Radiative decay and magnetic moment of neutrinos in matter

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Effects of a medium on the radiative decay and the magnetic moments of neutrinos are studied in a simple Feynman-diagram approach. It is shown that even though the radiative decay rate is dramatically increased in a medium, the magnetic moments, both diagonal and off diagonal, are not enhanced in the medium from their vacuum value due to the coherence condition of the medium effects. It is also shown that there cannot be any helicity flip in a medium due to the weak interactions alone, independently from the interaction with a magnetic field.

I. INTRODUCTION

The basic properties of neutrinos can be altered when they pass through a medium. Although the cross sections for their weak interactions with the medium are proportional to G_F^2, coherent effects in the medium are proportional to G_F and hence can be significant. The best known example is the Mikheyev, Smirnov, and Wolfenstein (MSW) effect,[1,2] which is a coherent effect of the medium that enhances (or suppresses) the effective masses (and consequently the effective mixing angles) of neutrinos.

This mechanism can, in principle, change other neutrino properties such as the radiative decay rate and the magnetic moments. The enchancement of the radiative decay rate in an electron-rich medium has been studied by D'Olivo, Nieves, and Pal[3] using a finite-temperature field-theoretical approach in which the internal electron lines are replaced by those modified by the medium. This replacement makes the Glashow-Iliopoulos-Maiani (GIM) mechanism, which suppresses the radiative decay in vacuum, inoperative for the matter contribution, leading to a dramatic enhancement of the decay rate in a medium. In this work we show that the result obtained in Ref. 3 can be derived in a simple and transparent approach using the usual Feynman diagrams.

The same approach has then been used to study possible changes of the effective diagonal and off-diagonal magnetic moments of the neutrinos in matter. This study is particularly important in view of a very interesting proposal to explain the solar-neutrino depletion that requires large diagonal[4] and/or transition[5] magnetic moments, of order $(10^{-11}-10^{-10})\ \mu_B$, where $\mu_B=e/2m_e$ is the Bohr magneton. Unfortunately, the value of the diagonal neutrino magnetic moment (for a Dirac neutrino) in the minimal extension of the standard model is extremely small,[6] in fact many orders of magnitude smaller than the required value for spin flip[4] and/or resonant spin-flavor flip[5] to take place in the Sun. Therefore it would be extremely interesting if the medium effects enhance the effective values of the magnetic moments as dramatically as the radiative decay rate in matter, say by 10 orders of magnitude. It is shown that, in contrast with the case of the radiative decay, the coherence condition severely restricts the enhancement of the effective magnetic moments of neutrinos. To the order in which the enhancement of the radiative decay rate is calculated, the magnetic moments are not enhanced at all. Although the enhancement in the next order may be nonvanishing, it is found to be even smaller than the vacuum value calculated in the simplest extension of the standard model,[6]

$$\mu(\nu_e)=\frac{3eG_F m_\nu}{8\sqrt{2}\pi^2}\simeq 3\times 10^{-19}\frac{m_\nu}{1\ \mathrm{eV}}\mu_B\ , \tag{1}$$

where m_ν is the mass of the electron neutrino.

These results are disappointing in light of the abovementioned proposal to explain the solar-neutrino depletion. Since the matter effects do not enhance the effective magnetic moments from their intrinsic vacuum value, our conclusion is that, in order for the magnetic moment scenario to succeed in explaining the solar-neutrino problem, the intrinsic vacuum value of the diagonal and/or off-diagonal magnetic moment itself must be as large as $(10^{-11}-10^{-10})\mu_B$. This does not imply, however, that matter effects are irrelevant; if the transition magnetic moments are indeed of the order $(10^{-11}-10^{-10})\mu_B$, a resonant spin-flavor flip[5] can occur in matter with effects as dramatic as in the case of the well-known MSW effect. However, it is necessary to invoke new physics well beyond the standard model to make the magnetic moments as large as $(10^{-11}-10^{-10})\mu_B$.[7]

We have also investigated the possibility whether the helicity can be flipped by the weak interactions when a neutrino is propagating through a medium, independently from the value of its magnetic and the presence of a magnetic field. It is shown that the order of magnitude of the ratio of the negative- and positive-helicity states of a neutrino, which is originally fixed by weak interactions, is not modified by the weak interactions with the medium.

II. ENHANCEMENT OF RADIATIVE DECAY IN A MEDIUM

When a neutrino propagates in a medium, its energy is changed by the coherent interactions with the particles of the medium; this fact can be expressed in terms of an index of refraction of the medium or in terms of a change in the effective mass of the neutrino. In order to preserve the coherence of the neutrino wave function, the additional potential energy, $V=V_C+V_N$, is given by the forward elastic scattering amplitude due to weak charged-current (V_C) and neutral-current (V_N) interactions with the particles of the medium.

The potential energy V_C for the electron neutrino can be calculated in a simple and transparent method[1,8] using the diagram shown in Fig. 1. The well known result is

$$V_C=\sqrt{2}G_F N_e \,,\qquad \Delta m^2_{\text{eff}}=2\sqrt{2}G_F E_\nu N_e \,. \tag{2}$$

For an appropriate value of the electron number density N_e, depending on the vacuum oscillation parameters (for two generations $\Delta m^2 \equiv m_2^2-m_1^2$ and $\sin\theta$, where θ is the mixing angle), two diagonal elements of the effective squared mass matrix in the flavor representation can be equal, leading to the well-known MSW resonance effect, which can explain the depletion of the observed flux of the solar neutrinos in a natural way for a wide range of values of the vacuum oscillation parameters.

In the original derivation of Eq. (2), the electrons in the medium were assumed to be nonrelativistic. This assumption is not necessary, as shown in the Appendix; a similar calculation was done in Ref. 9 in the framework of the finite-temperature field-theoretical approach.

Our study of the neutrino radiative decay in an electron-rich medium is based on a simple extension of

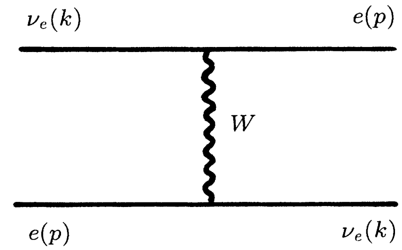

FIG. 1. Feynman diagram for coherent scattering of a neutrino off an electron.

the approach described above. We consider the process $\nu_2(k_2)\rightarrow\nu_1(k_1)+\gamma(q)$; the relevant diagrams are shown in Fig. 2. The decay amplitude is given by

$$M(k_1,k_2,q,p)=iJ_\mu(k_1,k_2,q,p)\epsilon^\mu(q)\mathcal{U}^*_{e1}\mathcal{U}_{e2} \,, \tag{3}$$

where $\mathcal{U}_{ei}$ are the elements of the mixing matrix in vacuum, defined by $\nu_\alpha=\mathcal{U}_{\alpha i}\nu_i$, with ν_α and ν_i, respectively, representing weak and mass eigenstates. For the two-generation case $\mathcal{U}^*_{e1}\mathcal{U}_{e2}=\sin\theta\cos\theta$. $\epsilon^\mu(q)$ is the polarization four-vector of the outgoing photon with four-momentum q, and p is the four-momentum of the electron; since we are calculating the coherent contribution of all the electrons of the medium, the initial and final electron four-momenta are equal.

The vertex function $J_\mu(k_1,k_2,q,p)$ is given by

$$\begin{aligned} J_\mu(k_1,k_2,q,p)=\frac{ieG_F}{\sqrt{2}}\Bigg[&\bar{u}_\nu(k_1)\gamma^\alpha(1+\gamma_5)u_e(p)\bar{u}_e(p)\frac{2p_\mu+\gamma_\mu\not{q}}{q^2+2p\cdot q}\gamma_\alpha(1+\gamma_5)u_\nu(k_2) \\ &+\bar{u}_\nu(k_1)\gamma^\alpha(1+\gamma_5)\frac{2p_\mu-\not{q}\gamma_\mu}{q^2-2p\cdot q}u_e(p)\bar{u}_e(p)\gamma_\alpha(1+\gamma_5)u_\nu(k_2)\Bigg] \,. \end{aligned} \tag{4}$$

After a Fierz transformation, Eq. (4) becomes

$$J_\mu(k_1,k_2,q,p)=\frac{-ieG_F}{\sqrt{2}}\bar{u}_\nu(k_1)\gamma^\alpha(1+\gamma_5)u_\nu(k_2)\bar{u}_e(p)\left[\frac{(2p_\mu+\gamma_\mu\not{q})\gamma_\alpha}{q^2+2p\cdot q}+\frac{\gamma_\alpha(2p_\mu-\not{q}\gamma_\mu)}{q^2-2p\cdot q}\right](1+\gamma_5)u_e(p) \,. \tag{5}$$

Since the external photon is on the mass shell, the invariant decay amplitude is given by

$$\begin{aligned} M&=\frac{eG_F\sin\theta\cos\theta}{2\sqrt{2}p\cdot q}\bar{u}_\nu(k_1)\gamma^\alpha(1+\gamma_5)u_\nu(k_2)\bar{u}_e(p)(\not{\epsilon}\not{q}\gamma_\alpha-\gamma_\alpha\not{q}\not{\epsilon})(1+\gamma_5)u_e(p) \\ &=\frac{eG_F\sin\theta\cos\theta}{\sqrt{2}p\cdot q}(q_\alpha\epsilon_\beta-\epsilon_\alpha q_\beta)\bar{u}_\nu(k_1)\gamma^\alpha(1+\gamma_5)u_\nu(k_2)\bar{u}_e(p)\gamma^\beta(1+\gamma_5)u_e(p) \,. \end{aligned} \tag{6}$$

In the rest frame of the medium we have

$$\bar{u}_e(p)\gamma^\beta(1+\gamma_5)u_e(p)\simeq\delta_{\beta0}u_e^\dagger(p)u_e(p)=\delta_{\beta0}N_e \,, \tag{7}$$

where N_e is the electron number density in the medium. Substituting Eq. (7) in Eq. (6), we obtain

$$M=\frac{eG_F N_e\sin\theta\cos\theta}{\sqrt{2}m_e q_0}(q_\alpha\epsilon_0-\epsilon_\alpha q_0)\times\bar{u}_\nu(k_1)\gamma^\alpha(1+\gamma_5)u_\nu(k_2) \,. \tag{8}$$

The amplitude in Eq. (8) satisfies (electromagnetic) gauge invariance. The decay rate $\Gamma_M(\nu_2\rightarrow\nu_1+\gamma)$ in an electron-rich medium can be calculated from Eq. (8) to be

$$\Gamma_M(\nu_2\rightarrow\nu_1+\gamma)=\frac{\alpha G_F^2 N_e^2}{2m_e^2}\left[m_2-\frac{m_1^2}{m_2}\right]\sin^2\theta\cos^2\theta\, F(v) \,, \tag{9}$$

where

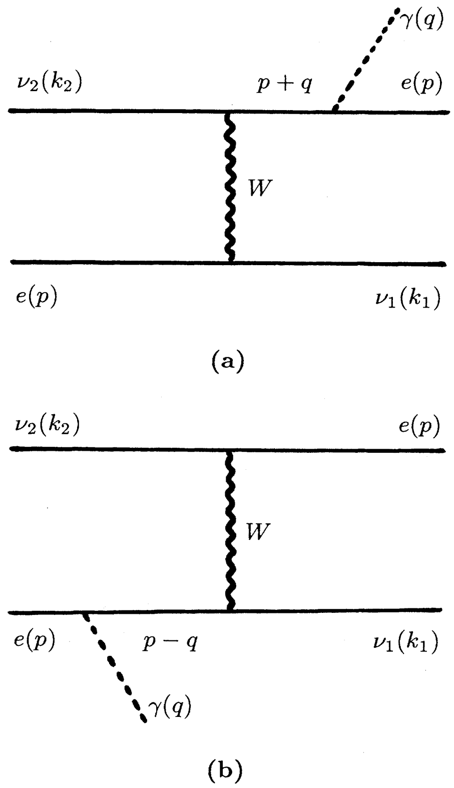

FIG. 2. Feynman diagrams for $\nu_2 \to \nu_1 + \gamma(q)$.

$$F(v) \equiv \sqrt{1-v^2}\left[\frac{2}{v}\ln\left[\frac{1+v}{1-v}\right] - 3 + \frac{m_1^2}{m_2^2}\right], \quad v \equiv \frac{|\mathbf{k}_2|}{E_2}. \tag{10}$$

Equation (9) is identical to the result obtained in Ref. 3 using the finite-temperature field-theoretical method. A comparison of the decay rate given by Eq. (9) (for $m_1 \ll m_2$) with the corresponding decay rate in a vacuum[10] (for the two-generation case)

$$\Gamma_V(\nu_2 \to \nu_1 + \gamma) = \frac{\alpha G_f^2}{2}\left[\frac{3}{32\pi^2}\right]^2 \times m_2^5 \sin^2\theta\cos^2\theta \left|\frac{m_\mu^2}{m_W^2} - \frac{m_e^2}{m_W^2}\right|^2 \tag{11}$$

yields a matter-induced enhancement factor given by

$$\frac{\Gamma_M}{\Gamma_V} \simeq 8.6\times 10^{23} F(v)\left[\frac{N_e}{10^{24}\ \mathrm{cm}^{-3}}\right]^2 \left[\frac{1\ \mathrm{eV}}{m_2}\right]^4. \tag{12}$$

Since $F(v)$ "is a very flat function of v, taking values between 1 and 1.55 for about the entire range of v,"[3] the enchancement factor in Eq. (12) seems enormous. However, $F(v)$ is very small for most physical neutrinos, except for the relic neutrinos. The neutrinos produced in the laboratory as well as those produced in the sun and supernovas are extremely relativistic. Therefore one obtains

$$F(v) \underset{v\to 1}{\longrightarrow} 4\frac{m_2}{E_2}. \tag{13}$$

For example $F(v) \sim 10^{-7}$ for $m_2 = 1$ eV and $E_2 = 10$ MeV. In spite of this, the decay rate is still enhanced significantly in the medium, although the enhanced decay rate is still too small to be of practical interest.

In deriving Eq. (9) we have used the vacuum values for the masses and the mixing angle of the neutrinos, but in a very dense medium it is necessary to use their effective values in the medium and also the effective mass of the photon.

III. MAGNETIC MOMENTS

The radiative decay rate of a neutrino in vacuum is characterized by an effective magnetic moment which happens to be highly suppressed due to the GIM mechanism. Since the enhancement of the radiative decay rate in a medium is due to the absence of the GIM mechanism, it is expected that the (diagonal) magnetic moment of a neutrino is also enhanced in the same medium to the order that we have considered in the previous section, i.e., $\sim G_F N_e$. In order to see if this enhancement is realized or not, we study the diagrams in Fig. 2 lead to an enhancement of the magnetic moment of a neutrino in the medium.

The effective magnetic moment of an electron neutrino in the medium can be calculated to the leading order from the diagrams in Fig. 2 with replacements of ν_2 and ν_1 by ν_e and $\epsilon^\mu(q)$ by an external electromagnetic field $A^\mu(q)$. The corresponding vertex function is still given by Eq. (5), where now $u_\nu(k_1)$ and $u_\nu(k_2)$ refer respectively to the final and initial states of the electron neutrino.

Since the contribution to the effective magnetic moment in the medium must be coherent with the neutrino propagation, it is necessary to take the limit $q^\mu \to 0$ in Eq. (5). Depending on how one approaches this limit, Eq. (5) yields different answers, because of its divergent nature. However, when $q^\mu = 0$, the internal electron lines with four-momenta $p+q$ in Fig. (2a) and $p-q$ in Fig. (2b) are on the mass shell; this means that, in the limit $q^\mu \to 0$, the physical effect of the process described in Fig. 2 is the same as that of Fig. 1, with a modification of the external electron lines by the electromagnetic field. That is, the electron number density N_e in Eq. (2) becomes, in principle, a function of the electromagnetic field.

Therefore, to the order in which we have considered the radiative decay, the magnetic moment is not enhanced in the medium although the radiative decay rate is dramatically enhanced. The difference is due to the requirement of coherence for the (diagonal) magnetic moment. Off-diagonal (transition) magnetic moments such as $\mu(\nu_e \to \nu_\mu)$ and $\mu(\nu_e \to \bar{\nu}_\mu)$, which may be generated in nonstandard models,[7] cannot be enhanced in the medium in the order of our interest, again due to the coherence condition.

In the next order of perturbation theory the external electrons in the diagrams of Fig. 2 can interact through a photon exchange and the resulting diagrams can contribute to the effective magnetic moment of the electron neutrino: since there is a loop, even in the limit $q^\mu \to 0$ the internal electron lines are off the mass shell [as in the diagrams which generate the magnetic moment of the electron neutrino in the simplest extension of the standard model, whose result is given in Eq. (1)]. The resulting amplitude is finite, but the contribution to the effective magnetic moment is at most, from dimensional arguments, of the order

$$\Delta\mu_{\rm eff} \sim \alpha \left[\frac{G_F N_e}{m_e} \right] \left[\frac{m_\nu}{m_e} \right] \mu_B$$

$$\simeq 10^{-27} \left[\frac{N_e}{10^{24}\ {\rm cm}^{-3}} \right] \left[\frac{m_\nu}{1\ {\rm eV}} \right] \mu_B \ , \tag{14}$$

which, in ordinary matter, is much smaller than the vacuum magnetic moment given in Eq. (1). In the supernova case, however, the matter contribution can be dominant over the vacuum magnetic moment.

IV. COHERENT NEUTRINO HELICITY FLIP IN MATTER

As we have seen in the previous section, the magnetic moment of a neutrino is not enhanced in matter, at least in the leading order in which the radiative decay rate is dramatically enhanced. Therefore, unless the vacuum value of the magnetic moment itself is of the order of $(10^{-11}-10^{-10})\mu_B$, spin flip and/or resonant spin-flavor flip due to the interaction with a magnetic field cannot take place in the sun. In this section we study whether the neutrino spin can be flipped by the interactions with a medium without considering the effect of a possible magnetic field.

The neutrinos produced by weak interactions propagate as a superposition of helicity eigenstates, the ratio of the positive- and negative-helicity components being of order m_ν/E_ν. This means that a relativistic neutrino is almost exactly left handed, with a small right-handed component proportional to m_ν/E_ν. Therefore, it is interesting to see if the neutrino helicity can be flipped by the coherent interactions with the medium and the ratio of the two helicity states is changed. To this purpose we consider the process described in Fig. 1 when the initial and final neutrino states are helicity eigenstates; the forward elastic scattering amplitude is given by

$$\mathcal{A}(\nu_e(h_i)\to\nu_e(h_f))$$

$$= \frac{G_F}{\sqrt{2}} \int d^3x \langle e(p,s_e)|\bar{e}(x)\gamma^\alpha(1+\gamma_5)e(x)|e(p,s_e)\rangle \times \langle \nu_e(k,h_f)|\bar{\nu}_e(x)\gamma_\alpha(1+\gamma_5)\nu_e(x)|\nu_e(k,h_i)\rangle \ , \tag{15}$$

where h_i and h_f are, respectively, the initial and final neutrino helicities and s_e is the electron polarization four-vector ($s_e \cdot p=0$, $s_e^2=-1$). Note that the spin of the electron (like its four-momentum) must be unchanged in the process because of the coherence requirement.

The electron matrix element is given by

$$\langle e(p,s_e)|\bar{e}(x)\gamma^\alpha(1+\gamma_5)e(x)|e(p,s_e)\rangle$$

$$= \bar{u}_e(p,s_e)\gamma^\alpha(1+\gamma_5)u_e(p,s_e)$$

$$= N_e {\rm Tr}\left[\frac{\not p+m_e}{2E_e}\frac{1-\gamma_5 \not s_e}{2}\gamma^\alpha(1+\gamma_5)\right]$$

$$= N_e \frac{p^\alpha - m_e s_e^\alpha}{E_e} \ . \tag{16}$$

The helicity-conserving neutrino matrix elements ($h_f=h_i$) are given by

$$\langle \nu_e(k,\pm)|\bar{\nu}_e(x)\gamma^\alpha(1+\gamma_5)\nu_e(x)|\nu_e(k,\pm)\rangle$$

$$= \bar{u}_\nu(k,\pm)\gamma^\alpha(1+\gamma_5)u_\nu(k,\pm)$$

$$= \frac{1}{\Omega}{\rm Tr}\left[\frac{\not k+m_\nu}{2E_\nu}\frac{1-\gamma_5 \not s_\nu(\pm)}{2}\gamma^\alpha(1+\gamma_5)\right]$$

$$= \frac{1}{\Omega}\frac{k^\alpha - m_\nu s_\nu^\alpha(\pm)}{E_\nu} \ , \tag{17}$$

where Ω is the normalization volume and $s_\nu(\pm)$ are the polarization four-vectors of the helicity eigenstates; for a relativistic neutrino

$$s_\nu^\alpha(\pm) = \left[\pm\frac{|\mathbf{k}|}{m_\nu}, \pm\frac{E_\nu}{m_\nu}\frac{\mathbf{k}}{|\mathbf{k}|}\right]$$

$$= \left[\pm\frac{E_\nu}{m_\nu}, \pm\frac{\mathbf{k}}{m_\nu}\right] + O\left[\frac{m_\nu^2}{E_\nu^2}\right] \ . \tag{18}$$

Here the helicity-conserving neutrino matrix elements are given by

$$\int d^3x \langle \nu_e(k,-)|\bar{\nu}_e(x)\gamma^\alpha(1+\gamma_5)\nu_e(x)|\nu_e(k,-)\rangle \simeq 2\left[1,\frac{\mathbf{k}}{E_\nu}\right] \tag{19a}$$

$$\int d^3x \langle \nu_e(k,+)|\bar{\nu}_e(x)\gamma^\alpha(1+\gamma_5)\nu_e(x)|\nu_e(k,+)\rangle = O\left[\frac{m_\nu^2}{E_\nu^2}\right] \ . \tag{19b}$$

In order to calculate the helicity-flip neutrino matrix elements we consider a neutrino propagating in the z direction with four-momentum $k^\alpha=(E_\nu,0,0,k)$. The Dirac spinors for the helicity eigenstates in the standard representation are given by

$$u_\nu(k,-)=\frac{1}{\sqrt{2E_\nu\Omega}}\begin{bmatrix}0\\ \sqrt{E_\nu+m_\nu}\\ 0\\ -\sqrt{E_\nu-m_\nu}\end{bmatrix},$$

$$u_\nu(k,+)=\frac{1}{\sqrt{2E_\nu\Omega}}\begin{bmatrix}\sqrt{E_\nu+m_\nu}\\ 0\\ \sqrt{E_\nu-m_\nu}\\ 0\end{bmatrix}. \tag{20}$$

Hence the helicity-flip neutrino matrix elements are

$$\int d^3x\,\langle \nu_e(k,\pm)|\bar{\nu}_e(x)\gamma^\alpha(1+\gamma_5)\nu_e(x)|\nu_e(k,\mp)\rangle = \frac{m_\nu}{E_\nu}(0,-1,\pm i,0)\ . \tag{21}$$

From Eqs. (15), (16), and (21) it is clear that in order to obtain a nonzero helicity-flip amplitude $\mathcal{A}(\nu_e(-)\to\nu_e(+))$ either $\mathbf{p}_e$ and/or $\mathbf{s}_e$ in Eq. (16) must have a nonvanishing component orthogonal to the neutrino direction of motion. Of course the net effect has to depend on a statistical average over all the electrons in the path of the neutrinos, so the requirement is actually that the medium has either a net momentum or a net spin perpendicular to the momentum of the neutrino. This requirement can be easily understood from the point of view of conservation of angular momentum. The neutrino states we are considering are eigenstates of J_z, the angular momentum in the z direction. In the individual scattering J_z has to be conserved, while the coherence of the process requires that the state of the electron remains unchanged. The only possibility for both of these requirements to be satisfied and for helicity flip to occur is that the electron is not an eigenstate of J_z. This can happen if $\mathbf{p}_e$ is not in the z direction, in which case the electron does not have a definite orbital angular momentum in the z direction, or when $\mathbf{s}_e$ is not in the z direction. From a macroscopic point of view the medium cannot be azimuthal-symmetric around the z direction, otherwise the neutrino cannot flip the spin.

From the neutrino matrix elements given in Eqs. (19) and (21) it is clear that, as expected, the helicity-conserving amplitude $\mathcal{A}(\nu_e(-)\to\nu_e(-))$ is dominant and the helicity-flipping amplitude $\mathcal{A}(\nu_e(-)\to\nu_e(+))$ is suppressed by a factor m_ν/E_ν, which is of course extremely small for relativistic neutrinos. Note that the suppression of the helicity-flipping amplitude is independent of the matter density; since both the helicity conserving and the helicity-flipping amplitudes are proportional to the matter density, there cannot be any resonant helicity-flipping transition. Therefore, the order of magnitude of the ratio of the helicity eigenstates that are produced by weak interactions (which is already $\sim m_\nu/E_\nu$) is not modified by the weak interactions with the medium. We emphasize here that this helicity flip has nothing to do with the magnetic moment and is independent from the presence of a magnetic field.

In summary we conclude that the medium effects, although they dramatically enhance the radiative decay rate as well as the neutrino oscillations, do not enhance the effective magnetic moment of neutrinos due to the coherence condition. Therefore the magnetic moment scenario can succeed in explaining the solar-neutrino problem only if the intrinsic vacuum value of the diagonal and/or off-diagonal magnetic moment is as large as $(10^{-11}-10^{-10})\mu_B$.[4,5] Finally, there cannot be any helicity flip in a medium due to the weak interactions alone, independently from the interaction with a magnetic field.

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APPENDIX: NEUTRINO POTENTIAL IN A MEDIUM

We consider an electron-neutrino propagating through a medium composed of electrons, protons, and neutrons. The effective charged-current weak Hamiltonian density is given by

$$\mathcal{H}_C(x)=\frac{G_F}{\sqrt{2}}J_C^\alpha(x)J_{C\alpha}^\dagger(x)\ ,$$
$$J_C^\alpha(x)=\bar{\nu}_e(x)\gamma^\alpha(1+\gamma_5)e(x)\ . \tag{A1}$$

The charged-current potential energy V_C is due to the weak charged-current interactions of the electron neutrino with the electrons of the medium [Fig. (1)]:

$$V_C=\int d^3p\, f(E_e,T)\left\langle \nu_e(k),e(p)\left|\int d^3x\,\mathcal{H}_C(x)\right|\nu_e(k),e(p)\right\rangle$$
$$=\frac{G_F}{\sqrt{2}}\int d^3p\, f(E_e,T)\int d^3x\,\langle e(p)|\bar{e}(x)\gamma^\alpha(1+\gamma_5)e(x)|e(p)\rangle\,\langle \nu_e(k)|\bar{\nu}_e(x)\gamma_\alpha(1+\gamma_5)\nu_e(x)|\nu_e(k)\rangle\ , \tag{A2}$$

where $f(E_e,T)$ is the statistical energy distribution of the electrons in a homogeneous and isotropic medium with temperature T (we work in the rest frame of the medium); $f(E_e,T)$ is normalized to

$$\int d^3p\, f(E_e,T)=1\ . \tag{A3}$$

The electron matrix element is given by the result in Eq. (16) averaged over the electron spin states:

$$\langle e(p)|\bar{e}(x)\gamma^\alpha(1+\gamma_5)e(x)|e(p)\rangle=N_e\frac{p^\alpha}{E_e}\ . \tag{A4}$$

The neutrino matrix element for a relativistic left-handed neutrino is given by Eq. (19a), and we obtain

$$V_C \simeq \sqrt{2} G_F N_e \int d^3p\, f(E_e,T) \left[1 - \frac{\mathbf{p}\cdot\mathbf{k}}{E_e E_\nu} \right]$$
$$= \sqrt{2} G_F N_e \; . \tag{A5}$$

Note that the potential energy and hence the change in the effective mass of the neutrino do not depend on the temperature of the medium, as long as the medium is isotropic and to the extent that the four-point interaction in Eq. (A1) is valid.

[1]L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[2]S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento C **9**, 17 (1986).

[3]J. C. D'Olivo, J. F. Nieves, and P. B. Pal, Phys. Rev. Lett. **64**, 1088 (1990).

[4]M. B. Voloshin, M. I. Vysotsky, and L. B. Okun, Yad. Fiz. **44**, 677 (1986) [Sov. J. Nucl. Phys. **44**, 440 (1986)]; M. B. Voloshin and M. I. Vysotsky, *ibid.* **44**, 845 (1986) [**44**, 544 (1986)]; L. B. Okun, *ibid.* **44**, 847 (1986) [**44**, 546 (1986)]; M. B. Voloshin, M. I. Vysotsky, and L. B. Okun, Zh. Eksp. Teor. Fiz. **91**, 754 (1986) [Sov. Phys. JETP **64**, 446 (1986)]; R. Barbieri and G. Fiorentini, Nucl. Phys. **B304**, 909 (1988).

[5]C. S. Lim and W. J. Marciano, Phys. Rev. D **37**, 1368 (1988); E. K. Akhmedov, Phys. Lett. B **213**, 64 (1988); E. Maina, *ibid.* **227**, 133 (1989); H. Minakata and H. Nunokawa, Phys. Rev. Lett. **63**, 121 (1989); J. Pulido, Phys. Rev. D **41**, 2956 (1990).

[6]K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. **45**, 963 (1980).

[7]For the latest models see, for example, K. S. Babu and R. N. Mohapatra, Phys. Rev. Lett. **63**, 228 (1989); **64**, 1705 (1990); G. Ecker, W. Grimus, and H. Neufeld, Phys. Lett. B **232**, 217 (1989); D. Chang, W. Y. Keung, and G. Senjanović, Phys. Rev. D **42**, 1599 (1990).

[8]H. A. Bethe, Phys. Rev. Lett. **56**, 1305 (1986).

[9]P. B. Pal and T. N. Pham, Phys. Rev. D **40**, 259 (1989).

[10]P. B. Pal and L. Wolfenstein, Phys. Rev. D **25**, 766 (1982).