate that additional smooth dependence on $s$ and $t$ is allowed: The Bessel function specifies mainly the location of the zeros. This specification agrees with the Regge-pole model for $\Delta\lambda=1$, so for such amplitudes the Regge energy behavior and phase are assumed, while for $\Delta\lambda \neq 1$ these are not specified.

It has been shown[3] that if $R\approx 1$ fm, then the above assumptions can account for the presence or absence of dips at $t\approx -0.6$ GeV$^2$ in many reactions. For the same $R$ it is predicted that $\operatorname{Im} f^s_{\Delta\lambda=0}=0$ at $t=t_c\approx -0.15$ GeV$^2$, which accounts naturally for the crossover phenomena in elastic reactions.[1,2] In what follows, the implications of such a zero for inelastic reactions will be examined.

To say anything about an inelastic cross section or polarization it is necessary to know the real as well as the imaginary parts of the amplitudes. Except for the $\Delta\lambda=1$ amplitudes having Regge behavior these real parts are not specified by the model. It appears, however, that the crossover points remain essentially fixed as the energy is varied, and so do such structures as the dip location in $\pi^- p \to \pi^0 n$. This implies that the radius $R$ in (1) is constant; therefore

$$\operatorname{Im} f^s_{\Delta\lambda=0}(s,t_c)=0 \quad \text{for all } s\,. \tag{2}$$

Fixed-$t$ dispersion relations can then give information about $\operatorname{Re} f^s_{\Delta\lambda=0}$ at $t=t_c$. The results should remain approximately valid if the crossover zero does move slightly with energy.

Such an analysis gives the strongest results for amplitudes which are odd under $s \leftrightarrow u$ crossing. In what follows, two examples will be considered: $\pi^- p\to\pi^0 n$ and $K_L^0 p \to K_S^0 p$. The polarization in $\pi^- p \to \pi^0 n$ is predicted to behave like const/$s$ at $t=t_c$, with the constant being calculated. The $K_L^0 p\to K_S^0 p$ cross section is predicted to exhibit structure at the same point. Amplitudes with no definite $s\leftrightarrow u$ symmetry are also discussed.

## $\pi^- p \to \pi^0 n$

In the reaction $\pi^- p\to \pi^0 n$ only $\rho$ exchange[5] is allowed. The observed cross section is well explained by assuming that the $f^s_{+-}$ amplitude dominates and has Regge behavior, with $R$ in Eq. (1) being about 1 fm.[3] Hence $t_c$ in Eq. (2) is expected to be about $-0.15$ GeV$^2$. Since the $f^s_{++}$ amplitude does not have the Regge phase due to absorption, some polarization is expected. To calculate the polarization at $t_c$, recall that

$$\begin{aligned} f^s_{++}(s,t) &= -\left(\frac{1+z_s}{2}\right)^{1/2}\left[A^-(\nu,t)+\frac{\nu-t}{4m}B^-(\nu,t)\right],\\ f^s_{+-}(s,t) &= -\left(\frac{1-z_s}{2}\right)^{1/2}\left(\frac{m^2}{4s}\right)^{1/2}\\ &\quad\times\left[\frac{\nu-t+4m^2}{2m^2}A^-(\nu,t)+\frac{\nu-t+4\mu^2}{2m}B^-(\nu,t)\right], \end{aligned} \tag{3}$$

where $\nu=s-u$, $z_s$ is the cosine of the scattering angle, and $A^-(\nu,t)$ and $B^-(\nu,t)$ satisfy

$$\begin{aligned} A^-(\nu,t)&=\frac{2\nu}{\pi}\int_{\nu_0}^{\infty} d\nu'\,\frac{1}{\nu'^2-\nu^2}\operatorname{Im}A^-(\nu',t),\\ B^-(\nu,t)&=\frac{2}{\pi}\int_{\nu_0}^{\infty} d\nu'\,\frac{\nu'}{\nu'^2-\nu^2}\operatorname{Im}B^-(\nu',t)\,. \end{aligned} \tag{4}$$

From $\operatorname{Im} f^s_{++}(s,t_c)=0$ it follows that

$$f^s_{++}(s,t_c)=\frac{2}{\pi}\,\frac{t_c}{4m}\left(\frac{1+z_s}{2}\right)^{1/2}$$

$$\times \int_{\nu_0}^{\infty} d\nu' \frac{1}{\nu'+\nu} \operatorname{Im} B^-(\nu', t_c), \quad \nu > 0. \tag{5}$$

To calculate the integral, take

$$f_{+-}^s(s,t) = (-t)^{1/2}\beta(t)\{\tan[\tfrac{1}{2}\pi\alpha(t)] + i\}\nu^{\alpha(t)}, \tag{6}$$

where $\alpha(t)$ is the effective $\rho$ trajectory; the form of $\beta(t)$ is irrelevant. Duality insures that this will give a good approximation to even the small-$\nu'$ part of the integration.[6] It yields

$$f_{++}^s(s,t_c) \sim \frac{4}{\sin[\pi\alpha(t_c)]} m t_c \beta(t_c) \nu^{\alpha(t_c)-1}. \tag{7}$$

Then since $|f_{++}^s|^2 \ll |f_{+-}^s|^2$, the polarization is

$$P(s,t_c) = \frac{2\operatorname{Im}(f_{++}^s f_{+-}^{s*})}{|f_{++}^s|^2 + |f_{+-}^s|^2}$$

$$\approx \frac{1}{\sin[\pi\alpha(t_c)]\{1+\tan^2[\tfrac{1}{2}\pi\alpha(t_c)]\}} \frac{8m(-t_c)^{1/2}}{\nu}, \tag{8}$$

which decreases like $s^{-1}$ as stated above. Taking $t_c = -0.15$ GeV$^2$ and $\alpha(t) = 0.58 + 1.03t$,[7] yields

$$P(s,t_c) = 8.2\% \text{ at } p_{\text{lab}} = 5.9 \text{ GeV}/c$$
$$= 4.3\% \text{ at } p_{\text{lab}} = 11.2 \text{ GeV}/c. \tag{9}$$

This is somewhat smaller than the measured values,[8] but the error bars are large and the theoretical result is dependent on the assumed value of $t_c$.

### $K_L^0 p \to K_S^0 p$

In the reaction $K_L^0 p \to K_S^0 p$ both $\omega$ and $\rho$ exchange are allowed,[9] the former contributing to $f_{++}^s$ and the latter mainly to $f_{+-}^s$.[10] In the approximation of $CP$ invariance

$$\langle K_S^0 p|S|K_L^0 p\rangle = \tfrac{1}{2}\langle K^0 p|S|K^0 p\rangle - \tfrac{1}{2}\langle \overline{K}{}^0 p|S|\overline{K}{}^0 p\rangle; \tag{10}$$

that is, the amplitudes are odd under $s$-$u$ crossing. As in the $\pi^- p \to \pi^0 n$ case, it follows that if $\operatorname{Im} f_{++}^s(s,t_c) = 0$, then $\operatorname{Re} f_{++}^s(s,t_c) \sim s^{\alpha-1}$, so that $|f_{++}^s|^2$ is negligible.

To see whether this result predicts a dip in the $K_L^0 p \to K_S^0 p$ cross section, one must estimate the relative magnitude of $|f_{++}^s|^2$ and $|f_{+-}^s|^2$ near $t_c$. To do this, consider the closely related reaction $K^- p \to K^- p$. The imaginary part of the $\omega + f^0$ contribution to its $f_{++}^s$ can be extracted from the difference of the $K^\pm p$ elastic cross sections.[2] From isospin invariance the $\rho + A_2$ contribution, which is mainly to its $f_{+-}^s$ amplitude, is given by $\frac{1}{4}d\sigma(K^- p \to \overline{K}{}^0 n)/dt$.[11] The sum of these contributions, shown in Fig. 1, evidently exhibits a shoulder but no real dip. Because $(\omega, f^0)$ and $(\rho, A_2)$ are exchange degenerate, the structure in the

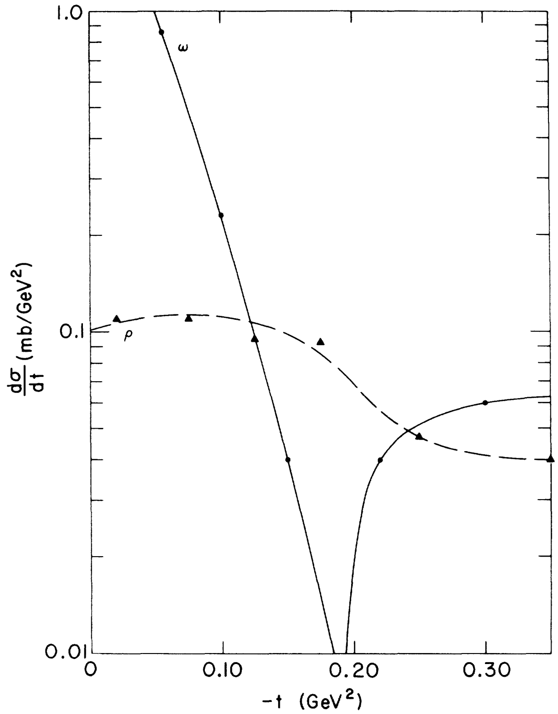


FIG. 1. Expected contributions of $\omega$ and $\rho$ trajectories to the $K_L^0 p \to K_S^0 p$ cross section. See text for explanation.

$K_L^0 p \to K_S^0 p$ cross section should be similar, there being no additional Clebsch-Gordan coefficients.

The Regge-pole model, however, is expected to be valid for $\rho$ and $A_2$ exchange, since they couple mostly to $\Delta\lambda = 1$. Their trajectories and residue functions should be exchange degenerate, so the $\rho$ contribution to $K_L^0 p \to K_S^0 p$ can be obtained from the $K^- p \to \overline{K}{}^0 n$ cross section. This contribution is

$$\tfrac{1}{4}\sin^2[\tfrac{1}{2}\pi\alpha_\rho(t)] \frac{d\sigma}{dt}(K^- p \to \overline{K}{}^0 n), \tag{11}$$

where the factor of $\frac{1}{4}$ comes from Clebsch-Gordan coefficients, and the $\sin^2[\frac{1}{2}\pi\alpha_\rho(t)]$ from the square of the signature factor. Therefore

$$\frac{d\sigma}{dt}(K_L^0 p \to K_S^0 p) - \tfrac{1}{4}\sin^2[\tfrac{1}{2}\pi\alpha_\rho(t)] \frac{d\sigma}{dt}(K^- p \to \overline{K}{}^0 n) \tag{12}$$

should be given essentially by the $\Delta\lambda = 0$ amplitude and should exhibit a dip at $t \approx -0.15$ GeV$^2$.

The $K_L^0 p \to K_S^0 p$ data[12] are not yet good enough to test this result: The statistics must be improved and the $t$ resolution improved to about 0.05 GeV$^2$.

## GENERAL CASE

The results which can be derived for a general reaction not allowing Pomeranchukon exchange are much weaker than those for reactions having $s$-$u$ antisymmetry. It is first necessary to assume that the absorption radii are the same in the $s$ and $u$ channels, so that

$$\mathrm{Im} f^s_{\Delta\lambda=0}(s, t_c) = \mathrm{Im} f^u_{\Delta\lambda=0}(s, t_c) = 0\,. \tag{13}$$

This is not unreasonable: For example, a variety of unrelated reactions exhibit dips at $t \approx -0.6$ GeV$^2$. The kinematic singularities having been divided out, a dispersion relation at fixed $t = t_c$ can then be written for any of the $s$-channel amplitudes with $\Delta\lambda = 0$. The right-hand cut will vanish, but the left-hand cut will receive contributions from $u$-channel amplitudes with $\Delta\lambda \neq 0$.

As $s \to \pm\infty$ with $t$ fixed, the $s$-$u$ crossing relation becomes[13]

$$f^s_{\lambda_c\lambda_d;\lambda_a\lambda_b}(s,t) \sim d^{J_c}_{\lambda_c'\,\lambda_c}(0)\, d^{J_c}_{\lambda_d'\lambda_d}(\pi)\, d^{J_d}_{\lambda_a'\lambda_a}(0)\, d^{J_b}_{\lambda_b'\lambda_b}(\pi)\, f^u_{\lambda_c'\lambda_d';\lambda_a'\lambda_b'}(s,t) = (-1)^{J_b+\lambda_b+J_d+\lambda_d} f^u_{\lambda_a,-\lambda_d;\lambda_c,-\lambda_b}(s,t)\,, \tag{14}$$

which is diagonal in the total helicity flip $|\Delta\lambda|$; the off-diagonal elements are $O(s^{-1})$. Hence the left-hand cut is $O(s^{\alpha-1})$, where $s^\alpha$ is the leading behavior of the $\Delta\lambda \neq 0$ amplitudes. (Possible logarithms have been ignored.) Because antisymmetry is lacking, however, a constant subtraction term can be added to the dispersion relation, so that

$$f^s_{\Delta\lambda=0}(s, t_c) \sim \mathrm{const} + O(s^{\alpha-1})\,. \tag{15}$$

The constant of course does not imply a fixed Regge pole: Only a single value of $t$ is being considered.

Since the $\Delta\lambda = 0$ amplitudes do not in general dominate near $t_c$, Eq. (15) does not restrict the energy dependence of the cross section. It does imply, for example, that the polarization in meson-baryon scattering behaves like $s^{-\alpha(t_c)} \approx s^{-0.3}$, but such a weak dependence would be very difficult to measure. It appears that experimentally significant consequences can be drawn only for reactions which are odd under $s$-$u$ crossing, so that the constant term in (15) is absent.

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