Properties of realistic cosmic-string loops

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We examine the angular momentum, gravitational radiation, number of kinks, and electromagnetic self-interaction for cosmic-string loops produced in a numerical simulation of loop fragmentation. The final distributions of both the string angular momenta and the power radiated in gravitational radiation appear to be independent of the initial conditions. The median specific angular momentum is $|\mathbf{J}|/\mu l^2 = 0.01$. For gravitational radiation, $\gamma \equiv (dE/dt)/G\mu^2$ is strongly peaked in the region $\gamma = 40$–60, with a distribution well fitted by $p(\gamma) = (1/\sqrt{2\pi}\sigma\gamma^2)\exp[-(1/\gamma-\mu)^2/2\sigma^2]$, with $\mu = 1.74\times 10^{-2}$ and $\sigma = 5.85\times 10^{-3}$. Most stable daughter loops have between 2 and 5 kinks. Electromagnetic self-interaction can give rise to exponentially growing ac currents on highly asymmetric "random" loop trajectories. This dynamo property is not correlated with either self-intersection of the loops or with their angular momentum.

I. INTRODUCTION

Cosmic-string loops can be divided into two classes: those which intersect themselves and fragment into smaller loops, and those which are not self-intersecting. The latter trajectories are the most interesting because they represent loops which are stable against fragmentation; a variety of such stable trajectories are known to exist.[1-4] Investigations of the properties of cosmic-string loops, such as calculations of the gravitational radiation from loops,[5] the angular momentum of loops,[2] and the electromagnetic self-interaction of loops,[6,7] have generally made use of these trajectories. However, all of these nonintersecting trajectories are quite simple and highly symmetric, and it is unlikely that they represent a reasonable approximation to stable loops which fragment from larger loops or infinite strings. In addition, all loops which form from self-intersection will contain kinks, which are points on the loop at which the tangent vector is discontinuous; all of the stable trajectories mentioned above contain no kinks.

Consequently, we have investigated a variety of loop properties using more realistic stable trajectories produced by a simulation of loop fragmentation. These trajectories are highly asymmetric and almost all contain kinks. In the next section, we review our numerical simulation of loop fragmentation. In Sec. III, we discuss the angular momentum of stable string trajectories produced by our simulation. In Sec. IV, we calculate the gravitational radiation for our stable daughter loops. In Sec. V, we examine the number of kinks found on the daughter loops. In Sec. VI, we discuss the electromagnetic self-interaction of the loops. Because electric currents are not well defined on strings with kinks, we examine only our initial parent loop trajectories, testing for unstable growing current modes. Our conclusions are summarized briefly in Sec. VII.

II. SIMULATION OF STRING FRAGMENTATION

For cosmic-string loops well inside the horizon, the expansion of the Universe can be neglected, and the string equation of motion takes the simple form

$$\partial_u \partial_v x^\mu = 0 \ . \tag{2.1}$$

If we choose $x^0 = \tau$, the equation of motion can be given in three-vector form:

$$\ddot{\mathbf{x}} = \mathbf{x}'' \ , \tag{2.2}$$

with the gauge conditions

$$\dot{\mathbf{x}}\cdot\mathbf{x}' = 0 \ , \tag{2.3}$$

$$\dot{\mathbf{x}}^2 + \mathbf{x}'^2 = 1 \ , \tag{2.4}$$

where $\dot{\mathbf{x}} = \partial\mathbf{x}/\partial\tau$ and $\mathbf{x}' = \partial\mathbf{x}/\partial\sigma$. An additional condition arises because the string trajectory represents a closed loop:

$$\mathbf{x}(\sigma + l, \tau) = \mathbf{x}(\sigma, \tau) \ , \tag{2.5}$$

where l is the total invariant length of the string loop. The general solution to Eqs. (2.2)–(2.5) is

$$\mathbf{x}=\tfrac{1}{2}[\mathbf{a}(\tau+\sigma)+\mathbf{b}(\tau-\sigma)]\ , \tag{2.6}$$

where $\mathbf{a}$ and $\mathbf{b}$ satisfy $\mathbf{a}'^2=\mathbf{b}'^2=1$ and $\mathbf{a}(\sigma+l)=\mathbf{a}(\sigma)$, $\mathbf{b}(\sigma+l)=\mathbf{b}(\sigma)$. Thus, $\mathbf{a}$ and $\mathbf{b}$ are arbitrary closed curves parametrized by their length.

Scherrer and Press[8] have developed a numerical simulation of loop fragmentation inside the horizon; we use this simulation to generate realistic nonintersecting cosmic-string trajectories. The simulation follows the analytic solution for the string trajectory forward in time, testing for self-intersections of the loop. When a self-intersection occurs, the $\mathbf{a}$ and $\mathbf{b}$ curves for the two daughter loops are generated by breaking the $\mathbf{a}$ and $\mathbf{b}$ curves of the parent loop at the points corresponding to the intersection and continuing these curves periodically in space; the $\mathbf{a}$ and $\mathbf{b}$ curves are no longer closed, because they correspond to loops with center-of-mass motion in the observer's frame (see Ref. 8 for details). This evolution continues until all of the loops are in stable non-self-intersecting trajectories. This simulation is particularly well suited to a study of loop properties, because each loop trajectory is always treated as a continuous function, rather than as a discrete set of points, so the smallest daughter loop trajectories can be described with the same degree of accuracy as the initial parent loop trajectory. The $\mathbf{a}$ and $\mathbf{b}$ curves for each daughter loop consist of pieces of the $\mathbf{a}$ and $\mathbf{b}$ curves for the original parent loop, patched together at the kinks. The disadvantage of such a simulation is uncertainty as to a realistic set of parent loops with which to begin. Following Scherrer and Press,[8] we use two very different sets of parent loops in order to see if the properties of the final daughter loops in the two cases have anything in common. We take the $\mathbf{a}$ and $\mathbf{b}$ curves to be of the form

$$\mathbf{a}_x(s)=\sum_{m=1}^{M} a_{xm}\cos(ms+\phi_{xm})\ , \tag{2.7}$$

and similarly for $\mathbf{a}_y$, $\mathbf{a}_z$, and $\mathbf{b}$. Note that s here does not give the length along the curve; $\sigma(s)$ is computed numerically. We study the same two sets of parent trajectories as those used by Scherrer and Press:[8] the ϕ_m's are taken to be random numbers chosen uniformly between 0 and 2π, $M=10$, and we examine (A) a set of trajectories with large high-frequency modes; the a_m's and b_m's are random numbers chosen uniformly between 0 and 1, and (B) a set of trajectories with little high-frequency power; the a_m's and b_m's are random numbers chosen uniformly between 0 and $1/m^2$. The loops in case A represent highly convoluted trajectories which fragment into a large number of daughter loops, while the case-B loops are smoother and fragment into a much smaller number of daughters.[8]

III. ANGULAR MOMENTUM

The angular momentum of a string loop is[2]

$$\mathbf{J}=\mu\int d\sigma\, \mathbf{x}(\sigma,\tau)\times\dot{\mathbf{x}}(\sigma,\tau)\ , \tag{3.1}$$

where μ is the mass per unit length of the string. The value of τ in this integral is arbitrary, since angular momentum is conserved. The magnitude of the angular momentum can be expressed in terms of a dimensionless specific angular momentum j defined by the relation $|\mathbf{J}|=j\mu l^2$. (Note that this definition of j differs from that of Turok[2] by a factor of 8π.) In terms of the $\mathbf{a}$ and $\mathbf{b}$ curves, this becomes

$$j=\frac{1}{4l^2}\left|\int d\sigma[\mathbf{a}(\sigma)\times\mathbf{a}'(\sigma)-\mathbf{b}(\sigma)\times\mathbf{b}'(\sigma)]\right|\ . \tag{3.2}$$

Consider first the case of string trajectories which lie entirely in a plane, so that $\mathbf{a}$ and $\mathbf{b}$ are also planar curves. The total area enclosed by the $\mathbf{a}$ curve is $A_{\rm a}=|\int d\sigma\, \mathbf{a}(\sigma)\times\mathbf{a}'(\sigma)|/2$, and similarly for $\mathbf{b}$. Thus, j for a two-dimensional string trajectory is simply a function of the areas $A_{\rm a}$ and $A_{\rm b}$ contained within the $\mathbf{a}$ and $\mathbf{b}$ curves:

$$j=\frac{1}{2l^2}|w_{\rm a}A_{\rm a}-w_{\rm b}A_{\rm b}|\ , \tag{3.3}$$

where w is the net winding number of the given curve, taken to be $+1$ for curves which wind in the counterclockwise direction, and -1 for curves which wind in the clockwise direction. [Equation (3.3) is strictly valid only for $\mathbf{a}$ and $\mathbf{b}$ curves which do not intersect themselves and so have a winding number of ± 1. However, it can be generalized to self-intersecting $\mathbf{a}$ and $\mathbf{b}$ curves: a self-intersecting planar curve can be broken down into a number of nonintersecting closed curves. Equation (3.3) can be applied to each curve, and the contributions from all of these curves can then be summed with the appropriate signs.]

The curve of fixed length which encloses the largest area is a circle, so the largest value of j is obtained when $\mathbf{a}$ and $\mathbf{b}$ are circles oriented in opposite directions. This corresponds to the degenerate trajectory of a rotating rod with a value of $j=1/4\pi$. For three-dimensional string trajectories, Eq. (3.3) is still valid, as long as $A_{\rm a}$ and $A_{\rm b}$ are taken to be the areas of the $\mathbf{a}$ and $\mathbf{b}$ curves projected into the plane perpendicular to $\mathbf{J}$. Thus, for every three-dimensional trajectory, there corresponds a two-dimensional trajectory with the same values for $A_{\rm a}$ and $A_{\rm b}$ and a smaller value for l, therefore, a larger value for j. The rotating rod trajectory therefore has the largest specific angular momentum of any cosmic-string trajectory: $j=1/4\pi$ is the upper limit on j. This is a well-known result from the theory of fundamental strings.[9]

We fragmented 20 parent loops of type A and 100 parent loops of type B. A histogram of the distribution of values for the specific angular momentum j of the parent loops is shown in Fig. 1. Typical values of j are quite different for the two cases; the highly convoluted loops have a much lower angular momentum. The distribution of values of j for the daughter loops is shown in Fig. 2. The distribution of angular momenta for the daughter loops is strikingly similar for the two cases studied, despite the fact that the parent loops begin with very different angular momenta. These results are important because they indicate that stable daughter loops have a characteristic angular momentum distribution indepen-

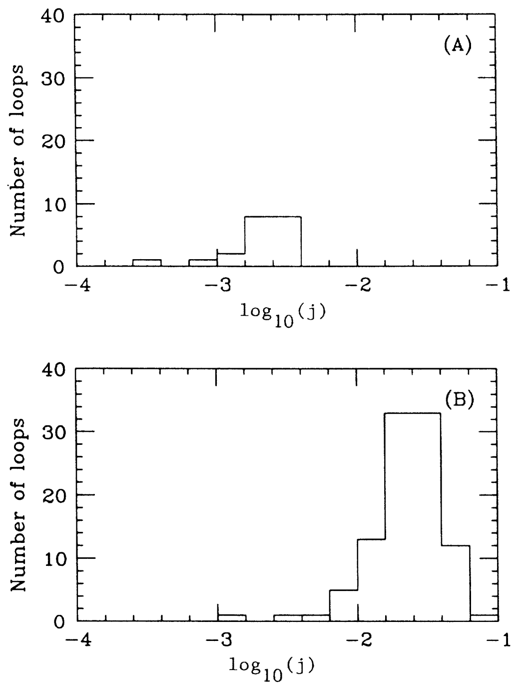

FIG. 1. The number of parent loops with a given value of specific angular momentum $j \equiv |\mathbf{J}|/\mu l^2$ for (A) case A, highly convoluted parent loops having large-amplitude high-frequency modes, and (B) case B, parent loops with little high-frequency power.

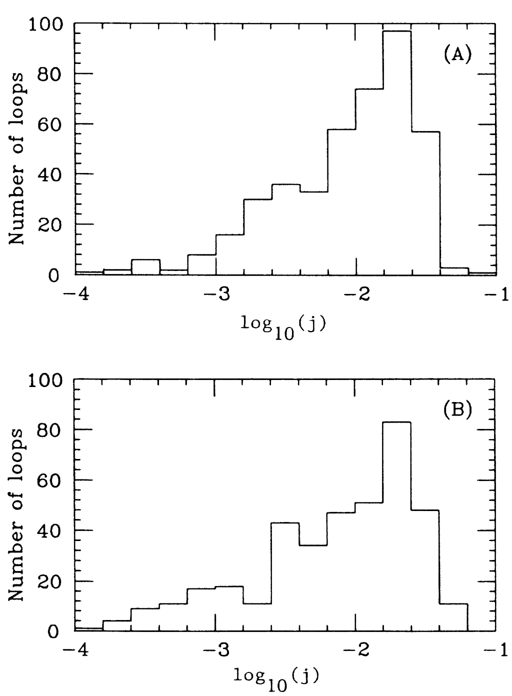

FIG. 2. The number of stable daughter loops with a given value of specific angular momentum $j \equiv |\mathbf{J}|/\mu l^2$ for (A) case A, highly convoluted parent loops having large-amplitude high-frequency modes, and (B) case B, parent loops with little high-frequency power.

dent of initial conditions. This, is turn, suggests that there exists a well-defined set of realistic nonintersecting loop trajectories produced by string fragmentation with properties independent of the fragmentation history. The median value of j for the two cases studied is $j = 1.1 \times 10^{-2}$ (case A) and $j = 1.0 \times 10^{-2}$ (case B).

IV. GRAVITATIONAL RADIATION

Here we outline the formalism we have used to compute the gravitational power radiated by the daughter loops. For a more detailed account, as well as an in-depth study of the gravitational back-reaction problem and the rounding of kinks, see Ref. 10.

Instead of computing the gravitational power in the standard fashion,[11] which uses the linearized form for the energy-momentum "tensor" of the gravitational radiation, we have proceeded by evolving the trajectory of the string loops. This includes the effects of back reaction, and by calculating the amount of energy lost by the string loops, the amount radiated into space by these loops is also determined. This formulation has the advantage that the change in the string trajectory is also computed, and the string evolution can be charted as it decays. From the energetics point of view, these two formulations are equivalent.[10]

We begin with the equations of motion for the string which is interacting with its own gravitational field. In the standard gauge, the self-interacting string obeys the equation

$$\partial_u \partial_v x^\mu = -\Gamma^\mu_{\alpha\beta}\partial_u x^\alpha \partial_v x^\beta \; . \tag{4.1}$$

Here u and v are light-cone coordinates, defined by $u \equiv \tau+\sigma, v \equiv \tau-\sigma$. Note that the noninteracting string simply obeys Eq. (2.1), $\partial_u \partial_v x^\mu = 0$, so the noninteracting string is composed of static left and right movers, as discussed in Sec. II. The gravitational field, given by the Christoffel affine connection

$$\Gamma^\mu_{\alpha\beta} \equiv \tfrac{1}{2} g^{\mu\gamma}(\partial_\alpha g_{\gamma\beta} + \partial_\beta g_{\gamma\alpha} - \partial_\gamma g_{\alpha\beta}) \; ,$$

is computed at the string position itself, i.e., on the string world sheet. In order to do this we use the linearized equations for the perturbed metric $h_{\mu\nu}$, which are[11]

$$\Box h_{\mu\nu} = -16\pi G(T_{\mu\nu} - \tfrac{1}{2}\eta_{\mu\nu}T^\alpha_\alpha) \; . \tag{4.2}$$

To lowest order, we can insert the trajectory of the unperturbed string loop, which we know, into Eq. (4.2). In the standard gauge, the energy-momentum tensor for the loop is given by[2]

$$T^{\mu\nu}(x^\alpha) = \mu \int du \, dv (\partial_u x^\mu \partial_v x^\nu + \partial_v x^\mu \partial_u x^\nu) \times \delta^4(x^\alpha - x^\alpha(u,v)) \; . \tag{4.3}$$

Inserting Eq. (4.3) into (4.2), we can compute the back-reaction force in (4.1). The integrated effect of the back reaction can be computed by integrating Eq. (4.1) over a period. Thus, a new trajectory is obtained, and it can be used to compute the back reaction for the next period. In such a manner, we can evolve the string loop in time.[10]

Here, however, we are more interested in describing the energetics of the string loop. First, we define the four-momentum of the string to be

$$P^{\mu} \equiv \int d^3x \; T^{\mu 0} \; , \tag{4.4}$$

where $T^{\mu\nu}$ is the energy-momentum tensor given in (4.3). We thus find the force density of the string to be

$$\partial_\nu T^{\mu\nu} = 2\mu \int du \; dv \; \partial_u \partial_v x^{\mu} \delta^4(x^{\alpha} - x^{\alpha}(u,v)) \; . \tag{4.5}$$

Now, combining (4.5) with (4.4) and invoking Gauss's divergence theorem, we obtain the important relation

$$\Delta P^{\mu} = 2\mu \int du \; dv \; \partial_u \partial_v x^{\mu} \; . \tag{4.6}$$

Here the change of the four-momentum of the string is defined over one period, so that the integration region in Eq. (4.6) is over the range corresponding to σ going from 0 to l and τ going from 0 to $l/2$, i.e., u and v ranging from 0 to l. Equation (4.6) is equivalent to the standard expression for the power radiated by a moving body, $\Delta E = -\Delta P^0$, where ΔE is the energy radiated by the string in a period.[10] Finally, noting that one period is simply $l/2$, we can compute the power. From dimensional considerations, or simply by combining Eq. (4.6) with Eqs. (4.1) and (4.2), the power can be written in the form $dE/dt \equiv \gamma G \mu^2$, where γ is a dimensionless number.

In Ref. 10, the numerical implementation of (4.6) for kinky strings is presented. The contribution to the radiated power is computed as a sum over straight string segments, where an analytical formula has been found for the contribution from each kink. Thus, $\partial_u \partial_v x^{\mu}$ is found over the u-v grid, and (4.6) integrated over a period. The radiated power is then computed, and we find γ. An investigation of the evolution of γ due to the change in the string trajectory from gravitational back reaction is given in Ref. 10 for simple string trajectories. Here we simply give the initial value of γ prior to any subsequent evolution from gravitational back reaction.

We have computed γ values for a total of 455 stable daughter loops; 240 of these are from fragmentation of type-A parent loops, and 215 from type-B parent loops. Note that γ is Lorentz invariant. Our γ values have been calculated in the parent loop center of mass, in which the daughter loops can have large velocities, but the results would be the same if calculated in the rest frame of the daughter loops. The loop lifetimes and the total energy radiated will, of course, depend on the frame of reference in the standard way. Histograms of the distribution of the values of γ are given in Fig. 3 for both cases. The values for γ, like those for angular momentum, are strikingly similar for the two cases examined. This lends further support to the view that the non-self-intersecting daughter loops form a well-defined set of loops with properties independent of their earlier evolution. Because of this, we have combined our results into a single graph of the distribution of γ values for all 455 loops (Fig. 4). This distribution has a mean of $\gamma = 61.7$ and a median of $\gamma = 55.4$; it is slightly skewed toward larger values of γ. Roughly half of the strings have γ values between 40 and 60, and 3/4 have γ values between 30 and 80. The relevant quantity for calculations of the gravitational radiation background is actually $1/\gamma$. A graph of the distribution of $1/\gamma$ is given in Fig. 5; this distribution is well fit in the vicinity of the peak by a Gaussian with a mean of $\mu = 1.74 \times 10^{-2}$ and a standard deviation of $\sigma = 5.85 \times 10^{-3}$. Hence, an excellent fit to the distribution of the values of γ is

$$p(\gamma)d\gamma = \frac{1}{\sqrt{2\pi}\sigma\gamma^2} e^{-(1/\gamma - \mu)^2/2\sigma^2} d\gamma \; , \tag{4.7}$$

where $\mu = 1.74 \times 10^{-2}$ and $\sigma = 5.85 \times 10^{-3}$. Equation (4.7) represents the best current estimate of the distribution of γ for realistic cosmic strings.[12] A graph of Eq. (4.7) is given in Figs. 4 and 5.

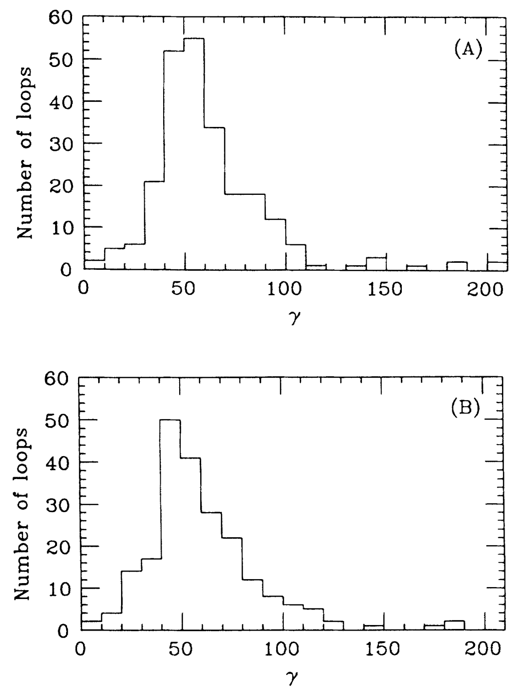

FIG. 3. The number of stable daughter loops with a given value of $\gamma = (dE/dt)/G\mu^2$ for gravitational radiation for (A) case A, highly convoluted parent loops having large-amplitude high-frequency modes, and (B) case B, parent loops with little high-frequency power.

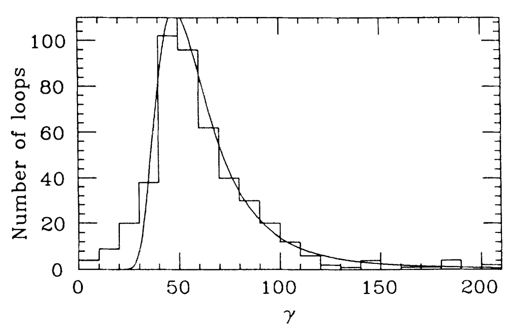

FIG. 4. The number of stable daughter loops with a given value of $\gamma = (dE/dt)/G\mu^2$, for all daughter loops. The solid curve is our analytic fit [Eq. (4.7)].

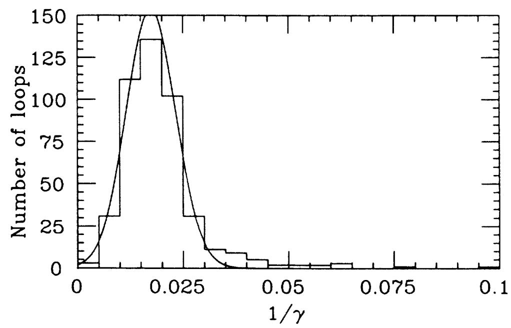

FIG. 5. The number of stable daughter loops with a given value of $1/\gamma = G\mu^2/(dE/dt)$ for all daughter loops. The solid curve is our analytic fit [Eq. (4.7)].

V. KINKS

The string fragmentation process leads to the formation of kinks, which are discontinuities in the tangent to the string direction; these kinks can be described in terms of kinks on the underlying **a** and **b** curves (see Refs. 8, 13, 14, and references therein). The fragmentation of a parent loop into two daughter loops produces a single kink on each of the daughter loops at the instant of fragmentation, but this kink evolves into a left-moving and a right-moving kink at later times, each of these kinks corresponding to a kink on either the **a** or **b** curve. Each fragmentation therefore leads to the formation of two kinks on each of the two daughter loops, for a total of four kinks. Thus, N fragmentations will produce $N_k = 4N$ kinks and $N_l = N+1$ daughter loops, so $N_k = 4(N_l - 1)$, and the mean number of kinks per loop approaches four after a large number of fragmentations.[15] Note that a string can have an odd number of kinks, if the fragmentation leaves a left-moving and right-moving kink pair on two different daughter loops. However, string trajectories with a single kink, while mathematically allowed by the solutions for string motion, are unphysical, since fragmentation produces a minimum of two kinks on each daughter loop.

In Fig. 6, we present a graph of the number of stable daughter loops with a given number of kinks for a set of 271 daughter loops from type-A fragmentations (crosses) and 230 daughter loops from type-B fragmentations (circles). This graph clearly shows the effects of the greater fragmentation of the type-A parent loops. The daughter loops from the type-B parent loops are most likely to have two kinks, with the probability for larger numbers of kinks decreasing with increasing kink number. A small number of these loops have no kinks, corresponding to parent loops which do not self-intersect at all. The daughter loops from the fragmentation of type-A parent loops are more nearly peaked around the expected mean of four kinks. In both cases, over 90% of the daughter loops have between two and five kinks, and the results do not show any bias in favor of an even number of kinks.

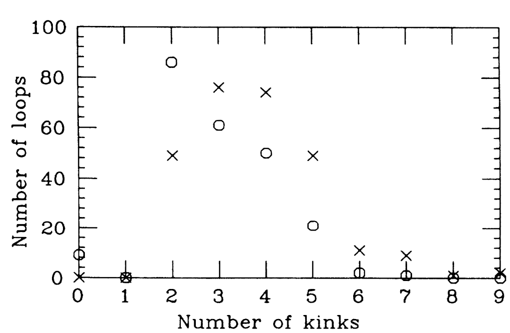

FIG. 6. The number of stable daughter loops with a given number of kinks for case A, highly convoluted parent loops (crosses) and case B, parent loops with little high-frequency power (circles).

VI. ELECTROMAGNETIC SELF-INTERACTION

Our discussions of angular momentum and gravitational radiation apply to all cosmic strings; in this section we consider the special case of superconducting cosmic strings.[16] Spergel, Press, and Scherrer[6,7] examined the electromagnetic self-interaction of superconducting cosmic strings to determine the behavior of ac currents on an isolated string with no external electromagnetic fields present. They were able to show that some loop trajectories produce growing ac currents; any small stray currents on these loops would grow exponentially until damped by other means. These dynamo trajectories convert the mechanical energy of the string into electromagnetic fields. However, Spergel, Press, and Scherrer examined only highly symmetric trajectories, and the loops with growing currents resembled the rotating rod trajectory discussed in Sec. III. It is therefore important to determine whether this current amplification is a property only of very symmetric loop trajectories.

The formalism developed in Ref. 7 cannot be applied directly to cosmic strings with kinks. The reason is that the instantaneous acceleration of a charge at the kink is infinite, so the kink will radiate an infinite amount of power. A string with a right-moving kink could support a right-moving current without infinite radiation as long as the charge at the kink were exactly zero; however, fragmentation produces, in general, two kinks moving in opposite directions, so any nonzero current will produce this infinite radiation. Physically, the most likely consequence of this is that the back reaction from the radiation rounds off the kink so that the radiation is no longer singular.

Although we cannot directly examine the electromagnetic self-interaction of our realistic loops with kinks, we can still address the question of whether a high degree of symmetry is required for a loop to act as a dynamo. We have examined the electromagnetic self-interaction of our random parent loops, which do not contain kinks. We have examined only the type-B loops, which more closely resemble the final nonintersecting loops. Also, a small fraction of the type-B parent loops are not self-intersecting, so they represent stable loops.

We tested a total of 100 self-intersecting type-B parent loop trajectories and 100 non-self-intersecting type-B trajectories. We determined the presence of growing ac

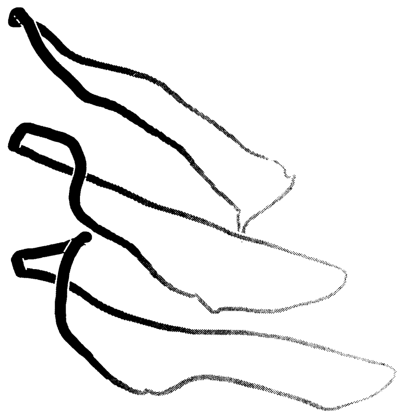

FIG. 7. Part of a typical loop trajectory with growing ac modes, at intervals of 1/16 of the loop period. This trajectory is self-intersecting.

modes using the numerical techniques discussed in Ref. 7: a matrix is constructed which multiplies the current vector at a given time to give the current vector one oscillation period later. If the matrix has positive eigenvalues, then there exist ac modes which grow exponentially with time. Of the loops sampled, we found seven growing ac modes among the self-intersecting trajectories, and four growing modes among the non-self-intersecting trajectories. Part of a typical dynamo loop trajectory is shown in Fig. 7. This trajectory displays no obvious symmetries. It appears that string dynamos can occur among realistic "random" string trajectories. There is also no correlation between this dynamo property and the question of whether the loop is self-intersecting, in agreement with the results of Ref. 7.

Because the dynamos discussed in Ref. 7 resemble the spinning rod trajectory, which has the highest possible angular momentum, it is reasonable to determine whether string dynamos have an unusually high angular momentum. We compared the specific angular momentum of a set of 14 string dynamos with a set of 100 nondynamos using the Kolmogorov-Smirnov test. The Kolmogorov-Smirnov test failed to find a difference between the angular momentum distributions for these two sets, with a significance level of 0.29. The median value of the specific angular momentum was $j=2.5\times10^{-2}$ for the nondynamos and $j=3.2\times10^{-2}$ for the dynamos. There is no clear correlation between the angular momentum of a loop and the probability that it will produce growing currents.

VII. CONCLUSIONS

Our results suggest the existence of a well-defined set of non-self-intersecting trajectories with properties independent of their fragmentation history. Both the distribution of the specific angular momentum and the distribution of the power radiated in gravitational radiation are extremely similar for daughter loops produced from two very different initial conditions. The number of kinks per daughter loop does appear to be sensitive to the initial conditions, but this is not surprising in view of the fact that it is directly correlated with the number of fragmentations which a parent loop undergoes.

Stable non-self-intersecting loops produced by our simulation of loop fragmentation have a characteristic angular momentum distribution with a median value of $|\mathbf{J}|/\mu l^2=0.01$, independent of the initial conditions.

When the power radiated by our stable daughter loops in gravitational radiation is expressed in terms of the parameter γ, where $dE/dt=\gamma G\mu^2$, the distribution of the values of $1/\gamma$ is well approximated by a Gaussian with a mean of 1.74×10^{-2} and a standard deviation of 5.85×10^{-3}. The distribution of values for γ is strongly peaked in the region from 40 to 60. These values are somewhat lower than those first derived for simple string trajectories,[5] where typical values were $\sim 50-100$.

The stable daughter loops typically have 2–5 kinks per loop, although the number of kinks per loop is smaller for the fragmentation of the type-B parent loops than it is for the type-A loops. We expect a mean of four kinks per daughter loop, but the type-B parent loops do not fragment enough to approach this value. Because of the large number of fragmentations of the type-A loops, the kink distribution in this case may be approaching a stable asymptotic form.

Although we cannot investigate the behavior of ac currents on loops with kinks, our investigations with kinkless parent loops indicate that the dynamo effect discovered in Refs. 6 and 7 does not require simple symmetric trajectories. We have observed growing ac modes on highly asymmetric "random" trajectories. Of course, the growth of such currents will be limited by plasma effects and vacuum breakdown.

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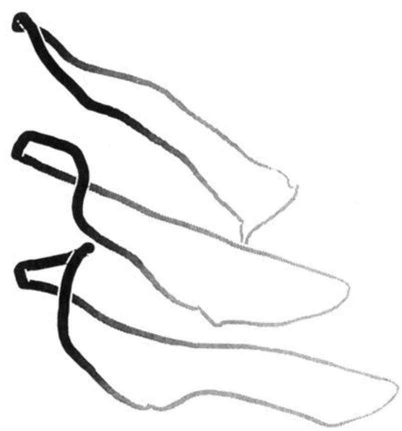

FIG. 7. Part of a typical loop trajectory with growing ac modes, at intervals of 1/16 of the loop period. This trajectory is self-intersecting.