and we can use it to construct the most general static solution by simple differentiation. This solution involves two arbitrary constants as in the metric given by Janis, Newman, and Winicour.[5]

To obtain the reduced ADM Hamiltonian we again impose the coordinate condition (22) and obtain for the action

$$I = 2\pi \int (\pi_\phi \dot{\phi} - \mathcal{H})\, dr\, dt\,, \tag{45}$$

where the Hamiltonian density $\mathcal{H}$ is given by

$$\mathcal{H} = 2\mu'\,. \tag{46}$$

The elimination of the Hamiltonian constraint can be effected by solving the first-order equation

$$4r\mu' + 2e^{2\mu} - 2 = \frac{1}{4r^2}\pi_\phi{}^2 + r^2\phi'^2\,, \tag{47}$$

and this gives us the reduced Hamiltonian density in terms of quadratures,

$$\mathcal{H} = \mathcal{H}_\phi + \frac{\exp[\int^r \mathcal{H}_\phi(\bar{r})\,d\bar{r}]}{2M - \int^r d\bar{r}\, \exp[\int^{\bar{r}} \mathcal{H}_\phi(r^*)\,dr^*]}\,, \tag{48}$$

where

$$\mathcal{H}_\phi = \frac{1}{2r}\left(\frac{1}{4r^2}\pi_\phi{}^2 + r^2\phi'^2\right) \tag{49}$$

and M is a constant of integration. Finally, by the use of similar methods we can obtain an ADM Hamiltonian for the massive scalar field where the $\exp\int \mathcal{H}(r)dr$ terms in Eq. (48) acquire the factor $(1 + \frac{1}{4}mr^2\phi^2)$ with m the mass of the scalar field.

The nonlocal nature of the Hamiltonian density in Eq. (48) is a discouraging feature of the present formulation. However, the explicit form of the ADM Hamiltonian is strongly dependent upon the choice of coordinate conditions and it is quite possible that alternative coordinate conditions can lead to a more tractable Hamiltonian.

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[1]P. A. M. Dirac, Proc. Roy. Soc. (London) A246, 333 (1958).

[2]R. Arnowitt, S. Deser, and C. W. Misner, in *Gravitation: An Introduction to Current Research*, edited by L. Witten (Wiley, New York, 1962), Chap. 7. This work will hereafter be referred to as ADM.

[3]C. W. Misner, in *Magic Without Magic—J. A. Wheeler 60th Anniversary Volume*, edited by J. Klauder (W. H. Freeman, San Francisco, to be published).

[4]K. Kuchař, Phys. Rev. D 4, 955 (1971).

[5]A. I. Janis, E. T. Newman, and J. Winicour, Phys. Rev. Letters 20, 878 (1968).

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Solutions of the Two-Body Problem in Classical Action-at-a-Distance Electrodynamics: Straight-Line Motion

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The equations of motion for two classical particles with equal charges and masses are solved for the case of motion confined to one dimension by the initial conditions. The charges interact via half-retarded plus half-advanced Liénard-Wiechert potentials. The equations, which are of differential-difference type, are solved by successive approximations and the solutions are compared with those obtained by perturbation theory. The results suggest the existence of a minimum distance of closest approach.

I. INTRODUCTION

Understanding of the classical relativistic two-body problem is progressing slowly. The particular example which has been studied most intensively is the system of two charged particles which interact via half-retarded plus half-advanced Liénard-Wiechert potentials. The equations of motion and conserved quantities for this system are derived from the time-symmetric Fokker action principle.[1] It is imperative that this problem be better understood at the classical level, not only because of the hope of finding alternative quantized theories of electromagnetism,[2] but also because of its relation to quantum field theories with an indefinite metric.[3]

The only known exact solutions are the circular orbits found by Schild.[4] Solutions are also known to approximate equations of motion which are power series in the small dimensionless numbers v/c, m_1/m_2, and e^2/rmc^2, where r is the interparticle distance.[5-7] Recently we have studied small perturbations to circular orbits without recourse to any of these expansions.[8] The present work is an investigation of the collision of two particles of equal charge and mass, initially moving toward each other along a straight line. We do not use any perturbation methods.

The equations of motion for this problem are difference-differential equations and hence are difficult to solve. They can be reduced to second-order differential equations by expansion in powers of v/c or e^2/rmc^2. For the latter case Hill[9] has computed the first three terms in the expression for the acceleration of the particles and finds that the third term is already quite complicated in structure, containing logarithms and many powers of velocity. Nevertheless, these approximate equations can be solved without difficulty by computer. Our approach is altogether different. It was first suggested by Synge[6] for retarded interactions and later by van Dam and Wigner[10] for time-symmetric interactions. The method is one of successive approximations and has, as far as we know, never been actually tried out.

The acceleration of particle one at a certain time depends upon its own position and velocity at that point in time and upon the position and velocity of particle two at an earlier and at a later point in time. These points are the intersections of the light cone centered at particle one with the world line of particle two. In order to find these points, one obviously needs to know the world lines, i.e., one needs the solution of the problem. To get around this difficulty we assume an arbitrary world line for particle one and its mirror image for particle two. We integrate the equations of motion to obtain a better approximation for the world line of particle one, and use its mirror image as the approximation for the world line of particle two and so forth. Whether or not this procedure would work was a matter of conjecture. To quote van Dam and Wigner[10]: "The procedure can be continued and will surely converge in many cases."

In fact we have found that this iteration procedure, when performed by computer, does not converge at all, even for very low velocities, in the case of opposite charges in nearly circular orbits. It does, however, converge for the problem described in this paper, provided the initial velocities are not too large. We define the low-energy regime by $1 \lesssim \gamma \lesssim 1.2$ $[\gamma \equiv (1-v^2/c^2)^{-1/2}]$ the intermediate-energy region by $1.2 \lesssim \gamma \lesssim 3.4$, and the high-energy region by $\gamma \gtrsim 3.4$. Both perturbation theory and our method work well at low energies. The iterative procedure also converges at intermediate energies where perturbation results are completely wrong. We know of no method for solving the equation of motion at high energy.

Some of the features of the solutions at intermediate energies, which correspond to distances of closest approach on the order of a classical electron radius e^2/mc^2, are surprising. We hope that our solutions will suggest new and more powerful analytic methods for solving this problem, which is a simple version of the unsolved classical electron-electron problem.

The fact that our method does not extend to high energies is very disappointing for the following reason: For almost circular orbits there exist more than one solution for the same initial conditions.[8] This fact is interpreted as evidence for the "corporate degrees of freedom" first suggested by Plass.[11] They make their appearance only at high energies and we had hoped to investigate their existence or absence in this problem. Until a better method of solution is found, that question remains open.

The plan of work is as follows: In Sec. II we present the equations of motion and the expressions for the constants of the motion. Section III deals with techniques of calculation and Sec. IV with results.

II. EQUATIONS OF MOTION AND CONSERVED QUANTITIES

The equations of motion derived from the Fokker action principle[1] are

$$m_i \ddot{x}_i^{\mu} = e_i F_{\nu}{}^{\mu} \dot{x}_i^{\nu},$$
$$F_{\mu\nu} \equiv \frac{\partial A_\nu}{\partial x_i^\mu} - \frac{\partial A_\mu}{\partial x_i^\nu}, \tag{2.1}$$
$$A^{\lambda}(x_i) \equiv e_j \int_{-\infty}^{\infty} \delta(x_{12}{}^2) \dot{x}_j^{\lambda} d\tau_j,$$

where $i=1$, $j=2$ or $i=2$, $j=1$ and $x_{12}{}^2 = (x_1 - x_2)^\mu \times (x_1 - x_2)_\mu$. The dots represent differentiation with respect to the appropriate proper time τ_i. The integral can be evaluated and yields one half the sum of the retarded and advanced Liénard-Wiechert potentials.

We now assume the special case $m_1 = m_2 = m$ and $e_1 = e_2 = e$. We further assume that the particles start off on the x axis traveling toward each other, and scatter in such a way that particle one is always on the positive x axis and particle two on the negative x axis. The equation of motion for particle one then does not contain the magnetic field and simplifies to the one-dimensional equation[9]

$$m\frac{dv_1}{dt}=(1-v_1{}^2)^{3/2}eE\,,$$
$$E=\tfrac{1}{2}e\left(\frac{1+v_2}{(x_1-x_2)^2(1-v_2)}\bigg|_{\rm ret}+\frac{1-v_2}{(x_1-x_2)^2(1+v_2)}\bigg|_{\rm adv}\right). \tag{2.2}$$

Equation (2.2), together with its counterpart for particle two, forms a system of two second-order difference-differential equations in two unknowns. However, since we always assume $x_2(t)=-x_1(t)$ and $v_2(t)=-v_1(t)$, we have in effect a single equation in a single unknown. This paper concerns the solution of this equation.

Invariance of the Fokker action under the inhomogeneous Lorentz transformations and dilatations implies the existence of 11 conserved quantities. Associated with dilatations is the scalar[12]

$$D(\tau_1,\tau_2)=\sum_{i=1}^{2}(P_i^\mu x_{i\mu}-m_i\tau_i)-e_1e_2\left\{\int_{\tau_1}^{\infty}d\tau_1'\int_{-\infty}^{\tau_2}d\tau_2'-\int_{-\infty}^{\tau_1}d\tau_1'\int_{\tau_2}^{\infty}d\tau_2'\right\}\times\delta'(x_{12}{}^2)(x_1-x_2)^\mu(x_1+x_2)_\mu\dot{x}_1^\nu\dot{x}_{2\nu}\,, \tag{2.3}$$

where

$$P_i^\mu\equiv m_i\dot{x}_i^\mu+e_iA^\mu(x_i)$$

and the prime on the δ function represents differentiation with respect to its argument. Equations (2.1) imply that $dD/d\tau_1=dD/d\tau_2=0$. Translation invariance guarantees conservation of the energy-momentum four-vector

$$P^\mu(\tau_1,\tau_2)=\sum_{i=1}^{2}P_i(\tau_i)+2e_1e_2\left\{\cdots\right\}\dot{x}_1^\nu\dot{x}_{2\nu}(x_1-x_2)^\mu\delta'(x_{12}{}^2)\,, \tag{2.4}$$

where the curly bracket is identical to the one in Eq. (2.3). Rotation invariance in Minkowski space yields the conserved angular momentum tensor

$$M^{\mu\nu}(\tau_1,\tau_2)=\sum_{i=1}^{2}(x_i^\mu P_i^\nu-x_i^\nu P_i^\mu)+2e_1e_2\left\{\cdots\right\}[\dot{x}_1^\rho\dot{x}_{2\rho}(x_1^\nu x_2^\mu-x_1^\mu x_2^\nu)\delta'(x_{12}{}^2)-\tfrac{1}{2}\delta(x_{12}{}^2)(\dot{x}_1^\nu\dot{x}_2^\mu-\dot{x}_1^\mu\dot{x}_2^\nu)]\,. \tag{2.5}$$

The opening term in the expansion of E in Eq. (2.2) about the equal-time point on the world line of particle two is given by[9]

$$E^{(1)}=\frac{e(1-v_1^2)}{(x_1-x_2)^2}\,, \tag{2.6}$$

which is evaluated at time t rather than at the retarded or advanced points. Since additional terms contain higher powers of $1/(x_1-x_2)$, this is the driving term in the asymptotic region. The solution of the asymptotic equation is[13]

$$x_1=Vt-\frac{e^2}{4V^2}(1-V^2)^{5/2}\ln\frac{t}{t_0}\,, \tag{2.7}$$

where t_0 is a constant of the motion and V is the asymptotic velocity. The fact that the logarithm survives has an amusing consequence. Let the asymptotic expression for a constant of the motion C be written $C=C_F+C_I$ where C_F is the expression for the case of free particles. Then, if C_F contains the position explicitly, it may happen that the logarithm in Eq (2.7) generates a time-dependent term which must be canceled by C_I. This asymptotically nonvanishing interaction contribution to a constant of the motion has been discussed[13] for $M_{\mu\nu}$, but we find that it also exists for D. In contrast, the asymptotic expression for the energy-momentum vector is simply

$$\lim_{t\to\pm\infty}P^\mu=\sum_{i=1}^{2}m\dot{x}_1^\mu, \tag{2.8}$$

with no interaction contribution.

We use the energy P^0 as a check on the correctness of the solutions of the equation of motion. Asymptotically it is given by

$$P^0(t_1=t_2=\pm\infty)=2m(1-V^2)^{-1/2}\,. \tag{2.9}$$

In general, the energy is given by Eq. (2.4). The δ function enables us to perform one integration and the special symmetry reduces the curly bracket to one integral. For simplicity we evaluate P^0 at the point of closest approach where $t_1=t_2=0$ and $v_1=v_2=0$. The notation is defined in Fig. 1. In the integrand $(x_1t_1v_1a_1)$ and $(x_2t_2v_2a_2)$ are understood to be evaluated at lightlike points, as indicated by the dotted line. It is typical that the integrals in the constants of the motion have finite range. After some algebra the expression for the energy becomes

$$P^0(t_1^0=t_2^0=0)=2m-2e^2[(t_2^r-t_1^0)-(x_2^r-x_1^0)v_2^r]^{-1}+e^2[(t_2^0-t_1^a)-(x_2^0-x_1^a)v_1^a]^{-1}$$

$$-e^2\int_0^{t_1^a} dt_1 \frac{(1-v_1v_2)(t_1-t_2)}{[(t_1-t_2)-(x_1-x_2)v_2]^2}\left[\frac{v_1a_2}{1-v_1v_2}+\frac{1}{t_1-t_2}-\frac{1-v_2{}^2+(x_1-x_2)a_2}{t_1-t_2-(x_1-x_2)v_2}\right]. \tag{2.10}$$

The second term arises from evaluating $eA^0(0)$, the third from end-point contributions from the double integral. For the special point at which we are evaluating P^0, the second term has twice the magnitude of the third term. If the world lines are solutions of the equations of motion, then numerical evaluation of Eqs. (2.9) and (2.10) must give the same result.

III. METHOD OF CALCULATION

Our starting point is the one-particle equation formed from Eq. (2.2). The natural units of length and time in this equation are e^2/mc^2 and e^2/mc^3. We set both equal to unity. Our method is to fix a distance d of closest approach (understood to occur at $t=0$) and assume that the acceleration is a symmetric function of time. Time is parametrized so that the steps increase in length when the particles are far apart. We let

$$t=T\tan\tfrac{1}{2}\theta\pi, \tag{3.1}$$

where T is a characteristic time and $-1<\theta<1$ is divided into several hundred steps of equal size. The velocity and acceleration change rapidly in the region $-T<t<T$ but the light cone of a particle within this time interval may intersect the world line of the other particle at times much larger than T.

After assuming an acceleration function we integrate to find the velocities and positions at the same points as chosen previously, namely the $2n-3$ points with $\theta=m/n$ and $m=0, \pm1, \pm2, \ldots, \pm(n-2)$. Our method of integration is to approximate the integral between m and $m+1$ by

$$\int_{t(m)}^{t(m+1)} f(t)dt \cong \sum_{k=-2}^{+2} w_k f(t(m+k))\frac{dt}{d\theta}(m+k), \tag{3.2}$$

where the weight functions for the equally spaced θ points were determined by a method discussed by Hamming.[14] We thus need to approximate the integrand at $m=n-1$ in order to arrive at the integral for $m=n-2$. For this we set $a(m-1)=0$ and $v(m-1)=v(m-2)$.

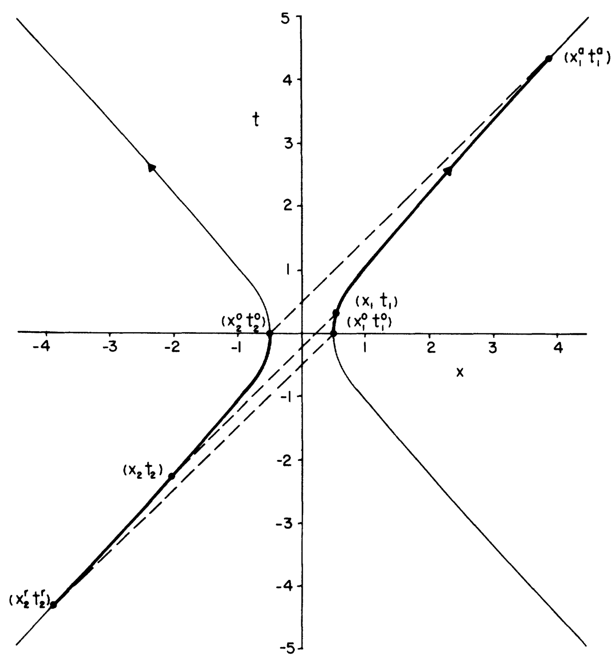

FIG. 1. The world lines of two equal-mass particles for straight-line motion with distance of closest approach $d=e^2/mc^2=1$ and a final velocity $V=0.888$. The labeling defines the notation of Eqs. (2.4) and (2.10).

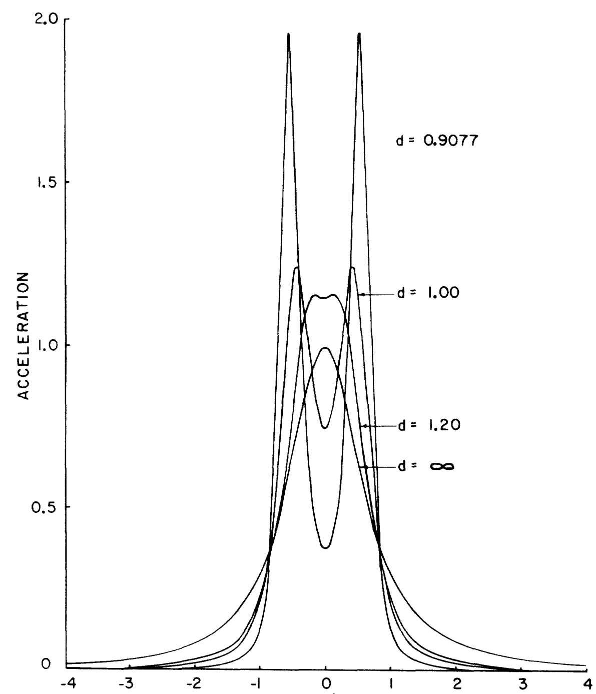

FIG. 2. Acceleration as a function of time for distance of closest approach $d=\infty$, 1.20, 1.00, and 0.9077. The acceleration plotted has been scaled by multiplying the true acceleration by d^2. The time plotted has been scaled by dividing the true time by $d^{3/2}$. The $d=\infty$ curve is valid (up to v^2/c^2 terms) for all nonrelativistic trajectories.

The next task is to determine the retarded and advanced times associated with each of these points. First the retarded and advanced times are bracketed by θ values of the form m/n, and then extrapolation [for $|\tau| > (n-2)/n$] or interpolation [for $|\tau| < (n-2)/n$] is used to find the position and velocity on the light cone. We base the extrapolation on a linear time dependence; the spatial interpolation[15] between points m and $m+1$ on a double Taylor series expansion which employs the values of x, v, a, $dt/d\theta$, and $d^2t/d\theta^2$ each evaluated at m and $m+1$; and the velocity interpolation[15] on the values of the velocity at the four points $(m-1, m, m+1, m+2)$ and the values of the acceleration at the points m and $m+1$.

Using Eq. (2.2) one can then compute a new acceleration for each of the original $2n-3$ points. For relatively low asymptotic velocities V it is sufficient to replace the original assumed acceleration curve by the newly computed one and to repeat the process over and over, keeping d fixed. For highly relativistic velocities, even for nearly convergent solutions, we damp the convergence by replacing the old acceleration by a linear combination of old and new. For our most highly relativistic convergent solution we found it necessary to use an acceleration which was 10% new plus 90% old.

The number of iterations needed for convergence varied from fewer than 10 for nonrelativistic speeds to 200 at the highest speeds.

For d larger than unity, our method is insensitive to the shape of the assumed acceleration curve provided the resulting first velocity integral does not exceed unity. For smaller d, the assumed curve has to be nearly correct to ensure convergence. Typically we assumed a shape given by scaling a solution with slightly larger d.

To check our calculations at low energies we made comparisons first with the nonrelativistic Newtonian equations, then with the Darwin expansion keeping terms in v^2/c^2, and finally with Hill's expansion in e^2/mc^2x, keeping three terms. (This differential equation was solved by using a modified Hamming routine, which is an improved version of the Runge-Kutta method.)

As a stringent test of the validity of our solutions we evaluated Eqs. (2.9) and (2.10). For $d = 1.0$ and $d = 0.9079$ the agreement of the two numbers was to within two parts in 10^4. The solution for $d = 0.9077$ seems to be accurate but for $d = 0.9075$ the procedure fails to converge.

IV. RESULTS AND CONCLUSIONS

We have computed a series of particle trajectories characterized by decreasing distances of closest approach d and increasing asymptotic ve-

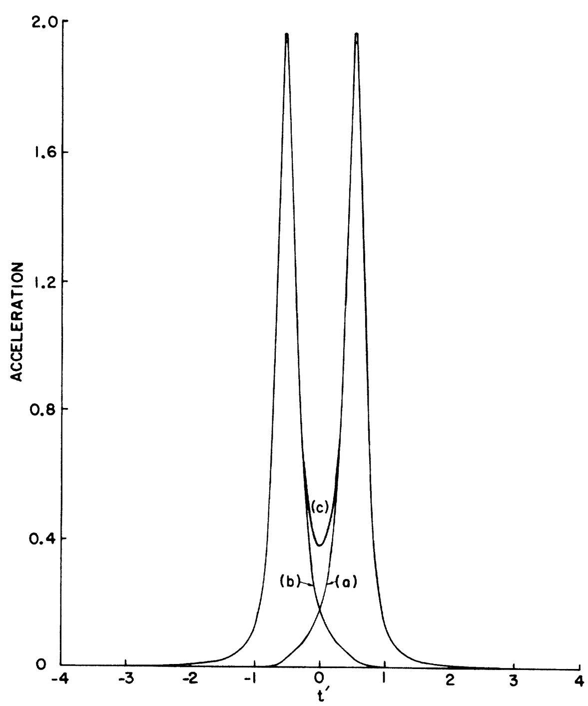

FIG. 3. The decomposition of the $d = 0.9077$ acceleration curve (c) into retarded (a) and advanced (b) contributions.

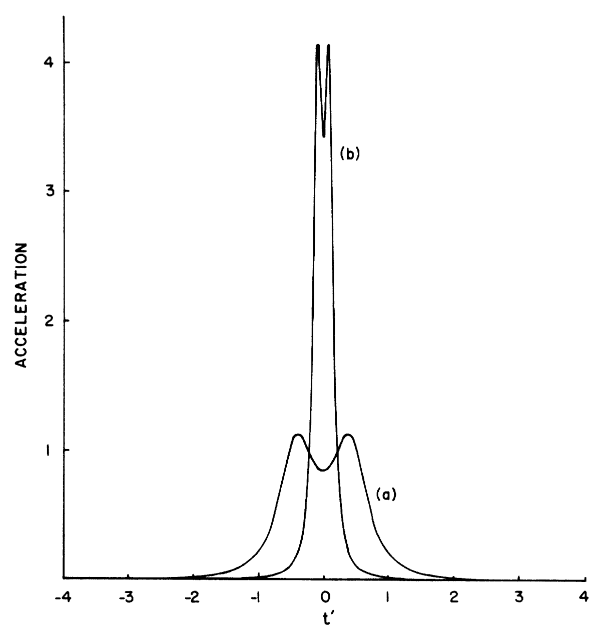

FIG. 4. A comparison of (a) the acceleration curve for the $d = 1$ "exact" solution with (b) the acceleration curve of the solution of Hill's third-order equation with $d = 0.8$. The final velocity V in each case is 0.8880.

locity V. Figure 1 shows the trajectories of two particles with $d=1$. In Fig. 2, we show the plots of acceleration vs time for $d=\infty$, 1.2, 1.0, and 0.9077. For ease of comparison we present only scaled quantities defined by $x'=x/d$, $v'=vd^{1/2}$, $a'=ad^2$, and $t'=t/d^{3/2}$.

The effect of scaling is evident in the nonrelativistic theory. The energy equation can be written

$$\left(\frac{dx}{dt}\right)^2+\frac{1}{2x}=\frac{1}{d}, \tag{4.1}$$

with a solution of the form $t=t(x,d)$. The scaled quantities, on the other hand, obey the equation

$$\left(\frac{dx'}{dt'}\right)^2+\frac{1}{2x'}=1, \tag{4.2}$$

with a solution of the form $t'=t'(x')$. Thus a single world line or acceleration function represents the nonrelativistic solution for all values of d. The curve in Fig. 2 labeled $d=\infty$ represents this universal curve. The other curves in Fig. 2 show the extent of the deviations due to relativistic corrections.

The most remarkable feature of the solutions is the development, at high energies, of the double-peaked acceleration curve. The peak with $t<0$ is due mainly to the advanced interaction with the other charge receding from the origin. The peak with $t>0$ is due mainly to the retarded interaction with the other charge approaching the origin. Figure 3 shows the decomposition of the total acceleration into two parts, one due to the retarded and the other due to the advanced interaction.

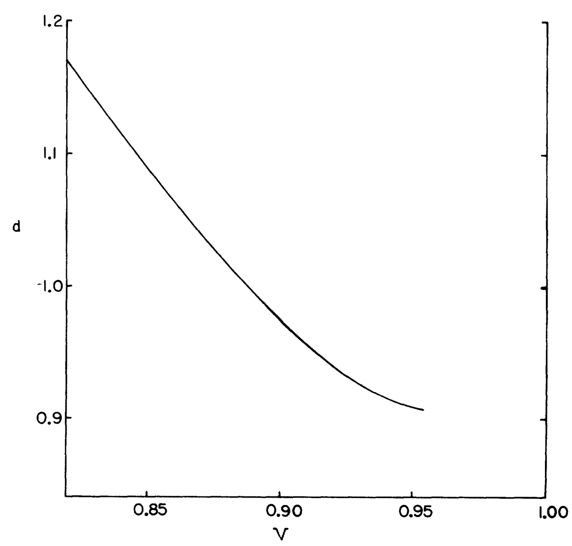

FIG. 5. Distance of closest approach d as a function of V. Our results end well before $V=1$, but they suggest that d will not go to zero as V approaches 1.

Comparison of our acceleration curve for $V=0.8880$ with the solution of Hill's third-order equation is shown in Fig. 4. The dip in the middle is evident even in the perturbation calculation, but the width is much too small.

An unexpected feature of the solutions is demonstrated by Fig. 5 which is a graph of d vs V. As V approaches unity, the distance of closest approach does not approach zero as expected. Although our iteration procedure stops converging at about $V\approx 0.9545$ ($d=0.9077$), the curve is consistent with the existence of a minimum distance of closest approach of $d\approx 0.9$.

A second comparison between Hill's solutions and ours is presented in Fig. 6, where the quantity V^2d is plotted against V^2. Equation (4.1) shows that in the nonrelativistic theory this quantity is unity. For small velocities all approximations, and of course the exact solution, converge to that limit. For higher asymptotic velocities the approximate solutions begin to deviate significantly from the exact solution. We find

$$V^2d=1-3V^2/4+O(V^4). \tag{4.3}$$

The question of why our iteration method eventually fails remains open. We present two speculations: One is that the higher V will correspond to larger rather than smaller d. This means that V is not a single-valued function of d, in which case an iterative procedure may be expected to work poorly near the minimum value of d. The second possibility is that additional solutions may exist for high V and that when they exist (as in the perturbed nonrelativistic circular orbits) they interfere with convergence.

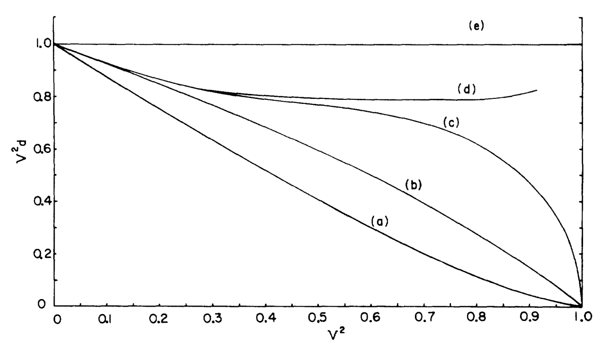

FIG. 6. The quantity V^2d as a function of V^2 for the exact solution (d) and for the solutions to Hill's equations of first (a), second (b), and third (c) order in e^2/rmc^2. In the Newtonian approximation (e) V^2d is unity. In the Darwin calculation to order v^2/c^2, we have $V^2d=1-\frac{3}{4}V^2$.

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[1]J. A. Wheeler and R. P. Feynman, Rev. Mod. Phys. 17, 157 (1945); 21, 425 (1949).

[2]F. Hoyle and J. V. Narlikar, Ann. Phys. (N.Y.) 54, 207 (1969); 62, 44 (1971); P. C. W. Davies, Proc. Cambridge Phil. Soc. 68, 751 (1970).

[3]E. C. G. Sudarshan, Fields and Quanta 2, 175 (1972).

[4]A. Schild, Phys. Rev. 131, 2762 (1963).

[5]C. G. Darwin, Phil. Mag. 39, 537 (1920).

[6]J. L. Synge, Proc. Roy. Soc. (London) A177, 118 (1940).

[7]E. H. Kerner, J. Math. Phys. 3, 35 (1962); 6, 1218 (1965).

[8]C. M. Andersen and H. C. von Baeyer, Phys. Rev. D 5, 802 (1972).

[9]R. N. Hill, J. Math. Phys. 8, 201 (1967).

[10]H. van Dam and E. P. Wigner, Phys. Rev. 138, B1576 (1965); 142, 838 (1966).

[11]G. N. Plass, Ph.D. thesis, Princeton, 1947 (unpublished).

[12]C. M. Andersen and H. C. von Baeyer, Am. J. Phys. 39, 914 (1971).

[13]R. N. Hill, J. Math. Phys. 11, 1918 (1970), his footnote 13.

[14]R. W. Hamming, in *Numerical Methods for Scientists and Engineers* (McGraw-Hill, New York, 1962), Sec. 10.4.

[15]R. W. Hamming, Ref. 14, Sec. 8.1.

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Gravitational Radiation from Relativistic Systems

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Gravitational radiation is calculated for two selected systems, a pair of masses interacting through scalar or vector fields, to illustrate the behavior of gravitational radiation when the emitting bodies have arbitrarily large velocities. Explicit expressions are obtained for the radiation potentials and radiated power, which can be used for any orbit or trajectory. Although the radiation exhibits the same relativistic peaking as is found in electromagnetism, the amount of energy radiated is found to depend critically on the kind of stresses binding the system. Numerical calculations are done for the cases of circular orbits and extreme hyperbolic trajectories, in order to show the transition from the nonrelativistic to extreme relativistic limits. The extreme hyperbolic case also allows comparison with previous results obtained for the gravitational deflection of a small mass in an unbound trajectory.

I. INTRODUCTION

The emission of gravitational waves from nongravitationally bound systems has been studied ever since Eddington first calculated the energy radiated by a spinning rod.[1] For nongravitationally bound systems the calculations are simplified by the fact that the linearized field equations[2] are a good approximation, whereas for gravitationally bound systems one must include the nonlinear terms in the field equations in order to have a consistent approximation scheme. However, work on both cases has tended to be concentrated in large part on systems for which the assumption of nonrelativistic motion holds, in which case the energy radiated, when expressed in terms of third time derivatives of the quadrupole tensor, has the same form for gravitationally bound as for nongravitationally bound systems.[2,3] Corrections to the quadrupole radiation, such as octupole contributions, are able to be calculated in either case, and in principle this allows one to discuss the gravitational radiation from bound systems in a power-series expansion in v^2/c^2.

For the case of large velocities ($v \to c$) such a power series does not adequately describe the dominant behavior of the radiation. Since the analogous situation in electromagnetism can be simply described in the relativistic limit, one may ask what gravitational radiation one might