

Batalin-Tyutin quantization of the CP^{N-1} model

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The CP^{N-1} model is quantized in the generalized canonical formalism of Batalin and Tyutin by converting the original second-class system to first class. Operator-ordering ambiguities present in the conventional quantization scheme of Dirac are thereby avoided. The first-class constraints, the involutive Hamiltonian, and the BRST charge are explicitly computed. The partition function is defined and evaluated in the unitary gauge.

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I. INTRODUCTION

The quantization of systems with second-class constraints has always been a challenging problem. One possible way is to employ the pioneering work of Dirac [1]. Another possibility is to interpret one-half of the second-class constraints as first class, and consider the other half as gauge-fixing conditions [2]. A more recent approach is the generalized canonical formalism developed by Batalin and Fradkin [3] and Batalin and Tyutin (BT) [4].

The object of this paper is to discuss the quantization of the CP^{N-1} model in the BT [4] formalism. It ought to be emphasized that the quantization of this model (which is an example of a second-class system) has been performed earlier, both by using the method of Dirac [5] as well as by splitting the second-class constraints into first-class constraints and gauge conditions [6,7]. In both these treatments, however, there are unpleasant features. The Dirac brackets, as computed in [5], are highly nontrivial. They are field dependent and also have a nonpolynomial structure. Consequently transition to the quantum theory is riddled with operator-ordering ambiguities. In the other treatment [6,7], on the other hand, there is an ambiguity in the splitting of constraints leading to a nonunique Hamiltonian. Moreover explicit computations [6,7] reveal that the modified Hamiltonian is *nonlocal*. All these bothersome features are bypassed in our analysis. Following BT [4] we convert the original second-class system into first class by introducing new fields in an extended phase space. The first-class constraints, the unitarizing Hamiltonian, and the Becchi-Rouet-Stora-Tyutin (BRST) charge are explicitly constructed. The phase-space partition function is defined.

It is explicitly computed in the unitary gauge [3] by doing the relevant momentum integrations. The original action (corresponding to the second-class theory) is reproduced in this gauge, but the measure is nontrivial. The connection of our results for an analogous computation done by us [8] in the case of the $O(N)$ nonlinear σ model is also discussed.

We mention that the basic idea of Batalin and Fradkin [3] or Batalin and Tyutin [4] of converting second-class systems to first class has been employed previously to quantize chiral gauge theories [9–11], the chiral boson theory [12], the massive Maxwell theory [10], the massive Yang-Mills theory [13], and the unitary gauge Abelian Higgs model [14]. Most of these examples, however, can be simply quantized by the conventional formulation of Dirac [1], since relevant Dirac brackets are simple and do not involve operator ordering ambiguities. The same, as we stressed earlier, is not true in the case of CP^{N-1} model. Furthermore, the quantization discussed in Ref. [10] is not a systematic application of the Batalin-Fradkin [3] or Batalin-Tyutin [4] method. Our analysis clearly demonstrates the necessity of a systematic approach, particularly when the algebra of constraints is field dependent as it is in the case of the CP^{N-1} model. Indeed the construction of the involutive Hamiltonian requires some involved algebra. Contrary to all previous computations [9–14], it contains an infinite number of terms. By proceeding systematically we find a remarkable series of cancellations which allows us to express this infinite series as an exponential. We feel that the insight gained from our analysis would be useful in quantizing other types of nonlinear models with a more involved constraint algebra.

The next section discusses the quantization of the model while our conclusions are presented in Sec. III.

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II. QUANTIZATION

In order to implement the generalized canonical quantization scheme of BT, it is necessary to specify the Hamiltonian together with its set of constraints. This is done by first considering the familiar form of the Lagrangian of the CP^{N-1} model,

$$\mathcal{L} = \partial_\mu z_\alpha^* \partial^\mu z^\alpha - \frac{2g}{N} (z_\alpha^* \partial^\mu z_\alpha) (z_\beta \partial_\mu z_\beta^*) - \lambda \left[z_\alpha^* z_\alpha - \frac{N}{2g} \right]. \quad (1)$$

The canonical momenta are

$$\begin{aligned} \Pi_\lambda &= \frac{\partial \mathcal{L}}{\partial \lambda} = 0, \\ \Pi_\alpha &= \frac{\partial \mathcal{L}}{\partial \dot{z}_\alpha} = M_{\alpha\beta} \dot{z}_\beta^*, \\ \Pi_\alpha^* &= \frac{\partial \mathcal{L}}{\partial \dot{z}_\alpha^*} = M_{\alpha\beta}^* \dot{z}_\beta^*, \end{aligned} \quad (2)$$

where

$$M_{\alpha\beta} = \delta_{\alpha\beta} - \frac{2g}{N} z_\alpha^* z_\beta \quad (3)$$

is a noninvertible matrix. The primary constraints of the theory are, therefore, given by

$$\begin{aligned} T_1 &= \Pi_\lambda \approx 0, \\ T_2 &= \Pi_\alpha z_\alpha + \Pi_\alpha^* z_\alpha^* \approx 0, \\ T_3 &= \Pi_\alpha z_\alpha - \Pi_\alpha^* z_\alpha^* \approx 0. \end{aligned} \quad (4)$$

The canonical Hamiltonian is

$$H_C = \int \left[\Pi_\alpha^* \Pi_\alpha - \partial_i z_\alpha^* \partial^i z^\alpha + \frac{2g}{N} (z_\alpha^* \partial^i z_\alpha) (z_\beta \partial_i z_\beta^*) + \lambda \left[z_\alpha^* z_\alpha - \frac{N}{2g} \right] \right]. \quad (5)$$

Time-conserving the primary constraints leads to the familiar CP^{N-1} model constraint

$$T_4 = |z|^2 - \frac{N}{2g} \approx 0 \quad (6)$$

and fixes λ in (5) as

$$\lambda = \frac{2g}{N} \left[\Pi_\alpha^* \Pi_\alpha + \partial_i z_\alpha^* \partial^i z^\alpha - \frac{4g}{N} (z_\alpha^* \partial^i z_\alpha) (z_\beta \partial_i z_\beta^*) \right]. \quad (7)$$

No further constraints are generated by this iterative scheme. Since the T_1 constraint does not involve any dynamical variable, we henceforth ignore it. Moreover inserting the value of λ from (7) in (5) we find

$$H_C = \int \left[(z_\alpha^* z_\alpha) (\Pi_\alpha^* \Pi_\alpha) - 2 \partial_i z_\alpha^* \partial^i z^\alpha \left[1 - \frac{g}{N} |z|^2 \right] - \frac{2g}{N} (z_\alpha^* \partial^i z_\alpha) (z_\beta \partial_i z_\beta^*) \left[\frac{4g}{N} z_\alpha^* z_\alpha - 3 \right] \right] \quad (8)$$

which, along with the constraints T_2, T_3, T_4 , are the basic inputs. The Poisson algebra of constraints

$$\{T_2, T_4\} = 2|z|^2 \delta(\mathbf{x} - \mathbf{y}), \quad (9)$$

with all other brackets being zero, reveals that T_2 and T_4 are second-class constraints while T_3 is first class. We now convert the second-class constraints into first class by following the systematic procedure of Ref. [4].

The Poisson brackets (PB) among the second-class constraints is compactly expressed as

$$\begin{aligned} \Delta_{ij}(x, y) &= \{\Theta_i(x), \Theta_j(y)\} \\ &= -2\epsilon_{ij} |z|^2 \delta(\mathbf{x} - \mathbf{y}) \\ &\quad (i, j = 1, 2) \quad (\epsilon^{12} = -\epsilon_{12} = 1), \end{aligned} \quad (10)$$

where we have made a change of notation,

$$\Theta_1(x) = T_2(x), \quad \Theta_2(x) = T_4(x). \quad (11)$$

The first-class constraints Θ'_i are then given by

$$\Theta'_i(z_\alpha, \Pi_\alpha, \phi^i) = \sum_{n=0}^{\infty} \Theta_i^{(n)}, \quad \Theta_i^{(n)} \sim (\phi)^n \quad (12)$$

subject to the boundary condition

$$\Theta_i^{(0)} = \Theta'_i(z_\alpha, \Pi_\alpha, 0) = \Theta_i, \quad (13)$$

where ϕ^i are the new dynamical variables in the extended phase space $(z_\alpha, \Pi_\alpha) \oplus (\phi^i)$ with the basic Poisson algebra [5]

$$\{\phi^i(x), \phi^j(y)\} = \omega^{ij}(x, y) \quad (14)$$

and ω is an antisymmetric invertible matrix,

$$\omega^{ij}(x, y) = -\omega^{ji}(y, x). \quad (15)$$

After (13), the next term in the series (12) is

$$\Theta_1^{(1)}(x) = \int dy X_{ij}(x, y) \phi^j(y), \quad (16)$$

where

$$\int dz' [X_{ij}(x, z) \omega^{jk}(z, z') X_{kl}(z', y)] = -\Delta_{il}(x, y) \quad (17)$$

with $\Delta_{il}(x, y)$ defined in (10).

An intelligent choice for $\omega^{ij}(x, y)$ and $X_{ij}(x, y)$, which considerably simplifies the algebra, satisfying (15) and (17) is

$$\begin{aligned} \omega^{ij}(x, y) &= 2\epsilon^{ij} \delta(\mathbf{x} - \mathbf{y}), \\ X_{ij}(x, y) &= \begin{bmatrix} 1 & 0 \\ 0 & -|z|^2 \end{bmatrix} \delta(\mathbf{x} - \mathbf{y}). \end{aligned} \quad (18)$$

Consequently,

$$\Theta_1^{(1)} = \phi^1, \quad \Theta_2^{(1)} = -|z|^2 \phi^2. \quad (19)$$

The other terms ($n > 1$) in the series (12), which are obtained by a recursion relation, vanish. Hence the first-class constraints are

$$\Theta'_1 = \Theta_1 + \phi^1, \quad \Theta'_2 = \Theta_2 - |z|^2 \phi^2, \quad (20)$$

which are strongly involutive,

$$\{\Theta'_i(x), \Theta'_j(y)\} = 0 \quad (21) \quad \text{subject to the initial condition}$$

Having found the first-class constraints we now compute the corresponding first-class Hamiltonian. This is given by

$$H'(z^\alpha, \Pi_\alpha, \phi^i) = \sum_{n=0}^{\infty} H'^{(n)}, \quad H'^{(n)} \sim (\phi)^n \quad (22)$$

$$H'^{(0)} = H'(z^\alpha, \Pi_\alpha, 0) = H_C \quad (23)$$

The general expression for $H'^{(n)}$ is given in Ref. [4]:

$$H'^{(n+1)} = -\frac{1}{n+1} \int dx dy dz [\phi^i(x) \omega_{ij}(x, y) X^{jk}(y, z) G_k^{(n)}(z)] \quad (n \geq 0), \quad (24)$$

where $\omega_{ij}(x, y)$ and $X^{jk}(y, z)$ are the inverse matrices of $\omega^{ij}(x, y)$ and $X_{jk}(y, z)$ respectively, defined in (18). The generating functional $G_k^{(n)}$ has a very simple form:

$$G_k^{(0)} = \{\Theta_k, H_C\}, \quad G_k^{(n)} = \{\Theta_k^{(1)}, H'^{(n-1)}\}_{(z_\alpha, \Pi_\alpha)} + \{\Theta_k, H'^{(n)}\}_{(z_\alpha, \Pi_\alpha)} \quad (n \geq 1), \quad (25)$$

which is a consequence of the judicious choice (18) so that the series (13) comprises only two terms Θ_i and $\Theta_i^{(1)}$. The remarkable algebraic simplification achieved in (25) can be appreciated by looking at the general structure for $G_k^{(n)}$ given in Eq. (2.54) of [4]. The symbol $\{\cdot, \cdot\}_{(z_\alpha, \Pi_\alpha)}$ appearing in (25) means that the relevant PB's have to be computed with respect to those variables. Using (18)–(25) all the terms in the series (24) may be evaluated. Interestingly, in contrast with (12), it turns out to be an infinite series. We find, however, that a remarkable sequence of cancellations occurs leading to the result

$$H' = H_C - \frac{g}{N} \int dx \phi^2 |z|^2 \Theta_2 + \frac{g}{2N} \int dx \phi^2 \phi^2 (|z|^2)^2 + \sum_{p=1}^{\infty} H^{(p)}, \quad (26)$$

where

$$H^{(p)} = \int dx_1 dx_2 \cdots dx_p \times \left[\frac{(-1)^p}{p!} \frac{1}{2} \left[\frac{\phi^1}{|z|^2} \right] (x_1) \left\{ \Theta_2(x_1), \frac{1}{2} \left[\frac{\phi^1}{|z|^2} \right] (x_2) \left\{ \Theta_2(x_2), \dots, \frac{1}{2} \left[\frac{\phi^1}{|z|^2} \right] (x_p) \left\{ \Theta_2(x_p), H_0 \right\} \right\} \right\} \right]_{p\text{-fold}} \quad (27)$$

and

$$H_0 = H_C - \frac{2g}{N} \int dx |z|^2 \Pi^* \Pi \quad (28)$$

is a function of z_α fields only. A compact way to express (27) is to use the functional Schrödinger representation $[\Pi_\alpha \rightarrow (-) \delta / \delta z_\alpha]$ so that

$$H^{(p)} = \frac{1}{p!} \int dx_1 \cdots dx_p \left[\frac{\phi^1}{2|z|^2} \left\{ z_\alpha \frac{\bar{\delta}_L}{\delta z_\alpha} + z_\alpha^* \frac{\bar{\delta}_L}{\delta z_\alpha^*} \right\} \right]^p H_0, \quad (29)$$

where $\bar{\delta}_L$ is the left derivative. Combining (29) with (26) yields the final Hamiltonian

$$H' = \frac{2g}{N} \int |z|^2 \Pi^2 - \frac{g}{N} \int \phi^2 |z|^2 \Theta_2 + \frac{g}{2N} \int \phi^2 \phi^2 (|z|^2)^2 + \int \exp \left[\int \frac{\phi^1}{2|z|^2} \left\{ z_\alpha \frac{\bar{\delta}_L}{\delta z_\alpha} + z_\alpha^* \frac{\bar{\delta}_L}{\delta z_\alpha^*} \right\} \right] H_0 \quad (30)$$

which is strongly involutive with the constraints Θ'_i :

$$\{H', \Theta'_i\} = 0. \quad (31)$$

We have thus converted the second-class system (with constraints Θ_1, Θ_2 and Hamiltonian H_C) to first class (with constraints Θ'_1, Θ'_2 , and Hamiltonian H').

Now we have to include the original first-class constraint T_3 (4) into our analysis. It is simple to see that the first-class nature of T_3 is preserved with respect to the new constraints Θ'_1, Θ'_2 :

$$\{T_3, \Theta'_i\} = 0 \quad (32)$$

which is a consequence of the fact that the new constraints Θ'_i differ from the old one Θ_i by fields (ϕ^1, ϕ^2) which have vanishing PB's with T_3 . It is also found that new Hamiltonian H' is in involution with T_3 . This is not obvious, but can be verified by an explicit calculation:

$$\{H', T_3\} = -\frac{8g}{N} \partial^i \left[\left(1 + \frac{\phi^1}{|z|^2} \right) (z^* \bar{\partial}_i z) \Theta'_1 \right]. \quad (33)$$

This completes the operatorial conversion of the original second-class theory to first class. Making a change of notation $T_3 \rightarrow \Theta'_3$, we can combine (21) and (32) as well as (31) and (33), to write the complete involution algebra as

$$\{\Theta'_\alpha, \Theta'_\beta\} = 0, \quad \alpha, \beta = 1, 2, 3, \quad (34)$$

$$\{H', \Theta'_\alpha\} = \int dy V_\alpha^\beta(x, y) \Theta'_\beta(y), \quad (35)$$

where

$$V_\alpha^\beta = - \left[\frac{8g}{N} \left[1 + \frac{\phi^1}{|z|^2} \right] (z^* \vec{\partial}_i z) \right] (y) \delta_{\beta 1} \delta_{\alpha 3} \partial_x^i \delta(\mathbf{x} - \mathbf{y}). \quad (36)$$

The above equations clearly illuminate the first-class nature of the system.

We next construct the BRST invariant Hamiltonian H_{BRST} and charge Q , which are given by

$$H_{\text{BRST}} = H' + \int dx P^3(x) V_3^1(x, y) \bar{C}_1(y), \quad (37)$$

$$Q = \int dx [C^\alpha(x) \Theta'_\alpha(x) + p_\alpha(x) P^\alpha(x)], \quad \alpha = 1, 2, 3, \quad (38)$$

where $(C^\alpha, \bar{P}_\beta)$ and $(P^\alpha, \bar{C}_\beta)$ form a pair of canonical ghost (antighost) having the opposite Grassmann parity as Θ'_α ,

$$\{C^\alpha(x), \bar{P}_\beta(y)\} = \{P^\alpha(x), \bar{C}_\beta(y)\} = \delta^\alpha_\beta \delta(\mathbf{x} - \mathbf{y}) \quad (39)$$

while (p_α, q^β) is a canonical multiplier set with the same Grassmann parity as Θ'_α ,

$$\{q^\alpha(x), p_\beta(y)\} = \delta^\alpha_\beta \delta(\mathbf{x} - \mathbf{y}). \quad (40)$$

Finally, the physical Hilbert space is defined by

$$Q|\text{phys}\rangle = 0, \quad |\text{phys}\rangle \neq Q|\cdots\rangle. \quad (41)$$

This completes the operator formation of the model. We now turn our attention to the partition function. Let us first define the gauge fermion operator ψ given in Ref. [3],

$$Z = \int [Dz_\alpha D\Pi_\alpha Dz_\alpha^* D\Pi_\alpha^* D\phi D\Pi_\phi] \delta(\Theta_1) \delta(\Theta_2) \delta(z_1 + z_1^*) \delta(\phi) \delta(\Pi_\phi) \delta(\Theta_3) \det|z_1| e^{iS} \quad (49)$$

with

$$S = \int (\Pi_\alpha \dot{z}^\alpha + \Pi_\alpha^* \dot{z}_\alpha^* + \Pi_\phi \dot{\phi} - H_C). \quad (50)$$

We next perform the momentum integrations. The Π_ϕ integral is trivial. The δ functions involving the original constraints are expressed by their corresponding Fourier transforms. Then the integrals over Π_α, Π_α^* are done. This yields, for the action,

$$S = \partial^\mu z_\alpha^* \partial_\mu z_\alpha - \frac{2g}{N} (z^* \partial^i z) (z \partial_i z^*) + \frac{N}{2g} \xi^2 + 2\xi \bar{z} z^* + \frac{N}{2g} \eta^2, \quad (51)$$

where ξ and η are the Fourier variables. The integral over η is a trivial Gaussian. Finally the ξ integration is done to yield

$$\psi = \int dx [\bar{P}_\alpha q^\alpha + \bar{C}_\alpha \chi^\alpha], \quad (42)$$

where χ_α is the Hermitian gauge-fixing function with the identical Grassmann parity as Θ_α and satisfies

$$\det|\{\chi_\alpha, \Theta'_\beta\}| \neq 0. \quad (43)$$

The complete unitarizing Hamiltonian H_U is now defined by

$$H_U = H_{\text{BRST}} + \{\psi, Q\}. \quad (44)$$

We next rename the variables ϕ^1 and ϕ^2 as

$$\phi^1 \rightarrow 2\phi, \quad \phi^2 \rightarrow \Pi_\phi, \quad (45)$$

so that (Π_ϕ, ϕ) may be considered as a canonically conjugate pair by virtue of (14) and (18). Then the partition function Z is given by

$$Z = \int [d\mu] e^{iS}, \quad (46)$$

where

$$S = \int [\Pi_\alpha \dot{z}_\alpha + \Pi_\alpha^* \dot{z}_\alpha^* + \Pi_\phi \dot{\phi} + C^\alpha \dot{\bar{P}}_\alpha + P^\alpha \dot{\bar{C}}_\alpha + p_\alpha \dot{q}^\alpha - H_U], \quad (47)$$

where the measure $[d\mu]$ includes all the variables appearing in the action. Different choices for the gauge function ψ can be done to explicitly evaluate the partition function. The final result for Z is, however, gauge independent by the Fradkin-Vilkovisky theorem [15,16].

Before concluding this section we shall work out Z in the “unitary gauge” [3,5], which amounts to choosing two of the gauge conditions to be the original second-class constraints:

$$\chi_1 = \Theta_1, \quad \chi_2 = \Theta_2, \quad \chi_3 = z_1 + z_1^*. \quad (48)$$

Making the change of variables $\chi_\alpha \rightarrow \chi_\alpha/\beta$, $p_\alpha \rightarrow \beta p_\alpha$, $\bar{C}_\alpha \rightarrow \beta \bar{C}_\alpha$ whose (super) Jacobian is unity, and then taking the limit $\beta \rightarrow 0$ [16], we obtain

$$S = \partial^\mu z_\alpha^* \partial_\mu z_\alpha - \frac{2g}{N} (z^* \partial^\mu z) (z \partial_\mu z^*) \quad (52)$$

while the measure is

$$[d\mu] = \delta(z_1 + z_1^*) \delta \left[|z|^2 - \frac{N}{2g} \right] \det|z_1| Dz_\alpha D\bar{z}_\alpha^*. \quad (53)$$

The expression for the original (classical) action (1) is seen to be reproduced. The term involving the Lagrange multiplier λ in (1) is manifested here through the δ function $\delta(|z|^2 - N/2g)$ appearing in the measure (49). Other forms of action with corresponding measures can be obtained by other choices of gauges.

III. CONCLUSION

We have systematically applied the generalized canonical formalism [3,4] to quantize the CP^{N-1} model, which

is an example of a second-class theory. The quantization of second-class systems, it may be recalled, is usually done by the method of Dirac [1]. In this particular case, however, the Dirac brackets are known [5] to be extremely complicated. They are field dependent and have a non-polynomial structure. Consequently transition to the quantum theory is plagued with severe operator ordering ambiguities. All these problems are bypassed here since the brackets are canonical. Moreover by converting the original second-class system to first class in an extended phase space, the quantization program simplifies since there already exists a well-established generalized canonical scheme [15,16] for quantizing such systems.

It is worthwhile to point out the difference of our results with other (non-Dirac) approaches, notably those presented in Refs. [6,7]. In these papers the pair of second-class constraints is interpreted as a combination of a first-class constraint and a gauge-fixing constraint. The Hamiltonian, consequently, gets modified. In fact, it turns out to be *nonlocal*. Our involutive Hamiltonian, on the contrary, is local. Moreover there is an arbitrariness in the decomposition of the second-class constraints into a first-class constraint and a gauge constraint, so that the corresponding Hamiltonian is not unique. The involutive Hamiltonian in our treatment is determined uniquely once the matrices ω_{ij} and X^{ij} (18) have been chosen. Although there is a “natural arbitrariness” in the choice of these matrices, these correspond to canonical transformation [4] in the extended phase space. Thus two involutive Hamiltonians resulting from a different choice of matrices ω_{ij}, X^{ij} are canonically equivalent.

The general philosophy of Batalin and Fradkin [3] and

Batalin and Tyutin [4] of converting second-class systems into first class ones has been used previously to discuss the quantization of the Proca model [10] and the chiral Schwinger model [9,10]. However these analyses are unsystematic and could be carried through principally because the algebra of constraints was very simple. In the present example where this algebra becomes field dependent, such an approach would be untenable. The construction of the involutive Hamiltonian drives home this point. This Hamiltonian comprises an infinite number of terms and only a systematic series of cancellations (whose origin is contained in the intelligent choice of ω^{ij} and X_{ij}) could enable us to express it as a closed (exponential) form.

Finally, we would like to make a connection of our result to those obtained previously by us [8] concerning the $O(N)$ -invariant nonlinear σ model. Following the BT prescription we had shown that there the partition function in the unitary gauge had the form

$$Z = \int dn^a \delta(n^2 - 1) \exp \left[i \int \frac{1}{4} \partial^\mu n^a \partial_\mu n^a \right], \quad (54)$$

where n^a are the σ model fields constrained by the δ function. For the special case of the $O(3)$ σ model and CP^1 model it can be seen from (49) and (53) that the action in two cases becomes identical, on using the Hopf map,

$$n^a = z^* \sigma^a z \quad (\sigma \text{ are Pauli matrices}) \quad (55)$$

but the measure does not agree. Contrary to the $O(3)$ example, the Faddeev-Popov determinant (50) for the CP^1 model is found to be nontrivial.

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