Thermalization of boson propagators in finite-temperature field theory

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Because photons and leptons do not equilibrate in a short-lived quark-gluon plasma they provide important signals of the plasma. Radiative corrections to their propagators are quite different. The propagator for the photon (and other bosons) develops a thermal form which is correct to lowest order in the fine-structure constant α but to all orders in the QCD coupling α_s. By contrast, conservation of fermion number prevents the propagator for electrons, muons, or very heavy quarks from thermalizing at all.

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I. INTRODUCTION

In perfect thermal equilibrium every species of particle is present in the ensemble: photons, neutrinos, electrons, muons, as well as strongly interacting particles. However, in systems such as stellar interiors or ultrarelativistic heavy-ion collisions not all species will be present. A species cannot be equilibrated if its mean free path is large compared to the system size or if its collision time is large compared to the system lifetime. Both conditions depend on how strongly that species is coupled to the other particles.

Denote the set of particles which are equilibrated by Ω. For example, in a QCD plasma above the deconfinement phase transition, the set Ω consists of quarks and gluons but does not contain leptons, photons, W's, Z's, or Higgs particles. Finite-temperature propagators contain a trace over the set Ω:

$$\mathrm{Tr}_\Omega\{T[\phi(x)\phi(y)]\exp(-\beta H)\}\ . \tag{1}$$

Even when the particles associated with the field ϕ are not in the equilibrated set Ω, this propagator is temperature dependent when computed beyond lowest order. Kinetic theory cannot predict the thermal structure of this propagator. It deals with real, on-shell particles in Fock space. It does not apply to intermediate states and especially not to virtual particles since they are a mathematical figment of Feynman-Dyson perturbation theory, which conserves energy in intermediate states at the expense of relaxing the mass-shell condition $E^2-\mathbf{p}^2=m^2$. (By contrast, Rayleigh-Schrödinger perturbation theory gives the same numerical results but is organized so that particles retain their physical masses in intermediate states but violate energy conservation.) Since the form of the propagators for unequilibrated particles (e.g., leptons, photons, Higgs particles) is not amenable to kinetic theory, it must be calculated. That is the purpose of this paper.

II. PROPAGATORS IN THERMAL FIELD THEORY

A. Feynman rules for equilibrated particles

The Feynman rules for finite-temperature calculations in real time require doubling the degrees of freedom [1–5]. Associated with each physical field is an auxiliary field. All propagators become 2×2 matrices in internal space. The free quark propagator is

$$S^{ab}(p)=V(p_0)\begin{bmatrix} S(p) & 0 \\ 0 & S(p)^* \end{bmatrix}V(p_0)\ , \tag{2}$$

where $S(p)=(\not{p}+m)/(p^2-m^2+i\epsilon)$ and S^* denotes conjugation of $i\epsilon$ but not of the Dirac matrices. The temperature dependence enters only through the matrix V:

$$V(p_0)=\begin{bmatrix} \sqrt{1-n_F} & -\epsilon(p_0)\sqrt{n_F} \\ \epsilon(p_0)\sqrt{n_F} & \sqrt{1-n_F} \end{bmatrix}\ , \tag{3}$$

where $n_F=1/[\exp(\beta|p_0|)+1]$ is the Fermi-Dirac function. The free gluon propagator in the Feynman gauge is

$$G^{ab}_{\mu\nu}(q)=-g_{\mu\nu}U(q_0)\begin{bmatrix} \Delta(q) & 0 \\ 0 & -\Delta^*(q) \end{bmatrix}U(q_0)\ , \tag{4}$$

where $\Delta(q)=1/(q^2+i\epsilon)$, the boson matrix is

$$U(q_0)=\begin{bmatrix} \sqrt{1+n_B} & \sqrt{n_B} \\ \sqrt{n_B} & \sqrt{1+n_B} \end{bmatrix}\ , \tag{5}$$

and $n_B=1/[\exp(\beta|q_0|)-1]$ is the Bose-Einstein function. It is a well-known feature of thermal field theories [1,5–7] that all the QCD corrections only change the form of the functions $S(p)$ and $\Delta(q)$ in the quark and gluon propagators. The matrices U and V are not changed. This property will be referred to as thermalization of the propagators.

B. Propagators for unequilibrated bosons

As a simple example let ϕ be a real, scalar field with a very weak coupling to the quarks:

$$\mathcal{L}_I=\lambda\bar{\psi}\psi\phi\ , \tag{6}$$

where $\lambda\ll1$. We ignore $SU(2)\times U(1)$ symmetry but will nevertheless refer to ϕ as a Higgs field. In a short-lived quark-gluon plasma the Higgs collision rate is too small for them to equilibrate; therefore Higgs scalars are not included in the trace over Ω. Consequently the free propagator has no thermal matrix U as in (4). It is

$$D^{ab}(k)=\begin{bmatrix}\Delta(k) & 0\\ 0 & -\Delta(k)^*\end{bmatrix}, \tag{7}$$

where $\Delta(k)=1/(k^2-m^2+i\epsilon)$. Note that since there has to be an auxiliary field, there is a nonzero D^{22} component. In lowest order, the vanishing of D^{12} and D^{21} decouple the Higgs field from thermal effects. However in higher orders thermal effects will emerge. The one-loop self-energy shown in Fig. 1 is temperature dependent because the quarks are thermalized:

$$\Pi^{ab}(k)=-i\lambda^a\lambda^b\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[S^{ab}(p)S^{ba}(p-k)]\ , \tag{8}$$

where the trace is over Dirac indices and $\lambda^a=\pm\lambda$ depending upon whether $a=1$ or 2. A remarkable thing happens when this is calculated. The result has the properties [1,5–7]

$$\Pi^{11}=-(\Pi^{22})^*\ , \tag{9}$$

$$\Pi^{12}=\Pi^{21}=-i\frac{\mathrm{Im}\Pi^{11}}{\cosh(k^0/2T)}\ . \tag{10}$$

Consequently it can be written in terms of one complex function Π as

$$\Pi^{ab}(k)=U^{-1}(k_0)\begin{bmatrix}\Pi & 0\\ 0 & -\Pi^*\end{bmatrix}U^{-1}(k_0) \tag{11}$$

with U the same matrix as (5). A one-gluon correction to the Higgs self-energy is shown in Fig. 2(a). The same proof [1,5–7] again guarantees that it is of the form (11) with a more complicated function Π. In fact, to first order in λ^2 but to all orders in the QCD coupling, α_s, the general self-energy shown in Fig. 2(b) will have the structure (11). However (11) is not true for higher-order corrections in the Higgs coupling λ. For example, the one-Higgs-boson correction shown in Fig. 3 will not satisfy (10) because the bare Higgs-boson propagator (7) is nonthermal.

Next we must see what happens when the approximate self-energy (11) and the bare propagator (7) are combined to form the full scalar propagator. The Schwinger-Dyson equation is

$$\begin{aligned}[D'^{-1}]^{ab}&=[D^{-1}]^{ab}-\Pi^{ab}\\ &=\begin{bmatrix}k^2-m^2+i\epsilon & 0\\ 0 & -k^2+m^2+i\epsilon\end{bmatrix}\\ &\quad -U^{-1}\begin{bmatrix}\Pi & 0\\ 0 & -\Pi^*\end{bmatrix}U^{-1}\ .\end{aligned} \tag{12}$$

The explicit $i\epsilon$ is now superfluous because $\mathrm{Im}\Pi<0$ [1,6].

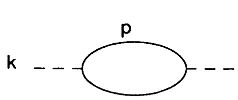

FIG. 1. Higgs-boson self-energy due to thermal quark loop.

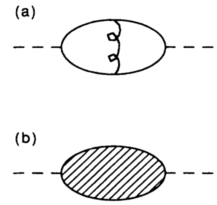

FIG. 2. Higgs-boson self-energy with (a) one-gluon correction and (b) all QCD corrections.

The trivial identity

$$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}=U^{-1}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}U^{-1} \tag{13}$$

means that (12) can be written

$$[D'^{-1}]^{ab}=U^{-1}\begin{bmatrix}k^2-m^2-\Pi & 0\\ 0 & -k^2+m^2+\Pi^*\end{bmatrix}U^{-1}\ . \tag{14}$$

The "full" propagator is then

$$D'^{ab}=U\begin{bmatrix}(k^2-m^2-\Pi)^{-1} & 0\\ 0 & -(k^2-m^2-\Pi^*)^{-1}\end{bmatrix}U\ . \tag{15}$$

This result will be referred to as thermalization of the propagator. It is accurate to all orders of QCD but only to order λ^2 in the Higgs coupling.

The same argument works for the photon propagator in a quark-gluon plasma. There is no thermal population of real photons so the free photon propagator is temperature independent. In the Feynman gauge

$$D^{ab}_{\mu\nu}(k)=-g_{\mu\nu}\begin{bmatrix}(k^2+i\epsilon)^{-1} & 0\\ 0 & -(k^2-i\epsilon)^{-1}\end{bmatrix}\ . \tag{16}$$

The one-loop vacuum polarization shown in Fig. 4(a) is

$$\Pi^{ab}_{\mu\nu}(k)=-ie^2\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}[\Gamma^a_\mu S^{ab}(p)\Gamma^b_\nu S^{ba}(p-k)]\ , \tag{17}$$

where $\Gamma^a_\mu=\pm\gamma_\mu$ depending upon whether $a=1$ or 2. The result is again of the form

$$\Pi^{ab}_{\mu\nu}(k)=U^{-1}(k_0)\begin{bmatrix}\pi_{\mu\nu} & 0\\ 0 & -\pi^*_{\mu\nu}\end{bmatrix}U^{-1}(k_0)\ . \tag{18}$$

All the higher-order gluon and quark corrections, Fig. 4(b), maintain this structure. Because of electric current

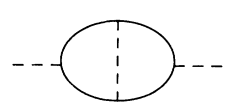

FIG. 3. A self-energy diagram that does not thermalize because of the nonthermal, bare Higgs-boson propagator.

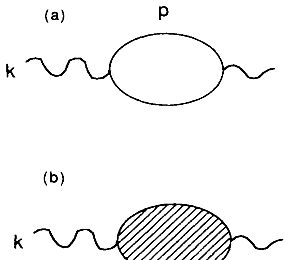

FIG. 4. Photon self-energy (a) due to thermal quark loop and (b) with all QCD corrections.

conservation, $k^\mu \pi_{\mu\nu}=0$. Therefore the vacuum polarization can be written as a sum of two tensors [1,8],

$$\pi_{\mu\nu}=P_{\mu\nu}\pi_T+Q_{\mu\nu}\pi_L \ , \tag{19}$$

one of which is transverse to the fluid velocity four-vector u^μ and one of which is longitudinal. These can be chosen as orthogonal ($PQ=0$) and idempotent ($PP=P$, $QQ=Q$). Their sum is $P_{\mu\nu}+Q_{\mu\nu}=-g_{\mu\nu}+k_\mu k_\nu/k^2$. Proceeding as before gives, for the full propagator,

$$D'^{ab}_{\mu\nu}=P_{\mu\nu}D'^{ab}_L+Q_{\mu\nu}D'^{ab}_T \ , \tag{20}$$

$$D'^{ab}_L=U\begin{bmatrix}(k^2-\pi_L)^{-1} & 0\\ 0 & -(k^2-\pi_L^*)^{-1}\end{bmatrix}U \ , \tag{21}$$

$$D'^{ab}_T=U\begin{bmatrix}(k^2-\pi_T)^{-1} & 0\\ 0 & -(k^2-\pi_T^*)^{-1}\end{bmatrix}U \ . \tag{22}$$

C. Propagators for unequilibrated fermions

The effective thermalization found above for boson propagators does not occur for fermion propagators. As a specific example, consider a very heavy quark, c or t, in a $T=200$ MeV plasma. With such a large mass M its collision rate will be too small for real c or t quarks to thermalize. Thus its free propagator is

$$S^{ab}(p)=\begin{bmatrix}S(p) & 0\\ 0 & S(p)^*\end{bmatrix} \ , \tag{23}$$

where $S(p)=(\not p+M)/(p^2-M^2+i\epsilon)$. The lowest-order self-energy due to a gluon loop shown in Fig. 5 is

$$\Sigma^{ab}(p)=ie^2\int\frac{d^4p}{(2\pi)^4}\Gamma^a_\mu S^{ab}(p)\Gamma^b_\nu D^{ba,\mu\nu}(p-k) \ . \tag{24}$$

Because $S^{12}=S^{21}=0$ the off-diagonal elements of the self-energy also vanish: $\Sigma^{12}=\Sigma^{21}=0$. This is an automatic consequence of fermion-number conservation. Every self-energy diagram for an out-of-equilibrium charm quark, no matter how complicated, satisfies $\Sigma^{12}=\Sigma^{21}=0$.

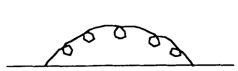

FIG. 5. Self-energy of an out-of-equilibrium heavy quark that will not be thermalized by the gluon correction.

This is true to all orders in all interactions and guarantees that the charm-quark self-energy does not have the thermal form

$$\Sigma^{ab}(p)\neq V^{-1}(p_0)\begin{bmatrix}\Sigma & 0\\ 0 & -\Sigma^*\end{bmatrix}V^{-1}(p_0) \ . \tag{25}$$

III. DISCUSSION

Physical interpretation. In a plasma of finite size and lifetime, thermalization of real particles and virtual particles is not synonymous. The Higgs boson and photon are examples of species that are not present in the heat bath (i.e., no real equilibration) but whose propagators are thermalized. The reason is that Higgs bosons and photons are evanescent; they continually disappear into equilibrated quark pairs and then reappear out of equilibrated quark pairs. The thermal nature of the quarks inevitably modifies the bosons. By contrast, unequilibrated fermions such as leptons and heavy quarks can never disappear. Consequently their nonthermal origins are not erased. The real distinction is not Bose versus Fermi but the existence of a conservation law. It is easy to construct a model in which a spinless boson carries a conserved quantum number. If there is no thermal population of these bosons, radiative corrections will fail to thermalize their propagator just as in Sec. II C.

Applications. One use for the thermalized boson propagator is in dilepton production. There the finite-temperature QCD effects on the photon propagator will produce a characteristic resonance in the dilepton mass [9,10]. In such cases it is kinematically impossible to be near the mass shell since $k^2>4m_l^2$. A much more troublesome situation occurs when photons or Higgs bosons cannot be kept away from their $T=0$ mass shell. Because the bare propagators (7) and (16) are diagonal but the self-energies (8) and (17) are not, higher-order insertions of the self-energies give nonanalytic terms involving products of $\Delta(k)$ with $\Delta(k)^*$. This is illustrated in the Appendix. The obvious escape is to sum up the insertions, which leads to the propagators (15) and (20) that cause no pinch singularities. However this reordering amounts to replacing the nonthermal, bare Higgs-boson propagator in Fig. 3 by a thermal, bare propagator.

Imperfect thermalization of quark and gluon propagators. In a plasma containing a thermal population of real u and d quarks and of real gluons, their propagators will be imperfectly thermalized. The problem is that the bare propagator for heavy quarks (23) is nonthermal. Consequently the lowest-order gluon self-energy due to one heavy-quark loop will be the same as at $T=0$:

$$\Pi^{ab}_{\mu\nu}(k)=\begin{bmatrix}\pi_{\mu\nu} & 0\\ 0 & -\pi^*_{\mu\nu}\end{bmatrix} \ . \tag{26}$$

This contribution cannot be discarded since its ultraviolet divergence contributes to the renormalization of α_s. For $k^2<4M^2$ (26) is real so that

$$\Pi^{ab}_{\mu\nu}(k)=U^{-1}(k_0)\begin{bmatrix}\pi_{\mu\nu} & 0\\ 0 & -\pi^*_{\mu\nu}\end{bmatrix}U^{-1}(k_0) \quad (k^2<4M^2) \ . \tag{27}$$

However, (26) cannot be written as (27) when $k^2 > 4M^2$. In a two-loop diagram, the momentum integrations over d^4k necessarily include $k^2 > 4M^2$. This will spoil the thermalization of the gluons and light quarks to two-loop order.

Finite size and lifetime of plasma. The kinematic effects of a finite-size L and finite-lifetime τ for the plasma can be incorporated as follows. Self-energies have the structure

$$\int_0^L dx^1 dx^2 dx^3 \int_0^\tau dx^0 \exp(-ip\cdot x) D(x) D(x) \; . \tag{28}$$

Expanding the propagators in a Fourier series on the finite interval makes the momenta integral multiples of $2\pi/L$ and the energies integral multiples of $2\pi/\tau$:

$$p^\mu = \left[\frac{2\pi}{\tau} n_0 \, , \frac{2\pi}{L} n_1, \frac{2\pi}{L} n_2, \frac{2\pi}{L} n_3 \right] \; . \tag{29}$$

The consequences of discretization are unimportant at high energy and momenta. For $L = \tau = 10$ fm, this requires that energies and momenta be much larger than 60 MeV. Of course, the same kinematics applies to thermal effects of all particles: light quarks, gluons, as well as mesons and baryons.

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APPENDIX: PINCH SINGULARITIES

The text has investigated how boson propagators develop a thermal structure if one sums up the self-energy insertions. This appendix shows the problems that occur if the resummation is not performed. Consider the quark self-energy graph shown in Fig. 6. Its value is

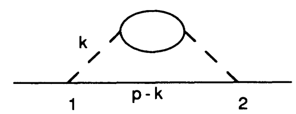

FIG. 6. Two loop contribution to quark self-energy that has pinch singularities because of the nonthermal, bare Higgs-boson propagator.

$$\lambda^4 \int \frac{d^4k}{(2\pi)^4} S^{12}(p-k) D^{1a}(k) \Pi^{ab}(k) D^{b2}(k) \; . \tag{A1}$$

The problem is that the bare Higgs-boson propagator (7) is diagonal, but the self-energy (8) is not. Consequently

$$\begin{aligned} D^{1a}(k)\Pi^{ab}(k)D^{b2}(k) &= -\Delta(k)\Pi^{12}(k)\Delta(k)^* \\ &= \Delta(k)[\Pi(k)-\Pi(k)^*]\Delta(k)^*\sqrt{n_B(1+n_B)} \; . \end{aligned} \tag{A2}$$

The product $\Delta\Delta^*$ results in a pinch singularity [1,2,5–7] in the complex k plane at $k^2 = m^2$. By contrast, a thermalized bare propagator would give

$$[\Delta(k)\Pi(k)\Delta(k) - \Delta(k)^*\Pi(k)^*\Delta(k)^*]\sqrt{n_B(1+n_B)} \; . \tag{A3}$$

This has no pinch singularities. It coincides with (A2) when Δ is real, as it is for $k^2 \neq m^2$. But at $k^2 = m^2$ the pinch singularity spoils analyticity and hence spoils causality. This problem only gets worse in higher order. For example, with two self-energy insertions the diagonal [e.g., (11)] part of the Higgs propagator,

$$\begin{aligned} D^{1a}(k)\Pi^{ab}(k)D^{bc}(k)\Pi^{cd}(k)D^{d1}(k) &= \Delta(k)\Pi^{11}(k)\Delta(k)\Pi^{11}(k)\Delta(k) \\ &\quad - \Delta(k)\Pi^{12}(k)\Delta(k)^*\Pi^{21}(k)\Delta(k) \; , \end{aligned} \tag{A4}$$

has pinch singularities.

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