Interference radiative phenomena in the production of heavy unstable particles

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We discuss the characteristic interference phenomena, including real as well as virtual radiative corrections, to the inclusive cross sections for production of heavy unstable systems such as $t\bar{t}$ or W^+W^- pairs. We demonstrate explicitly the (fortunate) cancellation between emissions associated with the production of unstable particles and with their decays, and between emissions accompanying the decay stages. As a result the total interference effect is shown to be of $O(\alpha_i\Gamma/M)$ or better (where $\alpha_i = \alpha$ or α_s as appropriate).

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I. INTRODUCTION

An important aim of present and future high-energy colliders is the detailed study of heavy unstable particles [such as W bosons, top quarks, supersymmetric particles] and the precise determination of their parameters, in particular their masses M. Such an ambitious program requires a detailed understanding of the production mechanisms and, in particular, the effects arising from their large widths, $\Gamma \sim 1$ GeV. For example, since the t quark is heavier than $\sim$ 110 GeV [1], then Γ_t is greater than the typical hadronic scale $\sim 1\ \mathrm{fm}^{-1}$ and it may decay before it has time to hadronize [2, 3].

Interesting phenomena occur in the (photon and/or gluon) radiative corrections to processes involving the production and decay of heavy unstable particles; see, e.g., Refs. [4–10]. The radiative pattern proves to be a rather complex issue, depending sensitively on the time scale of the emission compared to the production time ($\sim 1/M$) or to the lifetime of the unstable particle ($\sim 1/\Gamma$). One important question concerns the correct treatment of various interference effects. For the purposes of illustration it is sufficient to focus attention on two processes of particular interest:

$$e^+e^- \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b}, \tag{1}$$

$$e^+e^- \rightarrow W^+W^- \rightarrow f_1\bar{f}_2f_3\bar{f}_4. \tag{2}$$

For the first process we need to study the effects of (a) interference between gluon radiation accompanying $t\bar{t}$ production and emission from the t and b quarks at the decay stages and (b) interference between gluons radiated in the decay stages, including in each case virtual as well as real gluon emission. The second process (2) has similar QED radiative corrections (with photon emission replacing gluon emission) but requires in addition a study of (c) the interference between photon emission from the initial and final states.

We list below some examples which reveal the importance of a detailed understanding of interference effects in the interpretation of data from future experiments.

(i) The calculation of the contribution of the radiative corrections to the total cross section for each process.[1]

(ii) Interferences between gluon emissions and the possibility of so-called "color recoupling" in the process $e^+e^- \rightarrow W^+W^- \rightarrow 4$ jets close to threshold [11], and also in other reactions involving the production of W's, Z's, and quark pairs [12].

(iii) The topology of the final states in the process (1) and the possibility to make a distinction between the so-called "one-string" and "three-string" scenarios [13,14].

(iv) The so-called "b-quark attraction effect", namely, the possible increase of the amplitude of process (1) due to the enhancement of the b-quark wave function near the origin, in the field of the $\bar{t}$ quark [15].

(v) Interferences between $W/Z+\gamma$ production ($p\bar{p} \rightarrow W/Z+\gamma$) and radiative W/Z decays ($p\bar{p} \rightarrow W/Z \rightarrow f\bar{f}'\gamma$); see, for example, Ref. [16].

Motivated by the above examples, we shall consider interference radiative phenomena contributing to the inclusive production of heavy unstable particles. To our knowledge these interference effects have not been adequately discussed in the literature so far.

Our main conclusions are as follows.

(i) Only the emission of soft quanta (virtual as well as real) of energy $k^0 \lesssim \Gamma$ can lead to significant interference contributions.

(ii) In practically all the relevant cases of the inclusive production of heavy systems, the soft real emissions are cancelled by virtual emissions up to terms of $O(\alpha_i\Gamma/M)$ or better (where $\alpha_i = \alpha$ or α_s as appropriate). We emphasize such a cancellation is a specific feature of inclu-

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[1] Interference phenomena in $t\bar{t}$ production at threshold have recently been discussed in Ref. [10].

sive quantities. For instance, it does not appear when the interacting decay products have low relative velocity.

(iii) The only residual interference effect is the Coulomb interaction between slowly moving particles in the threshold region.

To be specific, in Sec. III we concentrate on the topical case of QED interference radiative corrections to the total cross section of the process (2); see Ref. [9]. Interference between amplitudes with (gluon) emission in the final state for process (1) is analogous to final state photon emission for process (2). The reason is that here the soft insertion technique (which we use throughout) is independent of the group structure of the gauge theory. Soft gluons and photons are treated in exactly the same way, and only differ in their overall couplings, color factors, and charges. In fact, when account is taken of the QCD group structure the interference could be even more suppressed. It is also worth mentioning that for $M_t \gtrsim$ 110 GeV there is no influence of the long-range nonperturbative QCD potential; see Refs. [17–20]. Our results can easily be extended to the case when more than two heavy unstable particles are produced in the intermediate state and when any number of relativistic particles appear in their decays. Finally we should emphasize that our analysis is completely perturbative. Thus our treatment of gluon emission only makes sense if its energy satisfies $k^0 \gtrsim 1\ \text{fm}^{-1}$.

The remainder of the paper is organized as follows. In Sec. II we study interference radiative effects in the production of a pair of massive stable particles. We use this example to develop techniques before turning in Sec. III to the more complicated case, which is our main concern, of interference radiative corrections associated with the production of heavy unstable particles. Section IV gives our conclusions.

II. MODEL EXAMPLE: SOFT RADIATION IN THE PRODUCTION OF STABLE MASSIVE PARTICLES

Although we are interested in the production of heavy *unstable* particles, it is useful to first gain insight by studying the simpler model example,

$$p_1 + p_2 \rightarrow q_1 + q_2, \tag{3}$$

representing, say, e^+e^- annihilation into a pair of massive charged *stable* particles, with masses M_1 and M_2, as shown in Fig. 1. In this way we can obtain the analytic structure of the interference effects without encountering the complications which occur due to the finite width effects of the produced particles. We then turn to the case of unstable heavy particles in the next section. There we will find we need only consider emissions of energy $k^0 \lesssim \Gamma$. The reason is that the large k^0 emissions push the final state far off the resonance energy for the nonradiative process and the interference contribution becomes negligible [7–9,21–23]; see Sec. III for more details. It therefore suffices to study just the effect of soft emissions on our illustrative inclusive process $e^+e^- \rightarrow M_1M_2X$ of (3). To be precise the aim of the model example is to expose the physical origin of the cancellation between the soft real emissions and the virtual contributions. In particular, we will highlight the special properties which arise in the threshold region.

Before we can begin the calculation of the radiative interference effects in the inclusive process (3), we must recall that the notion of interference is, in principle, gauge dependent. The radiative interference contribution we are concerned with can be decomposed into interference between emissions at physically different stages (production, decays) which themselves are all gauge invariant (see, for example, Refs. [7,21]). However, for technical reasons it is more convenient to perform the analysis in terms of interference contributions corresponding to Feynman diagrams. Since one diagram can contribute to emissions at different stages of the process, the explicit gauge invariance will now be lost. Therefore we need to specify the gauge. We found it convenient to use the Feynman gauge for the purposes of this paper. We may also repeat the analysis in the Coulomb gauge, which would have some advantages in the physical interpretation of the effects.

We begin with the contribution δ_{IF}, corresponding to the interference between diagrams with initial and final state radiation. An example of the interference between the *real* emission diagrams is illustrated pictorially in Fig. 2(a). (Throughout the paper it is implicit that we also include the complex conjugate diagrams.) As mentioned above we are only interested in soft photon emissions, with $k^0 \ll \sqrt{s}$. In the Feynman gauge we have

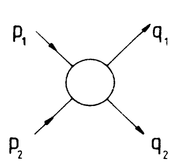

FIG. 1. Process (3) representing the production of massive stable particles of four-momenta q_1 and q_2.

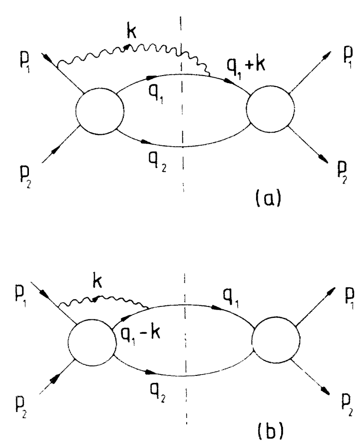

FIG. 2. An example of radiative contributions to the inclusive process (3) arising from interference between diagrams with initial and final state emission. We show (a) real and (b) virtual photon emission. The photons are real or virtual depending on the position of the vertical dot-dashed line.

$$\delta_{\rm IF}^{(r)}(p_1,q_1) = 4\pi\alpha e_{p_1}e_{q_1}\int \frac{d^3k}{(2\pi)^3 2\omega}\,\frac{2(p_1\cdot q_1)}{(p_1\cdot k)(q_1\cdot k)}, \tag{4}$$

where we have used the "soft insertion" technique which amounts to neglecting the k term in the numerator. Throughout this paper we take p_i and q_i to be the four-momenta of the incoming and outgoing on-shell particles respectively; see Fig. 2. Also e_p is used to denote the charge (in units of e) of the particle with momentum p.

We must also include the corresponding *virtual* emission contributions, of the type shown in Fig. 2(b). The interference term between the virtual emission diagram and the nonradiative Born diagram has the form

$$\delta_{\rm IF}^{(v)}(p_1,q_1) = 4\pi\alpha e_{p_1}e_{q_1}\int \frac{d^4k}{(2\pi)^4 i(k^2+i\epsilon)}\,\frac{4(p_1\cdot q_1)}{(-2p_1\cdot k+k^2+i\epsilon)(-2q_1\cdot k+k^2+i\epsilon)} + \text{c.c.}, \tag{5}$$

where the complex conjugate (c.c.) allows for interference between the Born and virtual diagrams, that is, the diagrams in the reverse order. As we are only concerned with soft emissions we may neglect the k^2 terms in the propagators of the radiating particles. Thus the only pole in the lower-half of the k^0 plane occurs at

$$k^0 = \omega - i\epsilon \quad \text{where} \quad \omega = |\mathbf{k}|. \tag{6}$$

Thus when we perform the dk^0 integration by closing the contour in the lower-half-plane we see immediately that

$$\delta_{\rm IF}^{(v)}(p_1,q_1) = -\delta_{\rm IF}^{(r)}(p_1,q_1).$$

This is an example of the well-known cancellation of the infrared divergency between the real and the virtual emission contributions [24]. In this case it is particularly evident, in contrast to the cancellation of some of the other radiative effects that we verify below.

Consider the interference between diagrams with radiation from the final state, Fig. 3(a). Here the virtual contribution, Fig. 3(b), is given by

$$\delta_{\rm FF}^{(v)}(q_1,q_2) = \int \frac{d^4k}{(2\pi)^4 i(k^2+i\epsilon)}\,\frac{4\pi\alpha e_{q_1}e_{q_2}4(q_1\cdot q_2)}{(2q_1\cdot k+k^2+i\epsilon)(-2q_2\cdot k+k^2+i\epsilon)} + \text{c.c.} \tag{7}$$

As in the previous case, the contribution of the photon pole, (6), is canceled by real emission. However, now there is an additional pole in the lower-half k^0 plane, even when we neglect the k^2 terms in the propagators of the radiating particles, located at

$$k^0 = \frac{\mathbf{k}\cdot\mathbf{q}_1}{q_1^0} - i\epsilon. \tag{8}$$

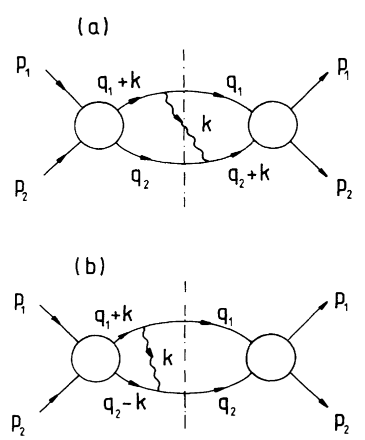

FIG. 3. As for Fig. 2 but showing interference between diagrams with final state emission.

Thus, on performing the dk^0 integration, we have an additional contribution proportional to the residue of this pole:

$$\text{Re}\int \frac{d^3k}{[\mathbf{k}^2-(\mathbf{k}\cdot\mathbf{q}_1)^2/(q_1^0)^2)]}\,\frac{1}{[q_2^0(\mathbf{k}\cdot\mathbf{q}_1)-q_1^0(\mathbf{k}\cdot\mathbf{q}_2)-i\epsilon]}, \tag{9}$$

where we take the real part on account of the c.c. contribution in (7). The integration is over all d^3k and, on using the substitution $\mathbf{k}\to-\mathbf{k}$, we see that the integral in (9) is pure imaginary. Therefore the additional term does not, in fact, contribute to $\delta_{\rm FF}(q_1,q_2)$.

We appear to have no contribution from interference effects between diagrams with soft final state emission. What has happened to the Coulomb effects [25] which we know to be present near threshold (when $|\mathbf{q}_{1,2}|\ll M_{1,2}$)? The dilemma is resolved if we note that the neglect of the k^2 terms in the propagators of charged particles in (7) is not valid in the threshold region. To be specific, when $|\mathbf{q}|\lesssim|\mathbf{k}|$ it is no longer possible to neglect $\mathbf{k}^2$ in comparison with $\mathbf{k}\cdot\mathbf{q}$. [Fortunately $(k^0)^2$ can always be dropped since it is small in comparison to k^0q^0.] Since the region $|\mathbf{k}|\sim|\mathbf{q}|$ plays a significant role in the Coulomb interaction, we need a more detailed treatment of the integral in (7) in the threshold case. Rather than (8), the additional pole is located at

$$k^0 = \frac{2\mathbf{k}\cdot\mathbf{q}_1 + \mathbf{k}^2}{2q_1^0} - i\epsilon. \tag{10}$$

Using the nonrelativistic approximation $q_i^0 \simeq M_i$, the "final-final state" interference contribution thus becomes

$$\begin{aligned}\delta_{\rm FF}^{(r)}(q_1,q_2) &+ \delta_{\rm FF}^{(v)}(q_1,q_2)\\ &= \delta_{\rm Coul}(q_1,q_2)\\ &\simeq 2{\rm Re}\int \frac{d^3k}{(2\pi)^3\mathbf{k}^2}\,\frac{8\pi\alpha e_{q_1}e_{q_2}}{\mathbf{q}^2/\mu - (\mathbf{k}+\mathbf{q})^2/\mu + i\epsilon}\end{aligned} \tag{11}$$

in the c.m. frame ($\mathbf{q} = \mathbf{q}_1 = -\mathbf{q}_2$), where $\mu = M_1M_2/(M_1+M_2)$ is the reduced mass of the produced M_1M_2 system and $|\mathbf{q}| \ll \mu$.

If we perform the d^3k integration we obtain

$$\delta_{\rm Coul}(q_1,q_2) \simeq -\frac{\alpha\pi}{v}e_{q_1}e_{q_2}, \tag{12}$$

which is the familiar expression for the first order Coulomb interaction between two slowly moving stable particles, where $v = |\mathbf{q}|/\mu$ is their relative velocity. We recognize the Coulomb correction (12) to be the leading order term in the α/v expansion of the famous Sommerfeld-Sakharov factor [25]

$$\begin{aligned}|\Psi_{q_1q_2}(0)|^2 &= \frac{X_{q_1q_2}}{1-\exp(-X_{q_1q_2})},\\ X_{q_1q_2} &= -e_{q_1}e_{q_2}\frac{2\pi\alpha}{v},\end{aligned} \tag{13}$$

where $\Psi_{q_1q_2}(0)$ is the final state wave function evaluated at the origin.

We emphasize that the Coulomb interaction is only important near threshold where it is intimately associated with soft radiation:

$$|\mathbf{k}| \sim |\mathbf{q}|, \qquad k^0 \sim \mathbf{q}^2/\sqrt{\mathbf{q}^2+\mu^2}; \tag{14}$$

see (10). As $|\mathbf{q}|$ increases the characteristic photon energy responsible for the Coulomb interaction no longer corresponds to soft emission. But, as mentioned above, we need only consider soft photons $k^0 \lesssim \Gamma$; the region $k^0 > \Gamma$ being suppressed by the requirement of resonance overlap (see Sec. III). Thus the Coulomb effect, (12), will only be observable if $\mathbf{q}^2 \lesssim \Gamma\mu$. At higher $\mathbf{q}^2$ this effect is negligible.

In summary, we have demonstrated the total cancellation of the order-α initial-final and final-final soft radiative interference effects in $e^+e^- \to M_1M_2$, except for a Coulombic contribution in the region of the M_1M_2 threshold. That is, the only nonvanishing contribution arises from a final-final state interference effect which is given by the (universal) Coulomb interaction between the slowly moving outgoing particles M_1 and M_2. Because the underlying Coulomb physics is quite different from the other radiative corrections, it is possible to treat the Coulomb effects separately; see, for example, Refs. [9, 17, 18].

III. INTERFERENCE EFFECTS IN $e^+e^- \to W^+W^-$

To discuss the subtleties of the interference radiative corrections in the case of unstable particles it is sufficient to consider, for example, process (2):

$$e^+e^- \to W^+W^- \to f_1\bar{f}_2f_3\bar{f}_4. \tag{15}$$

The application of the results to other cases is straightforward. In the presence of unstable particles the analytical structure of the interference contributions is complicated by the existence of additional poles arising from the various resonance propagators.

Recall that the dominant contribution from radiative emission can be divided into three parts of different physical origin, each of which is gauge invariant. The emissions can occur off the WW production process or from either of the W decay processes. Our interest here is focused on the interferences between the radiative amplitudes for these various emissions. The different types of interference contributions for process (15) are illustrated by the diagrams of Figs. 4–6. Note that in addition to the initial-final state interference of Fig. 4 (compare Fig. 2) and the final-final state interference of Fig. 5 (compare Fig. 3), we now also have the intermediate-final state interference contribution of Fig. 6. The latter enters both the production-decay and the decay-decay interference contributions.

Figures 4–6 do not cover all types of interference diagrams. For example, we could also have a diagram such as Fig. 7 in which a photon is exchanged between an intermediate unstable particle and one of its decay products. Figure 7 leads to a nonvanishing contribution due to emission in the course of W decay. This contribution is related to the gauge invariance of the emission at the decay stage and is incorporated into the physical decay width of the W. Figure 7 also contains a contribution from interference between emission from the W at the production stage (i.e., before its decay) and emission from one of its decay products. Now this latter contribu-

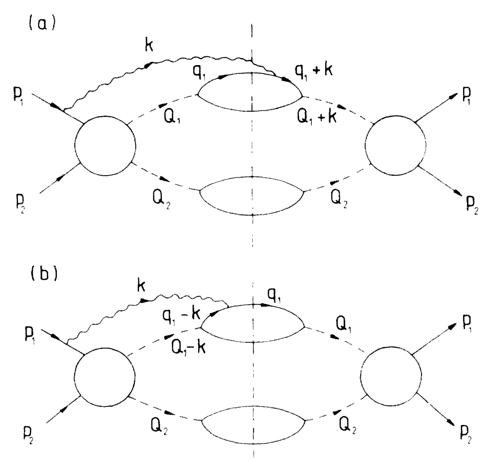

FIG. 4. An example of a (a) real and (b) virtual initial-final state interference contribution to the process $e^+e^- \to W^+W^-X \to 4fX$.

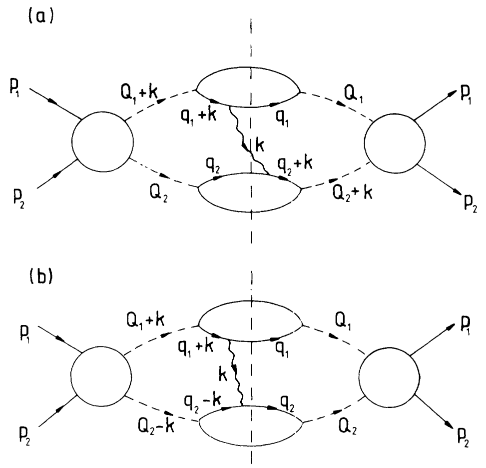

FIG. 5. An example of a (a) real and (b) virtual final-final state interference contribution to the process $e^+e^- \to W^+W^-X \to 4fX$.

tion is related to the gauge invariance of the interference between emissions both at the production and at the decay stage. As we noted above, we prefer to work with Feynman diagrams (losing, unfortunately, explicit gauge invariance), because the separation of the gauge invariant parts is rather cumbersome. Now Figs. 4–6 contribute only to interferences between emissions at physically different stages of the process. We examine these contributions in the Feynman gauge below and show that each vanishes to $O(\alpha\Gamma/M)$, or better. In the general case it is not so simple (but is manageable) to show explicitly the vanishing of the interference between emissions from physically different stages when a diagram, such as Fig. 7, has other (nonvanishing) contributions. However, for instance, for Fig. 7 it is relatively easy to show that the contributions arising from interference between emissions at the production and decay stages vanish. Here we have only the photon pole, (6), in the lower-half k^0 plane, and

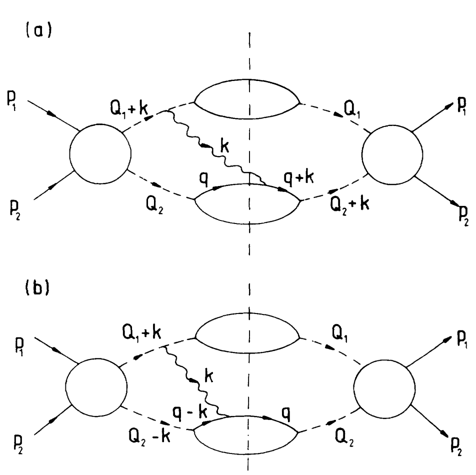

FIG. 6. An example of a (a) real and (b) virtual intermediate-final state interference contribution to the process $e^+e^- \to W^+W^-X \to 4fX$.

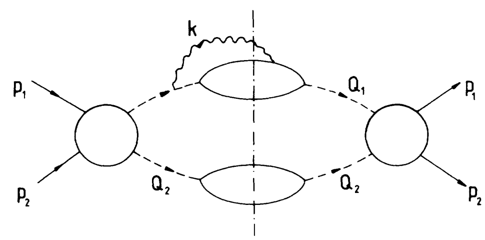

FIG. 7. A Feynman diagram which contributes both to interference between emissions from physically different (production and decay) stages of the process and to the W radiative decay process alone.

so the cancellation of the real and virtual contributions of soft photons is quite straightforward. Such a cancellation also occurs in the course of the W decay, and hard photons do not contribute to interference of emissions at the production and decay stages; so there is no contribution from the interferences of the type of Fig. 7 to the inclusive process (apart of course from that incorporated in the W decay width).

An important feature of radiation from unstable particles is that their width acts as a natural cutoff on the emitted energy, such that only radiation with $k^0 \lesssim \Gamma$ need be considered. The reason follows from inspection of the propagators of the unstable particles. The propagators contain a factor[2]

$$\frac{1}{D(P)} = \frac{1}{P^2 - M^2 + iM\Gamma}, \tag{16}$$

where P, M, and Γ are the four-momentum, mass, and width of the particle (which in our case is a W boson). We present the argument at the beginning of the following subsection using initial-final state interference as the example.

A. Initial-final state interference

An example of the initial-final state interference contribution, I_{if}, is shown in Fig. 4. Suppose, for simplicity, we are in the threshold region. If the photon energy $k^0 \gg \Gamma$ and the W boson with four-momentum Q_1 is close to its mass shell,

$$\frac{M\Gamma}{D(Q_1)} \approx \frac{1}{i}, \tag{17}$$

then in the interfering amplitude it will be far off shell:

$$\frac{M\Gamma}{D(Q_1+k)} \approx \frac{\Gamma}{2k^0 + i\Gamma}. \tag{18}$$

Integrating over Q_1^2 the interference contribution becomes

[2]We use the simplified form (16) of the resonance propagator; the approximation $M\Gamma$ of $\sqrt{P^2}\Gamma(P^2)$ gives a negligible $O(\Gamma^2/M^2)$ effect.

$$I_{if} \sim \mathrm{Re}\left\langle \frac{M^2\Gamma^2}{D(Q_1)D^*(Q_1+k)} \right\rangle \sim \frac{\Gamma^2}{(k^0)^2+\Gamma^2}. \tag{19}$$

Thus we expect the dominant interference contribution to come from radiated photons with $k^0 \lesssim \Gamma$. This is also valid for the other interference contributions (Figs. 5 and 6); see Refs. [8, 9].

Therefore we find only soft quanta (real or virtual) can lead to significant interference contributions between emissions at the various production and decay stages of the process. This phenomenon has been quite well understood since (at least) the early days of J/ψ physics [22, 23].

It is informative to trace the physical origin of this general result. When considering soft radiation ($k^0 \ll M$) the production and decay of heavy unstable particles can be regarded essentially as pointlike processes with a characteristic time scale

$$t_{\text{char}} \sim 1/M. \tag{20}$$

However, because of W decay, the processes are separated in time by intervals

$$\tau \sim 1/\Gamma. \tag{21}$$

Thus for $k^0 \gg \Gamma$ the relative phases of the emissions accompanying these processes are large, that is, $k^0\tau \gg 1$. As a consequence when we average over all the times t_i between WW production and the W decays, the interferences between the emissions will average to zero. To be explicit for the interference shown in Fig. 4 we have

$$I_{if} \sim \Gamma\,\mathrm{Re}\left(\int_0^\infty e^{-ik^0 t_1} e^{-\Gamma t_1} dt_1\right) = \frac{\Gamma^2}{(k^0)^2+\Gamma^2}. \tag{22}$$

An analogous result holds for the final-final state interferences, but there we must average over both the W decay times t_1 and t_2. Thus a new scale Γ for photon energies naturally arises. Only in the $k^0 \lesssim \Gamma$ domain might we anticipate a significant interference contribution.

Let us evaluate I_{if} (see Fig. 4) in this domain. As mentioned above the difference between the unstable and the stable case (of the previous section) arises from the propagators of the intermediate W bosons. We see we will have resonance factors $D(Q_i)$ or $D(Q_i \pm k)$ in the denominators of the integrands of the interfering amplitudes. The notation should be clear from Fig. 4; we always use Q_i to denote the four-momentum of the $f\bar{f}$ system or, to put it another way, the four-momentum of the nonradiative decay of intermediate particle i. Now we can still neglect terms of order k^2 in the denominators and since we have only one pole in the lower-half k^0 plane the contributions from real and virtual photons cancel each other. This can be seen with the help of Fig. 4. For real emission, Fig. 4(a), the photon line corresponds to a factor

$$-g^{\mu\nu}\frac{d^3k}{(2\pi)^3 d\omega} \tag{23}$$

in the Feynman gauge, while for virtual emission, Fig. 4(b), the corresponding factor is

$$\frac{g^{\mu\nu}d^4k}{(2\pi)^4 i(k^2+i\epsilon)}. \tag{24}$$

After taking the residue of the photon pole in the lower-half k^0 plane, (24) coincides with (23). Recall that p_i, q_i are defined as the four-momenta of the on-shell radiating particles.

From Fig. 4 we can make two important observations.

(i) The photon momentum enters the propagator of the radiating daughter particle with opposite signs ($q_1 \pm k$) in the real and virtual diagrams.

(ii) The momenta of the unstable intermediate particles are given in terms of the momenta (Q_i) of their on-shell decay products in a different way in the real and virtual contributions. For example for the real emission of Fig. 4(a) we have Q_1 and Q_1+k, whereas for the virtual contribution of Fig. 4(b) we have Q_1-k and Q_1 respectively.

It is observation (i) which guarantees the cancellation between the real and virtual pieces since at $k^2=0$ for the on-shell particle q_1 the corresponding propagators have opposite signs, $\pm 2k\cdot q_1$. There is also an important message from observation (ii). Namely, the cancellation is not local in momentum space. It occurs only for the inclusive cross section, when the integration is performed not only over the photon momentum but over the momenta of the decay products q_i as well.

The accuracy of the cancellation corresponds to that of the soft photon approximation and so is $O(k^0/M)$. The interference contribution itself is $\lesssim \alpha\Gamma/k^0$ and so the radiative correction I_{if} to the inclusive cross section is $O(\alpha\Gamma/M)$ or smaller. Such an estimate remains valid at the energies far above the production threshold.

B. Final-final state interference

We turn now to the interference between diagrams with (soft) emission from the final state particles; see Fig. 5. The virtual contribution of Fig. 5(b) is given by

$$\delta^{(v)}_{\rm FF}(q_1,q_2) = \int \frac{d^4k}{(2\pi)^4 i(k^2+i\epsilon)} \frac{4\pi\alpha e_{q_1}e_{q_2}4(q_1\cdot q_2)}{(2q_1\cdot k+i\epsilon)(-2q_2\cdot k+i\epsilon)} \frac{|D(Q_1)|^2|D(Q_2)|^2}{D(Q_1+k)D(Q_2-k)D^*(Q_1)D^*(Q_2)} + \text{c.c.} \tag{25}$$

The D factors in the numerator arise simply from the normalization of $\delta_{\rm FF}$ relative to the nonradiative cross section. We have assumed that the interacting particles with momentum q_1 and q_2 are relativistic so that the k^2 terms in their propagators can be neglected in comparison with the $2q_i \cdot k$ terms. This approximation cannot be justified if the interacting particles are nonrelativistic (in their c.m. frame) but the phase space of this kinematic configuration is negligibly small. Also when we evaluate (25) it will be sufficient to retain only the linear terms in k in the propagators $D(Q_1+k)$ and $D(Q_2-k)$.

The integrand in (25) has three poles in both the upper- and lower-half k^0 planes. Let us carry out the integration in the lower-half k^0 plane. It is straightforward to show, in an analogous way to before, the complete cancellation of the photon pole, (6), by the real emission contribution of Fig. 5(a). To evaluate the contribution of the remaining two poles it is convenient to shift the contour so that the photon pole lies in the upper-half k^0 plane by the substitution

$$k^2 + i\epsilon \;\rightarrow\; (k^0 - i\epsilon)^2 - \mathbf{k}^2. \tag{26}$$

Then after the cancellation we find the total final-final state interference correction is

$$\begin{aligned}\delta_{\rm FF}(q_1,q_2) &= \delta_{\rm FF}^{(v)}(q_1,q_2) + \delta_{\rm FF}^{(r)}(q_1,q_2)\\ &= \int \frac{d^4k}{(2\pi)^4 i} 4\pi\alpha e_{q_1} e_{q_2} 4(q_1\cdot q_2)|D(Q_1)|^2|D(Q_2)|^2\\ &\quad\times\left\{\frac{1}{[(k^0-i\epsilon)^2-\mathbf{k}^2](2q_1\cdot k+i\epsilon)(-2q_2\cdot k+i\epsilon)}\,\frac{1}{D(Q_1+k)D(Q_2-k)D^*(Q_1)D^*(Q_2)}\right.\\ &\quad\left.-\frac{1}{[(k^0+i\epsilon)^2-\mathbf{k}^2](2q_1\cdot k-i\epsilon)(-2q_2\cdot k-i\epsilon)}\,\frac{1}{D^*(Q_1+k)D^*(Q_2-k)D(Q_1)D(Q_2)}\right\},\end{aligned} \tag{27}$$

where we have written the complex conjugate term explicitly. It is now easy to show that the expression in curly brackets, and hence $\delta_{\rm FF}$, vanishes. If in the second term in curly brackets we reverse the signs of k^0 and $\mathbf{k}$, and then make the substitutions[3]

$$\begin{aligned} Q_1 &\rightarrow Q_1 + k,\\ Q_2 &\rightarrow Q_2 - k \end{aligned} \tag{28}$$

(which is permitted for an inclusive process), then it becomes identical to the first term but with opposite sign, and so the interference contribution vanishes. The result remains valid if the $\mathbf{k}^2$ term is kept in the propagators of the unstable particles.

C. Intermediate-final state interference

Finally we study the type of interference shown in Fig. 6, that is, interference $\delta_{\rm InF}$ between emission from one of the intermediate unstable particles and emission from a decay product of the other one. Recall that such contributions enter both the production-decay and the decay-decay interferences. The virtual contribution of Fig. 6(b) is given by

$$\delta_{\rm InF}^{(v)}(Q_1,q) \;=\; \int \frac{d^4k}{(2\pi)^4 i(k^2+i\epsilon)}\,\frac{4\pi\alpha e_{Q_1} e_q\, 4(Q_1\cdot q)|D(Q_1)|^2|D(Q_2)|^2}{(-2q\cdot k+i\epsilon)D(Q_1+k)D(Q_2-k)|D(Q_1)|^2 D^*(Q_2)} + \text{c.c.}, \tag{29}$$

where as before we have neglected the k^2 term in the propagator of the relativistic particle of four-momentum q. Also we can again drop the $(k^0)^2$ term in the propagators $D(Q_1+k)$ and $D(Q_2-k)$, but since the intermediate particles may be produced near threshold we should retain the $\mathbf{k}^2$ terms. Once more it is quite straightforward to trace the cancellation of the photon pole (6) in $\delta_{\rm InF}^{(v)}$ by the real contribution.

The main contribution from a resonance denominator $D(P)$, such as (16), comes from the region $P^2 \simeq M^2$ and

$$P^0 \;\simeq\; \sqrt{\mathbf{P}^2+M^2} \;\equiv\; \epsilon_{\mathbf{P}}. \tag{30}$$

Thus, neglecting $O(\Gamma/M)$ terms, we can rewrite $D(P)$ as

$$D(P) \;\simeq\; 2\epsilon_{\mathbf{P}}\left(P^0 - \epsilon_{\mathbf{P}} + \frac{iM\Gamma}{2\epsilon_{\mathbf{P}}}\right). \tag{31}$$

The integrand in (29) contains only one pole [additional to the photon pole of (6)] in the lower-half k^0 plane, aris-

[3]Note that the substitution (28) is not to be made in the D factors in the numerator of (27), which occur simply to normalize $\delta_{\rm FF}$ relative to the nonradiative cross section $\sim |D(Q_1)D(Q_2)|^{-2}$.

ing from the resonance denominator $D(Q_1+k)$, located at

$$k^0 = -Q_1^0 + \epsilon_{\mathbf{Q}+\mathbf{k}} - \frac{iM\Gamma}{2\epsilon_{\mathbf{Q}+\mathbf{k}}}, \tag{32}$$

where for convenience we are working in the c.m. frame with

$$\mathbf{Q}_1 = -\mathbf{Q}_2 \equiv \mathbf{Q}. \tag{33}$$

Carrying out the k^0 integration in the virtual contribution (29) [and adding the corresponding real emission contribution of Fig. 6(a)] gives

$$\begin{aligned}\delta_{\rm InF}(Q_1,q) &= \delta_{\rm InF}^{(v)}(Q_1,q) + \delta_{\rm InF}^{(r)}(Q_1,q)\\ &= -\int \frac{d^3k}{(2\pi)^3}\,\frac{4\pi\alpha e_{Q_1} e_q\, 4(Q_1\cdot q)|D(Q_1)|^2|D(Q_2)|^2}{[(k^0)^2-\mathbf{k}^2](-2q^0k^0+2\mathbf{q}\cdot\mathbf{k})(2\epsilon_{\mathbf{Q}+\mathbf{k}})^2\left(\sqrt{s}-2\epsilon_{\mathbf{Q}+\mathbf{k}}+\frac{iM\Gamma}{\epsilon_{\mathbf{Q}+\mathbf{k}}}\right)|D(Q_1)|^2D^*(Q_2)} + \text{c.c.},\end{aligned} \tag{34}$$

with k^0 given by (32) and

$$\sqrt{s} = Q_1^0 + Q_2^0. \tag{35}$$

To determine the inclusive radiative cross section we integrate over Q_1^0 [taking account of (35)] and over $\mathbf{Q}$. Because of the dominance of the contribution from the region $Q_1^0 \approx \epsilon_{\mathbf{Q}}$ we can set $Q_1^0 = \epsilon_{\mathbf{Q}}$ everywhere (apart from the resonance denominators) and integrate over Q_1^0 from $-\infty$ to $+\infty$. The integration over Q_1^0 can now be performed by taking the residue of the only pole located in the upper-half Q_1^0 plane, that is, the pole in $D^*(Q_1)$ at

$$Q_1^0 = \epsilon_{\mathbf{Q}} + \frac{iM\Gamma}{2\epsilon_{\mathbf{Q}}}. \tag{36}$$

Recall that the D factors in the numerator of (34) are associated with the nonradiative amplitude and so the result of the integration is

$$\begin{aligned}\int \frac{dQ_1^0 d^3Q\,\delta_{\rm InF}(Q_1,q)}{|D(Q_1)|^2|D(Q_2)|^2} &\simeq \frac{\pi}{M\Gamma}\int\frac{d^3k}{(2\pi)^3}\,\frac{d^3Q\,4\pi\alpha e_{Q_1}e_q\,4(Q_1\cdot q)}{[(k^0)^2-\mathbf{k}^2](2q^0k^0-2\mathbf{q}\cdot\mathbf{k})}\\ &\quad\times\frac{1}{(2\epsilon_{\mathbf{Q}+\mathbf{k}})^2(2\epsilon_{\mathbf{Q}})^2(\sqrt{s}-2\epsilon_{\mathbf{Q}+\mathbf{k}}+iM\Gamma/\epsilon_{\mathbf{Q}+\mathbf{k}})(\sqrt{s}-2\epsilon_{\mathbf{Q}}-iM\Gamma/\epsilon_{\mathbf{Q}})} + \text{c.c.},\end{aligned} \tag{37}$$

with k^0 given by (32) and (36), namely,

$$k^0 = \epsilon_{\mathbf{Q}+\mathbf{k}} - \epsilon_{\mathbf{Q}} - \frac{iM\Gamma}{2}\left(\frac{1}{\epsilon_{\mathbf{Q}+\mathbf{k}}} + \frac{1}{\epsilon_{\mathbf{Q}}}\right). \tag{38}$$

We can now readily show the inclusive form (37) vanishes. We make the substitutions

$$\mathbf{k} \to -\mathbf{k}, \qquad \mathbf{Q} \to \mathbf{Q}+\mathbf{k} \tag{39}$$

in the integrand in (37), which results in the interchanges

$$\begin{aligned}&\epsilon_{\mathbf{Q}+\mathbf{k}} \to \epsilon_{\mathbf{Q}}, \qquad \epsilon_{\mathbf{Q}} \to \epsilon_{\mathbf{Q}+\mathbf{k}},\\ &k^0 \to -k^{0*}.\end{aligned} \tag{40}$$

Note that we can neglect the variation of the $Q_1\cdot q$ factor in the soft photon approximation ($|\mathbf{k}| \ll M$). From these substitutions we see that the integral in (37) is equal to its complex conjugate but with the opposite sign. Thus both terms in (37) compensate each other and the intermediate-final state interference contribution vanishes (up to terms of order $\alpha\Gamma/M$).

D. Coulomb effects on interference near the production threshold

Until now we have used the soft insertion technique in which we systematically neglect the radiation momentum k everywhere except in the resonance propagators. Of course, the technique is valid when we consider radiative corrections to Born amplitudes, but it is not always correct in the general case. For example, such a procedure is not applicable if the unstable intermediate particles are produced near their threshold. In such a case the (multiple) Coulomb interaction[4] between these particles leads to an additional momentum-dependent "vertex function" f associated with each production "blob" in Figs. 4–6. For the production of a pair of unstable particles of mass M and width Γ it has been shown [18] that

[4]Coulomb effects are especially important for process (1) and are found to drastically modify the cross section near threshold [17, 18, 20].

$$f(E,\mathbf{p}) \;=\; \langle \mathbf{p}|(\hat{H}-E-i\Gamma)^{-1}|\mathbf{r}=0\rangle \left(\frac{\mathbf{p}^2}{M}-E-i\Gamma\right), \tag{41}$$

where E is the nonrelativistic energy of the pair in their c.m. frame ($E=\sqrt{(p_+ + p_-)^2}-2M$ with $p_\pm$ being the particle four-momenta), $|\mathbf{p}\rangle$ denotes a state of definite momentum $\mathbf{p}=\mathbf{p}_+ = -\mathbf{p}_-$, and $|\mathbf{r}\rangle$ a state of definite relative coordinate. We use the normalization $\langle \mathbf{p}|\mathbf{r}=0\rangle = 1$. $\hat{H}$ is the nonrelativistic Hamiltonian of the pair in their c.m. frame:

$$\hat{H} \;=\; \frac{\hat{\mathbf{p}}^2}{M}+V(\mathbf{r}),$$

with $\hat{\mathbf{p}}=-i\partial/\partial\mathbf{r}$ and where $V(\mathbf{r})$ is the interaction energy.

When we consider the radiative process, the vertex function f becomes dependent on the emission momentum k. Fortunately, however, the dependence on the k^0 component is such as to allow the integration over k^0 in the same way as before. Moreover, the product of the vertex functions corresponding to the two "blobs" in each interference diagram (Figs. 4–6) enters into the amplitude in a "Hermitian" form, and so our conclusion that the total radiative correction, $\delta=\delta^{(r)}+\delta^{(v)}$, is zero remains valid.

Let us verify these statements for the example of process (2) in the c.m. frame. The k^0 dependence can only enter the vertex function f through its dependence on E, and only in the case of initial-final state interference does E depend on k^0. To be precise, for the left blob of Fig. 4 we have $E=\sqrt{s}-k^0-2M$; in all other cases $E=\sqrt{s}-2M$. From (41) it is evident that this k^0 dependence will only give singularities in the upper-half k^0 plane and so we can perform the integration over k^0 exactly as before, leading again to the cancellation of the real and virtual contributions. The vertex functions appear in the integrands in the first terms of (27) and (37) as the product

$$F(s,\mathbf{Q},\mathbf{k}) \;=\; f(\sqrt{s}-2M,\mathbf{Q}+\mathbf{k})f^*(\sqrt{s}-2M,\mathbf{Q}), \tag{42}$$

and, of course, as the complex conjugate in the second term. If we change the sign of $\mathbf{k}$ and make the replacement (28) or, equivalently, if we make the substitution (39), then

$$F(s,\mathbf{Q},\mathbf{k}) \;\to\; F^*(s,\mathbf{Q},\mathbf{k}). \tag{43}$$

Thus if we repeat the procedure outlined above then we again see that the expressions for $\delta_{\rm FF}$ and $\delta_{\rm InF}$ (as well as $\delta_{\rm IF}$) still vanish. This is in accord with the claims of Ref. [10] for process (1) in the threshold region.

The soft insertion technique, with the radiation momentum k neglected everywhere except in the resonance propagators, may also be invalid away from threshold. For example, it sometimes fails in QCD at high energies [26]. This might make one doubt the validity of our conclusion that the interference contributions considered above are suppressed by $O(\Gamma/M)$ when, for example, the QCD interaction between the decay products of an unstable particle is taken fully into account. Although we do not at the moment have a rigorous proof of the suppression in this case, we have arguments to believe that it will be true for inclusive processes.

IV. CONCLUSIONS

There are several topical processes concerned with the production of heavy unstable particles (such as $t\bar{t}$, W^+W^-, etc.) for which a detailed knowledge of radiative effects is important. The various production-decay and decay-decay radiative interferences for a given process might, in principle, be expected to produce a complicated effect. Fortunately, we have shown explicitly that such interference corrections are canceled in the inclusive cross sections up to terms of relative order $\alpha\Gamma/M$ or better. The only exception is the interference contribution arising from the universal Coulomb interaction between slowly moving produced objects.

Our explicit calculations have been performed for photon emissions but apply equally well to gluon radiation. When account is taken of the group structure of the QCD gauge theory there could be an additional suppression of the interference effects. For example, the interference between gluon radiation accompanying the quark decay modes of the W bosons produced in process (2) is suppressed by $(C_F\alpha_s/N_C)^2\Gamma/M$.

The results of our analysis should make it easier to construct reliable Monte Carlo generators for the final state events in processes of the type of (1) and (2). For instance, if we consider the b jets emerging from $t\bar{t}$ production and decay, then the azimuthal asymmetry of the jets expected to be caused by gluon interference effects should, in practice, be rather small (of order $\alpha_s\Gamma/M$). In other words the so-called $b\bar{b}$ antenna will not develop a QCD cascade since the emission of energetic primary gluons is governed only by the $t\bar{t}$, tb, and $\bar{t}\bar{b}$ antennas (see also Ref. [8]).

There are also consequences for the total cross section of process (1), $e^+e^-\to t\bar{t}\to W^+W^-b\bar{b}$; see Refs. [17, 18, 20]. We believe the QCD radiative effects are correctly included by proper account of the running of α_s and the hard gluon corrections to the $\gamma(Z)t\bar{t}$ vertex [20], assuming that the physical top quark width incorporates the radiative effects associated with top decay. There should be no other substantial order α_s corrections (here our conclusion is in accord with Ref. [10]). So despite the cautions of the authors of Ref. [20], their final formula (6.10) has stood up to our scrutiny.

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