[24]R. A. Brandt, W.-c. Ng, and P. Vinciarelli (unpublished).
[25]J. B. Kogut and D. E. Soper, Phys. Rev. D 1, 2901 (1970); J. M. Cornwall and R. Jackiw, *ibid.* 4, 367 (1971).
[26]S. Coleman and S. Glashow, Phys. Rev. 134, B671 (1964); S. Coleman and H. Schnitzer, *ibid.* 136, B223 (1964).
[27]J. D. Bjorken, Phys. Rev. 148, 1467 (1966).

PHYSICAL REVIEW D VOLUME 5, NUMBER 10 15 MAY 1972

Model-Independent Deep-Inelastic Electroproduction Sum Rules*

David R. Palmer
Department of Physics and Astronomy, University of Rochester, Rochester, New York 14627
(Received 9 February 1972)

Using the most general form for the null-plane commutator of the electromagnetic current, sum rules are derived for electroproduction in the deep-inelastic region. The structure of these sum rules is discussed and a free, massless, fermion model is considered as an illustration.

I. INTRODUCTION

The observation[1] that Bjorken scaling[2] may be interpreted in terms of the behavior of current commutators on the light cone has revived interest in commutators restricted to a null plane.[3,4] In particular, Dicus, Jackiw, and Teplitz[4] have shown that null-plane current commutators can be used to derive improved fixed-mass sum rules as well as deep-inelastic sum rules. A crucial issue in the use of null-plane commutators is their model dependence; for they seem to be much more model-dependent than equal-time commutators. While this dependence presents an opportunity for testing various models experimentally, it does not always permit one to easily recognize aspects of a problem which are the result of more general considerations such as covariance and causality. In this paper we determine those characteristics of deep-inelastic electroproduction sum rules which are independent of any particular model for the null-plane commutators of the electromagnetic current. The complete set of sum rules is derived by assuming the most general form for the null-plane commutators. As an example of how particular models may be examined, we consider a free, massless, fermion model. It is shown that the condition for finite electromagnetic self-masses found by Pagels[5] is satisfied in this model.

After presenting several preliminaries related to the Compton amplitude in Sec. II, the sum rules are derived in Sec. III. In Sec. IV the fermion model is considered.

II. COMPTON AMPLITUDE

The forward virtual Compton amplitude for photons of mass squared q^2 scattering from an unpolarized hadronic target having momentum p is[6]

$$T_{\mu\nu}(p,q)=i\int d^4x\, e^{iqx}\langle p|T^*(V_\mu(x),V_\nu(0))|p\rangle$$

$$=A^2_{\mu\nu}T_2(q^2,\nu)+A^L_{\mu\nu}T_L(q^2,\nu)\,, \tag{2.1}$$

$$A^2_{\mu\nu}=\frac{1}{M^2}\left(p_\mu p_\nu-\frac{\nu}{q^2}(p_\mu q_\nu+p_\nu q_\mu)+\frac{\nu^2}{q^2}g_{\mu\nu}\right) \tag{2.2a}$$

$$A^L_{\mu\nu}=-g_{\mu\nu}+q_\mu q_\nu/q^2\,, \tag{2.2b}$$

where V_μ is the hadronic electromagnetic current, $\nu=p\cdot q$, M is the hadron's mass, and the hadronic state is normalized according to

$$\langle p|p'\rangle=\frac{E}{M}(2\pi)^3\delta^{(3)}(\vec{p}-\vec{p}\,')\,.$$

For q^2 spacelike, we assume[7] T_2 satisfies an unsubtracted dispersion relation in ν and T_L obeys a once-subtracted dispersion relation

$$T_2(q^2,\nu)=\frac{1}{\pi}\int_0^\infty d\nu'^2\frac{\operatorname{Im}T_2(q^2,\nu')}{\nu'^2-\nu^2-i\epsilon}\,, \tag{2.3a}$$

$$T_L(q^2,\nu)=T_L(q^2,0)+\frac{\nu^2}{\pi}\int_0^\infty\frac{d\nu'^2}{\nu'^2}\frac{\operatorname{Im}T_L(q^2,\nu')}{\nu'^2-\nu^2-i\epsilon}\,. \tag{2.3b}$$

Introducing the dimensionless functions

$$F_2(q^2,\omega)=\frac{\nu}{M\pi}\operatorname{Im}T_2(q^2,\nu)\,, \tag{2.4a}$$

$$F_L(q^2,\omega)=\frac{M}{\pi}\operatorname{Im}T_L(q^2,\nu), \tag{2.4b}$$

where $\omega=-q^2/\nu$, Eqs. (2.3) become

$$T_2(q^2,\omega)=\frac{M\omega^2}{q^2}\int_0^4\frac{d\omega'^2}{\omega'}\frac{F_2(q^2,\omega')}{\omega'^2-\omega^2+i\epsilon}, \tag{2.5a}$$

$$T_L(q^2,\omega)=T_L(q^2,\infty)-\frac{1}{M}\int_0^4 d\omega'^2\frac{F_L(q^2,\omega')}{\omega'^2-\omega^2+i\epsilon}. \tag{2.5b}$$

We will assume a smooth approach to scaling ($i=2, L$):

$$F_i(q^2,\omega)=F_i(\omega)+\left(\frac{M^2}{q^2}\right)G_i(\omega)+\left(\frac{M^2}{q^2}\right)^2H_i(\omega)+\cdots. \tag{2.6}$$

Here the quantities F_i are the celebrated scaling functions[2] and G_i and H_i characterize first and second-order corrections to scaling. The functions F_i, G_i, H_i will collectively be referred to as structure functions. We also write

$$MT_L(q^2,\infty)=s+\left(\frac{M^2}{q^2}\right)t+\left(\frac{M^2}{q^2}\right)^2u+\cdots, \tag{2.7}$$

where s, t, and u are dimensionless numbers. With Eqs. (2.5)–(2.7) we have

$$MT_2(q^2,\omega)=\left(\frac{M^2}{q^2}\right)\omega\mathfrak{F}_2(\omega)+\left(\frac{M^2}{q^2}\right)^2\omega\mathcal{G}_2(\omega)+\cdots, \tag{2.8a}$$

$$MT_L(q^2,\omega)=\mathfrak{F}_L(\omega)+\left(\frac{M^2}{q^2}\right)\mathcal{G}_L(\omega)+\left(\frac{M^2}{q^2}\right)^2\mathcal{H}_L(\omega)+\cdots, \tag{2.8b}$$

where

$$\mathfrak{F}_2(\omega)=\omega\int\frac{d\omega'^2}{\omega'}\frac{F_2(\omega')}{\omega'^2-\omega^2+i\epsilon}, \tag{2.9a}$$

$$\mathcal{G}_2(\omega)=\omega\int\frac{d\omega'^2}{\omega'}\frac{G_2(\omega')}{\omega'^2-\omega^2+i\epsilon}, \tag{2.9b}$$

and

$$\mathfrak{F}_L(\omega)=s-\int d\omega'^2\frac{F_L(\omega')}{\omega'^2-\omega^2+i\epsilon}, \tag{2.10a}$$

$$\mathcal{G}_L(\omega)=t-\int d\omega'^2\frac{G_L(\omega')}{\omega'^2-\omega^2+i\epsilon}, \tag{2.10b}$$

$$\mathcal{H}_L(\omega)=u-\int d\omega'^2\frac{H_L(\omega')}{\omega'^2-\omega^2+i\epsilon}, \tag{2.10c}$$

so that $\pi F_2(\omega)=-\operatorname{Im}\mathfrak{F}_2(\omega)$, $\pi F_L(\omega)=+\operatorname{Im}\mathfrak{F}_L(\omega)$, etc. The reason for keeping terms through order $(1/q^2)^2$ in Eqs. (2.8) will become clear. If it should turn out that the subtraction assumption is incorrect, then almost all of the following analysis remains unchanged if $\mathfrak{F}_2,\ldots,\mathcal{H}_L$ are suitably redefined.

When considering commutators restricted to a null plane, it is advantageous to use a basis consisting of the four numerical vectors $n^+, n^-, n^{1,2}$; n^+, n^- are lightlike, while $n^{1,2}$ are spacelike. Therefore, with $g^{\alpha\beta}=n^\alpha\cdot n^\beta$, we can have ($i,j=1,2$)

$$g^{++}=g^{--}=g^{+i}=g^{-i}=0,$$

$$g^{+-}=1,\quad g^{ij}=-\delta^{ij}.$$

The components of a four-vector x in this basis are $x^\alpha=x\cdot n^\alpha$ ($\alpha=+,-,1,2$), so that $x^\pm=x_\mp$. We will also use the notation $x_\perp=(x^1,x^2)$.

In Sec. III, sum rules are derived by considering the $q^-\to\infty$ limit of $T_{\mu\nu}(p,q)$. In this limit

$$T_2(q^2,\omega)\to-\frac{M}{q^-p^+}\mathfrak{F}_2(\omega_0)+\frac{M}{(q^-p^+)^2}\left(\frac{M^2}{\omega_0}\mathcal{G}_2(\omega_0)-\frac{1}{2}p_+p_-\omega_0\frac{d}{d\omega_0}[\omega_0\mathfrak{F}_2(\omega_0)]-p_\perp\cdot q_\perp\frac{d}{d\omega_0}[\omega_0\mathfrak{F}_2(\omega_0)]-q_\perp{}^2\frac{d}{d\omega_0}\mathfrak{F}_2(\omega_0)\right), \tag{2.11a}$$

$$\begin{aligned}T_L(q^2,\omega)\to{}&\frac{1}{M}\mathfrak{F}_L(\omega_0)+\frac{1}{q^-p^+M}\left(-\frac{M^2}{\omega_0}\mathcal{G}_L(\omega_0)+\frac{1}{2}p_+p_-\omega_0^2\frac{d}{d\omega_0}\mathfrak{F}_L(\omega_0)+p_\perp\cdot q_\perp\omega_0\frac{d}{d\omega_0}\mathfrak{F}_L(\omega_0)+q_\perp{}^2\frac{d}{d\omega_0}\mathfrak{F}_L(\omega_0)\right)\\
&+\frac{M}{(q^-p^+)^2}\left(\frac{M^2}{\omega_0^2}\mathcal{H}_L(\omega_0)-\frac{1}{2}p_+p_-\omega_0\frac{d}{d\omega_0}\mathcal{G}_L(\omega_0)+\frac{1}{8}\frac{(p_+p_-)^2}{M^2}\omega_0^3\frac{d^2}{d\omega_0^2}[\omega_0\mathfrak{F}_L(\omega_0)]\right)\\
&+\frac{p_\perp\cdot q_\perp}{(q^-p^+)^2M}\left(-M^2\frac{d}{d\omega_0}\mathcal{G}_L(\omega_0)+\frac{1}{2}p_+p_-\omega_0^2\frac{d^2}{d\omega_0^2}[\omega_0\mathfrak{F}_L(\omega_0)]\right)\\
&+\frac{q_\perp{}^2}{(q^-p^+)^2M}\left[-M^2\frac{d}{d\omega_0}\left(\frac{1}{\omega_0}\mathcal{G}_L(\omega_0)\right)+\frac{1}{2}p_+p_-\omega_0\frac{d}{d\omega_0}\left(\omega_0\frac{d}{d\omega_0}\mathfrak{F}_L(\omega_0)\right)\right]\\
&+\frac{(p_\perp\cdot q_\perp)^2}{(q^-p^+)^2M}\frac{1}{2}\omega_0\frac{d^2}{d\omega_0^2}[\omega_0\mathfrak{F}_L(\omega_0)]+\frac{q_\perp{}^2(p_\perp\cdot q_\perp)}{(q^-p^+)^2M}\frac{d}{d\omega_0}\left(\omega_0\frac{d}{d\omega_0}\mathfrak{F}_L(\omega_0)\right)+\frac{(q_\perp{}^2)^2}{(q^-p^+)^2M}\frac{1}{2}\frac{d^2}{d\omega_0^2}\mathfrak{F}_L(\omega_0),\end{aligned} \tag{2.11b}$$

where here, and in what follows, we have defined

$$\omega_0 = -2q^+/p^+ . \tag{2.12}$$

III. SUM RULES

The basic ingredient in the derivation of the following sum rules is a generalization by Cornwall and Jackiw[3] of the Bjorken-Johnson-Low theorem[8]

$$T_{\mu\nu}(p,q) \underset{q^-\to\infty}{\sim} (\text{polynomial in } q^-) - \frac{1}{q^-}\int d^4x\, e^{iqx}\delta(x^+)\langle p|[V_\mu(x), V_\nu(0)]|p\rangle + \cdots . \tag{3.1}$$

Equation (3.1) may be interpreted in the following way[9]: If one considers a noncovariant, τ-ordered product

$$T^\tau(V_\mu(x)V_\nu(0)) = \theta(x^+)V_\mu(x)V_\nu(0) + \theta(-x^+)V_\nu(0)V_\mu(x), \tag{3.2}$$

then

$$i\int d^4x\, e^{iqx}\langle p|T^\tau(V_\mu(x)V_\nu(0))|p\rangle \underset{q^-\to\infty}{\sim} -\frac{1}{q^-}\sum_{n=0}\left(\frac{i}{q^-}\right)^n \int d^4x\, e^{iqx}\delta(x^+)\langle p|[\partial_+^{\,n} V_\mu(x), V_\nu(0)]|p\rangle , \tag{3.3}$$

provided the commutators exist. The difference between the covariant ordered product and this noncovariant τ-ordered product is an object which can be shown, in general, to be expressible in terms of a δ function in x^+ and a finite number of derivatives of such a δ function. Therefore, with

$$T^*(V_\mu(x)V_\nu(0)) = \Delta_{\mu\nu}(x,0) + T^\tau(V_\mu(x)V_\nu(0)), \tag{3.4}$$

we have

$$\tilde{\Delta}_{\mu\nu}(q) \equiv i\int d^4x\, e^{iqx}\langle p|\Delta_{\mu\nu}(x,0)|p\rangle = \text{polynomial in } q^- . \tag{3.5}$$

Equation (3.1) follows immediately from Eq. (2.1) and Eqs. (3.3)–(3.5). The difference $\Delta_{\mu\nu}$, which plays a role analogous to the role played by the seagull term, can be constructed algebraically if one assumes some model for the commutators $[V_-(x), V_\nu(0)]\delta(x^+)$.

By causality, the commutator $[V_\mu(x), V_\nu(0)]$ vanishes for $x^2 = 2x^+x^- - x_\perp^2 < 0$. Therefore, on the plane $x^+=0$, the commutator has support concentrated at $x_\perp = 0$ and hence must be representable in terms of a δ function in $x_\perp$ and a finite number of derivatives of such a δ function. We can therefore generally write

$$[V_\mu(x), V_\nu(0)]\delta(x^+) = \sum_{n=0}\sum_{k_i=1,2}\Phi_{\mu\nu}^{k_1\cdots k_n}(x^-)\partial_{k_1}^{(x)}\cdots\partial_{k_n}^{(x)}\delta^{(2)}(x_\perp)\delta(x^+) , \tag{3.6}$$

where the Φ's depend only on x^- and the number of terms in the sum over n is finite. The quantity $\Phi_{\mu\nu}^{k_1\cdots k_n}$ (which is symmetric in $k_1\cdots k_n$) will be said to be of order n. We will use the notation

$$\phi_{\mu\nu}^{k_1\cdots k_n}(\omega) = M^{n-1}\int_{-\infty}^{+\infty} d\alpha\, e^{-i\omega\alpha/2}\left\langle p\left|\Phi_{\mu\nu}^{k_1\cdots k_n}\left(\frac{\alpha}{p^+}\right)\right|p\right\rangle , \tag{3.7}$$

so that

$$\frac{p^+}{M}\int d^4x\, e^{iqx}\delta(x^+)\langle p|[V_\mu(x), V_\nu(0)]|p\rangle = \sum_{n=0}\frac{q_{k_1}\cdots q_{k_n}}{M^n}\phi_{\mu\nu}^{k_1\cdots k_n}(\omega_0) . \tag{3.8}$$

The powers of M have been chosen so that the ϕ's are dimensionless. The dependence of the ϕ's on p_μ has not been explicitly indicated.

The sum rules are now obtained[10] by expanding $T_{\mu\nu}(p,q)$ as $q^-\to\infty$, using Eqs. (2.1), (2.2), and (2.11), and comparing with Eq. (3.1), where the polynomial in q^- is expressed in terms of $\tilde{\Delta}_{\mu\nu}$, and the null-plane commutator in terms of the ϕ's [Eqs. (3.5) and (3.8)]. The results are ($\hat{p}_\mu = p_\mu/M$)

$$\phi_{--}(\omega) = \tfrac{1}{4}(\hat{p}_-)^2\omega\mathfrak{F}_L(\omega), \tag{3.9a}$$

$$\phi_{-i}(\omega) = \tfrac{1}{2}\hat{p}_-\hat{p}_i\mathfrak{F}_2(\omega), \tag{3.9b}$$

$$\phi_{-i}^{l}(\omega) = \delta_{il}\hat{p}_-\left(\frac{1}{\omega}\mathfrak{F}_2(\omega) - \mathfrak{F}_L(\omega)\right), \tag{3.9c}$$

$$\phi_{-+}(\omega)=\hat{p}_-\hat{p}_+\left(\frac{1}{4}\omega^2\frac{d}{d\omega}\mathcal{F}_L(\omega)+\mathcal{F}_2(\omega)\right)-\frac{1}{2\omega}\mathcal{G}_L(\omega), \tag{3.9d}$$

$$\phi^{l}_{-+}(\omega)=\hat{p}_l\left(\frac{1}{2}\omega\frac{d}{d\omega}\mathcal{F}_L(\omega)+\frac{1}{\omega}\mathcal{F}_2(\omega)\right), \tag{3.9e}$$

$$\phi^{ll'}_{-+}(\omega)=\frac{\delta_{ll'}}{2\omega}\frac{d}{d\omega}[\omega\mathcal{F}_L(\omega)], \tag{3.9f}$$

$$\phi_{ij}(\omega)=\hat{p}_i\hat{p}_j\mathcal{F}_2(\omega)+\frac{\delta_{ij}}{\omega^2}[\omega\mathcal{G}_L(\omega)-\mathcal{G}_2(\omega)]+\delta_{ij}\hat{p}_+\hat{p}_-\left(\frac{1}{2}\frac{d}{d\omega}[\omega\mathcal{F}_2(\omega)]-\mathcal{F}_2(\omega)-\frac{1}{2}\omega^2\frac{d}{d\omega}\mathcal{F}_L(\omega)\right), \tag{3.9g}$$

$$\phi^{l}_{ij}(\omega)=(\hat{p}_i\delta_{lj}+\hat{p}_j\delta_{li})\frac{1}{\omega}\mathcal{F}_2(\omega)+\delta_{ij}\hat{p}_l\omega\frac{d}{d\omega}\left(\frac{1}{\omega}\mathcal{F}_2(\omega)-\mathcal{F}_L(\omega)\right), \tag{3.9h}$$

$$\phi^{ll'}_{ij}(\omega)=+\delta_{ij}\delta_{ll'}\frac{d}{d\omega}\left(\frac{1}{\omega}\mathcal{F}_2(\omega)-\mathcal{F}_L(\omega)\right)+(\delta_{il}\delta_{jl'}+\delta_{il'}\delta_{jl})\frac{1}{2\omega}\mathcal{F}_L(\omega), \tag{3.9i}$$

$$\phi_{+i}(\omega)=\frac{\hat{p}_i}{\hat{p}^+}\left[\frac{1}{2}\hat{p}_+\hat{p}_-\left(\frac{d}{d\omega}[\omega\mathcal{F}_2(\omega)]+\mathcal{F}_2(\omega)\right)-\frac{1}{\omega^2}\mathcal{G}_2(\omega)\right], \tag{3.9j}$$

$$\phi^{l}_{+i}(\omega)=\frac{\hat{p}_i\hat{p}_l}{\hat{p}^+}\frac{d}{d\omega}\mathcal{F}_2(\omega)+\frac{\delta_{il}}{\hat{p}^+}\left[\frac{\hat{p}_+\hat{p}_-}{2\omega}\left(2\mathcal{F}_2(\omega)+\omega^2\frac{d}{d\omega}\mathcal{F}_L(\omega)\right)-\frac{1}{\omega^2}\mathcal{G}_L(\omega)\right], \tag{3.9k}$$

$$\phi^{ll'}_{+i}(\omega)=\frac{1}{2\hat{p}^+}(\delta_{il}\hat{p}_{l'}+\delta_{il'}\hat{p}_l)\frac{d}{d\omega}\mathcal{F}_L(\omega)+\frac{\delta_{ll'}\hat{p}_i}{\hat{p}^+}\frac{d}{d\omega}\left(\frac{1}{\omega}\mathcal{F}_2(\omega)\right), \tag{3.9l}$$

$$\phi^{l_1l_2l_3}_{+i}(\omega)=\frac{1}{3\hat{p}^+}(\delta_{il_1}\delta_{l_2l_3}+\delta_{il_2}\delta_{l_1l_3}+\delta_{il_3}\delta_{l_1l_2})\frac{d}{d\omega}\left(\frac{1}{\omega}\mathcal{F}_L(\omega)\right), \tag{3.9m}$$

$$\phi_{++}(\omega)=\frac{1}{(\hat{p}^+)^2\omega^3}\mathcal{H}_L(\omega)-\frac{\hat{p}_+\hat{p}_-}{2\omega^2(\hat{p}^+)^2}\left(4\mathcal{G}_2(\omega)+\omega^2\frac{d}{d\omega}\mathcal{G}_L(\omega)\right)+\frac{1}{8}\frac{(\hat{p}_+\hat{p}_-)^2}{(\hat{p}^+)^2}\left(\omega^2\frac{d^2}{d\omega^2}[\omega\mathcal{F}_L(\omega)]+8\frac{d}{d\omega}[\omega\mathcal{F}_2(\omega)]\right), \tag{3.9n}$$

$$\phi^{l}_{++}(\omega)=-\frac{\hat{p}_l}{(\hat{p}^+)^2}\frac{1}{\omega}\frac{d}{d\omega}\mathcal{G}_L(\omega)+\frac{\hat{p}_l\hat{p}_+\hat{p}_-}{(\hat{p}^+)^2}\left(\frac{1}{2}\omega\frac{d^2}{d\omega^2}[\omega\mathcal{F}_L(\omega)]+2\frac{d}{d\omega}\mathcal{F}_2(\omega)\right), \tag{3.9o}$$

$$\phi^{ll'}_{++}(\omega)=-\frac{\delta_{ll'}}{(\hat{p}^+)^2}\frac{d}{d\omega}\left(\frac{\mathcal{G}_L(\omega)}{\omega^2}\right)+\frac{1}{2}\frac{\hat{p}_l\hat{p}_{l'}}{(\hat{p}^+)^2}\frac{d^2}{d\omega^2}[\omega\mathcal{F}_L(\omega)]+\frac{\delta_{ll'}\hat{p}_+\hat{p}_-}{2(\hat{p}^+)^2}\left[\omega\frac{d^2}{d\omega^2}\mathcal{F}_L(\omega)+4\frac{d}{d\omega}\left(\frac{1}{\omega}\mathcal{F}_2(\omega)\right)\right], \tag{3.9p}$$

$$\phi^{l_1l_2l_3}_{++}(\omega)=\frac{1}{3(\hat{p}^+)^2}(\delta_{l_1l_2}\hat{p}_{l_3}+\delta_{l_1l_3}\hat{p}_{l_2}+\delta_{l_2l_3}\hat{p}_{l_1})\frac{d^2}{d\omega^2}\mathcal{F}_L(\omega), \tag{3.9q}$$

$$\phi^{l_1l_2l_3l_4}_{++}(\omega)=\frac{1}{6(\hat{p}^+)^2}(\delta_{l_1l_2}\delta_{l_3l_4}+\delta_{l_1l_3}\delta_{l_2l_4}+\delta_{l_1l_4}\delta_{l_2l_3})\frac{d^2}{d\omega^2}\left(\frac{1}{\omega}\mathcal{F}_L(\omega)\right), \tag{3.9r}$$

and ($\hat{p}_\mu=p_\mu/M$, $\hat{q}_\mu=q_\mu/M$)

$$\tilde{\Delta}_{--}(q)=\tilde{\Delta}_{-i}(q)=0, \tag{3.10a}$$

$$\tilde{\Delta}_{-+}(q)=-\frac{1}{2M}\mathcal{F}_L(\omega_0), \tag{3.10b}$$

$$\tilde{\Delta}_{ij}(q)=\frac{\delta_{ij}}{M}\left(\mathcal{F}_L(\omega_0)-\frac{1}{\omega_0}\mathcal{F}_2(\omega_0)\right), \tag{3.10c}$$

$$\tilde{\Delta}_{+i}(q)=-\frac{1}{\hat{p}^+M\omega_0}[\hat{q}_i\mathcal{F}_L(\omega_0)+\hat{p}_i\mathcal{F}_2(\omega_0)], \tag{3.10d}$$

$$\tilde{\Delta}_{++}(q)=-\frac{\hat{q}^-}{\hat{p}^+M}\frac{1}{\omega_0}\mathcal{F}_L(\omega_0)-\frac{1}{(\hat{p}^+)^2M\omega_0}\left[-\frac{1}{\omega_0}\mathcal{G}_L(\omega_0)+\hat{p}_+\hat{p}_-\left(2\mathcal{F}_2(\omega_0)+\frac{1}{2}\omega_0^2\frac{d}{d\omega_0}\mathcal{F}_L(\omega_0)\right)\right.$$
$$\left.+\hat{p}_\perp\cdot\hat{q}_\perp\omega_0\frac{d}{d\omega_0}\mathcal{F}_L(\omega_0)+\hat{q}_\perp{}^2\omega_0\frac{d}{d\omega_0}\left(\frac{1}{\omega_0}\mathcal{F}_L(\omega_0)\right)\right]. \tag{3.10e}$$

With the aid of Eqs. (3.9), we have listed in Table I combinations of the structure functions and their derivatives which are determined from a given term in the null-plane commutators. We note the following characteristics:

(A) If terms represented by a zero in the table and terms of order greater than 4 are present in the null-plane commutators, they cannot contribute to the matrix element we are considering.

(B) Different models for the null-plane commutators are distinguished in that they predict the vanishing of one or more of the structure functions and/or relations between these functions. Any model can at most give information about the five functions F_L, F_2, G_L, G_2, and H_L. In order to obtain information about higher-order corrections to scaling, one must use a model for null-plane commutators of currents and derivatives of currents.

(C) The terms in the commutators which involve corrections to scaling and therefore should be related to the mechanism which breaks dilatation invariance are the most model-dependent terms ϕ_{ij}, ϕ_{-+}, ϕ_{+i}, ϕ^{l}_{+i}, ϕ_{++}, ϕ^{l}_{++}, and $\phi^{ll'}_{++}$.

(D) By considering little-group transformations, one can form the chain[11] ($(\mu\nu)$ refers to $[V_\mu(x), V_\nu(0)]\delta(x^+)$)

$$(++)\Rightarrow(+i)\Rightarrow\begin{matrix}(-+)\\(ij)\end{matrix}\Rightarrow(-i)\Rightarrow(--)\,, \tag{3.11}$$

indicating that covariance alone determines all the commutators to the right of a given commutator in terms of that commutator. As one can see from the table, this result is verified for the matrix element we are considering. The most model-dependent commutator is the (++) commutator and the least model-dependent is the (− −) one. Although the commutators (− +) and (ij) cannot be transformed into one another, and they both determine the $(-i)$ and (− −) commutators, (ij) is somewhat more model-dependent (e.g., ϕ_{ij} determines separately F_2 and $dF_L/d\omega$, whereas ϕ_{-+} determines only a combination of F_2 and $dF_L/d\omega$). This is reasonable, for the (− +) commutator can be directly related to the charge operator,[4] whereas the (ij) commutator cannot.

(E) With the exception of ϕ_{ij}, ϕ_{+i}, and ϕ_{++}, all the terms in the commutators can be determined from the relatively model-independent $(-\mu)$ commutators. This is an example of a more general result[9] which follows solely from covariance.

(F) Finally, from Eqs. (3.10), we see the difference between the covariant and τ-ordered product, $\Delta_{\mu\nu}$, can be determined from the $(-\mu)$ commutators. This also is an example of a more general result.

IV. MASSLESS FERMION MODEL

We now consider the sum rules which result from a particular model for the null-plane commutators, namely, the free, massless, fermion model.[12] In

TABLE I. The combinations of the structure functions and their derivatives which are determined from a given term in the null-plane commutators. The symbol $(\mu\nu)$ refers to the commutator $[V_\mu(x), V_\nu(0)]\delta(x^+)$, and the order of a term in a commutator is the number of derivatives of the two-dimensional, spatial δ function associated with that term.

$(\mu\nu)$	Order 0	1	2	3	4
$(--)$	F_L	0	0	0	0
$(-i)$	F_2	$\omega F_L + F_2$	0	0	0
$(-+)$	G_L, $\frac{1}{4}\omega^2 \frac{dF_L}{d\omega} - F_2$	$\frac{1}{2}\omega^2 \frac{dF_L}{d\omega} - F_2$	$\frac{d}{d\omega}(\omega F_L)$	0	0
(ij)	F_2, $\frac{dF_L}{d\omega}$, $\omega G_L + G_2$	F_2, $\frac{dF_L}{d\omega}$	F_L, $\frac{d}{d\omega}\left(\frac{1}{\omega}F_2\right)$	0	0
$(+i)$	G_2, $\omega\frac{dF_2}{d\omega} + 2F_2$	G_L, $\frac{dF_2}{d\omega}$, $\frac{1}{2}\omega^2\frac{dF_L}{d\omega} - F_2$	$\frac{dF_L}{d\omega}$, $\frac{d}{d\omega}\left(\frac{1}{\omega}F_2\right)$	$\frac{d}{d\omega}\left(\frac{1}{\omega}F_L\right)$	0
$(++)$	H_L, $\omega^2\frac{dG_L}{d\omega} - 4G_2$, $\omega^2\frac{d^2}{d\omega^2}(\omega F_L) - 8\frac{d}{d\omega}(\omega F_2)$	$\frac{dG_L}{d\omega}$, $\frac{1}{2}\omega\frac{d^2}{d\omega^2}(\omega F_L) - 2\frac{dF_2}{d\omega}$	$\frac{d}{d\omega}\left(\frac{1}{\omega}G_L\right)$, $\frac{d^2}{d\omega^2}(\omega F_L)$, $\frac{d}{d\omega}\left(F_L + \frac{2}{\omega}F_2\right)$	$\frac{d^2}{d\omega^2}F_L$	$\frac{d^2}{d\omega^2}\left(\frac{1}{\omega}F_L\right)$

such a model the current is given by

$$V_\mu(x)=\bar{\psi}(x)\gamma_\mu Q\psi(x),$$

where Q is the charge matrix. The null-plane commutators can be expressed in terms of a single, vector, bilocal operator $\mathcal{V}_\mu(x|0)$[13]:

$$[V_\mu(x),\ V_\nu(0)]\delta(x^+)=-2i\delta(x^+)\partial^\lambda\Delta(x;0)\mathcal{V}^\tau(x|0)s_{\mu\nu\lambda\tau},\tag{4.1}$$

$$s_{\mu\nu\lambda\tau}=g_{\mu\lambda}g_{\nu\tau}+g_{\mu\tau}g_{\nu\lambda}-g_{\mu\nu}g_{\lambda\tau},$$

where $\Delta(x;0)$ is the odd, invariant solution to the massless Klein-Gordon equation, normalized so that

$$\partial_t\Delta(x;0)|_{t=0}=-\delta^{(3)}(\vec{x}).$$

In terms of the fermion fields,

$$2i\mathcal{V}_\mu(x|0)=\bar{\psi}(x)\gamma_\mu Q^2\psi(0)-\bar{\psi}(0)\gamma_\mu Q^2\psi(x),\tag{4.2}$$

so that[14]

$$\mathcal{V}_\mu(0|0)=0.\tag{4.3}$$

Bilocal form factors are defined by the expression

$$\langle p|\mathcal{V}_\mu(x|0)|p\rangle=\frac{p_\mu}{M}V_1(x^2,p\cdot x)+x_\mu MV_2(x^2,p\cdot x).\tag{4.4}$$

If the expressions obtained for the ϕ's from Eqs. (3.7), (3.8), (4.1), and (4.4) are substituted into Eqs. (3.9), one finds the following sum rules:

$$\mathfrak{F}_L(\omega)=0,\tag{4.5a}$$

$$\mathfrak{F}_2(\omega)=\tfrac{1}{2}\omega\int d\alpha\, e^{-i\omega\alpha/2}\epsilon(\alpha)V_1(0,\alpha),\tag{4.5b}$$

$$\mathcal{G}_L(\omega)=-\omega^2\int d\alpha\, e^{-i\omega\alpha/2}|\alpha|\,V_2(0,\alpha),\tag{4.5c}$$

$$\mathcal{G}_2(\omega)=\omega\,\mathcal{G}_L(\omega)+\frac{1}{4}\omega^4\frac{d}{d\omega}\left(\frac{1}{\omega}\mathfrak{F}_2(\omega)\right)+\frac{i\omega^2}{M^2}\int d\alpha\, e^{-i\omega\alpha/2}|\alpha|\left(\frac{\partial V_1(x^2,\alpha)}{\partial(x^2)}\right)_{x^2=0},\tag{4.6a}$$

$$\mathcal{H}_L(\omega)=\frac{1}{4}\omega^3\frac{d\mathcal{G}_L(\omega)}{d\omega}+\omega^2\,\mathcal{G}_L(\omega)-\frac{1}{2}\omega^3\mathfrak{F}_2(\omega)-\frac{2i\omega^3}{M^2}\int d\alpha\, e^{-i\omega\alpha/2}\epsilon(\alpha)\alpha^2\left(\frac{\partial V_2(x^2,\alpha)}{\partial(x^2)}\right)_{x^2=0},\tag{4.6b}$$

and

$$\mathcal{G}_L(\omega)+2i\omega\int d\alpha\, e^{-i\omega\alpha/2}\epsilon(\alpha)V_2(0,\alpha)=\omega\mathfrak{F}_2(\omega).\tag{4.6c}$$

Equations (4.5) are the sum rules of Dicus, Jackiw, and Teplitz.[4] Equation (4.6c) is particularly interesting because it leads to a relation between $\mathcal{G}_L$ and $\mathfrak{F}_2$. Technically, it results from a comparison of the expressions for $\mathcal{G}_L$ and $\mathcal{G}_2$ which follow from Eqs. (3.9d), (3.9g), and (3.9j). Its ultimate origin, however, is current conservation. For, using current conservation and the Jacobi identity, it is straightforward to show that

$$\begin{aligned}i[M_{+i},[V_-(x),V_\nu(0)]\delta(x^+)]=&\{L_{+i}^{(x)}+x_i\partial_+\}[V_-(x),V_\nu(0)]\delta(x^+)\\&+g_{+\nu}[V_-(x),V_i(0)]\delta(x^+)-g_{i\nu}[V_-(x),V_\nu(0)]\delta(x^+)\\&-\partial_k\{x_i[V_k(x),V_\nu(0)]\delta(x^+)\}+\partial_-\{x_i[V_+(x),V_\nu(0)]\delta(x^+)\},\end{aligned}\tag{4.7}$$

where $M_{\mu\nu}$ are the usual infinitesimal generators of the Lorentz group and $L_{\mu\nu}^{(x)}=x_\mu\partial_\nu-x_\nu\partial_\mu$. Substituting Eq. (4.1) into Eq. (4.7) yields

$$\epsilon(x^-)\delta^{(2)}(x_\perp)\delta(x^+)[\partial_i\mathcal{V}_j(x|0)-\partial_j\mathcal{V}_i(x|0)]=0,\tag{4.8a}$$

$$|x^-|\delta^{(2)}(x_\perp)\delta(x^+)[\partial_j\mathcal{V}_-(x|0)-\partial_-\mathcal{V}_j(x|0)]=0,\tag{4.8b}$$

$$\epsilon(x^-)\delta^{(2)}(x_\perp)\delta(x^+)[\partial_+\mathcal{V}_-(x|0)+\partial_-\mathcal{V}_+(x|0)-\partial_k\mathcal{V}_k(x|0)]=0,\tag{4.8c}$$

for $\nu=i$ and

$$\epsilon(x^-)\delta^{(2)}(x_\perp)\delta(x^+)[\partial_+\mathcal{V}_i(x|0)-\partial_i\mathcal{V}_+(x|0)]=0,\tag{4.9a}$$

$$|x^-|\delta^{(2)}(x_\perp)\delta(x^+)[\partial_k\mathcal{V}_k(x|0)-2\partial_-\mathcal{V}_+(x|0)]=0,\tag{4.9b}$$

for $\nu=+$. Equations (4.8c) and (4.9a) combined give Eq. (4.6c) and the relation

$$\mathcal{G}_L(\omega)=\frac{2i\omega}{M^2}\int d\alpha\, e^{-i\omega\alpha/2}|\alpha|\left(\frac{\partial V_1(x^2,\alpha)}{\partial x^2}\right)_{x^2=0}+\frac{1}{2}\omega\mathfrak{F}_2(\omega).\tag{4.10}$$

Equation (4.10) was not obtained from the sum rules. Equations (4.8a), (4.8b) yield trivial identities, and Eq. (4.9b) gives nothing additional.

We see that Eqs. (4.6a), (4.6c), and (4.10) yield the differential equations[15,16]

$$2\,\mathcal{G}_2(\omega)=3\omega\,\mathcal{G}_L(\omega)-\omega^2\mathcal{F}_2(\omega)+\frac{1}{2}\omega^3\frac{d\mathcal{F}_2(\omega)}{d\omega}, \tag{4.11}$$

$$\omega\frac{d\mathcal{G}_L(\omega)}{d\omega}-2\,\mathcal{G}_L(\omega)=\omega^2\frac{d\mathcal{F}_2(\omega)}{d\omega}. \tag{4.12}$$

Substituting Eqs. (2.9a) and (2.10b) into Eq. (4.12) and letting $\omega\to\infty$ gives

$$t+\int_0^2 d\omega\,F_2(\omega)=0\,. \tag{4.13}$$

This relation, which relates an integral over the scaling function F_2 to a piece of the subtraction constant in the dispersion relation for T_L, is interesting because it is precisely the relation that Pagels[5] found must hold if electromagnetic self-masses of hadrons are to be finite.[17] The validity of Eq. (4.13) in a massless fermion model is consistent with the analysis of Bjorken[8] based on equal-time current commutators in the quark model.

ACKNOWLEDGMENTS

The author would like to thank Dr. D. A. Dicus and Dr. D. N. Levin for informative conversations and Dr. Dicus for his reading of the manuscript and comments.

---

*Work supported in part by the U. S. Atomic Energy Commission.

[1]R. A. Brandt, Phys. Rev. Letters 23, 1260 (1969); L. S. Brown, in lectures given at the Summer Institute for Theoretical Physics, Boulder, Colorado, 1969 (unpublished); H. Leutwyler and J. Stern, Nucl. Phys. B20, 77 (1970); R. Jackiw, R. Van Royen, and G. B. West, Phys. Rev. D 2, 2473 (1970).

[2]J. D. Bjorken, Phys. Rev. 179, 1547 (1969).

[3]J. M. Cornwall and R. Jackiw, Phys. Rev. D 4, 367 (1971); J. B. Kogut and D. E. Soper, *ibid.* 1, 2901 (1970); J. D. Bjorken, J. B. Kogut, and D. E. Soper, *ibid.* 3, 1382 (1971); R. A. Neville and F. Rohrlich, *ibid* 3, 1692 (1971); Y. Georgelin, J. Stern, and J. Jersák, Nucl. Phys. B27, 493 (1971); G. Domokos, in lectures given at the Summer Institute for Theoretical Physics, Boulder, Colorado, 1971 (unpublished). These articles contain references to earlier work.

[4]D. A. Dicus, R. Jackiw, and V. L. Teplitz, Phys. Rev. D 4, 1733 (1971).

[5]H. Pagels, Phys. Rev. 185, 1990 (1969).

[6]Our metric has signature $(+---)$.

[7]H. Harari, Phys. Rev. Letters 17, 1303 (1966).

[8]J. D. Bjorken, Phys. Rev. 148, 1467 (1966); K. Johnson and F. E. Low, Progr. Theoret. Phys. (Kyoto) Suppl. 37–38, 74 (1966).

[9]D. R. Palmer, unpublished.

[10]This is the same method as used in Ref. 4.

[11]Y. Georgelin, J. Stern, and J. Jersák, Ref. 3. In general, the parts of the commutator symmetric and antisymmetric in Lorentz indices should be considered separately.

[12]H. Fritzsch and M. Gell-Mann [talk presented at the Coral Gables Conference on Fundamental Interactions at High Energy, University of Miami, Miami, Florida, 1971 (unpublished)] have suggested that this model describes the most singular terms in the (covariant) asymptotic expansion of $[V_\mu(x),\ V_\nu(0)]$ as $x^2\to 0$.

[13]We ignore the axial-vector bilocal operator since it cannot contribute to the matrix element we are considering.

[14]When abstracting from the fermion model, this does not become an additional condition; it follows from Eq. (4.1) and covariance.

[15]Similar equations involving various structure functions have been proposed by K. T. Mahanthappa and T. Yao, Trieste Report No. IC/71/45 (unpublished); D. Dicus and V. Teplitz, Rochester Report No. UR-875-364 (unpublished).

[16]One would not expect such equations to be valid in a lowest-order perturbation calculation in a massless fermion theory, for there one would set the proton mass equal to zero and Eqs. (2.8) could no longer define the $\mathcal{G}$'s. The author thanks Dr. D. Dicus for a discussion of this point.

[17]If T_L is unsubtracted, t may be expressed in terms of G_L; our relations and results remain unchanged however.