Proton-Neutron Mass Difference in Configuration Space*

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We argue that the canonical structure of light-cone singularities should break down at distances of the order of 10^{-16} cm. We propose that this be the separation from the light cone of the boundary of a region where configuration-space structure functions effectively lose their x^2 dependence. A new formula for the proton-neutron mass difference is derived as a limit of configuration-space invariant amplitudes. The above conjecture is then used for a numerical estimate which happens to be in excellent agreement with the experimental value. Finally we exhibit bilocal operators in the quark-gluon model responsible for divergences in $\Delta I=1$ electromagnetic mass shifts.

I. INTRODUCTION

The electrodynamics of hadrons at very high energy and large momentum transfers, beyond the present limits of SLAC, should be pregnant with surprises.

The Bjorken scaling[1] of the electroproduction structure functions is today accounted for by several theoretical models: the light-cone-dominance model,[2] more mathematical in nature, and the parton model,[3] which appeals to physical intuition. Both, the light-cone-dominance model with *canonical* singularities, a version vigorously proposed by Brandt and Preparata,[2] and the parton model predict simple scaling to hold up at higher and higher energies and momentum transfers. There is no good reason that this should be the case.

In the light-cone language, experiments have been probing the commutator of two electromagnetic currents lightlike separated up to distances of the order of 10^{-15} cm, showing that a canonical structure can be consistently assumed up to such distances. We ask the question: When and how could this canonical structure break down?

In the parton picture, scaling results from uncorrelated scattering from pointlike sources. Of course, the fact that the parton appears pointlike to a photon with wavelength of the order of 10^{-15} cm is consistent with a parton size of 10^{-16} cm. This is the size that the parton would have in the philosophy of a model recently proposed by Brandt and Vinciarelli.[4] The parton is there thought to be a cluster of dyons (particles electrically and magnetically charged) held together by the exchange of a magnetically charged spin-one field, the particle dual to the W boson, of mass (equal to the W mass) $(\sqrt{2}\, e^2/G)^{1/2} \cong 110$ GeV. The size of the parton is taken to be the corresponding Compton wavelength $\lambdabar = (G\hbar^2/\sqrt{2}\, e^2c^2)^{1/2} \cong 10^{-16}$ cm.

Such a model predicts drastic changes in the behavior of the structure functions to happen as soon as the photon is able to resolve the parton structure.

If the photons were to see the magnetic fluctuations inside the parton, cross sections would presumably greatly increase, and there would appear violations of parity and time-reversal invariance.[5]

However, a breakdown in the interaction of the electromagnetic field with matter might then manifest itself. Such a possibility would conform to a principle of hierarchy of interactions: Very small distances are the natural playground of interactions from which the electromagnetic ones (typically long range) are excluded (*"Ubi mayor minor cessat"*). Such a possibility is also particularly attractive in the light of the observation that a "change" in the physics can be expected to occur in correspondence to some fundamental unit and that the proposed model possesses such a unit of length ($\lambdabar$) and already predicts an association between this and the breakdown of space-time symmetries and of invariance under charge conjugation: $\lambdabar$ is indeed the distance at which weak and "superweak" interactions take place.

Thus, in our opinion, at SLAC we have been witnessing the setting in of an *intermediate regime* (scaling) whose limits of validity are determined, on one side, by the parton mass (~0.50 GeV),[6] which breaks scaling in the low-energy region, and, on the other side, by the W-boson mass. We believe that $\lambdabar$ is the invariant distance from the light cone at which the canonical structure of commutators should be modified.

To be more specific about the character of such modification, let us adopt the language of operator-product expansions.[7] Then, at short invariant distance, we write

$$A(x)B(0) \underset{x^2\to 0}{\sim} \sum_d E(x^2;d)\,O^{(d)}(x|0), \tag{1.1}$$

where $E(x^2;d)$ are c-number functions singular as $x^2 \to 0$, while $O^{(d)}(x|0)$ are bilocal operators regular in the same limit when sandwiched between states. It is part of the philosophy of the expansion to separate small-distance effects, represented by the coefficient functions $E(x^2;d)$, from effects at ordinary distances (~10^{-14} cm), represented by the operators. Following Wilson,[8] we therefore confine the effects of the existence of a fundamental length ($\lambdabar$) to the singular coefficients $E(x^2;d)$ and consequently assume that the smooth operators $O^{(d)}(x|0)$ should reveal no surprises as $x^\mu \to 0$ beyond the region accessible at present.

The question we posed should then be rephrased: How is $E(x^2;d)$ going to be modified at small x^2?

It is certainly plausible and desirable that the canonical singularity be *smoothed out* when other interactions come into play. Is there a dynamical scheme which implements such a proposal?

One appealing possibility is that of summation techniques associated with nonpolynomial interactions.[9] Briefly, a nonpolynomial Lagrangian in a field $\phi(x)$ of the form

$$L_{\text{int}} = g:\sum_{n=0}^{\infty} a_n[\lambda\phi(x)]^n:$$

should (hopefully) yield Green's functions which are expandable in a double power series in g and λ (the "major" and the "minor" coupling constants). A summation technique has been developed which allows a perturbative expansion in g. The coefficients are, of course, an infinite series in λ, which can be summed exactly in many cases of interest. It happens that the gravitational, the weak, and the chiral-invariant strong interactions can be associated with Lagrangians nonpolynomial in their field variables. Now, it has been shown, for the gravitational[10] and the chiral[11] Lagrangians, that the light-cone singularities of the various Green's functions turn out to be regularized. For the weak interaction, the summation technique enables us to treat the situation of multiple emission of particles (W bosons) when the interaction becomes strong. We shall exhibit a model in which infinity suppression is possible. Such a model essentially reduces a logarithmically divergent quantity according to

$$\ln(0) \to \ln(Gm^2). \tag{1.2}$$

In such calculations, "honest" field theory has been employed, with no artificial cutoffs. However, a rule emerges which says that any divergent quantity should be regularized by the *effective* cutoff of Gm^2. We suspect that this might be true in nature.

It is then legitimate to *speculate* that the *coefficient functions* $E(x^2;d)$ *essentially lose their* x^2 *dependence for* $x^2 \lesssim \lambdabar^2$:

$$E(x^2;d) \cong E(\lambdabar^2;d), \quad x^2 \lesssim \lambdabar^2. \tag{1.3}$$

A direct experimental verification of such a conjecture will certainly not be possible for some years to come. Yet, we can presently study some implications of (1.3).

A very relevant one is its implication on the theory of electromagnetic mass differences,[12] in particular, the proton-neutron mass difference, to which we address ourselves here.

We shall be using configuration-space rather than momentum-space techniques. This choice, we believe, offers considerable advantages. And, in any case, the conventional decomposition into a Born-term, Regge, fixed-pole, and inelastic contributions, which comes in a package deal with the momentum-space approach, has not been seen in the past ten years to lead anywhere closer to a solution of the problem, because of the total lack of control on some of the terms.

In Sec. II, a new configuration-space formula for the nucleon self-mass is obtained. Direction independence in Minkowski space is exploited in Sec. III to enable us to relate the unknown functions in the formula to matrix elements of known bilocal operators. Up to this point, the formula contains meaningless divergent quantities. Our statement of smoothness (1.3), which cures them, is justified in Sec. IV in a specific dynamical framework embodying summation techniques in treating nonpolynomial interactions. This allows us to numerically evaluate the proton-neutron

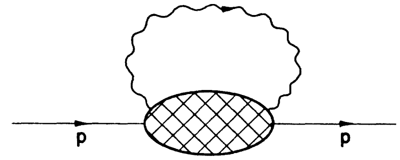

FIG. 1. Pictorial representation of the second-order self-energy function $\Sigma(p)$.

mass difference in Sec. V. Finally, in Sec. VI, algebraic properties of the naively divergent mass shifts are deduced from the quark-gluon model, and the empirical success of such relations is taken as support for the conjectured lack of correlation between the c-number and q-number parts of operator-product expansions.

II. FORMULA FOR THE NUCLEON ELECTROMAGNETIC SELF-MASS

The forward scattering of an electromagnetic current from a nucleon off its mass shell (momentum p) is described, in configuration space, by the amplitude

$$\hat{T}^{\mu\nu}(x^2, x\cdot p, p^2) = -i\int d^4y\, d^4z\, \bar{u}_p(y)\vec{\mathrm{D}}_y\, \langle 0|T(J^\mu(x)\psi(y)\bar{\psi}(z)J^\nu(0))|0\rangle \vec{\mathrm{D}}_z u_p(z).$$

The trace of this tensor, a quantity of relevance to the nucleon electromagnetic self-energy, will be related to the conventional scalar amplitudes free of kinematic singularities, $\hat{U}_1$ and $\hat{U}_2$,

$$\hat{T} = g_{\mu\nu}\hat{T}^{\mu\nu} = 3\Box\hat{U}_1(x^2, x\cdot p, p^2) - [\Box + 2(p\cdot\partial)^2]\hat{U}_2(x^2, x\cdot p, p^2). \tag{2.1}$$

The nucleon self-energy, in order α (see Fig. 1),

$$\Sigma(p^2) = -i(2\pi)^4\alpha\int d^4x\, D_0^c(x) g_{\mu\nu}\hat{T}^{\mu\nu}(x^2, x\cdot p, p^2), \tag{2.2}$$

can then be expressed as the sum of two terms

$$\Sigma(p^2) = \Sigma_1(p^2) + \Sigma_2(p^2), \tag{2.3}$$

where

$$\Sigma_1(p^2) = -i(2\pi)^4\alpha\int d^4x\, D_0^c(x)\Box[3\hat{U}_1(x^2, x\cdot p, p^2) - \hat{U}_2(x^2, x\cdot p, p^2)], \tag{2.4}$$

$$\Sigma_2(p^2) = 2i(2\pi)^4\alpha\int d^4x\, D_0^c(x)(p\cdot\partial)^2\hat{U}_2(x^2, x\cdot p, p^2), \tag{2.5}$$

and $D_0^c(x)$ is the causal solution of

$$\Box D_0^c(x) = \delta^4(x). \tag{2.6}$$

Now, integration by parts will give Eq. (1.4) a manifestly "local" form

$$\Sigma_1(p^2) = -i(2\pi)^4\alpha\int d^4x\, \delta^4(x)[3\hat{U}_1(x^2, x\cdot p, p^2) - \hat{U}_2(x^2, x\cdot p, p^2)], \tag{2.7}$$

the integral having support at $x = 0$. We further assume sufficient regularity to proceed setting

$$\Sigma_1(p^2) = \lim_{x\to 0\ (x^2\le 0)}\{-i(2\pi)^4\alpha[3\hat{U}_1(x^2, x\cdot p, p^2) - \hat{U}_2(x^2, x\cdot p, p^2)]\}. \tag{2.8}$$

Needless to say, in the absence of a "second-generation cutoff," expression (2.8), being somehow related to an expectation value of the product of two electromagnetic currents at the same space-time point, would be expected to diverge. However, we do have a "second-generation cutoff" by the weak interaction, embodied in our conjecture of smoothness in the behavior of the current commutator at very short distances [whose content is partially carried by Eq. (1.3)]. We shall later see how this condition is implemented in such a way as to obtain from (1.8) a well-defined contribution to the proton-neutron mass difference.

Unfortunately, we do not possess as much of a control on $\Sigma_2(p^2)$. Were we to take our numerical result, Eq. (5.15), more seriously than we do, we would here be arguing that $\Sigma_2(p^2 = m_N^2)$ (m_N being the nucleon physical mass) *must* be negligible with respect to $\Sigma_1(p^2 = m_N^2)$. The argument would presumably be based on the fact that Σ_2 does not have a pointlike support (which, by the way, is what makes dealing with Σ_2 not as simple as with Σ_1). Alternatively, we would resort to a dispersive approach to claim that $\Sigma_2(p^2)$ is "slowly varying" in between zero and the physical nucleon mass. The observation that the function vanishes identically at $p^2 = 0$ would then do the trick and enable us to give the nucleon electromagnetic self-mass the form

$$\delta m \cong \delta m_1 = \Sigma_1(m_N^2) = \lim_{x\to 0\ (x^2\le 0)}\{-i(2\pi)^4\alpha[3\hat{U}_1(x^2, x\cdot p) - \hat{U}_2(x^2, x\cdot p)]\}. \tag{2.9}$$

Since $\hat{U}_1(x^2, x\cdot p)$ are functions presently inaccessible experimentally, Eq. (2.9) cannot be used directly for a numerical evaluation. Fortunately, though, to this purpose, we can usefully exploit the freedom of the direction in the Minkowski space along which the limit in Eq. (2.9) is taken. In the following section, indeed, by taking the limit along a direction "almost" lightlike, we shall be able to express δm in terms of less mysterious quantities.

III. GOING ALONG THE LIGHT CONE

We start by defining the commutator-invariant functions in the usual way

$$\frac{1}{2\pi}\langle p|[J^\mu(x), J^\nu(0)]|p\rangle = [g^{\mu\nu}\Box - \partial^\mu\partial^\nu]\hat{V}_1(x^2, x\cdot p) + [-p^\mu p^\nu\Box + p\cdot\partial(\partial^\mu p^\nu + \partial^\nu p^\mu) - g^{\mu\nu}(p\cdot\partial)^2]\hat{V}_2(x^2, x\cdot p). \quad (3.1)$$

$\hat{V}_1(x^2, x\cdot p)$ and $\hat{V}_2(x^2, x\cdot p)$ are Fourier transforms of experimentally accessible quantities. To make connection with the time-ordered invariant functions, $\hat{U}_1(x^2, x\cdot p)$ and $\hat{U}_2(x^2, x\cdot p)$, we shall approach the light cone. Up to the boundaries of validity of a canonical structure of the singularities, we set

$$\begin{aligned}\hat{V}_1(x^2, x\cdot p) &\cong \epsilon(x\cdot p)\delta(x^2)g_1(x\cdot p) + \epsilon(x\cdot p)\theta(x^2)f_1(x\cdot p),\\ \hat{V}_2(x^2, x\cdot p) &\cong \epsilon(x\cdot p)\theta(x^2)f_2(x\cdot p).\end{aligned} \quad (3.2)$$

Then, from the DGS representations[13]

$$\begin{aligned}\hat{V}_i(x^2, x\cdot p) &= \int_0^\infty da\int_{-1}^{+1} db\,\sigma_i(a, b)e^{-ibx\cdot p}D(x; a+b^2),\\ \hat{U}_i(x^2, x\cdot p) &= \int_0^\infty da\int_{-1}^{+1} db\,\sigma_i(a, b)e^{-ibx\cdot p}D^c(x; a+b^2),\end{aligned} \quad (3.3)$$

we read off the transcription rule

$$D(x; m^2) \rightarrow D^c(x; m^2), \quad (3.4)$$

which, near the light cone, yields the correspondences:

$$\epsilon(x\cdot p)\delta(x^2) \rightarrow \frac{1}{2}\left[\delta(x^2) - \frac{i}{\pi}\frac{1}{x^2}\right], \quad (3.5)$$

$$\epsilon(x\cdot p)\theta(x^2) \rightarrow \frac{1}{2}\left[\theta(x^2) - \frac{i}{\pi}\ln(\tfrac{1}{4}m^2x^2)\right]. \quad (3.6)$$

These, applied to (3.2), give

$$\mathrm{Im}\hat{U}_1(x^2, x\cdot p) \cong [(1/x^2)g_1(x\cdot p) + \ln(\tfrac{1}{4}m^2x^2)f_1(x\cdot p)], \quad (3.7)$$

$$\mathrm{Im}\hat{U}_2(x^2, x\cdot p) = \ln(\tfrac{1}{4}m^2x^2)f_2(x\cdot p). \quad (3.8)$$

We should stress that, for our purposes, we need (3.7) and (3.8) to hold only for small values of $x\cdot p$. This remark justifies the "optimistic attitude" adopted in deriving them.

If now we *assume* the validity of (3.7) and (3.8) for arbitrary small x^2 and substitute them into (2.9), we obtain

$$\delta m_1 = -(2\pi)^4\alpha i \lim_{x\to 0(x^2\cong 0)}\{3g_1(x\cdot p)(1/x^2) + [3f_1(x\cdot p) - f_2(x\cdot p)]\ln(\tfrac{1}{4}m^2x^2)\}. \quad (3.9)$$

This expression would diverge quadratically if $g_1(0)\neq 0$, logarithmically otherwise (which would presumably be the case if the longitudinal scaling function were to vanish).[14] Of course, present experiments *do not* force on us such conclusions since the available energy is not high enough for them to offer information for $x^2 < 10^2\lambda\!\!\!^{-2}$. We are, therefore, free from this point of view, to *destroy the canonical structure* (3.2) by smoothing away the light-cone singularities at very short distances. Finiteness of the nucleon self-mass will then be ensured.

In Sec. IV the possibility is discussed that the theoretical framework of nonpolynomial Lagrangians might provide us in the future with a detailed dynamical answer to this problem.

IV. POSSIBLE MECHANISM FOR SMOOTHING LIGHT-CONE SINGULARITIES

Both the gravitational and the weak interactions[15] are expected to become important at very short distances (10^{-40} cm and 10^{-16} cm, respectively). Since they can all be cast in nonpolynomial forms,[16] we expect them to play the role of cutoff for the electromagnetic and the strong interactions.

It has been shown[10] that the Lagrangian for gravity and electrodynamics can be written approximately as

$$L = L_{\text{gravity}} + \bar{\psi}'(x)\gamma^\mu[i\partial_\mu - e_0 A_\mu(x)]\psi'(x) - \tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}e^{kh^{aa}} - m_0\bar{\psi}'(x)\psi'(x), \tag{4.1}$$

where $\psi'(x)$ is a *transformed* fermion field, kh^{ab} appears in the exponential parametrization of the gravitational field

$$g^{\mu\nu} = [\exp(k\gamma_{ab}h^{ab})]^{\mu\nu}, \tag{4.2}$$

and k is the Newtonian constant. In such nonpolynomial theories, the various Green's functions can be calculated by taking the S matrix up to $i^2T(L_{\text{int}}(x)L_{\text{int}}(0))$ (second order in the *major* coupling constant), but summing up all orders in k, the *minor* coupling constant appearing in the exponential. Such a summation takes into account approximately the effect of multiple emission and reabsorption of particles, and has been demonstrated to smooth away light-cone singularities in the interacting Green's functions. We can understand roughly how the intervention of nonpolynomial interactions can have a smoothing effect on light-cone singularities as follows. Suppose that the free propagator of a field $\phi(x)$ is

$$\Delta_F(x) = \langle 0|T(\phi(x)\phi(0))|0\rangle. \tag{4.3}$$

Then the exact two-point function of the theory is proportional to

$$T(\Delta_F(x)) = \left\langle 0\left|\frac{\delta^2 S}{\delta\phi(x)\delta\phi(0)}\right|0\right\rangle. \tag{4.4}$$

Imagine S to be calculated to second order in the major coupling constant, as alluded to above. For a *nonpolynomial* interaction term, $S_2(x)$ will be a *power series* in $\Delta_F(x)$:

$$S_2(x) = \sum_{n=0}^{\infty} c_n(k^2)^n[\Delta_F(x)]^n. \tag{4.5}$$

While each term in the series is singular as $x^2 \to 0$, it is certainly possible for the sum to be *nonsingular* as $\Delta_F(x) \to \infty$, which corresponds to $x^2 \to 0$. In fact, for many nonpolynomial interactions, this is exactly what happens after the series has been appropriately summed. Note that a polynomial interaction term yields an $S_2(x)$ that is a finite polynomial in $\Delta_F(x)$, and can thus never have the indicated regularizing effect.

In particular, the fermion electromagnetic mass shift has been calculated in this model.[10] The precise result is

$$\delta m = \frac{3\alpha}{4\pi} m_0 \ln(k^2 m_0^2). \tag{4.6}$$

All singular Green's functions turn out to be regularized in this theory, and a general rule emerges that the singularity

$$\lim_{x_\mu \to 0} \ln|x^2| \tag{4.7}$$

in a theory neglecting gravitational effects is actually replaced by $\ln(k^2m_0^2)$. The singularity consistently reappears when one sets $k \to 0$ in the regularized Green's functions. The calculation by itself never employs artificial cutoffs, yet the effect of the inclusion of gravitational effects on the purely electromagnetic interactions of the fermion has been to replace divergences of $\ln(0)$ by the finite quantity $\ln(k^2m_0^2)$. Thus the result of the gravitational perturbation is the institution of a fundamental unit of length that simulates an *effective* universal cutoff for all Green's functions. We expect this rule to work in general for nonpolynomial interactions (at least of this exponential form) and conjecture that the inclusion of the strong interactions for the fermion should not change the effective cutoff mandated by the nonpolynomial perturbation.

We now turn to the weak interaction of a W boson of mass M_W with a massive fermion. Suppressing inessential internal quantum numbers, we represent the weak Lagrangian schematically as

$$L_{\text{int}} = -g\bar{\psi}(x)\gamma^\mu(1-\gamma_5)\psi(x)W_\mu(x), \tag{4.8}$$

where $g^2 = Gm_0^2$, and G is the usual Fermi constant. The W^μ field can be decomposed meaningfully according to Stückelberg[17] as follows

$$W^\mu(x) = \phi^\mu(x) + \frac{1}{M_W}\partial^\mu B(x), \tag{4.9}$$

where $B(x)$ is a massive scalar field. Since physics is invariant under the addition of any total derivative to the Lagrangian, we can rewrite

$$\begin{aligned} L_{\text{int}} &= -g\bar{\psi}(x)\gamma^\mu(1-\gamma_5)\psi(x)\phi_\mu(x) + \frac{g}{M_W}\partial_\mu[\bar{\psi}(x)\gamma^\mu(1-\gamma_5)\psi(x)]B(x) \\ &= -g\bar{\psi}(x)\gamma^\mu(1-\gamma_5)\psi(x)\phi_\mu(x) - \frac{g}{M_W}2im_0\bar{\psi}(x)\gamma_5\psi(x)B(x). \end{aligned} \tag{4.10}$$

It is precisely the nonconservation of the weak current that accounts simultaneously for the nonrenormalizability of the usual polynomial formulation of the weak Lagrangian and for the emergence of its essentially nonpolynomial character. If such a transformation is applied to the usual quark-gluon model, for example, the interaction term involving the scalar daughter of the vector gluon decouples entirely from the Lagrangian.

The Lagrangian for electrodynamics and weak interactions is now

$$L=\bar{\psi}(x)\gamma^\mu[i\partial_\mu-e_0A_\mu(x)]\psi(x)-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu}-m_0\bar{\psi}(x)\psi(x)-g\bar{\psi}(x)\gamma^\mu(1-\gamma_5)\psi(x)\phi_\mu(x)-\frac{g}{M_W}2im_0\bar{\psi}(x)\gamma_5\psi(x)B(x). \quad (4.11)$$

The nonpolynomial character is revealed by the field transformation

$$\psi(x)\to\psi'(x)=e^{i\sqrt{G}\gamma_5B(x)}\psi(x), \quad (4.12)$$

$$A^\mu(x)\to A^{\mu\prime}(x)=e^{i\sqrt{G}\gamma_5B(x)}A^\mu(x). \quad (4.13)$$

Then the combined electromagnetic and weak Lagrangian becomes

$$L=L_{\text{free}}(\psi')+L_{\text{free}}(A^{\mu\prime})-g\bar{\psi}'(x)\gamma^\mu(1-\gamma_5)\psi'(x)\phi_\mu(x)-\frac{g}{M_W}2im_0\bar{\psi}'(x)\gamma_5e^{-i2\sqrt{G}\gamma_5B(x)}\psi'(x)B(x)$$
$$-e_0\bar{\psi}'(x)\gamma^\mu\psi'(x)A'_\mu(x)e^{-i\sqrt{G}\gamma_5B(x)}. \quad (4.14)$$

The electromagnetic interaction term now has an extra exponential dependent upon the field of the weak-interaction quantum. The Lagrangian in question becomes rather similar to the gravity-electromagnetism Lagrangian. We can therefore argue that the weak interaction here also introduces a fundamental unit of length which serves as an effective cutoff for divergent quantities.[18] It is our conjecture that, with the inclusion of weak interactions to second order in the major coupling constant, a divergent quantity like $\ln(0)$ would be regularized to $\ln(Gm_0^2)$. It is certainly desirable to have detailed dynamical calculations performed to verify this conjecture. We might add that the self-mass will be given by contributions from diagrams of the type shown in Fig. 2, which indicates the self-energy due to the emission and subsequent absorption at the same space-time points of one single photon and a multitude of weakly interacting quanta.

V. NUMERICAL EVALUATION

To compute numbers, we first have to relate our causal commutator functions in configuration space, $\hat{V}_1(x^2, x\cdot p)$ and $\hat{V}_2(x^2, x\cdot p)$, to the experimentalist's structure functions in momentum space $W_1(q^2,\nu)$ and $W_2(q^2,\nu)$. In terms of the tensor

$$C_{\mu\nu}(q^2,\nu)=\frac{1}{2\pi}\int d^4x\,e^{iqx}\langle p|[J_\mu(x),J_\nu(0)]|p\rangle, \quad (5.1)$$

the latter are defined by

$$C_{\mu\nu}(q^2,\nu)=\left(p_\mu-\frac{\nu q_\mu}{q^2}\right)\left(p_\nu-\frac{\nu q_\nu}{q^2}\right)W_2(q^2,\nu)-\left(g_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)W_1(q^2,\nu), \quad (5.2)$$

and the connection reads

$$W_1(q^2,\nu)=\int d^4x\,e^{iqx}[q^2\hat{V}_1(x^2,x\cdot p)-\nu^2\hat{V}_2(x^2,x\cdot p)], \quad (5.3)$$

$$W_2(q^2,\nu)=\int d^4x\,e^{iqx}[q^2\hat{V}_2(x^2,x\cdot p)]. \quad (5.4)$$

Then we have to supplement the experimental information, insufficient by itself, with a conjecture relating W_1 to W_2. We choose to impose the condition that the longitudinal structure function

$$W_L(q^2,\nu)=W_1(q^2,\nu)-\left(1-\frac{\nu^2}{q^2}\right)W_2(q^2,\nu) \quad (5.5)$$

vanishes in the A limit ($q^2\to\infty$ with $\omega=-q^2/2\nu$ fixed) fast enough to ensure

$$\nu W_L(q^2,\nu)\xrightarrow{A}0. \quad (5.6)$$

We should mention that our condition (5.6), being an asymptotic condition, considerably relaxes the much stronger Lee-Pagels[19,20] hope

$$W_L(q^2,\nu)=0, \quad (5.7)$$

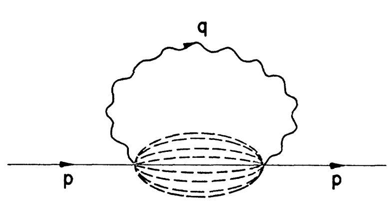

FIG. 2. Possible term in $\Sigma(p)$ where weak damping operates. Dotted lines represent multiple emission of the B field.

which is in agreement with the data. We now proceed to extract from (5.6) the information we need. From (5.5), (5.4), and (3.2), we obtain

$$\nu W_L(q^2,\nu)=\nu q^2\int d^4x\, e^{iqx}[\hat{V}_1(x^2,x\cdot p)-\hat{V}_2(x^2,x\cdot p)]$$
$$\overset{A}{\to}\nu q^2\int d^4x\, e^{iqx}\epsilon(x\cdot p)\{\delta(x^2)g_1(x\cdot p)+\theta(x^2)[f_1(x\cdot p)-f_2(x\cdot p)]\}. \tag{5.8}$$

Then, the usual rules

$$\int d^4x\, e^{iqx}\epsilon(x\cdot p)\delta(x^2)h(x\cdot p)\overset{A}{\to}\frac{i\pi}{\nu}\int d(x\cdot p)e^{-i\omega x\cdot p}h(x\cdot p), \tag{5.9}$$

$$\int d^4x\, e^{iqx}\epsilon(x\cdot p)\theta(x^2)h(x\cdot p)\overset{A}{\to}\frac{2\pi}{\nu^2}\int d(x\cdot p)e^{i\omega x\cdot p}(x\cdot p)h(x\cdot p),$$

and (5.6) imply

$$g_1(x\cdot p)=0,$$
$$f_1(x\cdot p)=f_2(x\cdot p). \tag{5.10}$$

It is worth pointing out that, since we shall use the constraints (5.10) only at $x\cdot p=0$, for our present purposes the condition (5.6) could have been further weakened. We are now ready to let our only experimental input come in the form of the Callan-Gross integral

$$f_2(0)=\frac{i}{(2\pi)^2}\int_0^1 d\omega\, F_2(\omega), \tag{5.11}$$

where $F_2(\omega)=\lim_A \nu W_2(q^2,\nu)$. Substituting (5.10) and (5.11) into (3.9), together with the imposition of our smoothness requirement, Eq. (1.3) [which, in this case, amounts to the rule (1.2)], gives

$$\delta m_1=2\alpha(2\pi)^2\ln(Gm_N{}^2)\int_0^1 d\omega\, F_2(\omega). \tag{5.12}$$

The value of the Callan-Gross integrals for the photon and the neutron may be read off from a recent phenomenological analysis [21]

$$\int_0^1 d\omega\, F_2^p(\omega)=0.1\bar{6}5\frac{m_N}{(2\pi)^3},$$
$$\int_0^1 d\omega\, F_2^n(\omega)=0.110\frac{m_N}{(2\pi)^3}. \tag{5.13}$$

These numbers and the well-known value of the Fermi coupling constant ($Gm_p{}^2=1.026\times10^{-5}$) yield the proton and neutron self-masses

$$\delta m_1^p=-4.08\text{ MeV},\quad \delta m_1^n=-2.74\text{ MeV}, \tag{5.14}$$

and the proton-neutron mass difference

$$\delta m^*=\delta m_1^p-\delta m_1^n=-1.34\text{ MeV}, \tag{5.15}$$

a result embarrassing when compared with the experimental value [22]

$$m_p-m_n=-1.293\pm0.01\text{ MeV}. \tag{5.16}$$

It appears difficult to quote errors on (5.14) and (5.15).

We should also emphasize again that Eq. (5.15) leaves out the term $\Sigma_2(m_N{}^2)$. When evaluated in momentum space, this piece receives contributions from both the scaling region, in the form of a Callan-Gross integral times a naively divergent quantity, and the low-energy region. It is our conjecture that these two contributions approximately cancel each other.

How does our result (5.15) compare with the conventional momentum-space analysis?

The Lee-Pagels conjecture, with the most recent data (5.13) and a weak-interaction effective cutoff, makes the inelastic contribution:

$$\delta m_\infty=-\tfrac{3}{2}\alpha(2\pi)^2\ln(Gm_N{}^2)\int_0^1 d\omega\, F_2^{p-n}(\omega)$$
$$\cong -1.0\text{ MeV}.$$

It seems, then, *if* such conjecture is right, that an essential cancellation between the Born-term,[23] the fixed-pole,[24] and the ordinary Regge contributions should occur. This does not seem impossible in the light of the analysis of Ref. 24, but, as pointed out there, much more detailed knowledge of various momentum-space quantities has to be gathered before one is able to draw any firm conclusion.

VI. SU(3) TRANSFORMATION PROPERTIES

In this section we shall see what can be learned about the algebraic patterns of electromagnetic mass differences. We assume that the singularities near the light cone are smoothed away one way or another, and we also assume that the algebraic properties of the bilocal operators multiplying these singularities are not changed in the process. We specialize to the quark-gluon model, and shall exhibit the explicit form of the bilocal *operators* associated with the leading singularities.

The mass shift is given by Eq. (2.2) taken on the mass shell ($p^2=m_N{}^2$):

$$\delta m=-i\alpha(2\pi)^4\int d^4x\, D_0^c(x)\hat{T}(x^2,x\cdot p,m_N{}^2). \tag{6.1}$$

In the usual framework of canonical singularities, one would conclude that the integrand is peaked (divergent) at $x^2=0$, and finite everywhere else.

Thus the *"divergent" piece* of the mass shift is calculable by taking $\hat{T}(x)$ to have the form valid near the light cone, i.e., the form given by operator-product expansions near the light cone:

$$T(J_\mu(x)J_\nu(0)) \underset{x^2\to 0}{\sim} (\partial_\mu\partial_\nu - g_{\mu\nu}\partial^2)\frac{1}{x^2-i\epsilon}\sum_n x^{\alpha_1}\cdots x^{\alpha_n}R_{0\alpha_1\cdots\alpha_n}(0)$$
$$+[g_{\mu\nu}\partial_\alpha\partial_\beta - g_{\alpha\nu}\partial_\beta\partial_\mu - g_{\alpha\mu}\partial_\beta\partial_\nu + g_{\alpha\mu}g_{\beta\nu}\partial^2]\ln(x^2-i\epsilon)\sum_n x^{\alpha_1}\cdots x^{\alpha_n}R^{\alpha\beta}_{2\alpha_1\cdots\alpha_n}(0). \tag{6.2}$$

The divergent piece is therefore proportional to the matrix elements of the infinite sum of local operators (which can be summed formally to give a bilocal operator). One of the ways to obtain explicit forms of the operators in the canonical quark-gluon model relevant to the divergent piece proceeds as follows. We consider for a moment the form of the *commutator* of the currents near the light cone:

$$\frac{1}{2\pi}\langle p|[J^\mu(x),J^\nu(0)]|p\rangle \underset{x^2\to 0}{\sim} (g^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu)\epsilon(x\cdot p)\theta(x^2)f_1(x^2,x\cdot p) + [-p^\mu p^\nu\partial^2 + p\cdot\partial(\partial^\mu p^\nu + p^\mu\partial^\nu) - g^{\mu\nu}(p\cdot\partial)^2]$$
$$\times\epsilon(x\cdot p)\theta(x^2)f_2(x^2,x\cdot p). \tag{6.3}$$

We have set the longitudinal scaling function equal to zero, in accordance with the model. Now $f_1(x^2, x\cdot p)$ and $f_2(x^2, x\cdot p)$ are the quantities multiplying the c-number "infinity." We have explicit knowledge of the light-cone commutators of the model from the technique of quantization in the infinite-momentum frame.[25] Taking $\mu=+$, $\nu=-$, and $x^+=0$,

$$[J^+(x),J^-(0)]_{x^+=0} = -\tfrac{1}{4}d^{QQc}\big(\partial_-\epsilon(x^-)\delta^2(\vec{\mathrm{x}})[V_c^-(x|0)-V_c^-(0|x)]$$
$$+\tfrac{1}{2}\partial_i\epsilon(x^-)\delta^2(\vec{\mathrm{x}})\{[V_c^i(x|0)-V_c^i(0|x)] - i\epsilon^{ij}[A_j^c(x|0)+A_j^c(0|x)]\}\big), \tag{6.4}$$

where

$$\{\tfrac{1}{2}\lambda_Q, \tfrac{1}{2}\lambda_Q\} = d^{QQc}\tfrac{1}{2}\lambda_c \tag{6.5}$$

and

$$\lambda_Q = \lambda_3 + (\tfrac{1}{3})^{1/2}\lambda_8\,. \tag{6.6}$$

Explicitly,

$$d^{QQc}V_c^\mu = \tfrac{8}{9}(\tfrac{3}{2})^{1/2}V_0^\mu + \tfrac{2}{3}J^\mu\,. \tag{6.7}$$

We define the matrix elements of the bilocal electromagnetic current $J^\mu(x|0)=\bar\psi(x)\gamma^\mu\lambda_Q\psi(0)$,

$$i\langle p|J^\mu(x|0) - J^\mu(0|x)|p\rangle = p^\mu A(x^2, x\cdot p) + x^\mu B(x^2, x\cdot p), \tag{6.8}$$

and it is easy to see (the singlet term does not contribute),

$$\partial_i p^i\pi x^- p^+(-2\pi)f_2(0, x^-p^+) = \langle p|\partial_i(-\tfrac{1}{12})J^i(0,x^-,\vec{0}|0)|p\rangle \tag{6.9}$$

and

$$\partial_-[\pi x^- 2\pi f_1(0,x^-p^+) + 2p^-\pi x^- p^+(-2\pi)f_2(0,x^-p^+)] = \langle p|\partial_-(-\tfrac{1}{6})J^-(0,x^-,\vec{0}|0)|p\rangle, \tag{6.10}$$

so that

$$\lambda f_2(0,\lambda) = \frac{-i}{24\pi^2}A(0,\lambda), \tag{6.11}$$

$$f_1(0,\lambda) = +\frac{i}{12\pi^2}B(0,\lambda). \tag{6.12}$$

The divergent piece of the mass shift can be put in the form

$$\delta m_{\mathrm{div}} = -\frac{2\pi^2}{3}\int d^4x\,D_0^c(x)\{3\partial^2\ln(-x^2+i\epsilon)A(0,\lambda) + [\partial^2 + 2(p\cdot\partial)^2]\ln(-x^2+i\epsilon)\,2B(0,\lambda)\}$$
$$= \alpha F[A(0,\lambda)] + \beta G[B(0,\lambda)], \tag{6.13}$$

where $F[A(0,\lambda)]$ and $G[B(0,\lambda)]$ indicate quantities dependent on $A(0,\lambda)$ and $B(0,\lambda)$, respectively, while α and β are "divergent" constants depending purely upon the c-number singular functions in the operator-product expansion.

It is clear that we have essentially a *universal* bilocal *operator* $J^\mu(x|0)$ whose matrix element between *arbitrary* particle states yields the coefficient multiplying a divergent constant. Since the bilocal electromagnetic current is an octet operator, it does not contribute to $\Delta I = 2$ mass differences. This means that the $\Delta I = 2$ mass differences receive contributions only from integration of finite (and fast decreasing at infinity) quantities over all space, and not from naively divergent quantities near the light cone. Thus, there are no subtleties at $x^\mu \to 0$ (or, equivalently, as $q^\mu \to \infty$) for this case, so that they are amenable to calculation by saturation with low-lying (in momentum space) states. For the $\Delta I = 1$ mass differences, our operator statement merely reproduces many well-known algebraic results (dependent only upon Clebsch-Gordan coefficients) also derived in the tadpole model.[26] Analogous conclusions were earlier obtained by Bjorken,[27] using the $q_0 \to i\infty$ method, and relating these coefficients to $[\dot{J}_i, J_k]$ equal-time commutators. Our configuration-space considerations using the canonical quark-gluon light-cone algebra is seen to be equivalent to this particular application of the Bjorken limit and equal-time commutators.

We might also mention that the *empirical* success of these algebraic relations supports the hypothesis that the c-number singular functions and the bilocal operators can be considered *separately* from each other. We see that the observed patterns of mass differences are well accounted for by assuming that the singular functions actually achieve a *finite* limit independent of the operators, and that the algebraic properties (for one) of the operators are *unchanged* at ultrashort distances.

Note added in proof. After this work was submitted for publication, we were informed by Roman Jackiw that he and Howard Schnitzer have independently obtained results [Phys. Rev. D 5, 2008 (1972)] that are essentially equivalent to our Sec. VI.

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Model-Independent Deep-Inelastic Electroproduction Sum Rules*

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Using the most general form for the null-plane commutator of the electromagnetic current, sum rules are derived for electroproduction in the deep-inelastic region. The structure of these sum rules is discussed and a free, massless, fermion model is considered as an illustration.

I. INTRODUCTION

The observation[1] that Bjorken scaling[2] may be interpreted in terms of the behavior of current commutators on the light cone has revived interest in commutators restricted to a null plane.[3,4] In particular, Dicus, Jackiw, and Teplitz[4] have shown that null-plane current commutators can be used to derive improved fixed-mass sum rules as well as deep-inelastic sum rules. A crucial issue in the use of null-plane commutators is their model dependence; for they seem to be much more model-dependent than equal-time commutators. While this dependence presents an opportunity for testing various models experimentally, it does not always permit one to easily recognize aspects of a problem which are the result of more general considerations such as covariance and causality. In this paper we determine those characteristics of deep-inelastic electroproduction sum rules which are independent of any particular model for the null-plane commutators of the electromagnetic current. The complete set of sum rules is derived by assuming the most general form for the null-plane commutators. As an example of how particular models may be examined, we consider a free, massless, fermion model. It is shown that the condition for finite electromagnetic self-masses found by Pagels[5] is satisfied in this model.

After presenting several preliminaries related to the Compton amplitude in Sec. II, the sum rules are derived in Sec. III. In Sec. IV the fermion model is considered.

II. COMPTON AMPLITUDE

The forward virtual Compton amplitude for photons of mass squared q^2 scattering from an unpolarized hadronic target having momentum p is[6]

$$T_{\mu\nu}(p,q)=i\int d^4x\, e^{iqx}\langle p|T^*(V_\mu(x),V_\nu(0))|p\rangle$$

$$=A^2_{\mu\nu}T_2(q^2,\nu)+A^L_{\mu\nu}T_L(q^2,\nu)\,, \tag{2.1}$$

$$A^2_{\mu\nu}=\frac{1}{M^2}\left(p_\mu p_\nu-\frac{\nu}{q^2}(p_\mu q_\nu+p_\nu q_\mu)+\frac{\nu^2}{q^2}g_{\mu\nu}\right) \tag{2.2a}$$

$$A^L_{\mu\nu}=-g_{\mu\nu}+q_\mu q_\nu/q^2\,, \tag{2.2b}$$

where V_μ is the hadronic electromagnetic current, $\nu=p\cdot q$, M is the hadron's mass, and the hadronic state is normalized according to

$$\langle p|p'\rangle=\frac{E}{M}(2\pi)^3\delta^{(3)}(\vec{p}-\vec{p}\,')\,.$$

For q^2 spacelike, we assume[7] T_2 satisfies an unsubtracted dispersion relation in ν and T_L obeys a once-subtracted dispersion relation

$$T_2(q^2,\nu)=\frac{1}{\pi}\int_0^\infty d\nu'^2\frac{\mathrm{Im}\,T_2(q^2,\nu')}{\nu'^2-\nu^2-i\epsilon}\,, \tag{2.3a}$$

$$T_L(q^2,\nu)=T_L(q^2,0)+\frac{\nu^2}{\pi}\int_0^\infty\frac{d\nu'^2}{\nu'^2}\frac{\mathrm{Im}\,T_L(q^2,\nu')}{\nu'^2-\nu^2-i\epsilon}\,. \tag{2.3b}$$

Introducing the dimensionless functions

$$F_2(q^2,\omega)=\frac{\nu}{M\pi}\mathrm{Im}\,T_2(q^2,\nu)\,, \tag{2.4a}$$