

Singlet Majoron model with hidden scale invariance

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We investigate an extension of the singlet Majoron model in which the breaking of dilatation symmetry by the mass parameters of the scalar potential is removed by means of a dilaton field. Starting from the one-loop renormalization group improved potential, we discuss the ground state of the theory. The flat direction in the classical potential is lifted by quantum corrections and the true vacua are found. Studying the finite temperature potential, we analyze the cosmological consequences of a Jordan-Brans-Dicke dilaton and show that the lepton number is spontaneously broken after the electroweak phase transition, thus avoiding any constraint coming from the requirement of the preservation of the baryon asymmetry in the early Universe. We also find that, contrary to the standard model case, the dilaton and/or neutrino cosmology do not impose any upper bound on the scale of the spontaneous breaking of scale invariance if the τ neutrino is sufficiently heavy.

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I. INTRODUCTION

One of the most remarkable properties of the standard model (SM) of electroweak interactions is that it is scale invariant up to the mass term in the Higgs sector, responsible for the spontaneous symmetry breaking, and a possible constant term related to the vacuum energy density.

Theories with mass parameters may still have *hidden* nonlinearly realized scale invariance [1], which may play a role in solving the cosmological constant problem [2] and can account for the hierarchy of scales observed in the SM between the weak scale and the Planck and/or the grand unification scale [3]. Also, it is intriguing that models which display a nonlinearly realized scale invariance in their low-energy effective theories occur in some unification schemes, including models based on compactification of higher dimensions [4] and string theories [5].

A trace of the scale invariance may still exist at low energies in the form of a pseudo Goldstone boson, the dilaton, which couples in a universal way to all mass terms and gains a mass due to the explicit breaking of classical scale invariance by quantum corrections [3]. Coupling a Jordan-Brans-Dicke (JBD) dilaton to the SM leads to a model equivalent under a conformal transformation to the SM coupled to a JBD gravity theory [6] and to interesting phenomenological consequences. In particular, Buchmüller and Busch [3] have shown that the existence of a JBD dilaton would impose an upper bound of about 100 GeV on the top quark mass, the precise value depending on the dilaton decay constant f , from the requirement that the ground state breaks the electroweak gauge symmetry.

The presence of a JBD dilaton would also have significant cosmological implications. For the range of cosmologically allowed neutrino masses [7] and for the dynamics of the electroweak phase transition, see Refs. [8,9], which has been shown to be first order and to occur

at the chiral-symmetry-breaking phase transition. In particular, McDonald [9] has recently emphasized that there is a serious problem with the energy density in the dilaton field following the electroweak phase transition, with most of the energy in the electroweak vacuum going into oscillations of the dilaton field at $T_{\text{QCD}} \sim 150$ MeV, causing the Universe to be matter dominated at nucleosynthesis and the standard calculation of element abundances to disagree with observations. To avoid this problem one has to impose the upper bound $f \lesssim 10^7$ GeV on the scale of the spontaneous breaking of scale invariance. Such a requirement is well in contrast to the natural identification $f \simeq M_P$ in models in which the hierarchy between the Planck scale M_P and the weak scale is explained by the underlying scale invariance [3] through the introduction of a JBD dilaton. Motivated by this troublesome inconsistency present in the SM, we investigate which kind of implications would have the coupling of a JBD dilaton to one of the most attractive extension of the SM, the singlet Majoron model (SMM) [10]. In the SMM a gauge singlet scalar and three right-handed neutrinos are introduced, and a global $U(1)_L$ group associated to the lepton number is spontaneously broken, giving rise to Majorana masses for the right-handed neutrinos. The model naturally incorporates the see-saw mechanism [11] and can therefore explain why left-handed neutrinos are much lighter than their right-handed counterparts. However, the realization that the baryon- (B -) and lepton- (L -) violating, while ($B-L$ -) conserving, quantum effects in the SM are efficient at high temperatures [12], gives rise to a very strong bound on the Majorana masses of light neutrinos, $m_\nu \lesssim 1$ eV [13]. The point is that the SM anomalous effects still allow a baryon asymmetry generated at some superheavy scale [14] to survive if the Universe had a nonvanishing primordial ($B-L$) asymmetry, but, if other interactions which violate B and L as well as the combination ($B-L$) are in equilibrium at temperatures above the Fermi scale, then no cosmological

baryon asymmetry can survive [13]. One has to require that the new L -violating interactions not to be in equilibrium at all temperatures at which anomalous interactions are still active, which leads to the above-mentioned strong bound on m_ν . A natural way to avoid such a constraint is to generate Majorana neutrino masses with a spontaneous L -number breaking at the electroweak scale or below. The conservation of the L number at higher scales prevents the existence of the dangerous $\Delta L=2$ effective operator $(m_\nu/\langle\phi\rangle^2)(L_L\phi)^2$, where ϕ is the standard Higgs scalar doublet and L_L is a lepton doublet, $L_L=(\nu_L, l_L)$. It has been recently shown that in the supersymmetrized version of the SMM [15] and in the triplet-singlet Majoron model [16] the phase transition leading to the L -number breaking can occur at temperatures below the weak scale, thus avoiding any constraint coming from the requirement of the preservation of the baryon asymmetry.

The aim of this paper is to show that the same feature is naturally achieved when a JBD dilaton is coupled to the SMM and that, contrary to what happens in the SM, no upper bound on the dilaton decay constant f comes from considerations about the JBD dilaton cosmology.

The paper is organized as follows. In Sec. II we describe the SMM with hidden scale invariance and derive the one-loop renormalization group improved effective potential, whose minimization allows us to break the vacuum degeneracy present at the classical level. In Sec. III we deal with the one-loop finite temperature effective potential and study the dynamics of the $SU(2)_L \otimes U(1)_Y$ and $U(1)_L$ phase transitions. In Sec. IV we present our conclusions.

II. THE SMM WITH HIDDEN SCALE INVARIANCE

The Lagrangian for the scale-invariant extension of the SMM is

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}e^{2\sigma/f}\partial_\mu\sigma\partial^\mu\sigma + (D_\mu\phi)^\dagger(D^\mu\phi) + (\partial_\mu S)^\dagger(\partial^\mu S) \\ & + i\bar{N}_R \not{\partial} N_R - (h_\nu \bar{L}_L \phi N_R + \frac{1}{2}g_R \bar{N}_R N_R^c S + \text{H.c.}) \\ & - V_0(\sigma, \phi, S), \end{aligned} \quad (1)$$

where

$$\begin{aligned} V_0(\sigma, \phi, S) = & \bar{a}^4 + \bar{m}_\phi^2(\phi^\dagger\phi) + \bar{m}_S^2(S^\dagger S) + \gamma(\phi^\dagger\phi)(S^\dagger S) \\ & + \frac{\lambda}{2}(\phi^\dagger\phi)^2 + \frac{\beta}{2}(S^\dagger S)^2 \end{aligned} \quad (2)$$

and

$$\begin{aligned} \bar{a}^4 & \equiv a^4 e^{4\sigma/f}, \quad \bar{m}_\phi^2 \equiv m_\phi^2 e^{2\sigma/f}, \\ \bar{m}_S^2 & \equiv m_S^2 e^{2\sigma/f}, \quad \lambda > 0, \quad \beta > 0. \end{aligned}$$

Here σ is the dilaton field, f its decay constant, ϕ is the scalar Higgs doublet, D_μ is the $SU(2)_L \otimes U(1)_Y$ gauge covariant derivative, S is the gauge singlet field carrying lepton number $L=2$, N_R is the gauge singlet right-handed neutrino ($N^c \equiv C\bar{N}^T$), L_L is the lepton doublet, and h_ν and g_R are 3×3 matrices of Yukawa couplings.

Because of the specific couplings of the Goldstone field

σ , the Lagrangian (1) is invariant under dilatations:

$$\begin{aligned} \delta\sigma & = \delta\alpha(f + x^\mu\partial_\mu\sigma), \quad \delta\phi = \delta\alpha(\phi + x^\mu\partial_\mu\phi), \\ \delta S & = \delta\alpha(S + x^\mu\partial_\mu S), \\ \delta\psi_{L(R)R} & = \delta\alpha(\frac{3}{2}\psi_{L(R)} + x^\mu\partial_\mu\psi_{L(R)}), \end{aligned} \quad (3)$$

where ψ denotes a generic fermion field in the Lagrangian (1). The classical equations of motion for the scalar fields read

$$\begin{aligned} D_\mu D^\mu\phi + \bar{m}_\phi^2\phi + \lambda(\phi^\dagger\phi)\phi + \gamma(S^\dagger S)\phi & = 0, \\ \partial_\mu\partial^\mu S + \bar{m}_S^2 S + \beta(S^\dagger S) + \gamma(\phi^\dagger\phi)S & = 0, \\ e^{2\sigma/f} \left[\partial_\mu\partial^\mu\sigma + \frac{2}{f}\partial_\mu\sigma\partial^\mu\sigma \right] + \frac{2}{f}\bar{m}_\phi^2(\phi^\dagger\phi) \\ & + \frac{2}{f}\bar{m}_S^2(S^\dagger S) + \frac{4}{f}\bar{a}^4 = 0. \end{aligned} \quad (4)$$

The existence of nontrivial constant solutions σ_0 , ϕ_0 , and S_0 constrains the allowed parameters present in V_0 . For instance, for $m_\phi^2 > 0$, $m_S^2 > 0$, and $\gamma > 0$, the only stationary points are $\phi_0=0$ and $S_0=0$, and σ_0 remains undetermined. In the following we shall choose m_ϕ^2 and m_S^2 both negative. In such a case, from Eq. (4) one discovers that the constant a^4 is fixed to be

$$a^4 = -\frac{1}{2} \frac{2\gamma m_\phi^2 m_S^2 - \beta m_\phi^4 - \lambda m_S^4}{\beta\lambda - \gamma^2} \quad (5)$$

and symmetry-breaking vacuum expectation values define in the valley floor a flat direction

$$\begin{aligned} \phi_0^\dagger\phi_0 \equiv v^2 & = \frac{\gamma\bar{m}_S^2 - \beta\bar{m}_\phi^2}{\beta\lambda - \gamma^2} > 0, \\ S_0^\dagger S_0 \equiv \bar{f}^2 & = \frac{\gamma\bar{m}_\phi^2 - \lambda\bar{m}_S^2}{\beta\lambda - \gamma^2} > 0, \\ e^{2\sigma_0/f} & = -\frac{\beta S_0^\dagger S_0 + \gamma\phi_0^\dagger\phi_0}{m_S^2} \\ & = -\frac{\lambda\phi_0^\dagger\phi_0 + \gamma S_0^\dagger S_0}{m_\phi^2}, \end{aligned} \quad (6)$$

with

$$\gamma^2 < \beta\lambda.$$

Note that $\gamma > 0$ should be bounded from above in order that symmetries are broken. (If $\gamma < 0$, the same condition is required for the potential to be bounded from below.)

The consistency requirements for the couplings present in V_0 imply that the classical energy density vanishes at the stationary points and the potential V_0 takes the special form

$$V_0(\sigma, \phi, S) = \frac{\lambda}{2} \left[\phi^\dagger \phi - \frac{e^{2\sigma/f}}{\beta\lambda - \gamma^2} (\gamma m_S^2 - \beta m_\phi^2) \right]^2 + \frac{\beta}{2} \left[S^\dagger S - \frac{e^{2\sigma/f}}{\beta\lambda - \gamma^2} (\gamma m_\phi^2 - \lambda m_S^2) \right]^2 \\ + \gamma \left[\phi^\dagger \phi - \frac{e^{2\sigma/f}}{\beta\lambda - \gamma^2} (\gamma m_S^2 - \beta m_\phi^2) \right] \left[S^\dagger S - \frac{e^{2\sigma/f}}{\beta\lambda - \gamma^2} (\gamma m_\phi^2 - \lambda m_S^2) \right]. \quad (7)$$

The degeneracy in the vacuum expectation values and the vanishing of the vacuum energy density are expected to disappear in quantum field theory where scale invariance is anomalous [17]. Thus, we consider the one-loop corrections to the potential V_0 which can be computed by standard methods [18]. Since the dilaton interactions are not manifestly renormalizable, we treat σ as a classical background field. A convenient choice of the renormalization conditions yields, in the Landau gauge $(z_\phi \equiv \phi^\dagger \phi, z_S \equiv S^\dagger S)$,

$$V(\sigma, z_\phi, z_S) = \bar{a}^4 + \bar{m}_\phi^2 z_\phi + \bar{m}_S^2 z_S + \gamma z_\phi z_S + \frac{\lambda}{2} z_\phi^2 + \frac{\beta}{2} z_S^2 \\ + \frac{1}{(8\pi)^2} \left\{ \frac{3}{2} (\bar{m}_\phi^2 + \lambda z_\phi + \gamma z_S)^2 \left[\ln \frac{(\bar{m}_\phi^2 + \lambda z_\phi + \gamma z_S)^2}{M^4} - 1 \right] \right. \\ + \frac{1}{2} (\bar{m}_S^2 + \lambda z_\phi + \gamma z_S)^2 \left[\ln \frac{(\bar{m}_S^2 + \lambda z_\phi + \gamma z_S)^2}{M^4} - 1 \right] + \frac{1}{2} m_+^4 \left[\ln \frac{m_+^4}{M^4} \right] - 1 \Bigg\} \\ + \frac{1}{2} m_-^4 \left[\ln \frac{m_-^4}{M^4} \right] - 1 + \frac{3}{2} g^4 z_\phi^2 \left[\ln \frac{g^2 z_\phi}{2M^2} \right] - \frac{1}{2} + \frac{3}{4} (g^2 + g'^2) z_\phi^2 \left[\ln \frac{(g^2 + g'^2) z_\phi}{2M^2} - \frac{1}{2} \right] \\ - 12 g_t z_\phi^2 \left[\ln \frac{g_t^2 z_\phi}{M^2} \right] - \frac{1}{2} - 2 \sum_{i=1}^3 g_{R_i} z_S^2 \left[\ln \frac{g_{R_i}^2 z_S}{M^2} \right] - \frac{1}{2} \Bigg\}, \quad (8)$$

where g and g' are the $SU(2)_L$ and the $U(1)_Y$ gauge couplings, respectively, g_t is the Yukawa coupling for the top quark, and the heaviest fermion in the SM and $m_\pm^2$ are given by

$$m_\pm^2 = \frac{1}{2} \left[\bar{m}_\phi^2 + \bar{m}_S^2 + 3\lambda z_\phi + 3\beta z_S + \gamma(z_\phi + z_S) \pm \sqrt{(\bar{m}_\phi^2 + 3\lambda z_\phi - \bar{m}_S^2 - 3\beta z_S + \gamma z_\phi - \gamma z_S)^2 + 16\gamma^2 z_\phi z_S} \right]. \quad (9)$$

The parameters a^4 , m_ϕ^2 , m_S^2 , λ , β , and γ are now renormalized parameters which depend on the renormalization mass M . Their M dependence can be read off from Eq. (8):

$$M \frac{da^4}{dM} = \frac{1}{16\pi^2} (2m_\phi^4 + m_S^4), \quad (10)$$

$$M \frac{dm_S^2}{dM} = \frac{1}{16\pi^2} (4\beta m_S^2 + \gamma m_S^2 + 3\gamma m_\phi^2) + 2\gamma_S m_S^2, \quad (11)$$

$$M \frac{dm_\phi^2}{dM} = \frac{1}{16\pi^2} (6\lambda m_\phi^2 + \gamma m_S^2 + \gamma m_\phi^2) + 2\gamma_\phi m_\phi^2, \quad (12)$$

$$M \frac{d\lambda}{dM} = \frac{3}{64\pi^2} [(g^2 + g'^2)^2 + 2g^4 - 16g_t^4 + 16\lambda^2 + \frac{8}{3}\lambda\gamma] \\ + 4\gamma\gamma_\phi, \quad (13)$$

$$M \frac{d\beta}{dM} = \frac{1}{32\pi^2} \left[8\gamma^2 + 20\beta^2 - \sum_{i=1}^3 g_{R_i}^4 + 12\beta\gamma \right] + 4\gamma_S\beta, \quad (14)$$

$$M \frac{d\gamma}{dM} = \frac{1}{16\pi^2} (3\lambda + \beta + 4\gamma)\gamma + 2(\gamma_\phi + \gamma_S)\gamma, \quad (15)$$

where γ_ϕ and γ_S are the anomalous dimensions of the Higgs boson and singlet fields

$$\gamma_\phi = \frac{1}{64\pi^2} (-9g^2 - 3g'^2 + 12g_t^2), \quad (16)$$

$$\gamma_S = \frac{1}{32\pi^2} \sum_{i=1}^3 g_{R_i}^2. \quad (17)$$

The constraint given by Eq. (5) can also be imposed on the renormalized quantities and, since the scale dependence of $(\beta\lambda - \gamma^2)a^4$ and

$$(2\gamma m_\phi^2 m_S^2 - \beta m_\phi^4 - \lambda m_S^4)$$

is different, the renormalization mass M remains fixed.

The minimization of the one-loop renormalization group effective potential allows us to eliminate the vacuum degeneracy described by Eq. (6). The analysis may be simplified considerably by expanding the complicated

terms in Eq. (8) in a series around $\gamma=0$ and retaining only the lowest-order terms. A straightforward calculation gives

$$\begin{aligned} (\phi^\dagger \phi)_0 &= \frac{1}{2\lambda} A_\phi \left[1 - \frac{\lambda}{16\pi^2} \ln \frac{A_\phi}{M^2} \right], \\ (S^\dagger S)_0 &= \frac{1}{2\beta} A_S \left[1 - \frac{\beta}{16\pi^2} \ln \frac{A_S}{M^2} \right], \end{aligned} \quad (18)$$

where

$$A_\phi = \exp \left[\frac{A_1^\phi - A_2^\phi}{B} \right] M^2,$$

$$A_S = \exp \left[\frac{A_1^S - A_2^S}{B} \right] M^2,$$

$$\begin{aligned} A_1^\phi &= K_\phi K_S \left[-\frac{3}{2\lambda} g^4 \ln \frac{g^2}{4\lambda} - \frac{3}{4\lambda} (g^2 + g'^2)^2 \ln \frac{g^2 + g'^2}{4\lambda} \right. \\ &\quad \left. + \frac{12}{\lambda} g_i^4 \ln \frac{g_i^2}{2\lambda} \right] \left[4\beta - \sum_{i=1}^3 \frac{g_{R_i}^4}{2\beta} \right], \end{aligned}$$

$$A_2^\phi = 8\gamma K_S^2 \sum_{i=1}^3 \frac{g_{R_i}^4}{2\beta} \ln \frac{g_{R_i}^2}{2\beta},$$

$$\begin{aligned} A_1^S &= 4K_\phi K_S \sum_{i=1}^3 \frac{g_{R_i}^4}{2\beta} \ln \frac{g_{R_i}^2}{2\beta} \left[4\lambda + \frac{3}{2\lambda} g^4 \right. \\ &\quad \left. + \frac{3}{4\lambda} (g^2 + g'^2)^2 - \frac{12}{\lambda} g_i^4 \right], \end{aligned} \quad (19)$$

$$\begin{aligned} A_2^S &= 2\gamma K_\phi^2 \left[-\frac{3}{2\gamma} g^4 \ln \frac{g^2}{4\lambda} - \frac{3}{4\lambda} (g^2 + g'^2)^2 \ln \frac{g^2 + g'^2}{4\lambda} \right. \\ &\quad \left. + \frac{12}{\lambda} g_i^4 \ln \frac{g_i^2}{2\lambda} \right], \\ B &= K_\phi K_S \left\{ \left[4\lambda + \frac{3}{2\lambda} g^4 + \frac{3}{4\lambda} (g^2 + g'^2)^2 - \frac{12}{\lambda} g_i^4 \right] \right. \\ &\quad \left. \times \left[4\beta - \sum_{i=1}^3 \frac{g_{R_i}^4}{2\beta} \right] - 4\gamma^2 \right\}, \end{aligned}$$

and

$$\begin{aligned} K_\phi &= \exp \left[-\frac{3g^4 \ln \frac{g^2}{4\lambda} + \frac{3}{2} (g^2 + g'^2)^2 \ln \frac{g^2 + g'^2}{4\lambda} - 24g_i^4 \ln \frac{g_i^2}{2\lambda}}{8\lambda^2 + 3g^4 + \frac{3}{2} (g^2 + g'^2)^2 - 24g_i^4} \right], \\ K_S &= \exp \left[\frac{4 \sum_{i=1}^3 g_{R_i}^4 \ln (g_{R_i}^2 / 2\beta)}{8\beta^2 - 4 \sum_{i=1}^3 g_{R_i}^4} \right]. \end{aligned} \quad (20)$$

The value of $\exp(2\sigma_0/f)$ can be read off from Eqs. (6) and (18)–(20) where all the parameters are meant to be evaluated at the scale M .

To find the ground-state energy and the dilaton mass, it is useful to define the new fields [1]

$$\tilde{\phi} \equiv e^{-\sigma/f} \phi, \quad \tilde{S} \equiv e^{-\sigma/f} S, \quad (21)$$

so that the one-loop effective potential (8) now reads

$$V(\sigma, \tilde{\phi}, \tilde{S}) = e^{4\sigma/f} \left[\bar{V}(\tilde{\phi}, \tilde{S}) + \Delta(\tilde{\phi}, \tilde{S}) \frac{\sigma}{f} \right], \quad (22)$$

where $\bar{V}$ is the one-loop effective potential without a dilaton field and Δ is the anomalous divergence of the dilatation current:

$$\Delta = -M \frac{\partial}{\partial M} \bar{V}(\tilde{\phi}, \tilde{S}). \quad (23)$$

Using the extremum conditions

$$\begin{aligned} \frac{\partial V}{\partial \tilde{\phi}} \bigg|_{\tilde{\phi}_0, \tilde{S}_0, \sigma_0} &= 0, \quad \frac{\partial V}{\partial \tilde{S}} \bigg|_{\tilde{\phi}_0, \tilde{S}_0, \sigma_0} = 0, \\ \frac{\partial V}{\partial \sigma} \bigg|_{\tilde{\phi}_0, \tilde{S}_0, \sigma_0} &= 0, \end{aligned}$$

and the fact that $\bar{V}$ is scale independent, $(d\bar{V}/dM)=0$, one easily derives the ground-state energy and the dilaton mass [3]:

$$\langle V \rangle_0 = \frac{1}{4} \left\langle M \frac{\partial \bar{V}}{\partial M} \right\rangle_0 = -\frac{1}{128\pi^2} \text{Str} \mathcal{M}^4, \quad (24)$$

$$m_\sigma^2 = -\frac{4}{f^2} \frac{1}{4} \left\langle M \frac{\partial \bar{V}}{\partial M} \right\rangle_0 = \frac{1}{8\pi^2 f^2} \text{Str} \mathcal{M}^4, \quad (25)$$

$$\text{Str} \mathcal{M}^4 = 6m_W^4 + 3m_Z^4 + m_+^4 + m_-^4 - 12m_t^4 - 2 \sum_{i=1}^3 m_{N_{R_i}}^4. \quad (26)$$

In Ref. [7] the upper bound $m_{N_R} \lesssim 130$ GeV on the largest right-handed neutrino mass was obtained, roughly requiring that $m_\sigma^2 > 0$ or, equivalently,

$$m_i^4 + \frac{1}{6} \sum_{i=1}^3 m_{N_{R_i}}^4 \lesssim (100 \text{ GeV})^4.$$

However, in Ref. [7] it was assumed that neutrino masses are present and the lepton number is broken at all scales. In the model under consideration the presence of the gauge singlet S is fundamental to provide neutrino masses via the spontaneous breaking of the global group $U(1)_L$ at the scale $\bar{f}$ and one has to require that

$$m_i^4 + \frac{1}{6} \sum_{i=1}^3 m_{N_{R_i}}^4 - \frac{1}{12} m_-^4 \lesssim (100 \text{ GeV})^4,$$

which does not give any particularly strong bound on the largest eigenvalue m_{N_R} . Nevertheless, in the see-saw mechanism one would obtain for the electron neutrino mass $m_{\nu_e} \sim m_D^2 / (g_R \bar{f})$, where in the simplest scenario one expects $m_{\nu_e} \sim m_e$. The current limit on m_{ν_e} would then imply that $\bar{f} \gtrsim 100$ GeV. If $\bar{f} \gg v$, then the ϕ and the S sectors would be effectively decoupled unless one imposes a fine-tuning on the involved parameters or, more correctly, a fine-tuning on the initial conditions for the renormalization group equations (10)–(15). Thus, in the following we shall assume that $\bar{f}$ and v are roughly the same scale, as well as m_ϕ^2 and m_S^2 .

We can embed our theory in a background spacetime described by a metric $g_{\mu\nu}$ and demand that the resulting theory be invariant under local rescalings of the metric

$$g_{\mu\nu} \rightarrow g'_{\mu\nu} = \exp[2\gamma(x)] g_{\mu\nu}.$$

In order that the potential term $\sqrt{-g} V(\sigma, \phi, S)$ be invariant, we demand that σ , ϕ , and S transform as $\sigma' = \sigma - f\gamma(x)$, $\phi' = \exp[-\gamma(x)]\phi$, and S'

$= \exp[-\gamma(x)]S$ [19]. The ϕ and the S kinetic terms used in Eq. (1) are not Weyl invariant. However, if we use for the terms involving the dilaton field σ and the Higgs doublet ϕ (and similarly for S) the Lagrangian

$$\begin{aligned} \mathcal{L} = \sqrt{-g} \left[\frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu \sigma - \frac{1}{f} g^{\mu\nu} \partial_\mu (\phi^\dagger \phi) \partial_\nu \sigma \right. \\ \left. + \frac{1}{f^2} g^{\mu\nu} \partial_\mu \sigma \partial_\nu (\phi^\dagger \phi) \right. \\ \left. + g^{\mu\nu} (D_\mu \phi)^\dagger (D_\nu \phi) - V(\sigma, \phi) \right], \end{aligned} \quad (27)$$

the classical theory is no longer invariant under dilations, which would require the kinetic term $\exp(2\sigma/f) \partial_\mu \sigma \partial^\mu \sigma$ for the dilaton field. However, a characteristic feature of the Lagrangian (27) is an approximate Weyl invariance in curved space, which is only broken by the kinetic terms of gravitational and dilaton fields:

$$\begin{aligned} \sqrt{-g} \left(\frac{1}{2} \kappa \mathcal{R} + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma \right) \\ = \sqrt{-\bar{g}} \left[D \bar{\mathcal{R}} + \frac{\omega}{D} \bar{g}^{\mu\nu} \partial_\mu D \partial_\nu D \right], \end{aligned} \quad (28)$$

$$\bar{g}_{\mu\nu} = e^{2\sigma/f} g_{\mu\nu},$$

where $\mathcal{R}$ is the curvature scalar, $\omega = f^2/4\kappa$, and

$$D = \frac{1}{2} \exp[-2\sigma/(f^2 + 6\kappa)^{1/2}]$$

is what is generally called the JBD dilaton. The theory is the Jordan-Brans-Dicke theory of gravity with the SMM as the matter sector [6]. In the next section we shall make use of kinetic terms such as those in Eq. (27) to find the Friedmann-Robertson-Walker (FRW) equation governing the dynamics of the dilaton field.

III. THE EFFECT OF DILATONS ON THE PHASE TRANSITIONS IN SMM

We first consider the finite temperature potential [20] of the SMM where the dilaton can be added in the standard manner [1]:

$$\begin{aligned} V_T(\sigma, z_\phi, z_S) = c\pi^2 T^4 + \bar{a}^4 + \bar{m}_\phi^2 z_\phi + \bar{m}_S^2 z_S + \gamma z_\phi z_S + \frac{\lambda}{2} z_\phi^2 + \frac{\beta}{2} z_S^2 + \frac{1}{6} \bar{m}_\phi^2 T^2 + \frac{1}{12} \bar{m}_S^2 T^2 + \alpha_\phi T^2 z_\phi + \alpha_S T^2 z_S \\ - \frac{1}{12\pi} \left[3(\bar{m}_\phi^2 + \alpha_\phi T^2 + \lambda z_\phi + \gamma z_S)^{3/2} + (\bar{m}_S^2 + \alpha_S T^2 + \beta z_S + \gamma z_\phi)^{3/2} \right. \\ \left. + m_+^2 + m_-^2 + 6 \left[\frac{g^2}{2} \right]^{3/2} z_\phi^{3/2} + 3 \left[\frac{g^2 + g'^2}{2} \right]^{3/2} z_\phi^{3/2} \right] \\ - \frac{1}{(8\pi)^2} \left[3(\bar{m}_\phi^2 + \alpha_\phi T^2 + \lambda z_\phi + \gamma z_S)^2 + (\bar{m}_S^2 + \alpha_S T^2 + \beta z_S + \gamma z_\phi)^2 \right. \\ \left. + m_+^4 + m_-^4 + \frac{3}{2} g^4 z_\phi^2 + \frac{3}{4} (g^2 + g'^2)^2 z_\phi^2 \right] \left[\ln \left[\frac{M^2}{A_b T^2} \right] + \frac{1}{2} \right] \\ - \left[12g_i^4 z_\phi^2 + 2 \sum_{i=1}^3 g_{R_i}^4 z_S^2 \right] \left[\ln \left[\frac{M^2}{A_f T^2} \right] + \frac{1}{2} \right], \end{aligned} \quad (29)$$

where

$$\begin{aligned}\alpha_\phi &= \frac{1}{24}(6\lambda + 2\gamma + \frac{9}{2}g^2 + \frac{3}{2}g'^2 + 6g_f^2), \\ \alpha_S &= \frac{1}{24} \left[4\beta + 4\gamma + \sum_{i=1}^3 g_{R_i}^2 \right], \\ A_f &= \pi^2 \exp(\frac{3}{2} - 2\gamma_E), \quad A_b = 16 A_f, \quad \gamma_E \simeq 0.57.\end{aligned}\quad (30)$$

The finite temperature potential V_T is given in an expansion of $(1/\pi)m_i/T$, up to terms of order $1/T$, where $m_i(\sigma, z_\phi, z_S)$ are the various particle masses as functions of the scalar fields. The constant c essentially counts the total number of degrees of freedom. We have replaced the bare mass parameters by the corresponding plasma masses [20] in V_T to avoid the latter being ill defined in $z_\phi = z_S = 0$.

The dilaton field σ is not in thermal equilibrium at temperatures below the scale f (see Enqvist in Ref. [8]). Since $|\dot{\sigma}/\sigma| \ll T$, from the point of view of finite temperature field theory the σ field will act as a constant [9]. The dynamics of the dilaton field is described by the FRW equation of motion

$$\begin{aligned}\ddot{\sigma} + 3H\dot{\sigma} + \frac{2}{f^2}\ddot{\sigma}(z_\phi + z_S) + \frac{2}{f^2}\dot{\sigma}(\dot{z}_\phi + \dot{z}_S) - \frac{1}{f}(\ddot{z}_\phi + \ddot{z}_S) \\ = -\frac{2}{f}e^{2\sigma/f}(2a^4e^{2\sigma/f} + m_\phi^2 z_\phi + m_S^2 z_S),\end{aligned}\quad (31)$$

where H is the Hubble constant. In writing Eq. (31) we have followed Ref. [9] and taken into account that σ does not fluctuate fast enough to be in thermal equilibrium with ϕ and S , so that its expectation value should not be determined by minimizing the finite temperature potential, but the $T=0$ potential where z_ϕ and z_S have to be replaced in Eq. (31) by their thermal average values $\langle z_\phi \rangle_T$ and $\langle z_S \rangle_T$, respectively. The value of σ at its minimum at a given temperature is therefore

$$\exp\left[\frac{2\langle\sigma\rangle}{f}\right] = k_\phi \langle z_\phi \rangle_T + k_S \langle z_S \rangle_T, \quad (32)$$

where

$$\begin{aligned}k_\phi &\equiv \frac{m_\phi^2(\gamma^2 - \beta\lambda)}{\lambda m_S^4 + \beta m_\phi^4 - 2\gamma m_\phi^2 m_S^2}, \\ k_S &\equiv \frac{m_S^2(\gamma^2 - \beta\lambda)}{\lambda m_S^4 + \beta m_\phi^4 - 2\gamma m_\phi^2 m_S^2}.\end{aligned}\quad (33)$$

In the range of validity of the high-temperature expansion of V_T , we can approximate $\langle z_\phi \rangle_T = \langle z_S \rangle_T \simeq T^2/12$ [21] and find that

$$\begin{aligned}V_T &= \bar{c}\pi^4 T^4 + a_\phi T^2 z_\phi + a_S T^2 z_S + \frac{\lambda}{2} z_\phi^2 + \frac{\beta}{2} z_S^2 \\ &\quad + \gamma z_\phi z_S + O(z_\phi^{3/2}, z_S^{3/2}),\end{aligned}\quad (34)$$

where

$$\begin{aligned}a_\phi &\equiv \alpha_\phi + \frac{m_\phi^2}{12}(k_\phi + k_S), \\ a_S &\equiv \alpha_S + \frac{m_S^2}{12}(k_\phi + k_S),\end{aligned}\quad (35)$$

and $\bar{c}$ can be easily inferred from Eqs. (29) and (32).

Since we are presently interested in the case $|\dot{\sigma}/\sigma| \gg H$, we expect that the value of $|\dot{\sigma}/\sigma|$ is set by the mass of σ at the minimum of its potential [9]:

$$m_\sigma^2 = \left. \frac{\partial^2 V}{\partial \sigma^2} \right|_{\sigma=\langle\sigma\rangle} \simeq \frac{\lambda}{f^2} \frac{T^4}{18}, \quad (36)$$

where we have assumed $\lambda \sim \beta$ and $m_\phi^2 \sim m_S^2$. Thus $|\dot{\sigma}/\sigma|$ will be much smaller than T if

$$T^2 \ll 18f^2/\lambda, \quad (37)$$

which is certainly satisfied for the range of temperatures under consideration (one can also easily show that the assumption $|\dot{\sigma}/\sigma| \gg H$ is well satisfied for $T \lesssim f$).

Looking at Eqs. (34) and (35), we discover that, if a_ϕ and a_S are both positive, the $SU(2)_L \otimes U(1)_Y$ and the $U(1)_L$ symmetries will be unbroken at any temperature. Moreover, we cannot conclude that, if a_ϕ and a_S are negative, the same symmetries are always broken since our finite temperature expansion is only valid for $a_\phi, a_S > 0$, so that no firm conclusion concerning the symmetry breaking can be drawn in this case. In the following we shall assume a_ϕ and a_S to both be positive, which is likely to be true if $\lambda \sim \beta$. Thus the finite temperature corrections generate a barrier at $z_\phi = z_S = 0$ which persists down to $T=0$.

Since in the SMM with dilaton the vacuum energy density vanishes at the tree level, we assume that the critical temperature T_c , which is defined by [8]

$$V_{T_c}(0) = V_{T_c}(v^2, \bar{f}^2), \quad (38)$$

is smaller than $v \sim \bar{f}$. If in the broken phase particle masses are either much smaller or much larger than the critical temperature, the difference in the finite temperature correction to the effective potential at $(z_\phi=0, z_S=0)$ and $(z_\phi=v^2, z_S=\bar{f}^2)$ is simply determined by the numbers of effectively massless particles in both phases [22] and T_c results

$$\begin{aligned}T_c &= \left[\frac{45}{1712\pi^4} \left(6m_W^4 + 3m_Z^4 + m_+^4 + m_-^4 - 12m_t^4 \right. \right. \\ &\quad \left. \left. - 2 \sum_{i=1}^3 m_{N_{R_i}}^4 \right) \right]^{1/4} \sim 10 \text{ GeV}.\end{aligned}\quad (39)$$

Using the numerical results found in Ref. [23], it is not difficult to convince oneself that the tunneling rate from the symmetric to the broken phase at $T < T_c$, which is given in a volume V by $\Gamma \simeq VT_c^4 \exp(-S_3/T)$, where S_3 is the bounce action, is extremely slow. Indeed, along the ray $\phi = R \cos\theta$ and $S = R \sin\theta$, where $\tan\theta = (\bar{f}/v)$, one

obtains the naive estimate

$$\frac{S_3}{T} = \frac{44(a_\phi^2 \cos^2 \theta + a_S^2 \sin^2 \theta)^{3/2} (4\pi)^2}{[G_1 \cos^3 \theta + (1 + 3^{3/2}) \beta^{3/2} \sin^3 \theta]^2} \gg 1, \quad (40)$$

where

$$G_1 \equiv 6(g^2/2)^{3/2} + 3[(g^2 + g'^2)/2]^{3/2} \simeq 1.52.$$

Thus neither the electroweak nor the lepton-number transitions are expected to occur at $T < T_c$. Nevertheless, as soon as the Universe cools down to a temperature below $T_{\text{QCD}} \simeq 150$ MeV, because of the dynamical breaking of the chiral symmetry and the appearance of a $\bar{\psi}\psi$ condensate, a new contribution to the finite temperature potential becomes effective:

$$\Delta V_T = -\varepsilon(T) z_\phi^{3/2}, \quad (41)$$

where [24]

$$\varepsilon(T) = g_t \langle \bar{\psi}\psi \rangle_0 \left[1 - \frac{N^2 - 1}{N} \frac{T^2}{12f_\pi^2} + O\left(\frac{T^4}{(12)^2 f_\pi^4}\right) \right], \quad (42)$$

and $\langle \bar{\psi}\psi \rangle_0 \simeq -(250 \text{ MeV})^3$, $N=6$, and $f_\pi=93$ MeV. At temperatures below T_{QCD} , a nonzero vacuum expectation value in the direction $z_S=0$ is induced at

$$\bar{z}_\phi^{1/2}(T) \simeq \frac{\varepsilon(T)}{2a_\phi} \frac{1}{T^2}. \quad (43)$$

The false vacuum ($z_\phi=0, z_S=0$) will roll down towards the new unstable vacuum [$z_\phi=\bar{z}_\phi^{1/2}(T), z_S=0$] very quickly, in a time (in Hubble time unit)

$$\left[\frac{\Delta t}{H^{-1}} \right] \simeq \frac{3}{2} \frac{g_*(T_{\text{QCD}})}{a_\phi} \left[\frac{T_{\text{QCD}}}{M_P} \right]^2 \ll 1, \quad (44)$$

where $g_*(T_{\text{QCD}})$ counts the effective degrees of freedom at the temperature T_{QCD} . When the temperature $T=T_*$, defined by¹

$$\varepsilon(T_*) = \frac{8\pi a_\phi^2}{G_1} T_*^3, \quad (45)$$

is reached, the metastability of the wrong unstable minimum $\bar{z}_\phi^{1/2}(T)$ is lost and the latter is free to roll down along the $z_S=0$ direction.²

¹Note that $\bar{z}_\phi^{1/2}(T_*)/T_* \simeq (4\pi)a_\phi/G_1 \lesssim 1/g^2$, so that the high-temperature expansion is still valid.

²Since the transition temperature is of order of Λ_{QCD} one may wonder about the size of perturbative and nonperturbative QCD corrections to our effective Coleman-Weinberg-type potential (8). Indeed, Flores and Sher [25] have argued that the transition to the global minimum in the SM with a Coleman-Weinberg-type potential does not take place since the coupling of the quartic term in the potential for the Higgs field changes sign for small values of ϕ . However, in the SM, as well as in the SMM, coupled to a JBD dilaton, the Higgs boson masses are large as required by the positivity of the dilaton mass.

Expanding now the potential about the value $(\phi_+, 0)$, we get

$$V_T(z_S) = (a_S T^2 + \gamma \phi_+^2) z_S + \text{other terms}. \quad (46)$$

If $\gamma < 0$ (looking at the renormalization group equation for γ one can infer that $\gamma=0$ is a fixed point so that γ does not change its sign with the scale M), from Eq. (46) we read off that the barrier along the z_S direction ceases to exist, and the lepton number is spontaneously broken, when ϕ_+ reaches the value $(a_S/\gamma)^{1/2} T$, which is likely to occur at $T < T_*$ since

$$[a_S T_*^2 + \gamma \bar{z}_\phi(T_*)] = a_S + \gamma(16\pi^2/G_1^2) a_\phi^2 > 0 \quad (47)$$

for reasonable values of the parameters. Thus, if $\gamma < 0$, it is the electroweak phase transition to drive the breaking of $U(1)_L$.

If $\gamma > 0$, from Eq. (46) one could conclude that a barrier in the z_S direction is always present. This is not the case. Indeed, Eq. (46) is only valid until the expression $\langle z_\phi \rangle_T = T^2/12$ may be used, i.e., until $\phi_+ \lesssim T$. As soon as $\phi_+ \gtrsim T$, Eq. (46) must be replaced by the more correct expression

$$V_T = \left[\left[\frac{\sum_{i=1}^2 g_{\bar{R}_i}^2}{24} + \frac{m_S^2}{12} k_S \right] T^2 + (\gamma + m_S^2 k_\phi) \phi_+^2 \right] z_S + \text{other terms}. \quad (48)$$

The presence of right-handed neutrinos is essential to keep the barrier in the z_S direction when ϕ_+ get values larger than T , i.e., when scalar fields no longer contribute to the $\sim T^2 z_S$ term. Assuming that $\lambda \sim \beta$ and $m_\phi^2 \sim m_S^2$, we conclude that, analogously to the case $\gamma < 0$, the $U(1)_L$ phase transition occurs after the electroweak transition, when ϕ_+ is of the order of

$$\left[\left[\sum_{i=1}^3 g_{\bar{R}_i}^2 - \lambda \right] / [24(\lambda - \gamma)] \right]^{1/2} T.$$

The change in the vacuum energy density as soon as the lepton number is spontaneously broken is given by

$$\Delta V \simeq a^4 e^{4(\sigma)/f} \simeq a^4 \frac{(\lambda \langle z_\phi \rangle + \gamma \langle z_S \rangle)^2}{m_\phi^4} \simeq \lambda T_{\text{QCD}}^4. \quad (49)$$

Thus the maximum reheating of the radiation energy density that could occur at this stage of the phase transitions is only $O(T_{\text{QCD}}^4)$, as it happens in the SM [9]. Most of the energy density of order $10^{-2} v^4$ will remain in the vacuum.

Quite surprisingly, solving the FRW equations for the dilaton and Higgs field system, one can show that in the SM case most of the electroweak phase transition goes into oscillations of the dilaton field after its long slow roll along the valley floor down to its $T=0$ minimum at σ_0 . The point is that quantum fluctuations around σ_0 get very small masses, of order v^2/f , and they can decay only into pairs of very light neutrinos with a rate $\Gamma_\sigma \sim m_\sigma^3/f^2$. The temperature at the end of the σ decays

is then

$$T_{\text{rh}} \simeq (M_P \Gamma_\sigma)^{1/2} \simeq (M_P m_\sigma^3 / f^2)^{1/2}.$$

To avoid the Universe being dominated by the energy stored in the dilaton field during the nucleosynthesis and causing the calculation of the light element abundances at present (which requires a radiation-dominated Universe [26] to be successful) to disagree with their observed values, one must impose the bound $T_{\text{rh}} \gtrsim 0.1$ MeV or, equivalently, the upper limit mentioned in the Introduction, $f \lesssim 10^7$ GeV.

In the model discussed in this paper the situation is completely different. Indeed, dilaton oscillations around the $T=0$ vacuum can decay very efficiently into a pair of Majorons, the massless Goldstone bosons associated to the breaking of the global $U(1)_L$ group.

Using the Cartesian decomposition of the gauge singlet field

$$S = \rho + \bar{f} + iJ, \quad (50)$$

where J is the Majoron, and writing

$$e^{2\sigma/f} = e^{2\langle\sigma\rangle/f} (1 + 2\sigma/f), \quad (51)$$

from Eq. (2) we read off the σJJ interaction term

$$\begin{aligned} \mathcal{L}_{\text{int}} &= \frac{2m_S^2}{f} e^{2\langle\sigma\rangle/f} \sigma JJ \\ &\simeq -2 \frac{\beta \bar{f}^2 + \gamma v^2}{f} \sigma JJ. \end{aligned} \quad (52)$$

If had we used the more common decomposition of S into polar coordinates, $S = (\rho + \bar{f}) \exp(iJ/\bar{f})$, the same interaction Lagrangian would have been inferred from the Majoron kinetic term

$$\mathcal{L}_{\text{kin}} = [1 + (\rho/\bar{f})]^2 \partial_\mu J \partial^\mu J, \quad (53)$$

which, after integrations by parts, yields the same interaction term of Eq. (52):

$$\frac{1}{\bar{f}} \partial_\mu \partial^\mu \rho JJ = \frac{m_S^2}{f} S e^{2\sigma/f} + \dots = \frac{2m_S^2}{f} e^{2\langle\sigma\rangle/f} \sigma JJ + \dots \quad (54)$$

The dilaton can decay into a pair of Majorons with a rate

$$\Gamma(\sigma \rightarrow JJ) = \frac{1}{8\pi} \frac{m_S^4}{f^2 m_\sigma} \simeq \frac{1}{8\pi} \beta^2 \frac{\bar{f}^2}{f}. \quad (55)$$

Since $\Gamma(\sigma \rightarrow JJ) \gg H$, we conclude that, as soon as the scalar fields relax to their $T=0$ minima after a long slow roll down along the valley floor given by Eq. (6), the Universe will be reheated to a temperature ~ 10 GeV if the τ neutrino is sufficiently heavy. Indeed, the pairs of Majorons produced by σ decays can rapidly begin to thermalize with the thermal bath at $T \simeq 150$ MeV through processes such as $J\nu_\tau \rightarrow J\nu_\tau$ or $JJ \rightarrow \nu_\tau \nu_\tau$. Since the rate of these scatterings is of order of $\Gamma \simeq 10^{-2} (m_{\nu_\tau}^2 / \bar{f}^4) T^3$ [27], the equilibrium condition $\Gamma \gg H$ is satisfied if

$$m_{\nu_\tau} \gg 1 \left[\frac{T}{150 \text{ MeV}} \right]^{-1/2} \left[\frac{\bar{f}}{100 \text{ GeV}} \right]^2 \text{ MeV}. \quad (56)$$

The presence of Majorons makes this heavy τ neutrino, for example, $m_{\nu_\tau} = (5-10)$ MeV, cosmologically harmless thanks to the fast decay $\nu_\tau \rightarrow \nu_{\mu,e} J$, in contrast with the situation discussed in Ref. [7], where neutrino masses were thought to be present at all scales through the seesaw mechanism and the rate of the only possible decay $\nu_\tau \rightarrow \nu_{\mu,e} \sigma$ was not large enough to avoid the decay products to close the Universe, unless $f \lesssim 10^9$ GeV, again in contrast with the natural identification $f \simeq M_P$.

In the present model, when the temperature drops down to $T \simeq m_{\nu_\tau}$ after the reheating, τ neutrinos rapidly decay and Majorons freeze out since their scatterings with ν_e 's and ν_μ 's are not enough fast enough to keep them in equilibrium. However, the fact that their number density is comparable to the number density of light neutrinos at the onset of nucleosynthesis is not in contrast with the usual upper bound on the effective number of light neutrinos at $T \simeq 1$ MeV, $N_\nu < 3.4$ [28], since now τ neutrinos are no longer present when light elements are produced. No upper bound on f has to be imposed since nucleosynthesis can now take place in a standard way when the Universe is radiation dominated.

We finally point out that the change in entropy from the initial to the final state is $S_f/S_i = (g_{*f} T_f^3 / g_{*i} T_i^3)$, where $g_{*i(f)}$ is the number of effectively massless particles in the final (initial) state, and $T_f (T_i)$ is the final (initial) temperature. With $T_i \sim T_{\text{QCD}} \sim 150$ MeV, $T_f \sim 10$ GeV, $g_{*i} \sim 114.25$, and $g_{*f} \sim 87$ one obtains $S_f/S_i \sim 4 \times 10^5$, which appears acceptable within the conventional framework of baryon number generation in the early Universe [14].

IV. CONCLUSIONS

The realization that any matter-antimatter asymmetry created at some superheavy scale [14] can be easily wiped out by B and L SM quantum effects [12], unless the baryon asymmetry is proportional to the combination $(B-L)$, makes us faced with the vital problem of avoiding that new L -violating interactions beyond the SM are in thermal equilibrium when the SM anomalous effects are still active. On the other hand, the SMM [10] naturally predicts L -violating interactions and it has been recently shown in Ref. [29] that the global $U(1)_L$ group is spontaneously broken before the electroweak phase transition so that a new mechanism to regenerate the baryon asymmetry at the Fermi scale has to be invoked, leading to a severe upper bound on the mass of the lightest neutral Higgs boson from the requirement that the SM anomalous effects are sufficiently suppressed after the accomplishment of the electroweak phase transition [30]. Motivated by these considerations, we have investigated an extension of the SMM with a dilaton field in which the breaking of the scale invariance by the mass parameters of the scalar potential is removed. Scale invariance remains broken by the dependence of the couplings on the renormalization mass, i.e., by the conformal anomaly,

and by the kinetic term of the dilaton. In curved space-time this theory is precisely the Jordan-Brans-Dicke theory of gravity with the SMM as matter sector [6].

Starting from the one-loop renormalization group improved potential we have discussed the ground state of the theory. The classical potential has a flat direction lifted by quantum corrections which have allowed us to break the vacuum degeneracy and to find the true minima of the model.

We have then analyzed the cosmological consequences of a JBD dilaton on the dynamics of the $SU(2)_L \otimes U(1)_Y$ and $U(1)_L$ phase transitions in the early Universe. We have concluded that the lepton number is spontaneously broken after the electroweak phase transition, when anomalous effects are no longer active, which permits us to avoid any strong bound on the couplings of the model.

Furthermore, we have shown that, contrary to the case of the SM coupled to a JBD dilaton, if a sufficiently τ neutrino exists, $m_{\nu_\tau} = (5-10)$ MeV, the scale f of the spontaneous breaking of scale invariance receives no limit from considerations on the dilaton and/or neutrino cosmology, so that we can still identify the scale f with the Planck scale as required to achieve a scalar-tensor theory of gravity and to provide an explanation for the mass hierarchy between the Planck and Fermi scale [3].

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