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# Chronology protection in two-dimensional dilaton gravity

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The global structure of a $(1+1)$-dimensional compact universe is studied in a two-dimensional model of dilation gravity. First we give a classical solution corresponding to the spacetime in which a closed timelike curve appears, and show the instability of this spacetime due to the existence of matter. It is also shown that if quantum effects are included our spacetime does not possess closed timelike curves unless the parameters in the model are fine-tuned.

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Callan, Giddings, Harvey, and Strominger [1] (CGHS) provided a useful toy model of $(1+1)$-dimensional gravity. They attempted to analyze the back reaction of the Hawking radiation [2] in the two-dimensional analogue of black hole geometry in a consistent way by the use of this model. In their first paper, CGHS developed their original scenario as follows: First introducing $(1+1)$-dimensional dilation gravity, CGHS found the solutions corresponding to black hole formation, and showed the occurrence of Hawking radiation in the spacetime background they obtained. Next, taking into account the quantum effect of such quantized conformal matter fields as the Polyakov term [3], they analyzed the back reaction of the Hawking effect in a leading semiclassical approximation consistently. However, their first scenario on the quantum black hole remains to be elaborated. Up to the present time, many people have studied the behavior of quantum black holes in this model [4,5].

On the other hand, in general relativity, there are other interesting problems which must be resolved, taking into account quantum effects. For example, since Morris, Thorne, and Yurtsever pointed out the possibility of making a time machine [6], quantum effects on the spacetime in which closed timelike curves (CTC) appear have been discussed by several authors [7–11]. One of the most important problems in this subject is whether or not a CTC spacetime continues to exist if quantum effects are taken into account. Hawking suggested the possibility that the laws of physics may prevent the appearance of closed timelike curves (the so-called "chronology protection conjecture"). In four dimensions the analysis of this problem so far has been done only in fixed background spacetimes. So it will be interesting to treat such spacetimes including back reaction in closed form even in lower dimensions.

In this paper we will investigate the quantum back reaction problem on the stability of a CTC spacetime in the two-dimensional model of dilaton gravity. Following the CGHS scenario, we apply this model to a compact one-dimensional universe. In the first half of this paper we give a classical solution corresponding to a CTC spacetime: an analogue of the Misner universe. Then we show the disappearance of this spacetime due to the existence of conformal matters even if the parameters are fine-tuned. In the second half we investigate whether or not the extension to such a CTC spacetime can be made if quantum back reaction is taken into account.

We consider the $(1+1)$-dimensional renormalizable theory of gravity coupled to a dilaton scalar field $\phi$ and $N$ massless conformal fields $f_i$. The classical action is

$$S=\frac{1}{2\pi}\int d^2x\sqrt{-g}\left[e^{-2\phi}[R+4(\nabla\phi)^2-4\lambda^2]-\frac{1}{2}\sum_{i=1}^{N}(\nabla f_i)^2\right], \tag{1}$$

where $R$ is the scalar curvature and $\lambda^2$ is a cosmological constant. This model differs from the original CGHS model in the sign of the cosmological term.

The equations of motion derived from (1) are

$$\begin{aligned}
&0=-4\partial_+\partial_-\phi+2\partial_+\partial_-\rho+4\partial_+\phi\partial_-\phi-\lambda^2e^{2\rho}\ ,\\
&0=\partial_+\partial_-\phi-\partial_+\partial_-\rho\ ,\\
&0=\partial_+\partial_-f_i\ ,
\end{aligned} \tag{2}$$

in the conformal gauge: $g_{\mu\nu}dx^\mu dx^\nu=-e^{2\rho}dx^+dx^-$, where $x^\pm=t\pm x$. In addition the following constraints have to be imposed:

$$4e^{-2\phi}(\partial_\pm^2\phi-2\partial_\pm\rho\partial_\pm\phi)=\sum_{i=1}^{N}\partial_\pm f_i\partial_\pm f_i\ . \tag{3}$$

In the following we adopt the periodic boundary condition that the spacetime point $(t,x)$ is identified with $(t,x+L)$ and the initial condition that the Universe

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starts from a static cylinder spacetime endowed with the usual Minkowski metric at past infinity. Then the general form of the solutions is given by

$$e^{-2\phi}=u_{+}+u_{-}+e^{-2\lambda t}\,,\qquad e^{2\rho}=e^{-2\lambda t}e^{2\phi}\,, \tag{4}$$

where $u_+$ and $u_-$ are chiral periodic functions which satisfy the following equations from the constraints (3):

$$0=\partial_\pm^2 u_\pm+\lambda\partial_\pm u_\pm+\tfrac{1}{2}\partial_\pm f\partial_\pm f\,. \tag{5}$$

If there is no matter field, general solutions satisfying the periodic boundary condition depend only on time:

$$e^{2\phi}=(M+e^{-2\lambda t})^{-1}\,,\qquad e^{2\rho}=e^{-2\lambda t}e^{2\phi}\,, \tag{6}$$

where $M$ is an arbitrary constant and corresponds to an initial value for $\phi$ imposed at some past time. We classify the behavior of the solutions into three types with respect to the sign of $M$.

When $M$ equals 0, the solution becomes an analogue of the linear dilaton vacuum solution in the CGHS model. The world is a static cylinder spacetime.

When $M$ is negative, we see from (6) that the observers meet a singularity in finite proper time. From the expression for the scalar curvature $R$.

$$R=-\frac{4\lambda^2 M}{M+e^{-2\lambda t}}\,, \tag{7}$$

we can see that this singularity is a true singularity. In fact this singularity is the same as the one in the $(1+1)$-dimensional black hole treated by CGHS.

On the other hand when $M$ is positive, the space collapses to zero volume in finite proper time (as coordinate time $t$ goes to $+\infty$). But from (7) the scalar curvature still retains a finite value $-4\lambda^2$ at the point. Hence we expect that the spacetime can be extended. In fact, if one defines the coordinates

$$\eta=-e^{-2\lambda t}\,,\qquad \psi=t\pm x\,, \tag{8}$$

the metric becomes

$$ds^2=\frac{-1}{M-\eta}\left[\eta\, d\psi^2+\frac{1}{\lambda}d\psi\, d\eta\right]\,, \tag{9}$$

which is analytic in the extended manifold defined by $\psi$ and by $-\infty<\eta<M$.

The behavior of the extended manifold is shown in Fig. 1. The region $\eta<0$ is isometric to the previous manifold. The region $\eta>0$ is an extended part, where the closed timelike curves appear, because the roles of $t$ and $x$ are interchanged. It should be noted that the spacetime has a naked singularity on $\eta=M$ where the dilaton field becomes $+\infty$. The surface $\eta=0$ is the boundary of the Cauchy development, that is, the Cauchy horizon, where the dilaton remains finite. This extension is achieved the same way as in the case of Misner space (we have its two-dimensional version when $\lambda=0$) and Taub-NUT (Newman-Unti-Tamburino) space [12]. Hawking used Misner space to discuss the chronology protection conjecture [10]. In the rest of the present paper we investigate the conjecture in this model both at the classical and the quantum levels.

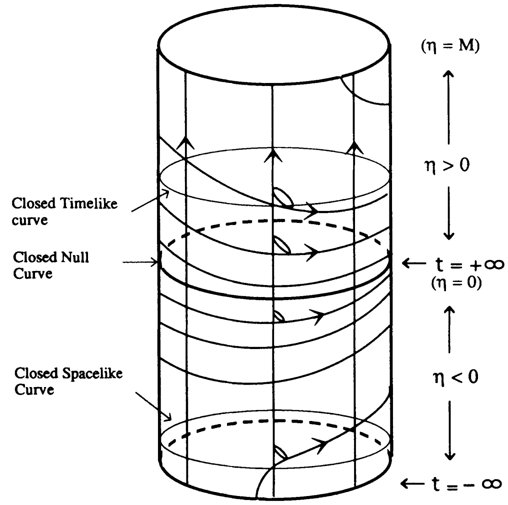


FIG. 1. Misner-type universe. $t=+\infty$ ($\eta=0$) is a closed null geodesics. The region $\eta<0$ is globally hyperbolic spacetime, and the region $\eta>0$ has closed timelike curves.

Next we consider the universe with classical massless scalar fields. The solutions of (3) satisfying the periodic boundary condition always exist for an arbitrary configuration of the scalar fields. We expand the scalar fields in Fourier series:

$$f(x^+,x^-)=f_+(x^+)+f_-(x^-)\,,\qquad f_\pm=\alpha_\pm+\left[\frac{\varepsilon}{2}\right]^{1/2}x^\pm+\sum_{n=1}^{\infty}\left[a_n^\pm\sin\frac{2\pi n}{L}x^\pm+b_n^\pm\cos\frac{2\pi n}{L}x^\pm\right]\,, \tag{10}$$

where $\alpha_\pm, a_n^\pm$, and $b_n^\pm$ are the expansion coefficients. We obtain the solution as

$$e^{-2\phi}=M-\left[\frac{\varepsilon}{2\lambda}+\frac{1}{2\lambda}\Sigma\right]t+e^{-2\lambda t}+(\text{oscillation part})\,,\qquad \Sigma=\sum_{(\cdot)=\pm}\sum_{n=1}^{\infty}\left[\frac{2\pi n}{L}\right]^2[(a_n^{(\cdot)})^2+(b_n^{(\cdot)})^2]\,, \tag{11}$$

where the fourth term on the right-hand side of the first equation in (11) is the oscillation part, that is, the sum of the trigonometric functions. From (11) it should be noted that any classical configuration of matter fields makes a finite contribution to the term proportional to time, which causes a divergence of the scalar curvature. Therefore the universe inevitably meets a singularity at $t=\infty$ and cannot extend to the region with closed timelike curves.

From now on we study how the classical solutions change if we include back reaction. So far several authors have pointed out the divergence of the quantum vacuum expectation value of the stress tensor at the Cauchy horizon even in lower dimensions and suggested that the behavior of the spacetime drastically changes [8,13,14]. In two dimensions, the quantum effect of massless matter fields is completely determined by the conformal anomaly [3,15] and the backreaction can be included consistently as follows.

The quantum effective action is the sum of the classical action (1) and the Polyakov term induced by the $N$ matter fields:

$$S_{\text{quantum}} = -\frac{\kappa}{8\pi}\int d^2x\sqrt{-g(x)} \times \int d^2x'\sqrt{-g(x')}R(x) \times G(x,x')R(x') , \tag{12}$$

where $\kappa$ is $N/12$ and $G(x,x')$ is a Green's function of the scalar fields. We assume that $\kappa$ is to be a large number and use the $1/N$ expansion. Further we add the following term introduced by Russo *et al.* [16] to the above action:

$$S'_{\text{quantum}} = -\frac{\kappa}{8\pi}\int d^2x\sqrt{-g}\; 2\phi R . \tag{13}$$

Making the field redefinition

$$\begin{aligned} \chi &= \sqrt{\kappa}\left[\rho - \frac{1}{2}\phi + \frac{1}{\kappa}e^{-2\phi}\right] , \\ \Omega &= \sqrt{\kappa}\left[\frac{1}{2}\phi + \frac{1}{\kappa}e^{-2\phi}\right] , \end{aligned} \tag{14}$$

we see that the semiclassical equations of motion are simplified:

$$\begin{aligned} \partial_+\partial_-\chi &= \frac{\lambda^2}{\sqrt{\kappa}}\exp\left[\frac{2}{\sqrt{\kappa}}(\chi-\Omega)\right] , \\ \partial_+\partial_-(\chi-\Omega) &= 0 , \end{aligned} \tag{15}$$

and

$$-\tfrac{1}{2}\partial_\pm f\cdot\partial_\pm f + \kappa t_\pm(x_\pm) = -\partial_\pm\chi\partial_\pm\chi + \partial_\pm\Omega\partial_\pm\Omega + \sqrt{\kappa}\partial_\pm^2\chi , \tag{16}$$

where the centered dot denotes the sum over $i$ and $t_\pm$ are arbitrary chiral functions to be determined by the boundary conditions. In this case the classical matter fields $f_i$ are interpreted as the macroscopic behavior of quantized fields. For an arbitrary macroscopic part of quantized matter fields as shown in (10), the solution of Eqs. (15) and (16) is given by

$$\begin{aligned} \sqrt{\kappa}\chi &= M - \frac{2\kappa}{\lambda}\left[\frac{\lambda^2}{4} - t_{\text{VEV}} + \frac{\varepsilon+\Sigma}{4\kappa}\right]t + e^{-2\lambda t} \\ &\quad + (\text{oscillation part}) , \\ \sqrt{\kappa}\Omega &= M + \frac{2\kappa}{\lambda}\left[\frac{\lambda^2}{4} + t_{\text{VEV}} - \frac{\varepsilon+\Sigma}{4\kappa}\right]t + e^{-2\lambda t} \\ &\quad + (\text{oscillation part}) , \end{aligned} \tag{17}$$

where $t_{\text{VEV}} \equiv t_\pm$ is determined to be $\pi^2/L^2$ due to the Casimir effect, and $\Sigma$ is the same as in (11). It should be noted that the term in the first parentheses in the second equation of (17) is equal to the quantum expectation value of the matter stress tensor $\langle T_{++}\rangle$ at $t=\infty$.

From equations (14) and (17) we can see the qualitative behavior of $\phi$ as a function of time $t$, when the position $x$ is fixed (Fig. 2).

We examine whether a quantum version of a CTC spacetime is realized in this model. From (14) and (17), we obtain $2\rho = 2\phi - 2\lambda t$. To extend the spacetime to the region with CTC's, $2\rho$ must become linear in $t$ as $t\to\infty$ in the conformal flat gauge, and also the coefficient of the linear term must be negative. By comparing $\Omega-\phi$ and $\Omega-t$ relations, we recognize that there are two distinct types of solutions as follows. One is realized in case (i): $M > \sqrt{\kappa}\Omega_{\text{cr}}$ in the parameters

$$\begin{aligned} a_n^{(\pm)} &= b_n^{(\pm)} = 0 \;\; \forall n , \\ \varepsilon &= \kappa\lambda^2 + 4\kappa t_{\text{VEV}} , \end{aligned} \tag{18}$$

where $\Omega_{\text{cr}}$ is the local minimum of $\Omega$ at $\phi=\phi_{\text{cr}}$ [Fig. 2(a)]. The first condition in (18) restricts the universe to a homogeneous one. The second condition means that the matter energy density $\langle T_{\mu\nu}n^\mu n^\nu\rangle$ along the comoving observer line, whose tangent vector is $n^\mu = e^{-\rho}(1,1)$, becomes 0 as $t\to\infty$, because the matter energy density behaves as

$$\left\{\left[\frac{\lambda^2}{4} + t_{\text{VEV}} - \frac{\varepsilon}{4\kappa}\right]f(t) + \frac{e^{-2\lambda t}}{t}g(t)\right\}e^{-2\rho}$$

where $f(t), g(t)$ are finite at $t=\infty$. The other is realized in case (ii): $\Omega_{\min} = \Omega_{\text{cr}}$ [$\Omega_{\min}$ is the local minimum of $\Omega$ in the $\Omega-t$ relation shown in Fig. 2(b)]. $a_n^{(\pm)} = b_n^{(\pm)} = 0$ ($\forall n$) and some appropriate negative $M$ is chosen. In this case the matter energy density diverges at $t=\infty$, al-

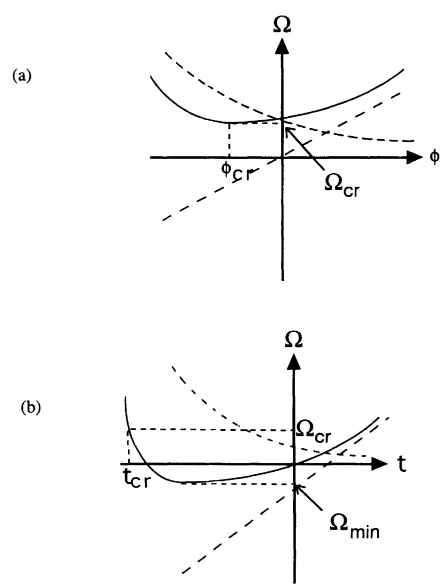


FIG. 2. (a) $\Omega$-$\phi$ relation, (b) $\Omega$-$t$ relation.

though the spacetime curvature remains finite.

In case (i), the value of the dilaton at $\eta=0$ can be adjusted to be so small that the semiclassical approximation is valid. On the other hand, in case (ii) the spacetime can be extended over $\phi=\phi_{\rm cr}$ to the strong coupling region smoothly, and the appearance of a CTC spacetime may occur in the limit of $t=\infty$ where $\phi$ becomes infinite. For the latter case, the analysis with full quantization is needed. The statement that whether or not CTC's appear in case (ii) may become meaningless.

Finally we conclude that the spacetime cannot be extended across $\eta=0$ with any configuration of quantized matter fields except the fine-tuned example as in (18). Thus it can be said that chronology protection holds in a *weaker* sense at the semiclassical level including back reaction.

In order to determine whether or not chronology protection holds in a *strong* sense, we must go on to extend the classical and semiclassical treatments of the compact universe in this paper to the full quantization of two-dimensional dilaton gravity (for example, see [17]). For the existence of any consistent solution in the extended region ($\eta>0$) inevitably depends on the information from the naked singularity ($\eta=M$), whose neighborhood is the very strong coupling region. For further analysis, we need a fully quantized theory. It is important to construct physical states corresponding to the classical and semiclassical behavior discussed in the present paper.

Recently another interesting case has been reported in [18], in which a two-dimensional analogue of inflation is treated. The CGHS scenario has been shown to be extensively useful for the study of back reaction problems in general relativity.

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