

Two-dimensional O(3) model coupled to two-dimensional quantum gravity

C. F. Baillie

Computer Science Department, University of Colorado, Boulder, Colorado 80309

D. A. Johnston

*Laboratoire de Physique Théorique et Particules Élémentaires, Université Paris Sud, Batiment 211, F-91405 Orsay, France**
and Department of Mathematics, Heriot-Watt University, Edinburgh, EH14 4AS, Scotland†

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It has been suggested that the peak in the specific heat observed numerically for random surface actions with extrinsic curvature on dynamical lattices might be the result of a low mass bound state in an asymptotically free theory, rather than the signal for a real phase transition. The O(3) model on a fixed lattice displays just such behavior, but in general peaks appear to be weakened when the models concerned are put on dynamical lattices (i.e., coupled to two-dimensional quantum gravity). We therefore perform simulations of the O(3) model on dynamical ϕ^3 graphs to see if there is still a peak in the specific heat and compare the results with those on fixed lattices.

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There has been a considerable amount of numerical work devoted to exploring the issue of whether a non-trivial continuum limit exists for actions of the form

$$S = \sum_{\langle ij \rangle} (\mathbf{X}_i - \mathbf{X}_j)^2 + \lambda \sum_{\Delta_i, \Delta_j} (1 - \mathbf{n}_i \cdot \mathbf{n}_j) \quad (1)$$

on dynamically triangulated random surfaces where the $\mathbf{n}_i$ are the unit normals on neighboring triangles. The theory apparently has a low λ crumpled phase and a large λ smooth phase similar to those displayed by similar actions on *fixed* triangulated surfaces [1]. The dynamical lattice simulations are of interest for constructing well-defined lattice versions of string theory [2–5]. The initial simulations in [2] saw what appeared to be a second-order transition on small lattices, but later work [3–5] with larger lattices and better statistics has shown that the transition, if it exists, is most probably higher order. Indeed, it was observed in [4] that the data were not inconsistent with a crossover as the correlation length reached the size of the surfaces simulated or with a low-mass bound state in an asymptotically free theory. The further simulations of [5] tend to exclude the first hypothesis, but the second remains a possibility.

The original one-loop calculation of Polyakov [6] for the random surface action with extrinsic curvature suggested that the extrinsic curvature coupling was asymptotically free and hence there was no phase transition. The second (extrinsic curvature) part of the action in Eq. (1) is, in effect, a σ model living on the dynamically triangulated surface subject to the constraint that the $\mathbf{n}$'s be normals to the surface. If one forgets these constraints for a surface embedded in three dimensions, which is where most of the numerical simulations have been done, it is a two-dimensional (2D) O(3) model

$$S = -\lambda \sum_{\langle ij \rangle} \mathbf{n}_i \cdot \mathbf{n}_j \quad (2)$$

with $|\mathbf{n}_i|^2 = 1$, which is known to be asymptotically free. The constraints on the normals in [6] modified the numerical coefficient in the one-loop β function by comparison with the O(3) model but did not change its sign. A more recent calculation by Polyakov for a surface embedded in four dimensions with an action incorporating a topological term has in fact shown that the constraints on the normal vectors are softened by renormalization and may thus, in this case, be irrelevant anyway [7]. Intriguingly, a calculation of the renormalization flow of the coupling has recently been carried out for a “standard” O(N) model coupled to 2D gravity [8] where it was found that the result without gravity was renormalized by a factor $(2/Q\alpha)$, where $Q = \sqrt{(26-N)/3}$ and $\alpha = (\sqrt{26-N} - \sqrt{2-N})/\sqrt{12}$. The rescaling becomes complex for the region $N > 2$ (i.e., $C > 1$) where the model without gravity is asymptotically free, which is yet another manifestation of the $C = 1$ barrier in continuum 2D quantum gravity. It is, however, unclear what to expect from a simulation of the O(3) model on dynamical lattices from these analytical arguments as the results apply in the far ultraviolet and it is not clear that the simulations are accessing this regime. The very strong finite-size effects that are put forward in [9] as a possible explanation for the apparent absence of trouble at $C = 1$ in multiple Ising models and percolation could also be coming into play here.

It was suggested in [5] that the 2D O(3) model might serve as a qualitative model for the behavior of discretized random surface actions with extrinsic curvature at the crumpling transition. For a fixed lattice 2D O(3) model one observes a peak in the specific heat that increases with system size for small lattices before saturating even though the model is known to be asymptotically free. The peak is explained by the excitation of a low-mass state, the σ particle [10,11], which contributes when its mass is comparable to the inverse correlation length

*Address from September 1993 to September 1994.

†Permanent address.

of the lattice $O(3)$ model. A similar peak is observed in $SU(2)$ lattice gauge theory, and it appears in both these cases that there are complex zeros lurking near the real axis which do not converge to it in the infinite-volume limit [12]. In general [13] transitions appear to be weakened when models are transcribed from fixed to dynamical lattices so it is *a priori* possible that the already weak peak in the specific heat of the 2D $O(3)$ model might get washed out completely when the model is put on a dynamical lattice thus invalidating the qualitative parallels that have been drawn between the *fixed* lattice 2D $O(3)$ model and the dynamically triangulated random surface action of Eq. (1).

In this paper we conduct a simulation of the 2D $O(3)$ model on dynamical ϕ^3 graphs, which is therefore one step closer to the simulations of the surface action in Eq. (1) than the fixed lattice $O(3)$ model simulations. In our case we are neglecting “only” the Gaussian term and the constraints on the vectors $\mathbf{n}_i$. The simulations can also be regarded as a test of whether anything catastrophic happens to the lattice $O(N)$ model for $N > 2$. We choose to employ a simple Metropolis update for the spins, which at first sight might appear rather perverse in view of the excellent cluster update methods that are now available [14]. There are two reasons for this choice: Firstly, the simulations of Eq. (1) which we are attempting to approximate employed local algorithms as cluster updates appear to be very difficult, if not impossible, to implement for actions containing extrinsic curvature terms especially on dynamically triangulated surfaces; secondly, a simulation of the $O(3)$ model on dynamical ϕ^3 graphs using the Wolff cluster algorithm revealed that there are equilibration problems between the mesh and the spins on dynamical triangulations that first appear at quite modest β values. This is peculiar to dynamical lattices, as a simulation of the $O(3)$ model on a fixed ϕ^3 graph gave identical results for the Metropolis and Wolff algorithms. Rather counterintuitively the solution to the problem when using the Wolff algorithm appears to be the use of *fewer* flip moves per Wolff sweep as β increases.

For the measurements reported here we simulate the $O(3)$ model on dynamical ϕ^3 graphs of spherical topology with a fixed number N of points and no tadpole or self-energy insertions which would correspond to degenerate triangulations on the direct lattice. We simulate graphs with N from 100 to 10 000 points for a range of β values from 0.05 to 10.0. For each data point we carried out 10 000 Metropolis equilibration sweeps, followed by 50 000 Metropolis measurement sweeps. We used the usual rule of thumb of carrying out N flip moves per Metropolis sweep, which was sufficient to ensure equilibration between the lattice and spin model. We measured the usual spin model properties: the energy

$$E = \frac{1}{N} \left\langle \sum_{\langle ij \rangle} \mathbf{n}_i \cdot \mathbf{n}_j \right\rangle, \quad (3)$$

specific heat

$$C = \lambda^2 N (\langle E^2 \rangle - \langle E \rangle^2), \quad (4)$$

and susceptibility. The Metropolis acceptance was also measured during the course of the simulation and the proposed moves adjusted so that it remained in the region of 50% as β was varied. For the lattice itself we measure AL and AF which relate to the acceptance of the flip moves on the graphs. A flip can be forbidden either from constraints arising from the graph (i.e., no tadpoles and no self-energy bubbles) or from the energy change in the spin model induced by the reconnection of the vertices. AL measures the fraction of randomly selected links which pass the first test and could be flipped according to the graph constraints, and AF measures the fraction of the links satisfying the graph constraints that actually are flipped, i.e., pass the Metropolis test using the $O(3)$ model energy change. We also measure the fraction of rings of length 3, PR3, which serves as an indicator of the local curvature distribution in the ϕ^3 graph.

Remembering that each point on the lattice is trivalent and taking into account our summation conventions we would expect the energy to approach $4/3$ for asymptotically large β . We find that this is, indeed, the case for the Metropolis algorithm for both fixed and dynamical ϕ^3 graphs, although the approach is rather slower on dynamical graphs and does not reach $E \simeq 1.33$ until $\beta \simeq 10$. The Wolff algorithm with the number of flips per measurement sweep taken as N fails to reach this asymptotic value completely. We are of course principally interested in the behavior of the specific heat curve on dynamical lattices and this is shown in Fig. 1 for β up to 4.0. It is clear from the graph that the specific heat looks very similar to that of the fixed-lattice $O(3)$ model. The peak in the specific heat thus still mimics rather well the behavior of the dynamically triangulated random surface action in Eq. (1). The peak grows for relatively small numbers of points but saturates with larger N —the points for $N = 10\,000$ in Fig. 1 are rather poorly equilibrated and are probably best discarded in making comparisons. In addition to showing that the $O(3)$ model on dynamical lattices still produces similar qualitative behavior to the random surface action, the simulation provides another example of numerical work venturing

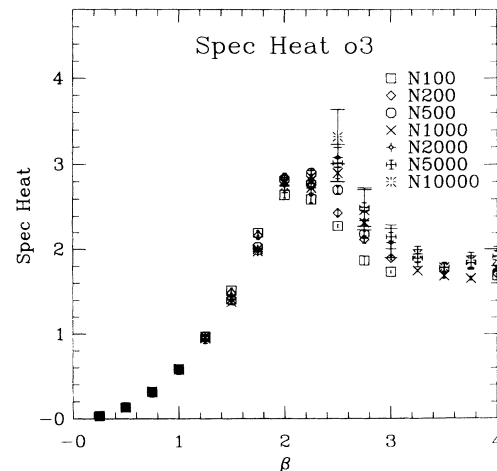


FIG. 1. The specific heat for the various graph sizes simulated using a Metropolis algorithm for the spins.

into regions where continuum analytical calculations suggest trouble (in this case the complex scaling found in [8] for $N > 2$) and finding nothing untoward. Simulations and series extrapolation of multiple Ising models on dynamical lattices which also have $C > 1$, at least naively, also fail to show any pathologies [15], whereas continuum calculations produce complex critical exponents [16].

The behavior of the dynamical ϕ^3 graphs is shown in Fig. 2 where AF, AL, and PR3 are plotted for $N = 5000$. There is a drop in the flip acceptance AL that is associated with the graph constraints at the same position as a peak in the probability of rings of length three, PR3, at a β value slightly below the peak in the specific heat. There is also a dip in the actual flip acceptance AF that is slightly shifted from this. These characteristics are again surprisingly similar to those observed in earlier random surface simulations, where similar peaks and dips are also manifest. This behavior is not totally universal for all models on dynamical lattices, as AF and AL look rather different for the XY and SOS (solid-on-solid) models [17], though Potts and multiple Potts models on dynamical lattices which have genuine phase transitions give results similar to those seen here for a model which presumably does not.

In summary, we have added to the repertoire of models that have been simulated on dynamically triangulated lattices or their duals by simulating a model that is asymptotically free on a fixed lattice. We found that the O(3) model still possessed a peak in its specific heat that eventually stopped growing for large N and displayed very similar lattice characteristics to random surface actions too. These results, like the fixed-lattice O(3) results in [5], show that it is not impossible that the observed crumpling "transition" for fluid random surfaces is a lattice artifact in an asymptotically free theory. In addition the results here are another example of a simulation in the region beyond the $C = 1$ barrier of the continuum formulation which have found no obvious pathologies in either the graphs or the spin model. From the algorithmic point of view a comparison of the Metropolis algorithm

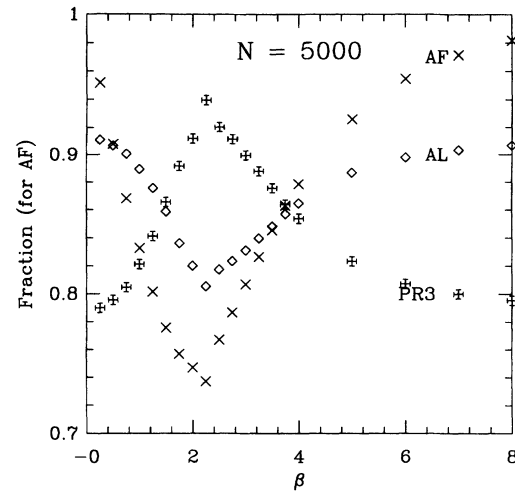


FIG. 2. The flip acceptances AL, AF and the probabilities of rings of length three, PR3, for a graph with $N = 5000$.

used here and a Wolff algorithm simulation revealed that there were unexpected equilibration problems on dynamical lattices for the Wolff algorithm as β was increased. In view of the results of [8] it would be most interesting to carry out a careful measurement of scaling on a dynamical lattice close to $\beta = 0$ (where the Wolff algorithm is still well behaved) in order to see how the renormalization group flow compared with the fixed lattice model, and we are currently doing this.

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