Thermodynamics of four-fermion models in the δ expansion

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Using the linear δ expansion we investigate the free energy of the 1+1 and 2+1 dimensional Gross-Neveu model. In both cases the large-N result is exactly reproduced at first order in δ. However, when higher-order corrections are included our results for the phase transition differ both quantitatively and qualitatively from the large-N predictions. In particular, our result for the critical temperature in 1+1 dimensions is in better agreement with a general theorem for phase transitions in one space dimension.

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I. INTRODUCTION

In this paper we apply the linear δ expansion [1,2] to study the effective potential (free energy) of the Gross-Neveu model [3]. In particular, we estimate the critical temperature (θ_c) for restoration of chiral symmetry in 1+1 and 2+1 dimensions. This model is very similar to the BCS theory of superconductivity in that dynamical symmetry breaking occurs and results in the formation of bound states which are analogous to Cooper pairs. The Gross-Neveu model is given by the Lagrangian density

$$\mathcal{L}=i\bar{\psi}\partial\!\!\!/\psi+\frac{g^2}{2N}(\bar{\psi}\psi)^2 , \tag{1.1}$$

and has a discrete chiral symmetry $\psi\to\gamma_5\psi$ which forbids the presence of a mass term for the fermions. However, chiral symmetry is broken in this model by radiative corrections which give rise to a fermion mass M_F at zero temperature.

The thermodynamics of (1.1) has been extensively discussed in both 1+1 and 2+1 dimensions, respectively [4,5], within the context of the $1/N$ expansion where N is the number of fermion fields. A leading-order investigation of both cases has been performed by Okopińska [6] with an extended version of the Gaussian effective potential ([6] and references therein). In 1+1 dimensions it was found that a second-order phase transitions occurs for $\theta_c=0.57M_F^r$, where M_F^r is the zero temperature renormalized mass. However, it is well known [7] that in 1+1 dimensions the critical temperature for restoration of chiral symmetry is rigorously zero. This can be very easily seen from a simple argument. In 1+1 dimensions it is energetically favorable to have kink configurations, which implies that the sign of the order parameter alternates over small regions of space. The average over space of these kink configurations then makes the average order parameter zero. Thus we have no symmetry breaking in 1+1 dimensions at finite temperature (Landau's theorem [8]). However, this effect is missed by the $1/N$ expansion. We shall see in this work that at higher orders the δ expansion predicts a lower value for the critical temperature, which is in better agreement with the observation above.

We also study the 2+1 dimensional model. Since it is not plagued by Landau's theorem the critical temperature for chiral symmetry restoration is finite. In fact, the $1/N$ expansion predicts a second-order phase transition at $\theta_c=(2\ln2)^{-1}M_F^r$, where M_F^r is the normalized fermion mass at zero temperature [5]. In 2+1 dimensions the Gross-Neveu model is analog to a BCS type theory of superconductivity in two spatial dimensions. Therefore, it will be interesting to see what happens beyond the large N limit.

The linear δ expansion, just like its precursor, the logarithmic δ expansion [9], is an approximation scheme devised to study some nonperturbative aspects of quantum field theory. It has been successfully applied to the study of symmetry breaking at zero temperature in the case of both scalar [2,11,12] and four-fermion theories [1,12,13]. In the finite temperature domain the δ expansion was employed by Bender and Rebhan [10] to the scalar theory.

The structure of this paper is as follows. In Sec. II we shall summarize the δ expansion technique and the finite temperature formalism within the context of the Gross-Neveu model in $d+1$ dimensions. In Sec. III we discuss the free energy of the order parameter field σ_c in 1+1 dimensions and show our results for the critical temperature. Section IV covers the details of the 2+1 dimensional case. Section V contains our conclusions. There is also an appendix giving the temperature dependent Feynman integrals that we evaluated numerically.

II. δ EXPANSION AND FINITE TEMPERATURE FORMALISM

We are interested in the finite temperature properties of the Gross-Neveu model which has the action given by (1.1) above. Since we are interested in the case of dynamical symmetry breaking we introduce an auxiliary field σ which is given by

$$\sigma=\frac{g}{N}\bar{\psi}\psi , \tag{2.1}$$

at the classical level. The expectation value of this operator is the order parameter for dynamical symmetry breaking. So, we replace (1.1) by

$$\mathcal{L}=i\bar{\psi}\partial\!\!\!/\psi-\frac{N}{2g}\sigma^2-\sigma\bar{\psi}\psi\ . \tag{2.2}$$

It can be easily verified that (2.2) is indeed equivalent to (1.1) by substituting (2.1), which is the classical equation for the σ field, into (2.2).

We shall now give a very brief review of the imaginary time formalism of finite temperature theory (for a more complete discussion see [14]). At nonzero temperature the partition function of a field theory with Hamiltonian H is given by

$$Z=\mathrm{tr}\exp(-\beta H)\ , \tag{2.3}$$

with $\beta=1/\theta$, θ being the temperature. One can rewrite (2.3) as a path integral after obtaining the Hamiltonian from (2.2) and by replacing the time interaction in the action by an integration over the temperature parameter, i.e.,

$$i\int_{t'}^{t''}dt\rightarrow\int_0^\beta d\tau\ . \tag{2.4}$$

To obtain (2.4) one introduces heuristically a variable $\tau=it=ix_0$ and takes the limits of integration to be $t'=0$ and $t''=-i\beta$. The field ψ is understood to be a function of τ and $\mathbf{x}$ antiperiodic in $0<\tau<\beta$

$$\psi(0,\mathbf{x})=-\psi(\beta,\mathbf{x})\ . \tag{2.5}$$

This result is a modification of the zero temperature Feynman rules, in that the integral over the p_0 component of momentum is replaced by a summation over a discrete variable. That is,

$$\int dp_0\rightarrow i\theta\sum_n\ , \tag{2.6a}$$

$$p_0\rightarrow i\omega_n\ , \tag{2.6b}$$

where, for fermions

$$\omega_n=(2n+1)\pi\theta\ . \tag{2.6c}$$

We shall now summarize the linear δ expansion and set up the calculation. A fuller discussion of the linear δ expansion can be found in Refs. [12,13]. To start, we introduce a variational parameter μ and a bookkeeping parameter δ (to be set equal to one at the end of the calculation) into the Lagrangian (2.2), thus replacing $\mathcal{L}$ by $\mathcal{L}_\delta$, where

$$\mathcal{L}_\delta=\bar{\psi}(i\partial\!\!\!/)\psi-\frac{N}{2g}\sigma^2-\delta\sigma\bar{\psi}\psi-(1-\delta)\mu\bar{\psi}\psi\ . \tag{2.7}$$

Notice that when $\delta=0$ we obtain a free massive theory, and that at $\delta=1$ we recover the original Lagrangian (2.2). To calculate the effective potential (V_δ) we perturb in powers of δ up to some finite order and set $\delta=1$. It should be noticed that the effective potential will be a function of μ even at $\delta=1$. This can be understood in the following way. Since our free theory is massive we work with a propagator that is μ dependent. On the other hand, the $-\delta\mu\bar{\psi}\psi$ term is treated only approximately as an interaction. This means that up to finite order in δ only partial cancellation of the μ-dependent contributions will occur (obviously, if we could calculate to all orders in δ and μ dependence would drop out completely). We see that $V_{\delta=1}$ is dependent on an arbitrary parameter μ. There are many ways of selecting μ [1,15–18]. We shall fix it using the principle of minimal sensitivity (PMS) introduced in [17], i.e., we fix μ to be a solution of

$$\left.\frac{\partial V_{\delta=1}}{\partial\mu}\right|_{\bar{\mu}}=0\ . \tag{2.8}$$

This procedure forces the solution to be at least locally (at the stationary point) independent of μ. Recent investigations [19] prove the convergence of the optimized linear δ expansion (at least in low dimensions) and give the method a calculational basis. It is important to note that (2.8) makes μ a function of the mean field σ_c, so that vacuum graphs which are usually dropped by normalizing the generating functional must now be kept. In fact all μ-dependent contributions which are *a priori* field independent must be retained [2,12].

The aim of this work is to see explicitly how the introduction of terms which are neglected in a large N calculation affect the critical temperature and the character of the phase transition. Therefore, it will be sufficient to work with dimensionless quantities without addressing the renormalization problem here. Discussions of renormalization within the δ expansion can be found in Refs. [2,11,20,21].

III. THE 1+1 DIMENSIONAL GROSS-NEVEU MODEL

In this section we give the calculation of the effective potential of the order parameter field σ_c in 1+1 dimensions. Given the Lagrangian (2.7) one can easily obtain the zero temperature effective potential to $O(\delta)$ [12,13]

$$N^{-1}V_\delta(\sigma_c,\mu)=\frac{\sigma_c^2}{2g}+i\int d^2p\,\ln(p^2-\mu^2)+2\delta i\int d^2p\frac{\mu(\mu-\sigma_c)}{p^2-\mu^2+i\epsilon}\ . \tag{3.1}$$

Using the translation rules to finite temperature given by (2.6) we obtain the finite temperature effective potential, which is given by

$$N^{-1}V_\delta(\sigma_c,\mu,\theta)=\frac{\sigma_c^2}{2g}-\theta\sum_n\int dp_1\ln(\omega_n^2+E_\mu^2)+2\delta\theta\sum_n\int dp_1\frac{\mu(\mu-\sigma_c)}{\omega_n^2+E_\mu^2}\ , \tag{3.2}$$

where $E_\mu^2=p_1^2+\mu^2$.

One may perform the sum over the Matsubara frequencies using the well-known relation [14]

$$\theta\sum_n\ln(\omega_n^2+E_\mu^2)=E_\mu+2\theta\ln[1+\exp(-E_\mu/\theta)]+c\ , \tag{3.3}$$

where c is a E_μ-independent constant. After summing over the frequencies one finds that the effective potential cleanly separates into two pieces, one being temperature dependent (V_δ^θ) and the other being temperature independent, (V_δ^0). So, at first order in δ we have

$$N^{-1}V_\delta(\sigma_c,\mu,\theta)=V_\delta^0(\sigma_c,\mu)+V_\delta^\theta(\sigma_c,\mu,\theta) , \tag{3.4}$$

with

$$V_\delta^0(\sigma_c,\mu)=\frac{\sigma_c^2}{2g}-\int dp_1\left[E_\mu-\delta\frac{1}{E_\mu}\mu(\mu-\sigma_c)\right] , \tag{3.5}$$

and

$$V_\delta^\theta(\sigma_c,\mu,\theta)=-2\theta\int dp_1\left[\ln[1+\exp(-E_\mu/\theta)]+\delta\frac{\mu(\mu-\sigma_c)}{\theta E_\mu[1+\exp(E_\mu/\theta)]}\right] . \tag{3.6}$$

The temperature-dependent integrals can be evaluated analytically and take on a simple form in the limit $\theta \gg \mu$. However, since we will fix μ using the PMS condition (2.8) this is not appropriate and we shall evaluate these integrals numerically. The zero temperature integrals are simple and on evaluation yield

$$V_\delta^0(\sigma_c,\mu)=\frac{\sigma_c^2}{2g}-\frac{\Lambda}{2\pi}X(\mu)+\frac{1}{4\pi}\mu^2Y(\mu)-\delta\frac{1}{2\pi}\mu(\mu-\sigma_c)Y(\mu) , \tag{3.7}$$

where

$$X(\mu)=(\Lambda^2+\mu^2)^{1/2} , \tag{3.8}$$

$$Y(\mu)=\ln\left[\frac{\mu}{\Lambda+X(\mu)}\right]^2 , \tag{3.9}$$

and Λ is an ultraviolet cutoff inserted on the "spatial momentum" p_1. The temperature-dependent part can be written as

$$V_\delta^\theta(\sigma_c,\mu,\theta)=-\theta^2\frac{2}{\pi}I_0^\theta(y)-\delta\frac{2}{\pi}\mu(\mu-\sigma_c)I_1^\theta(y) , \tag{3.10}$$

where the finite temperature integrals $I^\theta(y)$ can be found in the Appendix. Setting $\delta=1$ and applying the PMS condition we obtain the temperature independent result

$$\bar{\mu}=\sigma_c , \tag{3.11}$$

Figure 1 shows the resulting effective potential for different temperatures. In fact, we have plotted the dimensionless effective potential

$$\tilde{V}_\delta=N^{-1}\frac{V_\delta}{\Lambda^2} , \tag{3.12}$$

as a function of the dimensionless auxiliary field $\tilde{\sigma}_c=\sigma_c/\Lambda$.

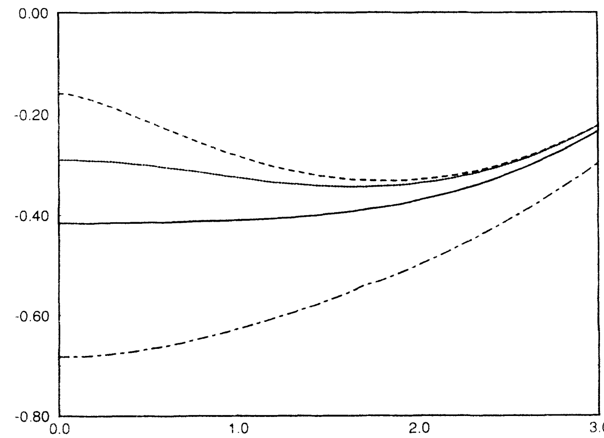

FIG. 1. The $O(\delta)$ dimensionless effective potential, for $g=6$, plotted as a function of $\tilde{\sigma}_c$ for $\tilde{\theta}=0$ (dashed line), $\tilde{\theta}=0.5$ (dotted line), $\tilde{\theta}=\tilde{\theta}_c=0.67$ (continuous line), and $\tilde{\theta}=1.0$ (dot-dashed line).

At $O(\delta)$ the (dimensionless) critical temperature is

$$\tilde{\theta}_c=0.36\tilde{M}_F ,$$

where $\tilde{M}_F=1.85$ is the dimensionless fermionic mass at zero temperature. At the critical temperature the free energy has only one minimum (at $\sigma_c=0$) characterizing a second-order phase transition. This is just the large N result.

At the next order the effective potential receives contribution from the one-loop (vacuum, tadpole, and self-energy) diagrams which can be written as

$$\delta^2\theta\sum_n\int dp_1\left[\frac{2\mu^2}{(\omega_n^2+E_\mu^2)^2}-\frac{1}{\omega_n^2+E_\mu^2}\right](\mu-\sigma_c)^2 . \tag{3.13}$$

There is also a contribution coming from the vacuum two-loop diagram

$$\delta^2\frac{g}{N}\mu^2\left[\theta\sum_n\int dp_1\frac{1}{\omega_n^2+E_\mu^2}\right]^2 . \tag{3.14}$$

The temperature-independent part now becomes

$$V_\delta^0(\sigma_c,\mu)=\frac{\sigma_c^2}{2g}-\frac{\Lambda}{2\pi}X(\mu)+\frac{1}{4\pi}\mu^2Y(\mu)-\delta\frac{1}{2\pi}\mu(\mu-\sigma_c)Y(\mu)+\delta^2\frac{1}{4\pi}(\mu-\sigma_c)^2\left[Y(\mu)+\frac{\Lambda}{X(\mu)}\right]+\delta^2\frac{g}{N}\frac{\mu^2}{16\pi^2}[Y(\mu)]^2 , \tag{3.15}$$

and the temperature-dependent part is given by

$$V_\delta^\theta(\sigma_c,\mu,\theta)=-\theta^2\frac{2}{\pi}I_0^\theta(y)-\delta\frac{2}{\pi}\mu(\mu-\sigma_c)I_1^\theta(y)+\delta^2\frac{1}{\pi}(\mu-\sigma_c)^2\left[I_1^\theta(y)+\frac{\mu^2}{\theta^2}[I_2^\theta(y)+I_3^\theta(y)]\right]+\delta^2\frac{g}{N}\frac{\mu^2}{\pi^2}I_1^\theta(y)\left[I_1^\theta(y)+\frac{1}{2}Y(\mu)\right] . \quad (3.16)$$

In (3.15) and (3.16) one should notice the appearance of $1/N$ terms at $O(\delta^2)$. This illustrates the fact the diagrams which belong to different orders of the $1/N$ expansion can contribute to the same order in a δ expansion calculation. If we take the limit $N\to\infty$ in (3.15) and (3.16) the PMS sets $\bar{\mu}=\sigma_c$ and the large N result is again reproduced. As emphasized in [12], this will happen at any order in δ, provided we stay within the large N limit.

We now choose $N=3$, $g=6$ and perform the temperature-dependent integrals numerically. In agreement with the original PMS philosophy [17] the arbitrary parameter μ is fixed for each different temperature and, as opposed to (3.11), turns out to be temperature dependent. One then obtains the results displayed in Fig. 2. The critical temperature is now

$$\tilde{\theta}_c=0.25\tilde{M}_F ,$$

where $\tilde{M}_F=2$ is the dimensionless $O(\delta^2)$ fermionic mass at zero temperature. If one recalls Landau's theorem, namely, that in 1+1 dimensions the critical temperature for symmetry restoration is zero, one will see that our $O(\delta^2)$ is an improvement over that given by the leading-order $1/N$ result. At this order not only the critical temperature values has decreased but also the character of the phase transition has changed. At the critical temperature, the free energy has two minima (at $\tilde{\sigma}_c=0$ and $\sigma_c=1.7$) with value $\tilde{V}_\delta=-0.287$. This is typical of a first-order phase transition. To see whether the $O(\delta^3)$ corrections will improve the critical temp result or make it worse we now calculate the $O(\delta^3)$ corrections. The contributions at $O(\delta^3)$ from the one-loop diagrams are

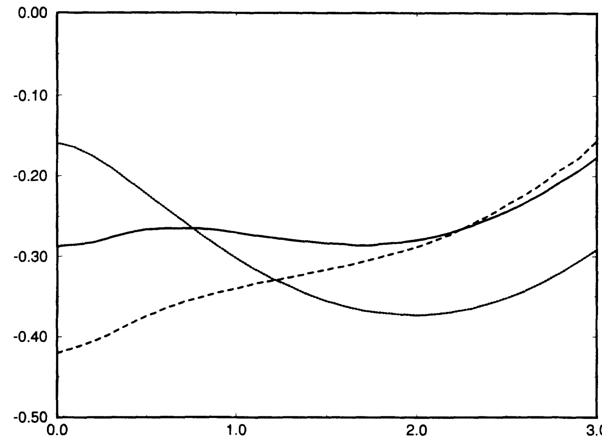

FIG. 2. The $O(\delta^2)$ dimensionless effective potential for $g=6$, $N=3$ and the different temperatures $\tilde{\theta}=0$ (dotted line), $\tilde{\theta}=\tilde{\theta}_c=0.5$ (continuous line), and $\tilde{\theta}=0.7$ (dashed line).

$$-\delta^3 2\mu(\mu-\sigma_c)^3\theta\sum_n\int dp_1\frac{1}{(\omega_n^2+E_\mu^2)^2}+\delta^3\frac{8}{3}\mu^3(\mu-\sigma_c)^3\theta\sum_n\int dp_1\frac{1}{(\omega_n^2+E_\mu^2)^3} . \quad (3.17)$$

The two-loop vacuum and tadpole contributions are given by

$$-\delta^3\frac{g}{N}2\mu\theta^2\left[\sum_n\int dp_1\frac{1}{\omega_n^2+E_\mu^2}-\sum_n\int dp_1\frac{2\mu^2}{(\omega_n^2+E_\mu^2)^2}\right]\times\left[\sum_n\int dp_1\frac{1}{\omega_n^2+E_\mu^2}\right](\mu-\sigma_c) . \quad (3.18)$$

Then, at $O(\delta^3)$ one obtains

$$V_\delta^0(\sigma_c,\mu)=\frac{\sigma_c^2}{2g}-\frac{\Lambda}{2\pi}X(\mu)+\frac{1}{4\pi}\mu^2Y(\mu)-\delta\frac{1}{2\pi}\mu(\mu-\sigma_c)Y(\mu)+\delta^2\frac{1}{4\pi}(\mu-\sigma_c)^2\left[Y(\mu)+\frac{\Lambda}{X(\mu)}\right]+\delta^2\frac{g}{N}\frac{\mu^2}{16\pi^2}[Y(\mu)]^2-\delta^3\frac{1}{2\pi}(\mu-\sigma_c)^3\frac{\Lambda}{\mu X(\mu)}\left[1-\frac{2\Lambda^2+3\mu^2}{3[X(\mu)]^2}\right]-\delta^3\frac{g}{N}\frac{1}{8\pi^2}\mu(\mu-\sigma_c)Y(\mu)\left[Y(\mu)+\frac{2\Lambda}{X(\mu)}\right] , \quad (3.19)$$

and

$$V_\delta^\theta(\sigma_c,\mu,\theta)=-\theta^2\frac{2}{\pi}I_0^\theta(y)-\delta\frac{2}{\pi}\mu(\mu-\sigma_c)I_1^\theta(y)+\delta^2\frac{1}{\pi}(\mu-\sigma_c)^2\left[I_1^\theta(y)+\frac{\mu^2}{\theta^2}[I_2^\theta(y)+I_3^\theta(y)]\right]+\delta^2\frac{g}{N}\frac{\mu^2}{\pi^2}I_1^\theta(y)\left[I_1^\theta(y)+\frac{1}{2}Y(\mu)\right]-\delta^3\frac{1}{\pi}\frac{\mu}{\theta^2}(\mu-\sigma_c)^3[I_2^\theta(y)+I_3^\theta(y)]+\delta^3\frac{1}{2\pi}\frac{\mu^3}{\theta^4}(\mu-\sigma_c)^3[I_4^\theta(y)-I_5^\theta(y)+2I_6^\theta(y)+I_7^\theta(y)]-\delta^3\frac{g}{N}\frac{1}{\pi^2}\mu(\mu-\sigma_c)I_1^\theta(y)\left[Y(\mu)+2I_1^\theta(y)+\frac{\Lambda}{X(\mu)}\right]-\delta^3\frac{g}{N}\frac{1}{2\pi^2}\frac{\mu^3}{\theta^2}(\mu-\sigma_c)[I_2^\theta(y)+I_3^\theta(y)][Y(\mu)+4I_1^\theta(y)] . \quad (3.20)$$

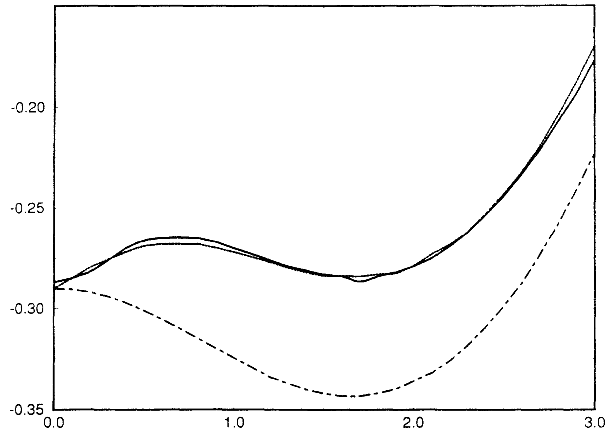

FIG. 3. The $O(\delta)$ (dot-dashed line), $O(\delta^2)$ (continuous line), and $O(\delta^3)$ (dotted line) for $\tilde{\theta}=0.5$, $g=6$, and $N=3$.

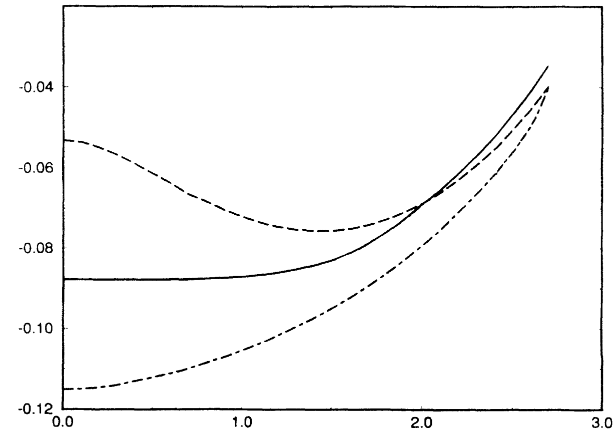

FIG. 4. The $O(\delta)$ dimensionless effective potential, for $g=20$, plotted as a function of $\tilde{\sigma}_c$ for the zero temperature (dashed line), for the critical temperature $\tilde{\theta}_c=0.495$ (continuous line), and for $\tilde{\theta}=0.6$ (dot-dashed line).

Performing the interactions in (3.20) and applying PMS numerically to (3.19) we obtain

$$\tilde{\theta}_c=0.24\tilde{M}_F \ ,$$

where $\tilde{M}_F=2.08$ is the $O(\delta^3)$ fermion mass at zero temperature. At this order, the phase transition is again of the first kind. In Fig. 3 we compare the $O(\delta)$, $O(\delta^2)$, and $O(\delta^3)$ results for the free energy for $\tilde{\theta}=0.5$, which is the critical temperature obtained at $O(\delta^2)$.

IV. THE 2+1 DIMENSIONAL GROSS-NEVEU MODEL

In this section we calculate the free energy of the 2+1 dimensional Gross-Neveu model. The diagrams which contribute to a given order of the effective potential in 1+1 dimensions will also contribute in 2+1 dimensions. Therefore, the actual evaluation of the effective potential is essentially the same in both cases. Following Sec. III, one then arrives at the $O(\delta^3)$ effective potential in 2+1 dimensions

$$N^{-1}V_\delta=V_\delta^0(\sigma_c,\mu)+V_\delta^\theta(\sigma_c,\mu,\theta) \ , \tag{4.1}$$

where

$$\begin{aligned}V_\delta^0(\sigma_c,\mu)=&\frac{\sigma_c^2}{2g}-\frac{1}{6\pi}[X^3(\mu)-\mu^3]+\delta\frac{1}{2\pi}\mu(\mu-\sigma_c)[X(\mu)-\mu]+\delta^2\frac{1}{4\pi}(\mu-\sigma_c)^2[X(\mu)-\mu]\left[\frac{\mu}{X(\mu)}+1\right]\\&+\delta^2\frac{1}{8\pi^2}\frac{g}{N}\mu^2[X(\mu)-\mu]^2-\delta^3\frac{1}{4\pi}(\mu-\sigma_c)^3\frac{1}{X(\mu)}\left[X(\mu)-\mu-\frac{1}{3X^2(\mu)}[X^3(\mu)-\mu^3]\right]\\&-\delta^3\frac{1}{4\pi^2}\frac{g}{N}\mu(\mu-\sigma_c)[X(\mu)-\mu]^2\left[1-\frac{\mu}{2X(\mu)}\right] \ ,\end{aligned} \tag{4.2}$$

and

$$\begin{aligned}V_\delta^\theta(\sigma_c,\mu,\theta)=&-\frac{\theta^3}{\pi}I_0^\theta(y)-\delta\frac{\theta}{\pi}\mu(\mu-\sigma_c)I_1^\theta(y)+\delta^2\frac{1}{2\pi}(\mu-\sigma_c)^2\left[\theta I_1^\theta(y)+\frac{\mu^2}{2\theta}[I_2^\theta(y)+I_3^\theta(y)]\right]\\&+\delta^2\frac{\theta}{2\pi^2}\frac{g}{N}\mu^2I_1^\theta(y)[\theta I_1^\theta(y)-X(\mu)+\mu]-\delta^3\frac{1}{2\pi}\frac{\mu}{\theta}(\mu-\sigma_c)^3[I_2^\theta(y)+I_3^\theta(y)]\\&+\delta^3\frac{1}{2\pi}\frac{\mu^3}{\theta^3}(\mu-\sigma_c)^3[I_4^\theta(y)-\tfrac{1}{3}I_5^\theta(y)+\tfrac{2}{3}I_6^\theta(y)+I_7^\theta(y)]+\delta^3\frac{\theta}{\pi^2}\frac{g}{N}\mu(\mu-\sigma_c)I_1^\theta(y)[X(\mu)-\mu-\theta I_1^\theta(y)]\\&+\delta^3\frac{1}{2\pi^2}\frac{g}{N}\mu^3(\mu-\sigma_c)[I_2^\theta(y)+I_3^\theta(y)]\left[I_1^\theta(y)-\frac{[X(\mu)-\mu]}{2\theta}\right]+\delta^3\frac{\theta}{4\pi^2}\frac{g}{N}\mu^2(\mu-\sigma_c)\frac{I_1^\theta(y)}{X(\mu)}[X(\mu)-\mu] \ .\end{aligned} \tag{4.3}$$

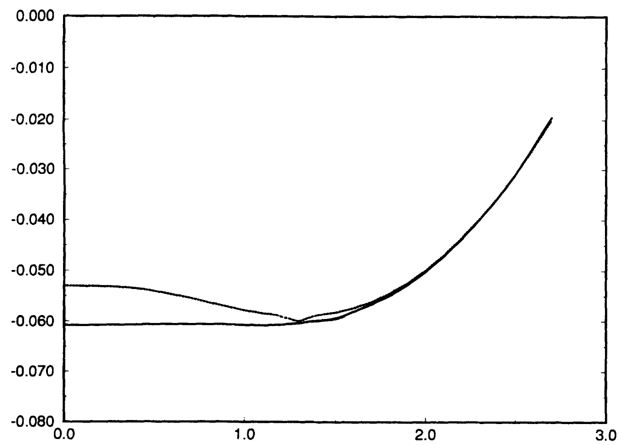

FIG. 5. The $O(\delta^2)$ dimensionless effective potential, for $g=20$ and $N=3$. The dotted line is the zero temperature result and the continuous line is the critical temperature ($\tilde{\theta}_c=0.3$) result.

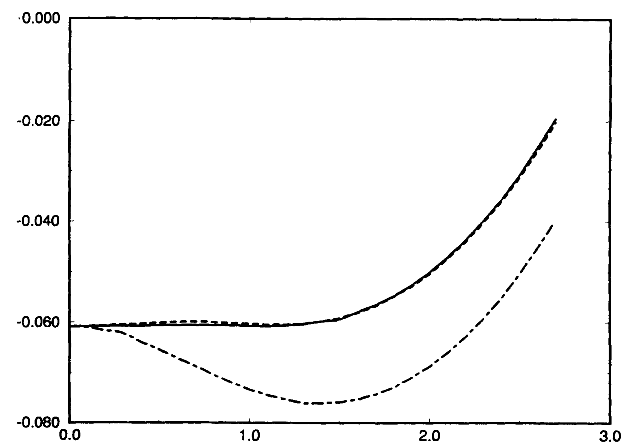

FIG. 6. The $O(\delta)$ (dot-dashed line), $O(\delta^2)$ (continuous line), and $O(\delta^3)$ (dashed line) for $\tilde{\theta}=0.3$, $g=20$, and $N=3$.

Optimization of (4.1) at $O(\delta)$ gives the temperature independent result $\bar{\mu}=\sigma_c$. Figure 4 shows the dimensionless effective potential for different temperatures and $g=20$. At this order the critical temperature is

$$\tilde{\theta}_c=0.35\tilde{M}_F ,$$

where the $O(\delta)$ fermion mass at zero temperature is $\tilde{M}_F=1.4$ and the phase transition is of second order.

The $O(\delta^2)$ result is shown in Fig. 5 for $N=3$, $\tilde{\theta}=0$, and for the critical temperature $\tilde{\theta}_c=0.3$. In terms of the $O(\delta^2)$ zero temperature fermion mass $\tilde{M}_F=1.3$ the critical temperature is

$$\tilde{\theta}_c=0.23\tilde{M}_F .$$

The free energy has minima at $\sigma_c=0$ and $\sigma_c=1.1$ (with value $\tilde{V}_\delta=-0.0608$) characterizing a first-order phase transition.

The $O(\delta^3)$ calculation gives

$$\tilde{\theta}_c=0.22\tilde{M}_F ,$$

where the $O(\delta^3)$ zero temperature fermion mass is $\tilde{M}_F=1.3$. The phase transition is also of first order.

The free energy results at $O(\delta)$, $O(\delta^2)$, and $O(\delta^3)$ are displayed in Fig. 6 for the $O(\delta^2)$ critical temperature.

V. CONCLUSIONS

We have used the linear δ expansion to calculate the finite temperature effective potential of the Gross-Neveu model in order to investigate the restoration of chiral symmetry in $1+1$ and $2+1$ dimensions.

In both cases the large N result is already recovered at first order in δ. At higher orders, the δ expansion brings into the calculation terms which are neglected in a large N approximation and significant changes occur. The first important result occurred for the $1+1$ dimensional case where we have explicitly seen that, in agreement with Landau's theorem, the δ expansion predicts [at $O(\delta^2)$] a smaller value for the critical temperature than the one predicted in a large N calculation. The phase transition appears as being of the first kind.

Then, in order to check how the inclusion of higher order terms affect these results we have evaluated the $O(\delta^3)$ contributions. The character of the phase transition remains unchanged (first kind). The decrease of the critical temperature value, although very small, is consistent with the δ expansion correctly taking into account the effects of unsuppressed "kink" configurations.

Also in $2+1$ dimensions the δ expansion results indicate that the value of the temperature at which chiral symmetry restoration occurs is smaller than the value predicted by the large N approximation. Our results show a first-order phase transition. Superconductors, on the other hand, exhibit a second-order phase transition. So, in this respect, it seems that the $2+1$ dimensional Gross-Neveu model is more closely related to the BCS theory of superconductivity in two spatial dimensions at $O(\delta)$ (which is equivalent to the large N limit) where the phase transition is of second order.

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APPENDIX

Here we list the temperature dependent integrals which have been evaluated numerically. Below, $d\Omega$ corresponds to dx in $1+1$ dimensions and to xdx in $2+1$ dimensions, whereas $x=p/\theta$ and $y=\mu/\theta$.

$$I_0^\theta(y)=\int_0^\infty d\Omega \ln[1+\exp-(x^2+y^2)^{1/2}] , \tag{A1}$$

$$I_1^\theta(y)=\int_0^\infty d\Omega \frac{1}{(x^2+y^2)^{1/2}[1+\exp(x^2+y^2)^{1/2}]} , \tag{A2}$$

$$I_2^\theta(y)=\int_0^\infty d\Omega \frac{1}{(x^2+y^2)^{3/2}[1+\exp(x^2+y^2)^{1/2}]} , \quad \text{(A3)}$$

$$I_3^\theta(y)=\int_0^\infty d\Omega \frac{\exp(x^2+y^2)^{1/2}}{(x^2+y^2)[1+\exp(x^2+y^2)^{1/2}]^2} , \quad \text{(A4)}$$

$$I_4^\theta(y)=\int_0^\infty d\Omega \frac{1}{(x^2+y^2)^{5/2}[1+\exp(x^2+y^2)^{1/2}]} , \quad \text{(A5)}$$

$$I_5^\theta(y)=\int_0^\infty d\Omega \frac{\exp(x^2+y^2)^{1/2}}{(x^2+y^2)^{3/2}[1+\exp(x^2+y^2)^{1/2}]^2} , \quad \text{(A6)}$$

$$I_6^\theta(y)=\int_0^\infty d\Omega \frac{\exp[2(x^2+y^2)^{1/2}]}{(x^2+y^2)^{3/2}[1+\exp(x^2+y^2)^{1/2}]^3} , \quad \text{(A7)}$$

$$I_7^\theta(y)=\int_0^\infty d\Omega \frac{\exp(x^2+y^2)^{1/2}}{(x^2+y^2)^2[1+\exp(x^2+y^2)^{1/2}]^2} . \quad \text{(A8)}$$

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