Cosmic magnetic field imprints on cosmic radiation

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We investigate the gravitational properties of photons traveling through magnetic fields. We find that, although no gravitational radiation is emitted by a photon in vacuum, a graviton forms inside a photon moving through a magnetic field. The properties of this graviton and its back reaction on the photon are explored. It is found that, in view of this effect, magnetic fields can "absorb" light in a frequency-independent fashion. Hence, we conclude that a distinctive anisotropy in the cosmic radiation effective temperature would be associated with a cosmic magnetic field. Three cosmic radiation experiments based on this effect are proposed. One of them can potentially yield a strict enough bound on the primordial magnetic field intensity to prove the need for a dynamo.

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I. INTRODUCTION

Magnetic fields (B fields) are ubiquitous in the Universe (see [1–3] for reviews). Their presence in planets and stars is well established. On a larger scale it is known that galaxies are endowed with magnetic fields of the order of a few μG coherent on the scale of several kpc. Quasars and their jets are known to contain particularly strong large scale B fields, reaching intensities of the order of 10^3 G. It is conceivable that magnetic fields also pervade much larger structures, such as clusters or walls, and we may even speculate the existence of an intercluster uniform magnetic field, forming a fundamental constituent of the Universe.

The origin of the cosmic B fields is not completely understood. Various theories of B-field formation have been proposed. They all rely on the existence of a seed or primordial B field, which is then amplified. The seed field can be explained in the inflationary scenario, from Planck epoch quantum fluctuations [4,5]. It can also be generated in the wiggly cosmic string scenario [6], elaborating on an earlier idea by Harrison [7]. When structures start to grow in the matter era several mechanisms of primordial B-field amplification can then come into play. Amplification by collapse, or B-field line dragging, is a linear amplification process which is always present. It relies on the assumption that the Universe is a good conductor, with the consequent "freezing-in" of the B fields [1,8,9]. This mechanism is present even when the collapse of structures goes nonlinear. However, as soon as nonlinear hydrodynamic processes switch on, other, more efficient, mechanisms of amplification can be of relevance—the so-called dynamos [1,2]. These are positive feedback processes, transferring angular momentum energy into a seed B field. Different dynamo models, with different efficiencies, have been considered. All in all our understanding of nonlinear astrophysical processes is not good enough for the correct model to be selected.

It appears, however, that a direct measurement of the seed B-field intensity would be crucial in deciding which is the best-fit theory. The seed B-field intensity is usually characterized by the ratio r of the average magnetic energy density and the cosmic microwave background radiation (CMBR) energy density. We have $r = \frac{\overline{B}^2/8\pi}{\rho_\gamma} = \left(\frac{\overline{B}(t_0)}{3.2\times10^{-6}\text{ G}}\right)^2$. This ratio stays conveniently constant throughout expansion. Crude estimates show that r has to be as large as 10^{-8} for line dragging amplification to be sufficient for explaining the observed B fields, making dynamos redundant. On the other hand most dynamos could account for, say, the galactic B field, with a value of r as low as 10^{-34}. Hence the importance of a direct measurement of r: it would tell us the linear input into the nonlinear regime, making up for our lack of understanding of the actual nonlinear astrophysical processes.

This brings up the issue of the visibility of magnetic fields. If these were visible we would have a chance of directly probing the primordial B field. Indeed we do have a light source behind, so to speak, the primordial B field: the CMBR. However, the linearity of Maxwell equations implies that B fields are invisible. Quantum corrections to electromagnetism (vacuum polarization) would render the B fields visible, but only very dimly at the energy scales we are considering. More commonly, media are used in astrophysics to paint B fields, as the media optical properties can be B-field dependent (e.g., Faraday rotation). It is, however, common to admit (even though this statement has been disputed), that the CMBR, after recombination, is effectively in vacuum. We will thus assume that the effect of media is irrelevant in whatever process we will consider (even though in Sec. IX we will also examine the implications of relaxing this assumption).

In this paper gravitation is used as the source of the electromagnetic nonlinearity needed for making B fields visible. In Secs. II–V we rederive, correct, and extend some results previously published in [10, 11] concerning the metric of a photon. These results show that gravity does render B fields visible. In Sec. II we explain why is it that a photon in vacuum does not emit gravitational radiation. New results are derived, in the Arnowitt-Deser-

Misner (ADM) framework, showing that, while a scalar wave superposes on the photon and a gravitational shock wave develops around it, no actual gravitational energy flux exists. These results are then rederived in Sec. III for the case of a photon immersed in a B field. The exterior metric is found unchanged, but now we find that a graviton forms inside the photon. Various properties of this graviton are explored and it is proved that the "graviton pumping" process entails energy transfer. Hence, somehow, the photon has to give up some of its energy. To find out how it does so, in Sec. IV we study the back reaction of the graviton metric on the photon. We conclude that no color or phase shift occur. Instead, a color-independent photon annihilation probability accounts for the graviton energy. This means that magnetic fields are not colored. They are hazy. In Sec. V all these results are extended with few surprises for the case of a cosmological background. It should be said that not all the results in Secs. II–V are totally new. However the method used is new, a few crucial mistakes in the work of [11] have been corrected, and a more complete answer to the problem is achieved.

In Secs. VI–IX we apply graviton pumping to the cosmic B fields as illuminated by the CMBR. We introduce the decisive distinction between CMBR effective and color temperature anisotropies, enabling the distinctive hallmarks of graviton pumping to stand out. Three carefully devised CMBR experiments are described, aimed at measuring r. These may potentially yield the strictest and most reliable bound on r, easily good enough to prove the need for a dynamo. Finally, in a concluding section, we confront the method proposed for measuring r with previous work on the subject. We point out several advantages of our method relative to previously used ones, but we also spell out the cosmological assumptions we have used, and show how their breakdown would irremediably invalidate our conclusions.

In this paper we use units such that $c = G = \hbar = k_B = 1$, the metric has signature $+2$, and the Cartesian coordinates, wherever used, are ordered like t, z, x, y.

II. THE METRIC OF A PHOTON IN VACUUM: WHY DOES A PHOTON NOT SHINE GRAVITATIONALLY?

For reasons to be made clear later, we shall adopt a wave packet picture of the photon [12]. We consider a linearly polarized plane electromagnetic (EM) wave which is confined inside a finite region defined by the window function $W(z-t, x, y)$. The electric and magnetic fields are given by

$$\mathbf{E} = \mathbf{e}_x A e^{ik\cdot x} W, \tag{1}$$

$$\mathbf{B} = \mathbf{e}_y A e^{ik\cdot x} W, \tag{2}$$

with $k = \omega(-dt + dz)$. The stress energy tensor of the EM field is

$$T_{\mu\nu} = F_{\mu\alpha} F_\nu{}^\alpha - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \tag{3}$$

and so the wave packet is characterized by the stress energy tensor components

$$\begin{aligned} \rho = -T^0_0 &= A^2 e^{2ik\cdot x} W, \\ \mathbf{\Sigma} = T^0_i &= A^2 e^{2ik\cdot x} W \mathbf{e}_z, \\ T_{ij} &= A^2 e^{2ik\cdot x} W \mathbf{e}_z \otimes \mathbf{e}_z . \end{aligned} \tag{4}$$

We will develop a dual view of the photon: the "inside" and the "outside" pictures. The "inside" picture consists of the piece of plane wave inside the wave packet and simulates the "inside" of a photon. The "outside" picture refers to the far away gravitational effects of the photon. From the outside point of view, one can approximate the stress energy tensor (4) by

$$\tilde{T}_{\mu\nu} = \delta(x)\delta(y)\delta(t-z) \int d^3x\, T_{\mu\nu} \tag{5}$$

implementing a point particle picture for the photon. The "shell region" of the photon will be ignored throughout, together with any of its effects. The total energy of the wave packet is $E = A^2\Omega/2$ where $\Omega = \int d^3x\, W$. Therefore, in the quantum picture, the wave packet contains $N = E/(\hbar\omega)$ coherent photons. Nevertheless, recall that the classical limit requires $N \gg 1$ [13], so the word photon ($N = 1$) is being used here in a somewhat metaphoric sense.

We shall now compute the linearized metric for the photon model proposed. We start with the inside picture. Using the notation and results revised in [14] we first perform the geometrical splitting of the stress energy tensor (4). One finds that an EM plane wave is a pure scalar:

$$\begin{aligned} \rho &= A^2 e^{2ik\cdot x} W, \\ \mathbf{\Sigma} &= \boldsymbol{\nabla}\Sigma, \quad \Sigma = \frac{A^2 e^{2ik\cdot x}}{2i\omega} W, \\ p &= \frac{1}{3} A^2 e^{2ik\cdot x} W, \\ \Pi^{TL}_{ij} &= \Delta_{ij}\Pi^T, \quad \Pi^T = \frac{A^2 e^{2ik\cdot x}}{-4\omega^2} W, \end{aligned} \tag{6}$$

a fact which is independent of the wave polarization state. This result supports the widely spread view that an EM wave, containing no tensor component, does not radiate gravitationally. The situation is however more complex. The linearized Einstein equations for this source can be found from the gauge-invariant formalism including seeds [14] by setting $h = 0$. They are

$$-\Delta\,\Phi = 4\pi\rho, \tag{7}$$

$$\dot{\Phi} = 4\pi\Sigma, \tag{8}$$

$$\Delta\,(\Phi + \Psi) = 8\pi\Delta\Pi_T, \tag{9}$$

$$-\ddot{\Phi} = 4\pi[p + (2/3)\Delta\Pi_T] . \tag{10}$$

The inside solution is

$$\Psi = \Phi = \frac{\pi A^2 e^{2ik\cdot x}}{\omega^2} W \tag{11}$$

showing that, contrary to popular belief, there can be scalar waves. It is indeed true that only tensor gravitational waves exist in vacuum. However, inside suitable matter configurations, the scalar metric perturbations can also be oscillatory.

Outside the wave packet the geometrical splitting approach is no longer simple, as vector and tensor modes come about. It is then easier to solve for the metric in the Lorentz gauge. The answer is the well-known Aichelburg-Sexl metric [10].

In order to clarify the physical interpretation of the result found we proceed to study the energetics of the metric. This can be approached by means of the effective stress energy tensor for gravitational waves, as defined in [15]:

$$T^{(\mathrm{GW})}_{\mu\nu} = \frac{1}{32\pi}\left\langle \bar{h}_{\alpha\beta,\mu}\bar{h}^{\alpha\beta}{}_{,\nu} - \frac{1}{2}\bar{h}_{,\mu}\bar{h}_{,\nu} - 2\bar{h}^{\alpha\beta}{}_{,\beta}\bar{h}_{\alpha(\mu,\nu)}\right\rangle . \tag{12}$$

Here angular brackets denote spatial averaging over several wavelengths, a procedure which, as is well known, produces a gauge-invariant gravitational stress energy tensor. In [15] only tensor contributions to $T^{(\mathrm{GW})}_{\mu\nu}$ are considered. We have shown that scalar (and indeed vector) waves exist inside appropriate configurations of matter. We will therefore calculate the scalar and vector wave contributions to $T^{(\mathrm{GW})}_{00}$ (the only component of $T^{(\mathrm{GW})}_{\mu\nu}$ we will need in this paper). Expressing the result in terms of gauge-invariant variables we find

$$T^{(\mathrm{GW})}_{00} = \frac{1}{8\pi}\left\langle -6\dot{\Phi}^2 + \dot{H}^T_{ij}\dot{H}^{Tij} + 3\Psi^i\partial_i\Phi\right\rangle . \tag{13}$$

In addition to the usual transverse traceless (TT) contribution, we find a negative scalar contribution and a vector-scalar contribution (but no purely vector terms). Applying this result to the interior metric of the photon we get

$$T^{(\mathrm{GW})}_{00} = -\frac{9\pi}{8}\frac{A^4}{\omega^2}W . \tag{14}$$

Hence the total gravitational energy inside the photon is the constant $-\frac{9\pi}{8}\frac{A^4}{\omega^2}\Omega$. Also, the total gravitational energy contained in the Aichelburg-Sexl metric is a constant (in as much as it is a constant for the Schwarzchild metric). This clarifies the sense in which it is true that a photon does not emit gravitational radiation. A photon is indeed surrounded by a gravitational shock wave (the Aichelburg-Sexl metric) and it contains a scalar gravitational wave inside itself. In this sense a photon is actually the source of gravitational waves. Nevertheless, these waves contain a fixed amount of energy, to be interpreted as a constant gravitational self-energy. In no way is gravitational energy being radiated away or is the photon energy itself being depleted by gravitational processes. It is in this sense that one can say that a photon does not shine gravitationally.

III. THE METRIC OF A PHOTON IN A B FIELD: PUMPING GRAVITONS INTO PHOTONS

We now investigate the effects of a magnetic field on the metric of a photon. We should mention that similar results apply to electric fields. However, such a set up has little cosmological relevance. Let us assume that the wave packet considered in Sec. II enters a B field $\mathbf{B} = B(z,t)\Theta(z)\mathbf{e}_y$ at time $t = 0$. This B field may vary in space and time, but only on scales much larger than the photon wavelength and period. More precisely $\frac{dB/dz}{\omega} \ll 1$ and $\frac{dB/dt}{\omega} \ll 1$. We can then, to leading order in WKB, use the wave packet equation of motion, $t = z$, to eliminate t in $B(z,t)$. Hence we will always take $B = B(z) = B(z,z)$. The linearity of Maxwell's equations implies that the photon EM fields and the B field superpose. However, the EM stress energy tensor is quadratic in the fields. As a result, when the photon and the B field are superposed, their stress energy tensors do not just add up. An interference term $T^{\mathrm{int}}_{\mu\nu}$ comes about, of the form:

$$\rho^{\mathrm{int}} = ABe^{ik\cdot x}W,$$
$$\mathbf{\Sigma}^{\mathrm{int}} = ABe^{ik\cdot x}W\mathbf{e}_z,$$
$$T^{\mathrm{int}}_{ij} = ABe^{ik\cdot x}W\,\mathrm{diag}\,(1,1,-1) . \tag{15}$$

In the outside picture of the photon we can ignore this interference term. Indeed $T^{\mathrm{int}}_{\mu\nu} \propto e^{ik\cdot x}$ and so it integrates out to zero at the level of $\tilde{T}_{\mu\nu}$. Consequently, the exterior metric of a photon in a B field is still the Aichelburg-Sexl metric, and so, again, no gravitational radiation is emitted by the photon. In contrast, the inside picture is considerably modified. Performing the geometrical splitting of (15) inside W one finds scalar and tensor components:

$$\begin{gathered}\rho^{\mathrm{int}} = ABe^{ik\cdot x}W,\\ \mathbf{\Sigma}^{\mathrm{int}} = \boldsymbol{\nabla}\Sigma^{\mathrm{int}},\ \ \Sigma^{\mathrm{int}} = \frac{ABe^{ik\cdot x}}{i\omega}W,\\ p^{\mathrm{int}} = \tfrac{1}{3}ABe^{ik\cdot x}W\\ \Pi^{TL\mathrm{int}}_{ij} = \Delta_{ij}\Pi^{T\mathrm{int}} + \Pi^{T\mathrm{int}}_{ij},\ \ \Pi^{T\mathrm{int}} = -\frac{ABe^{ik\cdot x}}{\omega^2}W,\\ \Pi^{T\mathrm{int}}_{ij} = ABe^{ik\cdot x}W\,\mathrm{diag}\,(0,1,-1) .\end{gathered} \tag{16}$$

The scalar part adds to the inside values of Ψ and Φ for a photon [see (11)] the extra term

$$\Psi^{\mathrm{int}} = \Phi^{\mathrm{int}} = \frac{4\pi ABe^{ik\cdot x}}{\omega^2}W . \tag{17}$$

On top of this we now have a tensor contribution. The tensor linearized Einstein equations are [14]

$$\Box\, H^{T\mathrm{int}}_{ij} = -8\pi\Pi^{T\mathrm{int}}_{ij} . \tag{18}$$

These can be solved with the ansatz

$$H^{T\mathrm{int}}_{ij} = h^{\mathrm{int}}(z)e^{-i\omega t}W\,\mathrm{diag}\,(0,1,-1) . \tag{19}$$

Equation (18) is then trivially satisfied outside W, and it becomes the condition

$$\left(\frac{\partial^2}{\partial z^2} + \omega^2\right)h^{\mathrm{int}}(z) = -8\pi ABe^{i\omega z} , \tag{20}$$

for $z > 0$, inside W. Using the Wronskian method and discarding solutions of the homogeneous equation one finally gets

$$h^{\rm int}(z) = \frac{4\pi i A e^{i\omega z}}{\omega} \int_0^z B\, d\tilde{z}\ . \tag{21}$$

Hence we find superposed on the EM wave packet a TT gravitational wave packet with spin $+2$ with respect to the x, y axis, a phase shift of $\pi/2$ relative to the EM wave, and an amplitude $\propto \int_0^z B\, d\tilde{z}$. The physical picture is that magnetic fields have the ability to create gravitons inside photons. This is the main reward for not having neglected the inside solution for a photon. Gravitons are massless, and so they are bound to move at the speed of light. As a result, one could never hope to find a driven (that is a nonvacuum) graviton, say, inside a star. Driven gravitons can only arise as inside solutions for sources moving at the speed of light. In the case of photons, we have shown that magnetic fields work as a mechanism for pumping gravitons into photons. Note that after a photon has come out of a B field, the gravitons formed inside it remain there. In such a situation the energy in the gravitons stays constant. We are therefore in the presence of a cumulative effect. Nevertheless, it should also be noted that the sign of B in formula (19) is relevant (gravitons formed at different points interfere). Consequently, B fields can also destroy gravitons existing inside photons. In particular, if $\int_i^f B\, d\tilde{z} = 0$, there are no gravitons at the end of the photon path. It is clear that the energy in these gravitons is not a fixed gravitational self-energy. Hence one may expect the photon energy itself to be depleted by this process. A quick calculation reveals that

$$\rho^{\rm int}_{\rm TT} = \frac{1}{8\pi}\left\langle \dot{H}^T_{ij}\dot{H}^{Tij}\right\rangle = 2\pi A^2\left(\int_i^f B\, dz\right)^2 W\ , \tag{22}$$

and so, the EM wave packet, between emission and absorption, has to lose the energy fraction

$$\frac{\delta E}{E} = -4\pi\left(\int_i^f B\, dz\right)^2 . \tag{23}$$

Determining whether this energy fluctuation comes from redshift, from a variation in the number of photons inside the wave packet, or from a combination of both, is the issue to be addressed in Sec. IV. We should finally mention that the scalar wave (17) also contains energy:

$$\rho^{\rm int}_{\rm scl} = -12\pi\frac{B^2}{\omega^2}W\ . \tag{24}$$

However the energy depletion in connection with it is not cumulative, and so, whenever a uniform component exists in the B field, we find the suppression factor

$$\frac{\rho^{\rm int}_{\rm scl}}{\rho^{\rm int}_{\rm TT}} < \left(\omega\int_i^f dz\right)^{-2} \ll 1\ . \tag{25}$$

Hence the scalar wave will be ignored in the rest of this paper.

We are now in position to explain why we have chosen a wave packet model for the photon. The null fluid model used by Aichelburg and Sexl would never capture the subtleties which we have just described. Indeed one has to realize that a photon is made up of EM fields for the interference term (15) to appear in the stress energy tensor. On the other hand, the model used by the author in [11] (an infinite plane wave) applies only for coherent photons. The effect described clearly has nothing to do with coherence among different photons. In fact, we have shown that it would equally happen for a bunch of incoherent wave packets, each of which could contain just a single photon (despite the fact that this would contradict the classical limit, as pointed out before).

IV. THE TT WAVE BACK REACTION ON THE EM WAVE: BRIGHTNESS VARIATION BUT NO REDSHIFT OR COHERENCE LOSS

Since the wave packet energy is $E = A^2\Omega/2$ energy conservation and (23) tell us that

$$\frac{\delta A}{A} = \frac{1}{2}\frac{\delta E}{E} = -2\pi\left(\int_i^f B\, dz\right)^2 . \tag{26}$$

However, an estimate of the photon redshift or phase shift due to the TT wave emission requires a more detailed analysis of the back reaction on the EM wave. This is of particular relevance for the self-consistency of formula (19). The reason why gravitons generated at different points have their amplitudes summed up (interference) is because their phases are locked to the photon phase (by a $\pi/2$ difference). Therefore, for as long as the photon keeps coherence with itself, so will the gravitons associated with them. (In this respect we are in contradiction with the author in [11] who claims that gravitons produced at different points are incoherent if the B fields at these points are uncorrelated. We have shown that this is not the case.) If, however, a considerable phase shift occurs as a back reaction on the EM wave, this will be transferred to the gravitons, and affect their interference. It is therefore essential to prove that no significant photon loss of coherence (phase shift) will come about.

To estimate the back reaction of the TT wave on the EM wave, we write Maxwell equations for a field configuration of the form $\mathbf{E} = E_x\,\mathbf{e}_x$, $\mathbf{B} = B_y\,\mathbf{e}_y$, in the presence of a TT spin $+2$ wave $H^T_{ij} = \mathrm{diag}\,(0, \phi, -\phi)$ where $\phi = \phi(z,t)$. Transforming from tetrad to coordinate components, one finds that the electromagnetic field tensor has the form

$$F^{\mu\nu} = (1-\phi)\begin{pmatrix} 0 & 0 & E_x & 0\\ 0 & 0 & B_y & 0\\ -E_x & -B_y & 0 & 0\\ 0 & 0 & 0 & 0\end{pmatrix}\ , \tag{27}$$

whereas its dual is

$$\tilde{F}^{\mu\nu} = (1+\phi)\begin{pmatrix} 0 & 0 & 0 & B_y\\ 0 & 0 & 0 & -E_x\\ 0 & 0 & 0 & 0\\ -B_y & E_x & 0 & 0\end{pmatrix}\ . \tag{28}$$

Since the determinant of the metric is unperturbed, the Maxwell equations in vacuum $(\sqrt{-g}F^{\mu\nu})_{,\nu}$ $=$

$(\sqrt{-g}\tilde{F}^{\mu\nu})_{,\nu}=0$ become simply

$$\begin{aligned}\frac{\partial E_x}{\partial x}&=0,\\ \frac{\partial}{\partial t}[E_x(1-\phi)]+\frac{\partial}{\partial z}[B_y(1-\phi)]&=0,\\ \frac{\partial B_y}{\partial y}&=0,\\ \frac{\partial}{\partial t}[B_y(1+\phi)]+\frac{\partial}{\partial z}[E_x(1+\phi)]&=0\,.\end{aligned}\tag{29}$$

In solving these equations perturbatively one separates the EM field into a primary field E_x^0, B_y^0, satisfying the nongravitational Maxwell equations, and a secondary field δE_x, δB_y, resulting from the interaction between the primary fields and ϕ described in (29). After some algebraic manipulation one is led to the wave equations

$$\begin{aligned}\Box\,\delta E_x&=-(E_x^0\phi)^{\cdot\cdot}-2(B_y^0\phi)^{'\cdot}-(E_x^0\phi)'',\\ \Box\delta B_y&=(B_y^0\phi)^{\cdot\cdot}+2(E_x^0\phi)^{\cdot'}+(B_y^0\phi)''\,,\end{aligned}\tag{30}$$

where a dot denotes $\partial/\partial t$ and a prime denotes $\partial/\partial z$. In our problem ϕ is given by (19) and the primary fields are $E_x^0=Ae^{ik\cdot x}W$, $B_y^0=B+Ae^{ik\cdot x}W$. Insertion into the right-hand side of (30) shows that the primary EM wave does not generate secondary fields, but the background B field gives

$$\delta E_x=\delta B_y=-4\pi\,Ae^{ik\cdot x}\left[\int_i^f B\left(\int_i^z B d\tilde{z}\right)dz-\frac{i}{\omega}\int_i^f B^2dz\right].\tag{31}$$

Now since $E_x^0=Ae^{ik\cdot x}$ we have

$$\delta E_x=\delta Ae^{ik\cdot x}+iAe^{ik\cdot x}[\delta\phi_0+\delta\omega(z-t)]\,,\tag{32}$$

and similarly

$$\delta B_y=\delta Ae^{ik\cdot x}+iAe^{ik\cdot x}[\delta\phi_0+\delta\omega(z-t)]+\delta B\,.\tag{33}$$

Therefore

$$\frac{\delta A}{A}=-2\pi\left(\int_i^f B\,dz\right)^2,\tag{34}$$

$$\delta\phi_0=4\pi\int_i^f\frac{B^2}{\omega}\,dz,\tag{35}$$

$$\frac{\delta\omega}{\omega}=0,\tag{36}$$

$$\delta B=0\,.\tag{37}$$

Thus, we have not only corroborated the energy conservation arguments leading to (26), but also shown that no redshift occurs. Therefore, since $E=N\hbar\omega$, the energy fluctuation is all due to a change in the number of photons contained in the wave packet: $\frac{\delta E}{E}=2\frac{\delta A}{A}=\frac{\delta N}{N}$. Whenever $N=1$ for each wave packet, $\delta N/N$ is to be interpreted as a photon annihilation probability. Also the phase shift in the photon is of the order $\delta\phi_0\approx\frac{\delta A}{A}/(\omega\int_i^f dz)\ll\frac{\delta A}{A}\ll 1$. Therefore no loss of coherence is brought about by graviton generation and so formula (19) is self-consistent. Finally note the expected result that no trail is left on the background B field by the passage of the wave packet.

The results we have just finished proving could have been derived in a quantum language, using the S-matrix formalism. Then it would all boil down to writing the photon-graviton vertex induced by the background magnetic field. In a quantum picture what happens is then the conversion of photons into gravitons with a probability amplitude $\propto\int_i^f B\,dz$.

As a side remark we point out, from inspection of Eqs. (30), that it is possible to generate secondary EM waves for which $E_x\neq B_y$. This "strange" light could, for instance, be used to trace shadow light [16]. Consider a shadow photon traveling through a shadow B field. If a nonshadow B field is also present, Eqs. (30) can be used to prove that a (nonshadow) secondary light wave with $E_x\neq B_y$ would be generated. This curiosity is probably of little practical use.

V. GRAVITON PUMPING IN AN EXPANDING BACKGROUND

It is obvious that some of the results presented in Secs. II–IV cannot be completely extended to an expanding background. For instance, the Aichelberg-Sexl metric of a cosmological photon has to be cut off at horizon distances and adjusted to the photon changing energy at each Hubble expansion. Also the energy in the scalar wave does not remain constant, but scales like a^{-3} (where a is the expansion factor of the Universe). Thus, the cosmological expansion will make a photon in "vacuum" radiate gravitationally. However, this sort of gravitational radiation, present for all bodies with an expansion-resisting internal constitution (any self-gravitating body), is suppressed by a typical factor of $O(\Omega^{1/3}H)$, where $H=\frac{1}{a}\frac{da}{dt}$ (with t the proper time) is the Hubble constant (see [17] for an example). We will thus concentrate on the graviton generating process studied in Secs. III and IV. It is not difficult to devise a heuristic argument showing how to adjust formula (19) to an expanding background. In a flat background we saw that gravitons generated at different points interfere. Furthermore each element is given by

$$dH_{ij}^T=\frac{4\pi iAe^{ik\cdot x}}{\omega}B\,dz\,W\,\mathrm{diag}(0,1,-1)\,.\tag{38}$$

If $dzH\ll 1$, this formula still applies in an expanding background. Also each of these elements is treated by cosmological expansion as a free graviton. Hence, with the definition $ds^2=a^2(\eta_{\mu\nu}dx^\mu dx^\nu+2H_{ij}^T dx^i dx^j)$, each dH_{ij}^T gets redshifted and has its amplitude multiplied by $1/a$. Therefore, at point z, the element $dH_{ij}^T(z,z_e)$ produced at point z_e has the value

$$dH_{ij}^T=\left(\frac{4\pi iA(z_e)e^{ik\cdot x}}{\omega(z_e)}B(z_e)\,dz(z_e)W\times\mathrm{diag}(0,1,-1)\right)\frac{a(z)}{a(z_e)}\,.\tag{39}$$

Since the wavelength of all the gravitons is the same at all times, and working on the assumption that no additional phase shift is brought about by the cosmological expansion, we can expect these gravitons still to interfere. That is

$$H_{ij}^T(z) = \int_0^z dH_{ij}^T(z, z_e) \ . \tag{40}$$

Recalling that $A(z_e) = A(z)[a(z)/a(z_e)]^2$ and $\omega(z_e) = \omega(z)[a(z)/a(z_e)]$ we finally obtain

$$H_{ij}^T(z) = \frac{4\pi i A(z) e^{ik\cdot x}}{\omega(z)} \left(\int_0^z B(z_e) dz_e \right) W \, \mathrm{diag}(0,1,-1) \ . \tag{41}$$

This heuristic guess can be made rigorous by direct substitution of (41) in the equation

$$\ddot{H}_{ij}^T + 2h\dot{H}_{ij}^T - \Delta^2 H_{ij}^T = 8\pi a^2 A B e^{ik\cdot x} \ , \tag{42}$$

where an overdot refers to conformal time and $h = \dot{a}/a = H/a$ (see [14] for a full explanation of the notation used). One has naturally to assume that $H/\omega \ll 1$.

Extending the argument in Sec. IV to an expanding background is straightforward in view of the conformal invariance of Maxwell equations. Putting

$$\begin{aligned} & ds^2 = a^2(\eta_{\mu\nu} dx^\mu dx^\nu + 2H_{ij}^T dx^i dx^j) \ , \\ & \mathbf{E} = \tilde{\mathbf{E}}/a^2 \ , \quad \mathbf{B} = \tilde{\mathbf{B}}/a^2 \ , \\ & \mathbf{A} = \tilde{\mathbf{A}}/a^2 \ , \quad \omega = \tilde{\omega}/a \ , \quad \phi = \tilde{\phi} \ , \end{aligned} \tag{43}$$

the argument in Sec. IV holds for the tilded quantities, but now with

$$H_{ij}^T = \frac{4\pi i \tilde{A} e^{ik\cdot x}}{\tilde{\omega} a} \left(\int \frac{\tilde{B} d\tilde{z}}{a} \right) W \, \mathrm{diag}(0,1,-1) \ . \tag{44}$$

The calculations yield

$$\begin{aligned} \frac{\delta A}{A} &= \frac{\delta N}{N} = \frac{\delta \tilde{A}}{\tilde{A}} = -2\pi \left(\int_i^f B \, dz \right)^2 , \\ \delta\phi_0 &= \delta\tilde{\phi}_0 \\ &= 4\pi \left[\int_i^f \frac{B^2}{\omega} dz - \int_i^f B \frac{H}{\omega} \left(\int_0^z B(\hat{z}) d\hat{z} \right) dz \right] , \\ \frac{\delta\omega}{\omega} &= \frac{\delta\tilde{\omega}}{\tilde{\omega}} = 0 \ . \end{aligned} \tag{45}$$

The extra term in $\delta\phi$ is negligible because $H/\omega \ll 1$. Everything else looks very much the same. Hence only very minor modifications to the results proved so far come about in an expanding background. The reason why this extension is not totally trivial lies in the absence of conformal invariance in the equations. Both Maxwell's equations and the free gravitational radiation equations in the $H/\omega \ll 1$ approximation are conformally invariant. Still their coupling through the source term of (42) breaks conformal invariance. We have however shown that in the limit $H/\omega \ll 1$ this novelty does not entail major surprises.

VI. ANISOTROPIES IN THE CMBR EFFECTIVE AND COLOR TEMPERATURE

In this section we look closer at perturbations on a cosmological photon Planck spectrum. Consider a Planck spectrum at temperature T. The number density of photons per unit of frequency and solid angle moving along a direction (θ, ϕ) and with frequency ν is

$$N(\nu, \theta, \phi) = N(\nu) = \frac{\nu^2}{e^{\nu/T} - 1}. \tag{46}$$

Let us calculate the effect on this spectrum of a perturbed expanding background, such that a density $\delta N(\nu, \theta, \phi) = \delta N(\theta, \phi)$ of photons is produced, a peculiar redshift $\delta\nu$ is induced, and an initial volume dV and solid angle $d\Omega$ of photons is changed by δdV and $\delta d\Omega$. Then, if a denotes the expansion factor of the Universe, one can write, for the perturbed quantities,

$$\tilde{\nu} = \frac{\nu}{a}\left(1 + \frac{\delta\nu}{\nu}\right) , \tag{47}$$

$$d\tilde{V} = dV \, a^3 \left(1 + \frac{\delta \, dV}{dV}\right) , \tag{48}$$

$$d\tilde{\Omega} = d\Omega \left(1 + \frac{\delta \, d\Omega}{d\Omega}\right) , \tag{49}$$

and since

$$\tilde{N}(\tilde{\nu}) \, d\tilde{\nu} \, d\tilde{V} \, d\tilde{\Omega} = N(\nu) \, d\nu \, dV \, d\Omega \left(1 + \frac{\delta N}{N}\right) , \tag{50}$$

we have

$$\begin{aligned} \tilde{N}(\nu) = & \left(1 - 3\frac{\delta\nu}{\nu} - \frac{\delta N}{N} - \frac{\delta dV}{dV} - \frac{\delta d\Omega}{d\Omega}\right) \\ & \times \frac{\nu^2}{e^{\nu/[\frac{T}{a}(1+\frac{\delta\nu}{\nu})]} - 1} \ . \end{aligned} \tag{51}$$

It is important to realize the difference between the effective temperature T_e and the color temperature T_c. T_e is related to the total power or intensity radiated P, according to the Stephan-Boltzmann law $P = \sigma T_e^4$. T_c, on the other hand, concerns the shape of the spectrum, and it can be obtained from the spectrum peak $\lambda_{\max}$ by Wien's displacement law $\lambda_{\max} T_c \approx 0.29$ cm deg (see [18]). These temperatures are equal for a perfect blackbody spectrum, but not for a generally perturbed spectrum. In fact, with these definitions, and from (51), we find

$$T_c = \frac{T}{a}\left(1 + \frac{\delta\nu}{\nu}\right) , \tag{52}$$

$$T_e = \frac{T}{a}\left[1 + \frac{1}{4}\left(\frac{\delta\nu}{\nu} + \frac{\delta N}{N} - \frac{\delta dV}{dV} - \frac{\delta d\Omega}{d\Omega}\right)\right] . \tag{53}$$

The unperturbed cosmological expansion redshifts T_e and T_c equally, preserving $T_e = T_c$ (a property to be expected from the conformal invariance of electromagnetism, but certainly not true if this invariance is broken by quantum effects [19]). Hence it makes sense to talk about CMBR effective and color temperature anisotropies. They are

$$\frac{\delta T_c}{T_c} = \frac{\delta\nu}{\nu}, \tag{54}$$

$$\frac{\delta T_e}{T_e} = \frac{1}{4}\left(\frac{\delta\nu}{\nu} + \frac{\delta N}{N} - \frac{\delta dV}{dV} - \frac{\delta d\Omega}{d\Omega}\right). \tag{55}$$

Here, we will not consider the $\frac{\delta dV}{dV}$ and $\frac{\delta d\Omega}{d\Omega}$ contributions, which are computed and analyzed in [19]. Suffice it to say that they contain gravitational and Doppler aberration components. $\frac{\delta\nu}{\nu}$ is the famous redshift effect computed in [20–23]. It contains gravitational, Doppler, and intrinsic photon density δ^γ contributions. The latter also contributes to $\frac{\delta N}{N}$ according to $\frac{\delta N}{N} = \frac{3}{4}\delta^\gamma\,|_i$. Therefore, excluding δ^γ perturbations, for which $\frac{\delta T_c}{T_c} = \frac{\delta T_e}{T_e}$, all the effects studied by the authors of [20] are associated with temperature anisotropies for which $\frac{\delta T_e}{T_e} = \frac{1}{4}\frac{\delta T_c}{T_c}$ (this statement will be revised in [19]). Now from (45) we notice that B-field perturbations are unique in that they induce effective temperature anisotropies,

$$\frac{\delta T_e}{T_e} = \frac{1}{4}\frac{\delta N}{N} = -\pi\left(\int_i^f B\,dz\right)^2, \tag{56}$$

without distorting the color temperature. Therefore, putting aside $\frac{\delta dV}{dV}$, $\frac{\delta d\Omega}{d\Omega}$, and δ^γ, one can separate B-field perturbations from other types of perturbations simply by comparing effective and color temperature anisotropy maps. Constructing these maps should be trivial data analysis, as long as the observations are performed outside the atmosphere so that the observed spectrum vertical scale is reliable.

It should be said that the $T_e - T_c$ distinction is not to be classified as a spectral distortion indicator. Rather, it refers to two different aspects of the Planck spectrum not often conveniently separated.

VII. COSMOLOGICAL BOUNDS: THE UNIFORM COMPONENT

For simplicity, let us model the cosmological B field as a uniform field with an intensity characterized by r (as defined in the Introduction) permeating the present Hubble radius of the Universe. Using the cosmic B-field direction to set up a polar coordinate system centered at the Earth, the expected CMBR temperature anisotropies are

$$\frac{\delta T_c}{T_c} = 0,$$

$$\frac{\delta T_e}{T_e} = -\pi(\sin^2\theta)\,P_{\hat\theta}^2\left(\int_{\rm LS}^0 |\overline{B}|\,d\lambda\right)^2. \tag{57}$$

Here $P_{\hat\theta}$ is the $\hat\theta$-tetrad component of the light's polarization vector. Several distinctive hallmarks of this effect stand out. It affects T_e but not T_c. The $\frac{\delta T_e}{T_e}$ fluctuations depend on the polarization. A particular angular orientation in the sky is selected. These peculiar features will be used in the rest of this section in different types of proposed CMBR experiments aimed at constraining r.

A. $\frac{\delta T_e}{T_e}$, polarization-blind experiment

Consider first a CMBR polarization-blind experimental setup selecting $\frac{\delta T_e}{T_e}$ fluctuations unmatched by $\frac{\delta T_c}{T_c}$. Then, averaging over polarizations,

$$\frac{\delta T_e}{T_e} = -\frac{C(r)}{2}\sin^2\theta \quad\text{with } C(r) = \pi\left(\int_{\rm LS}^0 |\overline{B}|\,d\lambda\right)^2. \tag{58}$$

Convenient units can be introduced in this formula by recalling that the Planck magnetic field is $B_P = 1.2\times10^{57}$ G and 1 light year $= 5.8\times10^{50}L_P$:

$$C(r) = 7.4\times10^{-13}\left(\int_{\rm LS}^0 \frac{|\overline{B}|}{G}\,\frac{d\lambda}{\text{light year}}\right)^2. \tag{59}$$

Then, using

$$\frac{|\overline{B}|}{G} = 3.2\times10^{-6}\sqrt{r}\left(\frac{a(t_0)}{a(t)}\right)^2 \tag{60}$$

and putting $t_0 = 1.5\times10^{10}$ yr, $z_{\rm LS} = 1000$, and assuming $z_{\rm LS} < z_{\rm eq}$, one gets

$$C(r) = 7.4\times10^{-23}\left(\frac{t_0}{y}\right)^2(1+z_{\rm LS})r = 16.7r, \tag{61}$$

where $z_{\rm LS}$ stands for the redshift of the last scattering surface. Expanding (58) in spherical harmonics one finds monopole and quadrupole components:

$$\frac{\delta T_e}{T_e} = \sum_{lm} C_m^l(r)Y_m^l(\theta,\phi),$$

$$C_0^0(r) = -\frac{2\sqrt{4\pi}}{3}\frac{C(r)}{2},$$

$$C_0^2(r) = \frac{1}{3}\sqrt{\frac{16\pi}{5}}\frac{C(r)}{2}. \tag{62}$$

The monopole $C_0^0(r)$ has observable consequences. Monopole harmonics are indeed unobservable (and in fact gauge dependent) if T_e and T_c are considered separately. However, the quantity

$$\delta = \frac{T_c - T_e}{\overline{T}} = \frac{T_c}{T_e} - 1 = \frac{\delta T_c}{T_c} - \frac{\delta T_e}{T_e} \tag{63}$$

is observable and gauge invariant, and it contains a monopole contribution from $C_0^0(r)$. The measurement of this monopole, not being a differential measurement, can at most have an accuracy comparable to the accuracy of T, that is 10^{-3}. Hence, assuming a negative result ($\delta < 10^{-3}$), we get $r < 10^{-4}$. Note that the monopole component of δ has to be greater than or equal to zero and measures how grey the radiation is.

The quadrupole $C_0^2(r)$ provides a tighter bound. The absence of $m \neq 0$ quadrupole harmonics in (62) marks the well-defined orientation of B-field-induced anisotropies (in contrast with a stochastic quadrupole). Hence the quadrupole observed by COBE ($C^2_{[\mathrm{COBE}]} \approx 6 \times 10^{-6}$, according to [24]), assuming it propagates to $\frac{\delta T_e}{T_e}$ (affected by a factor of 1/4), cannot be due to a B field. This implies that $r < 1.6 \times 10^{-7}$.

B. CMBR polarization

Define the degree of linear polarization as

$$\alpha(\theta,\phi,\nu) = \frac{N^{\hat\theta} - N^{\phi}}{N^{\hat\theta} + N^{\phi}}, \tag{64}$$

where $N^{\hat\theta}$ and N^{ϕ} are photon number densities. Hence,

$$\alpha = \alpha(\theta,\phi) = 2\left(\frac{\delta T_e^{\hat\theta}}{T_e^{\hat\theta}} - \frac{\delta T_e^{\phi}}{T_e^{\phi}}\right), \tag{65}$$

and we see that whenever the effective temperature fluctuation is polarization dependent a nonvanishing α occurs. From (57) and (62) we see that B fields induce a monopole and a quadrupole polarization:

$$\alpha = \sum_{lm} \alpha_m^l(r) Y_m^l(\theta,\phi), \tag{66}$$

$$\alpha_0^0(r) = -\frac{4\sqrt{4\pi}}{3} C(r), \tag{67}$$

$$\alpha_0^2(r) = \frac{2}{3}\sqrt{\frac{16\pi}{5}}\, C(r)\ . \tag{68}$$

The large-scale CMBR degree of linear polarization has been experimentally constrained in [25], with the result $|\alpha| < 10^{-5}$. Hence we can conclude that $r < 10^{-6}$. The bound [25] is expected to improve considerably in the near future.

C. Kinematic quadrupole deflection

It is well known that the Earth's peculiar velocity v causes a Doppler shift in the CMBR temperature. To second order in v the kinematic anisotropies in $\frac{\delta T_c}{T_c}$ and $4\frac{\delta T_e}{T_e}$ are

$$C_0^0(v) = -\frac{v^2}{6}\sqrt{4\pi}, \tag{69}$$

$$C_0^1(v) = v\sqrt{\frac{4\pi}{3}}, \tag{70}$$

$$C_0^2(v) = \frac{v^2}{3}\sqrt{\frac{16\pi}{5}}\ . \tag{71}$$

We shall concentrate on (71). We point out that both this effect and (57) have a well-defined orientation marked by the absence of $m \neq 0$ harmonics. However their preferred direction may be not the same. As a result, when we combine $C_0^2(v)$ with $C_0^2(r)$ (to be treated as a perturbation), we may find that the kinematic quadrupole direction in $\frac{\delta T_c}{T_c}$ and in $\frac{\delta T_e}{T_e}$ are not the same. This effect we shall call kinematic quadrupole deflection and we shall proceed to quantify it.

Let $\mathbf{e}_{z'}$, $\mathbf{e}_{z''}$, and $\mathbf{e}_z$ be polar axes associated with polar coordinate systems (θ',ϕ'), (θ'',ϕ''), and (θ,ϕ), and let $\psi = \mathrm{ang}(\mathbf{e}_{z'},\mathbf{e}_{z''})$. Then, if $\phi = Y_0^2(\theta',\phi') + \epsilon Y_0^2(\theta'',\phi'')$, with $\epsilon \ll 1$, we have

$$\phi = Y_0^2(\theta,\phi) + O(\epsilon)\frac{Y_2^2(\theta,\phi) + Y_{-2}^2(\theta,\phi)}{2}, \tag{72}$$

where $\mathbf{e}_z$ is obtained by rotating $\mathbf{e}_{z'}$ towards $\mathbf{e}_{z''}$ through the angle

$$\delta\psi = \frac{\epsilon}{2}\sin(2\psi)\ . \tag{73}$$

This statement can be easily proved if quadrupole harmonics are seen as quadratic forms on a sphere. Then all we have to do is to add the two corresponding forms and diagonalize the result, to first order in ϵ. Applying this theorem to (62) and (71) we derive the kinematic quadrupole deflection angle:

$$\delta\psi = \frac{C(r)}{v^2}\sin(2\psi)\ . \tag{74}$$

A negative result on $\delta\psi$ provides an upper bound on r dependent on ψ (the angle subtended by the B field and v). This bound would be tightest for $\psi = \frac{\pi}{4}, \frac{3\pi}{4}$, and nonexistent for $\psi = 0, \frac{\pi}{2}$. However, it is pure nonsense to constrain the intensity of the B field on the assumption that we know its direction. If we do not know $|\overline{B}|$ we do not know its direction either. The sensible thing to do is to admit ignorance on ψ, and average the intensity of this undetected effect over all possible directions. Thus

$$\langle\delta\psi\rangle = \frac{2C(r)}{3v^2}\ . \tag{75}$$

Assume now an upper bound on $\delta\psi$ of the same order of the accuracy in the determination of the direction of v. From [24], $\delta\psi < 10^{-2}$, leading to

$$r < 3 \times 10^{-9}\ . \tag{76}$$

This is the best bound on r obtained so far. It cannot be tightened with improved $\frac{\delta T}{T}$ accuracy (once this goes past $\approx 10^{-6}$). The quality of the bound is directly linked to the accuracy in the determination of the kinematic quadrupole direction in $\frac{\delta T_c}{T_c}$ and $\frac{\delta T_e}{T_e}$. The author does not see why 1 arc min accuracy levels have not yet been achieved. This would bring the bound on r to $r < 10^{-10}$.

VIII. A REMARK ON THE B-FIELD RANDOM COMPONENT

If the cosmic B fields are frozen in the matter, as soon as matter and radiation decouple and structures start to grow, the B field develops a random component. Typically $\frac{\delta B}{B} \approx \frac{\delta\rho}{\rho}$, but the random component sign and direction is also random. As a result the constructive effect of these fluctuations is balanced by their destructive effect in formula (41). Hence, fluctuations tend to be erased from $\frac{\delta T_e}{T_e}$. The only exceptions are very large scales, which, at the beginning of the photon path, have their constructive and destructive phases at different expansion times (and so they do not quite cancel). However, a quick calculation from (41) shows that even $\frac{\delta T_e}{T_e}$ from these fluctuations is affected by a suppression factor of the order of $\frac{\delta\rho}{\rho}(\lambda, t_{\rm LS}) \approx 10^{-3}$. Therefore the B-field random component, at least in the linear regime, can safely be ignored.

IX. CONCLUSIONS

Other methods to constrain r have been considered in the past [3]. The most direct consists of trying to measure the current intercluster uniform B field $[\overline{B}(t_0)]$ by means of the Faraday rotation measures (FRM's) of light from distant (but astrophysical) objects ([3] and references cited therein). This method has proved to be highly vulnerable to the complex problems of separating intercluster from intrinsic and galactic FRM's. It also relies on not easily testable assumptions on the intercluster medium (such as the electron number density). Altogether these difficulties explain why this method has led some groups to conclude the strictest upper bound on r ever ($r < 4 \times 10^{-11}$ according to [27]), when it has taken others to the claim of a positive detection ($r \approx 4 \times 10^{-7}$ in [28]). Another, more indirect, method to measure r is based on the study of the B-field back reaction on the cosmological metric. Several anisotropic Bianchi cosmologies are known to accommodate a cosmic B field. These provide a link between r and the cosmological anisotropy, allowing for bounds on the anisotropy to be converted into bounds on r. Two types of bound, based on the CMBR polarization and $\frac{\delta T}{T}$ fluctuations, have been obtained. It is known that anisotropic models imprint a quadrupole $\frac{\delta T}{T}$ on the CMBR. This can be used to set $r < 10^{-9}$, but the exact value of the bound depends on the particular cosmological model used (see [3] for a summary of results). Also anisotropic models tend to polarize the CMBR, leading to $r < 10^{-9}$. In this case the quantitative aspects depend not only on the cosmological model used, but also depend on the details of the decoupling process and on the ionization history of the Universe (again see [3] for revision). The conclusion is that the bounds provided by this approach are very model dependent. In any case it has to be said that the stability and full consequences of these cosmological models have not yet been thoroughly explored. Finally one should mention that there are a few very indirect bounds on r obtained by means of various types of theoretical considerations. These are reviewed in [3], and an example of recent work of this type is [29] where the effect of primordial B fields on the big bang nucleosynthesis has been used to set $r < 4 \times 10^{-6}$.

In contrast with these methods, graviton pumping has the virtue of being a direct experimental method relying on very undemanding assumptions. These are (1) plain hot big bang model of the Universe (back reaction on the cosmological metric disregarded), (2) CMBR effectively in vacuum one expansion time after decoupling, (3) $z_{\rm LS} = 10^3$ or thereabout. Putting aside these requirements, the bounds obtained from graviton pumping are very robust. They are independent of the decoupling details or any assumptions on the present intercluster medium. They result from an in-vacuum, linear calculation, unrestrictedly valid for as long as a linear, in-vacuum regime exists for the CMBR. This method also has the advantage of bearing very distinctive marks, allowing the effect on which it is based to be separated from any other CMBR distorting effects. If the CMBR data analysis does provide us with the results predicted in Sec. VII, graviton pumping would yield the safest and strictest bounds on r. For the particular method described in Sec. VII C, the bound expected ($r < 3 \times 10^{-9}$) is already good enough to predict the need for a dynamo ($r < 10^{-8}$).

We should however make the proviso that the assumptions stated above may be wrong. For instance, it has been mentioned that even though CMBR photons move through a transparent medium after decoupling, some forms of "transparent interaction" may survive. Zeldovich and Novikov [26] even state that these interactions could change the speed of light, destroying the resonance on which graviton pumping is based. This claim may be somewhat extreme. We explore instead the effects of the mildest form of transparent interaction, one which would change at random the photon phase at every distance ξ along its path. Gravitons pumped at different ξ-sized segments would therefore add incoherently, and formula (57) would no longer be valid. In the limit $\xi \ll t_{\rm LS}$ we would have instead

$$\begin{aligned}\frac{\delta T_e}{T_e} &= -\pi(\sin^2\theta)\, P_{\hat{\theta}}^2 \xi \int_{\rm LS}^0 |\overline{B}|^2\, d\lambda \\ &= 2.1 \times 10^{-6} \frac{\xi}{\text{light year}} r\,. \end{aligned} \tag{77}$$

The anisotropies would thus get suppressed by a factor $\chi = 1.25 \times 10^{-7}(\xi/\text{light year})$. Undoubtedly even the mildest form of transparent interaction would be disastrous. However all standard order of magnitude estimates point towards a noninteracting CMBR after decoupling. Perhaps we should not worry too much.

The breakdown of assumption (3) would also be catastrophic. Not only would we lose a few orders of magnitude in the final result [cf. (61)], but also, if $z_{\rm LS} \approx 10$, the CMBR would have practically always lived in the nonlinear regime of structure formation (a regime totally overlooked in this paper). Nevertheless, it should be said in general that if such were the case then the whole trend ruling how CMBR calculations and observations are done would have to change anyway. Indeed we do not have a nonlinear regime analytic model with which to predict the statistical properties of the matter field and CMBR fluctuations. On the other hand, we do have something much better. Nonlinear structures are usually individually visible, if not in the optical, at least in the radio domain. In general they are a lot more accessible to detailed study. Hence, we could always elect remarkably well-known nonlinear structures, work out for them the CMBR fluctuations predicted by some effect under study, and then look out for these fluctuations in the appropriate sky positions. This attitude towards CMBR observation and analysis is what is in fact done for the Sunyaev-Zeldovich effect (an effect involving purely nonlinear structures). It should be obvious that if $z_{\rm LS} \approx 10$ any statistical predictions on $\frac{\delta T}{T}$ would be inappropriate. Instead, one could use $\frac{\delta T}{T}$ maps as a telescope for observing and studying nonlinear objects, some of which might have hitherto escaped optical and radio observations. In the particular case of the effect studied in this paper, the CMBR fluctuations are negligible for most nonlinear objects. The remarkable exceptions are quasars and their jets, and the accretion discs of active galactic nuclei, for which $\frac{\delta T_e}{T} \approx 10^{-6}$. Confirming this effect on a particularly well-known object would open the doors for its use as telescope giving the magnetic face of some nonlinear structures.

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