PHYSICAL REVIEW D VOLUME 49, NUMBER 2 15 JANUARY 1994

Two-dimensional quantum dilaton gravity and the positivity of energy

S. P. de Alwis*
Department of Physics, Box 390, University of Colorado, Boulder, Colorado 80309
(Received 4 March 1993)

Using an argument due to Regge and Teitelboim, an expression for the ADM mass of two-dimensional quantum dilaton gravity is obtained. By evaluating this expression we establish that the quantum theories that can be written as a Liouville-like theory have a lower bound to energy, provided there is no critical boundary. This fact is then reconciled with the observation made earlier that the Hawking radiation does not appear to stop. The physical picture that emerges is that of a black hole in a bath of quantum radiation. We also evaluate the ADM mass for the models with RST boundary conditions and find that negative values are allowed. The Bondi mass of these models goes to zero for large retarded times, but becomes negative at intermediate times in a manner that is consistent with the thunderpop of RST.

PACS number(s): 04.60.+n, 97.60.Lf

In Ref. [1] [Callan-Giddings-Harvey-Strominger (CGHS)], a theory of dilaton gravity coupled to matter was proposed. Subsequently it was argued in [2,3] that the quantum version of the theory must be a conformal field theory (CFT), and furthermore that it can be transformed into a solvable Liouville-like theory. In this paper we address the question of the positivity of energy in this theory. It has been claimed [4] that these theories are sick since they do not have a lower bound to energy. Since all quantum dilaton gravity theories that have been studied so far can be transformed into the Liouville-like theory (because they all have zero-field space curvature) it is important to check whether this is indeed the case. Related to this statement about the absence of a lower bound to energy is the observation [3,2,5] that the black hole solution of the quantum theory seems to radiate forever.

Recent work by Park and Strominger [6] seems to indicate that there might be a resolution of this problem. They established a positivity theorem using arguments derived from supersymmetry considerations, for fields satisfying certain asymptotic conditions. However, the expressions for the Arnowitt-Deser-Misner (ADM) mass given there do not give well-defined answers for the *solutions* of the classical theory since the asymptotic conditions of the theorem are violated. Furthermore the situation for the Liouville-like theory was left unresolved and it was not clear why there appeared to be a conflict with previous work [4].

In this work we resolve these issues. Indeed we are able to do so directly for the quantum theory since the complete space of solutions to the Liouville-like theory is known. We derive an expression for the ADM and Bondi mass using arguments given by Regge and Teitelboim [7]. Our argument shows that the expression used in [8,5] actually needs to be modified. We also show that the positivity theorems apply to the mass as defined by this procedure provided there is no critical boundary in the space-time such as the one in Ref. [9]. Furthermore unless one imposes boundary conditions on some critical line [9,10], corresponding to $r=0$ in four dimensions, there will be a contribution from the negative infinite end of the space.[1] Classically when the matter stress tensor has zero expectation value (giving static solutions) the ADM mass is zero, while for the dynamic solutions (i.e., in the presence of collapsing matter) the mass is positive if the incoming matter has positive energy. In the quantum case, in the no-boundary theory, the static solutions again have zero mass. In the dynamical case with collapsing matter, there is an infinite contribution from the negative end of the space; i.e., this theory describes black hole collapse in an infinite bath of radiation. The Bondi mass can also be defined. Again in the theory without a boundary the Bondi mass is infinite (and positive) at any finite retarded time, but at positive infinite retarded time, it becomes equal to the energy of the collapsed matter. This is consistent with the fact that the ADM mass in this model is infinite. The Hawking radiation in this picture is just this infinite bath which comes to an end at infinite retarded time leaving behind the original mass which collapsed. This is the way that the formalism resolves the positivity of energy with Hawking radiation which lasts for an infinitely long time. It does not reduce the energy from M_0 to negative infinity but from positive infinity to M_0. This is perhaps not a situation that can give any insight into the four-dimensional physics of black hole evaporation, but within the well-defined for-

*Electronic address: dealwis@gopika.colorado.edu

[1]This expression is given already in the second and third papers of [2] but it was evaluated only at one end of the space.

malism of this theory of two-dimensional quantum gravity, with no phenomenological boundary, it seems to be the only interpretation possible.

With the boundary conditions of Russo, Susskind, and Thorlacius (RST) [9] on the other hand, we find in the quantum theory, that static solutions with negative or zero masses are allowed. In the dynamic case the ADM mass is just that of the collapsing matter and thus is positive if the latter is positive. We also calculate the Bondi mass with RST boundary conditions. Here we find that at negative infinite retarded time, it goes to the mass of the collapsing matter, in agreement with the ADM mass of the model, decreases with increasing time, and for positive infinite retarded time, goes to zero. However, at an intermediate time the energy becomes negative, and is discontinuous across a certain null line; i.e., we encounter the thunderpop of RST [9]. The existence of negative Bondi energy is of course consistent with our previous result that there are negative energy static solutions.

The Liouville-like action for quantum dilaton gravity [2,3,5] is[2]

$$S=\frac{1}{4\pi}\int d^2\sigma[\mp\partial_+X\partial_-X\pm\partial_+Y\partial_-Y+2\lambda^2e^{\mp\sqrt{2/N}(X\mp Y)}]+S^f+S^{b,c}\,, \tag{1}$$

where S^f is the conformal matter action, and $S^{b,c}$ is the reparametrization ghost action. The quantum theory is then given by functionally integrating with the naive, translationally invariant, measures. The field variables X,Y are related to the original variables ϕ (the dilaton) and ρ (half the logarithm of the conformal factor) that occur in the CGHS action gauge fixed to the conformal gauge ($g_{\alpha\beta}=e^{2\rho}\eta_{\alpha\beta}$), by the relations

$$Y=\sqrt{2N}\left[\rho+N^{-1}e^{-2\phi}-\frac{12}{N}\int d\phi\, e^{-2\phi}\bar{h}(\phi)\right], \tag{2}$$

$$X=2\left[\frac{12}{N}\right]^{1/2}\int d\phi\, P(\phi)\,, \tag{3}$$

where

$$P(\phi)=e^{-2\phi}[(1+\bar{h})^2-Ne^{2\phi}(1+h)]^{1/2}\,, \tag{4}$$

N being the number of matter fields. In the above, the functions $h(\phi)$, $\bar{h}(\phi)$ parametrize quantum (measure) corrections that may come in when transforming to the translationally invariant measure (see [2,3,5] for details). The statement that the quantum theory has to be independent of the fiducial metric (set equal to η in the above) implies that this gauge fixed theory is a conformal field theory. The above solution to this condition was obtained by considering only the leading terms of the β-function equation but it was conjectured in [2,3] (because of its resemblance to the Liouville theory) that it is an exact solution to the conformal invariance conditions.[3] It should be noted here that this latter statement is strictly valid only when P has no zeros. When N is greater than 24 this implies some restrictions on the possible quantum corrections, but as shown in [5] there is a large class which satisfies these conditions.

Let us calculate the time translation generator by following the argument of Regge and Teitelboim [7]. This goes as follows. Suppose the Hamiltonian of the theory is given as the integral of the stress tensor over a spatial slice, $H=\int_\Sigma d\Sigma^\mu T_{0\mu}$, where as usual time derivatives of the fields have been eliminated in favor of canonical momenta π. Then if one is to get Hamilton's equations of motion one should be able to write $\delta H=\int d\Sigma^\mu(A_\mu\delta\pi+B_\mu\delta\phi)$. In general, however, when the space-time is not spatially closed there will be a boundary term on the right-hand side of this equation, so in order to get Hamilton's equations one should redefine H by adding a boundary term whose variations will cancel this extra term. The resulting expression is the generator of time translations. In addition in a generally covariant theory, there are constraints which imply that the total stress tensor (for matter plus gravity) is (weakly) zero, so that the total energy of a solution is given by evaluating just this boundary term. By Hamilton's equations it is indeed conserved. In 3+1 dimensions this boundary is the two-sphere at infinity and for asymptotically flat solutions the corresponding energy is just the ADM energy. In our two-dimensional case the boundary of the one-space is the set of points $\sigma=\pm\infty$. To obtain the required expression let us write down the Hamiltonian as the space integral of T_{00}:

$$H_0=\frac{1}{2}\int d\sigma\left[2(\Pi_X^2-\Pi_Y^2)+\frac{1}{2}(X'^2-Y'^2)+2\left[\frac{N}{12}\right]^{1/2}Y''-4\lambda^2e^{\sqrt{12/N}(X+Y)}\right]+H^f+H^{b,c}\,,$$

where Π_X, Π_Y are momenta canonically conjugate to X,Y and a prime denotes differentiation with respect to the spacelike coordinate σ. Then we have, for the variation,

$$\delta H_0=\frac{1}{2}\int d\sigma\left\{4(\Pi_X\delta\Pi_X-\Pi_Y\delta\Pi_Y)-\left[X''+4\lambda^2\left[\frac{12}{N}\right]^{1/2}e^{\sqrt{12/N}(X+Y)}\right]\delta X+\left[Y''-4\lambda^2\left[\frac{12}{N}\right]^{1/2}e^{\sqrt{12/N}(X+Y)}\right]\delta Y\right\}+\delta H^f+\delta H^{b,c}+\frac{1}{2}\left[X'\delta X-Y'\delta Y+2\left[\frac{N}{12}\right]^{1/2}\delta Y'\right]_{-\infty}^{\infty} \tag{5}$$

[2] We will work with N, the number of matter fields, very large, so that the difference between N and $\kappa=(24-N)/6$ will be ignored.
[3] This statement has now been proved in [10].

assuming that the matter and ghost configurations are bounded. Thus we need to add a boundary term and redefine the Hamiltonian as

$$H=H_0+H_\partial ,$$

such that δH_∂ cancels the boundary term in (5). H_∂ cannot be defined for arbitrary configurations. But for the space of configurations which either vanish asymptotically or go to an arbitrary solution of the equations of motion, this may be defined. This is because the general solution [3,2] to the equations of motion is given by

$$\begin{aligned}X=&-\left[\frac{12}{N}\right]^{1/2}[u_+(\sigma^+)+u_-(\sigma^-)]\\&+\lambda^2\left[\frac{12}{N}\right]^{1/2}\int^{\sigma^+}d\sigma^+e^{g_+(\sigma^+)}\int^{\sigma^-}d\sigma^-e^{g_-(\sigma^-)}\\=&-Y+\left[\frac{N}{12}\right]^{1/2}(g_++g_-) ,\end{aligned} \tag{6}$$

where $u_\pm(\sigma^\pm)$ are arbitrary chiral functions to be determined by the constraints and $g_\pm(\sigma^\pm)$ reflect the arbitrariness in the choice of conformal frame.[4] For this space of configurations and variations within this space, we have

$$H_\partial=-\left[\frac{N}{12}\right]^{1/2}\left[\frac{1}{2}g'(\sigma)X-X'\right]_\partial , \tag{7}$$

where $g=g_++g_-$. Now since $H_0=0$ (weakly) is a constraint of the theory the energy is entirely given by the boundary term. We should, however, measure energy relative to the linear dilaton vacuum (LDV) which corresponds to the solution with $u_\pm=0$ in (6). Defining $\Delta X=X-X_0$ where X_0 is the LDV solution, we have our final expression for the ADM energy:

$$E_{\rm ADM}=-\left[\frac{N}{12}\right]^{1/2}\left[\frac{1}{2}g'(\sigma)\Delta X-\Delta X'\right]_{-\infty}^{\infty} . \tag{8}$$

It is instructive to express this in terms of the original variables of CGHS using (2) and dropping all quantum corrections, i.e., $O(Ne^{2\phi})$ terms. Then we get

$$E_{\rm ADM}=\Delta[e^{-2\phi}(\lambda+2\phi')]_{-\infty}^{\infty} . \tag{9}$$

At this point it is incumbent upon us to discuss in what coordinate system this expression is expected to be valid. As we observed after (5) the above results are valid provided that the matter and ghost configurations are bounded. In the quantum theory we cannot make this assumption in every conformal frame. The reason is that the stress tensors of matter, dilaton gravity and ghosts are not tensors separately due to quantum anomalies (Schwartz derivative terms).[5] It is only the total stress tensor that is a tensor and it is zero in every frame. *Thus we take the point of view that the expressions for the quantum ADM and Bondi masses are correct only in asymptotically Minkowski coordinates.*

Let us evaluate (8) for the static quantum solution [3,2]

$$X=-\left[\frac{12}{N}\right]^{1/2}\frac{M_0}{\lambda}+X_0 , \tag{10}$$

where M_0 is a constant, and X_0 is the quantum solution corresponding to the linear dilaton vacuum of the classical theory which is given by

$$X_0=\left[\frac{12}{N}\right]^{1/2}\left[e^{\lambda(\sigma^+-\sigma^-)}-\frac{N}{24}\lambda(\sigma^+-\sigma^-)\right] . \tag{11}$$

Thus in the formula for the ADM energy (8), we must set $\Delta X=-\sqrt{12/N}M_0/\lambda$. The result is

$$E_{\rm ADM}=0 .$$

It should be noted that this result is true even in the classical limit and is due to the contribution from the negative end of the space. This is of course consistent with positive energy theorems since this configuration is obtained in the situation where the expectation value of the matter stress tensor is zero. In the earlier computation of the ADM energy [2,4] (following [8,1]) only the contribution from the $\sigma\to\infty$ was kept. However, the time translation generator that one gets from the Regge-Teitelboim argument requires the evaluation of the integral at both ends $\sigma=\pm\infty$. In addition expression (9) differs from the expressions given in [8] and [1]. In fact this can be calculated directly from the classical action using the Regge-Teitelboim argument to get exactly the same result as (9). The original expression actually does not give a well-defined answer for the ADM mass of the static black hole since the solution does not go to the LDV as $\sigma\to-\infty$.

The apparent existence of negative mass static solutions was pointed out as a problem for the Liouville-like theory [4] since these solutions are nonsingular (unlike the corresponding static solutions which had timelike singularities, and hence could be ruled out on physical grounds). Our result shows, however, that the correct mass for such configurations is always zero.

To get nonzero mass one has to put in matter. For simplicity of presentation we will explicitly consider only the case of an incoming shock wave of f matter $T^f_{++}=(ae^{\lambda\sigma_0^+}/\lambda)\delta(\sigma^+-\sigma_0^+)$, $T^f_{--}=0$ [1], but the generalization to arbitrary bounded configurations is trivial. The conformal frame in which this solution is asymptotically Minkowski is related to the σ frame by $\bar\sigma^+=\sigma^+$, $\bar\sigma^-=-(1/\lambda)\ln(e^{-\lambda\sigma^-}-a/\lambda)$.

Then the solution is (see, for example, [5])

[4]In Kruskal-like coordinates ($x^\pm$ of Ref. [1]) the latter are zero while in the asymptotically Minkowski coordinates $[\sigma=\pm(1/\lambda)\ln(\pm\lambda x^\pm)]$ $g_\pm=\pm\lambda\sigma^\pm$. These coordinates were called $\hat\sigma$ in Ref. [2,5] while the Kruskal coordinates were called σ. In this paper we stick to the notation of [1].

[5]The point is that the normal ordering that is necessary to define the stress tensor is frame dependent.

$$-\left[\frac{N}{12}\right]^{1/2}X=\begin{cases}\dfrac{M}{\lambda}+e^{\lambda(\bar\sigma^+-\bar\sigma^-)}-\dfrac{N}{24}\ln\left[\dfrac{e^{\lambda\bar\sigma^-}}{\lambda}\left[\dfrac{e^{-\lambda\bar\sigma^-}}{\lambda}+\dfrac{a}{\lambda^2}\right]\right] & \text{for } \sigma^+>\sigma_0^+\ ,\\ e^{\lambda\bar\sigma^+}\left[e^{-\lambda\bar\sigma^-}+\dfrac{a}{\lambda^2}\right]-\dfrac{N}{24}\ln\left[\dfrac{e^{\lambda\bar\sigma^+}}{\lambda}\left[\dfrac{e^{-\lambda\bar\sigma^-}}{\lambda}+\dfrac{a}{\lambda^2}\right]\right] & \text{for } \sigma^+<\sigma_0^+\ .\end{cases} \tag{12}$$

In the above we have set $M=ae^{\lambda\sigma_0^+}$ the mass of the classical black hole. In evaluating the ADM mass this needs to be compared with the LDV solution in the same conformal frame, i.e., (11) with σ replaced by $\bar\sigma$. Then we get

$$E_{\text{ADM}}=M+\frac{N\lambda}{24}\ln\left[1+\frac{a}{\lambda}e^{\lambda(\bar\tau-\bar\sigma)}\right]\Bigg|_{\bar\sigma\to-\infty}+\frac{N}{24}\lambda\ . \tag{13}$$

Thus the quantum anomaly in the dynamical solution gives an infinite contribution to the ADM mass from the negative infinite end of the spacelike line. It should be noted that the classical solution has a well-defined mass equal to M in these coordinates. The source of the infinite radiation bath is quantum mechanical ($N-N\hbar$ in the above). It comes from the fact that due to the quantum anomaly the solution in the region $\bar\sigma^+<\bar\sigma_0^+$ does not go to the LDV as $\bar\sigma\to-\infty$ once we insist that the solution for $\sigma^+>\sigma_0^+$ is asymptotically Minkowski. It should be noted that the energy though infinite is positive.

If we want to represent the situation corresponding to the absence of a radiation bath we have to set a boundary as done in [9]. We should stress here that to do this we need not necessarily use the RST choice of functions h, $\bar h$ in (4). The physics may be discussed entirely in terms of the X and Y fields of the Liouville-like model and the difference between the two types of models lies in whether one chooses to set a boundary or not.

The RST boundary condition puts $\partial_\pm X=f=0$, on a critical curve $X=X_c$, which is regarded as the left boundary of space-time, wherever it is timelike. Let us first consider the static solutions (10) and (11). The RST boundary is at

$$X=X_c=-\left[\frac{12}{N}\right]^{1/2}\left[\frac{N}{24}-\frac{N}{24}\ln\frac{N}{24}\right]\ .$$

For $M_0=0$, i.e., for the LDV, this is the line $\sigma=(1/2\lambda)\ln(N/24)$. For $M_0>0$ on the one hand there is no solution to the boundary curve equation and it is not clear how to evaluate E_{ADM}. On the other hand for $M_0<0$ there is a boundary, and since from the boundary conditions there will be no contribution to the ADM mass from this end of the space one gets

$$E_{\text{ADM}}=M_0\ . \tag{14}$$

In other words there is no positivity in the theory with boundary. This is of course consistent with the fact that there is a negative energy thunderpop [9] in the dynamical solutions of this theory and as we shall see later, the Bondi mass also reflects this phenomenon.

Let us now consider the collapse scenario. From the explicit solution it may be seen that the boundary curve $X=X_c$ is timelike for $\sigma^+<\sigma_0^+$. Hence the boundary condition may be imposed there, and in particular it follows that there is no contribution to the ADM mass at the left end of the space. It should be stressed that this is the case since the boundary is given by a fixed value of X (at which $\partial_\pm X=0$) and thus its contribution cancels between the collapse solution and the LDV. Thus with the RST boundary conditions,

$$E_{\text{ADM}}=ae^{\lambda\sigma_0^+}\equiv M\ . \tag{15}$$

If incoming energy is positive ($a>0$) then the ADM mass is positive. Clearly this generalizes easily to arbitrary bounded configurations of matter so that for these dynamical solutions we have explicitly demonstrated the positive energy theorem for quantum dilaton gravity. It should be pointed out that in deriving the ADM mass for the theory with RST boundary conditions, we need only the conditions at the timelike boundary for $\sigma^+<\sigma_0^+$. In order to obtain the complete physical picture one needs to impose boundary conditions on the critical curve when it becomes timelike again somewhere above the line $\sigma^+=\sigma_0^+$. As explained by Russo, Suskind, and Thorlasius [9] this leads to a thunderpop—a burst of negative energy to $\mathcal{I}_L^+$. We will rediscover this effect when we calculate the Bondi mass.

Let us now discuss the issue of Hawking radiation. As discussed at length in Sec. V of [5] the usual calculation of Hawking radiation [1] cannot really be justified in a situation in which one takes back reaction and the constraints of general covariance into account. Since this point does not seem to be widely appreciated it is perhaps necessary to reiterate it here. The constraints tell us that (the expectation value of) the total stress tensor is zero; i.e.,

$$T^{XY}_{\pm\pm}+T^{f}_{\pm\pm}+T^{b.c.}_{\pm\pm}=0\ , \tag{16}$$

where the last term is the contribution of the reparametrization ghosts, together with the equation of motion for the conformal factor $T_{+-}=0$. The usual arguments are equivalent to the following. Since only f matter is propagating only the f stress tensor should contribute to radiation. Now we have the following anomalous transformation law for the stress tensor, from the σ coordinate system which covers the whole space, to the $\bar\sigma$ system which covers only the region outside the horizon:

$$\bar T^f_{--}(\bar\sigma^-)=T^f_{--}(\sigma)+\frac{N}{24}\lambda^2\left[1-\frac{1}{[1+(a/\lambda)e^{\lambda\bar\sigma}]^2}\right]\ ,$$

where the second term on the right-hand side (RHS) is the Schwartz derivative for the transformation from σ to $\bar{\sigma}$. Then it is argued that σ^- is the appropriate coordinate on $\mathcal{I}_{\mathcal{R}}^-$ and since there should be no incoming radiation on this null line one must set $T^f_{--}(\sigma)=0$ so the Hawking radiation observed on $\mathcal{I}_{\mathcal{L}}^+$ is just given by the Schwartz derivative term. However this argument ignores the constraint equation (16). In fact the counting of propagating degrees of freedom merely tells us that there are N of them, but this does not imply that the energy propagated is given just by the f stress tensor. In fact the physical propagating state must be dressed by X, Y fields and possibly ghost fields as well. For the above result to be compatible with the constraints there has to be an inflow of X, Y, b, c field energy to compensate for the outflow of f energy.

Thus we believe [5] that the evaluation of the Hawking radiation must be done by calculating the Bondi mass of the system. This quantity, like the ADM mass, could be nonzero for open systems even though the expectation value of the total stress tensor is zero. Since it must give us the total energy minus the energy that has been radiated away up to a given retarded time, it must be evaluated on a line which is asymptotic to $\bar{\sigma}^-=\text{const}$ at $\sigma^+\to\infty$ (i.e., on $\mathcal{I}_{\mathcal{R}}^+$) and to $\bar{\sigma}^+=\bar{\sigma}_1^+<\bar{\sigma}_0^+$ on[6] $\sigma^-\to\infty$ (i.e., on $\mathcal{I}_{\mathcal{L}}^+$). Then, in analogy with the expression (8) for the ADM mass, we have, for the Bondi mass,

$$E_{\text{Bondi}}(\bar{\sigma})=\left[\frac{N}{12}\right]^{1/2}[-\lambda\Delta X +(\partial_+\Delta X-\partial_-\Delta X)]^{\bar{\sigma}^+=+\infty}_{\bar{\sigma}^-=\infty,\bar{\sigma}^+=\bar{\sigma}_1^+} . \tag{17}$$

We have to substitute in the above the difference between the dynamical solution evaluated in the $\bar{\sigma}$ frame and the LDV solution. Then we get

$$E_{\text{Bondi}}(\bar{\sigma})=M_0-\frac{N}{24}\ln\left[1+\frac{a}{\lambda}e^{\lambda\bar{\sigma}^-}\right]-\frac{N}{24}\frac{\lambda}{[1+(\lambda/a)e^{-\lambda\bar{\sigma}^-}]} -\left[-\frac{N}{24}\ln\left[1+\frac{a}{\lambda}e^{\lambda\bar{\sigma}^-}\right]-\frac{N}{24}\frac{\lambda}{[1+(\lambda/a)e^{-\lambda\bar{\sigma}^-}]}\right]_{\bar{\sigma}^-\to\infty} . \tag{18}$$

This gives an infinite Bondi mass for any finite value of $\bar{\sigma}^-$, but this is a reflecting of the fact that the ADM mass is infinite. On the other hand in the limit $\bar{\sigma}^-\to\infty$ the Bondi mass becomes equal to M_0, i.e., the initial incoming matter energy. In [5] by contrast the Bondi mass was identified (just as with the ADM mass) as the value evaluated at one end, i.e., $\mathcal{I}_{\mathcal{R}}^+$. However, that led to the result that the Hawking radiation did not stop and the Bondi mass decayed indefinitely. In view of our discussion of the ADM mass, we believe now that the problem was the fact that both ends of the line (spacelike in the case of ADM, lightlike in the Bondi case) need to be considered. This is the correct time translation generator in the ADM case and the object that satisfies positivity.

We have shown that quantum CGHS theory is soluble and satisfies positivity of energy. Unfortunately it is not possible to get much insight into four-dimensional black hole physics from it, since the conformal invariance forces us to a situation where one has to start with an infinite bath of radiation. It is clear that to make contact with four-dimensional physics one should have a timelike boundary (corresponding to the origin of polar coordinates in 3+1 dimensions) as is done in the models with the so-called quantum singularity [11,12]. However, in that case it is not clear whether conformal invariance, which is a consequence of general covariance, can be preserved. Within the strict interpretation of the formalism of two-dimensional quantum dilaton gravity, the theory without boundary seems to be the only consistent one.

However, for phenomenological purposes and to check that (17) gives the physical picture that one expects in the Hawking evaporation of black holes let us evaluate the Bondi mass with RST boundary conditions. In this case the lower limit in (17) is replaced by an point on the critical curve $X=X_c$ for $\bar{\sigma}^+<\bar{\sigma}_0^+$. Then there is no contribution from this end to the energy. At the upper end, however, there are now two regions to consider. Calling the point where the apparent horizon $(\partial_+X=0)$ and the critical curve intersect $(\bar{\sigma}_s^+,\bar{\sigma}_s^-)$, we have the region $\bar{\sigma}^-<\bar{\sigma}_s^-$ (region I of RST; see [10] for figure) and the region between the timelike boundary and $\bar{\sigma}^-=\bar{\sigma}_s^-$ (region II of RST). In region II the black hole has decayed and the solution is taken to be the LDV. In region I the solution is the collapse solution for $\bar{\sigma}^+>\bar{\sigma}_0^+$. Thus we have

$$-\left[\frac{N}{12}\right]^{1/2}\Delta X=\begin{cases}0 \quad \text{in II} ,\\ \dfrac{M}{\lambda}-\dfrac{N}{24}\ln\left[1+\dfrac{a}{\lambda}e^{\lambda\bar{\sigma}^-}\right] \quad \text{in } I .\end{cases}$$

Computing the Bondi mass from (17) we have

[6]One may also take the line to be asymptotic to a spacelike line at the $\bar{\sigma}\to-\infty$ end.

$$E_{\text{Bondi}}(\sigma^-)=\left\{M_0-\frac{N}{4}\lambda\ln\left[1+\frac{a}{\lambda}e^{\lambda\bar\sigma^-}\right]-\frac{N}{4}\frac{\lambda}{1+(\lambda/a)e^{-\lambda\bar\sigma^-}}\right. \text{ in I },\tag{19}$$

Thus we seem to have a physical picture of Hawking radiation when the RST boundary conditions are imposed. For $\bar\sigma^-\to-\infty$, $E_{\text{Bondi}}\to M$; while at late retarded times $E_{\text{Bondi}}\to 0$. However, there is an unphysical feature in the model in that some time before $\bar\sigma^-=\bar\sigma_s^-=-\ln[(a/\lambda)(1-e^{-4M/\lambda N})]$ the Bondi energy goes negative. Indeed the energy flow is discontinuous at $\bar\sigma^-=\bar\sigma_s^-$:

$$E_{\text{Bondi}}(\bar\sigma_s^-+0)=0\ ,$$

$$E_{\text{Bondi}}(\sigma_s^--0)=-\frac{N}{4}(1-e^{-4M/\lambda N})\ .$$

This is just the effect of the thunderpop of RST [9] and is caused by the fact that when the collapse solution is matched to the LDV along the null line $\bar\sigma^-=\bar\sigma_s^-$, the result is continuous but not smooth.

In conclusion, what we have shown is that in the absence of a boundary in the two-dimensional space one is forced to an interpretation where the black hole is immersed in an infinite bath of quantum radiation. However, although both the ADM and Bondi masses are infinite (the latter actually goes to the collapsing mass at infinite time) they are positive. On the other hand if RST boundary conditions are imposed one has a physical picture of black hole evaporation at the price, however, of a negative energy thunderpop. The latter is consistent with the fact there are negative energy static solutions in the theory.

While this paper was being prepared for publication the author had a discussion on the present work with A. Bilal and I. Kogan, during which they showed me a paper [13] by themselves in which formula (9) is derived in a somewhat different manner. This work was partially supported by Department of Energy Contract No. DE-FG02-91-ER-40672.

[1] C. G. Callan, S. B. Giddings, J. A. Harvey, and A. Strominger, Phys. Rev. D **45**, R1005 (1992).
[2] S. P. de Alwis, Phys. Lett. B **289**, 278 (1992); **300**, 330 (1993).
[3] A. Bilal and C. Callan, Nucl. Phys. **B394**, 73 (1993).
[4] S. Giddings and A. Strominger, Phys. Rev. D **47**, 2454 (1993).
[5] S. P. de Alwis, Phys. Rev. D **46**, 5429 (1992).
[6] Y. Park and A. Strominger, Phys. Rev. D **47**, 1569 (1993); A. Bilal, *ibid.* **48**, 1665 (1993).
[7] T. Regge and C. Teitelboim, Ann. Phys. (N.Y.) **88**, 286 (1974).
[8] E. Witten, Phys. Rev. D **44**, 314 (1991).
[9] J. Russo, L. Susskind, and L. Thorlacius, Phys. Rev. D **46**, 3444 (1992); **47**, 533 (1993).
[10] S. P. de Alwis, Phys. Lett. B (to be published).
[11] T. Banks, A. Dabholkar, M. R. Douglas, and M. O'Loughlin, Phys. Rev. D **45**, 3607 (1992).
[12] J. G. Russo, L. Susskind, and L. Thorlacius, Phys. Lett. B **292**, 13 (1992); L. Susskind and L. Thorlacius, Nucl. Phys. **B382**, 123 (1992).
[13] A. Bilal and I. I. Kogan, Phys. Rev. D **47**, 5408 (1993).