Modeling parton spin-transfer densities

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(Received 24 October 1988; revised manuscript received 20 November 1989)

We present models for the spin-weighted quark and gluon distribution in a longitudinally polarized proton. The models embody different underlying concepts concerning the application of spin-dependent forces in the nonperturbative or "soft" regime of chromodynamics. The different spin densities are used to investigate a possible long-range experimental program for measuring various hard-scattering processes using polarized protons.

I. INTRODUCTION

The measurement of spin-related observables in processes involving polarized protons provides an outstanding opportunity to enhance our knowledge of chromodynamics. Within the framework of the QCD-aided parton model, the well-known spin structure of various hard-scattering subprocesses can be used to probe correlation between the proton spin and the spin of its constituents. Several papers[1-4] have provided phenomenological estimates for spin-weighted inclusive scattering processes based on simple models for the parton spin densities. Until recently, however, little attention had been devoted to the question of what a measurement of a spin-weighted parton distribution tells us about proton structure.

The situation has now changed dramatically. The measurement of the longitudinal spin-spin asymmetry in deep-inelastic leptoproduction[5] has recently been extended to small values of the Bjorken-x variable by the European Muon Collaboration[6] (EMC). The EMC results suggest a strong negative polarization for the sea of $q\bar{q}$ pairs and have generated a spate of theoretical papers.[7-13] The theoretical attention serves to emphasize the importance of understanding how the angular momentum of the proton is shared among the spin of its constituents and orbital angular momentum.

The polarization of the gluons in a longitudinally polarized proton has received particular attention. Many of the possibilities discussed theoretically raise new experimental possibilities. It now becomes important to assign a high priority to a comprehensive experimental program involving polarized protons aimed at answering the many questions raised by these new results.

We would like to discuss the information gleamed from the EMC experiment in terms of a future experimental program aimed at separately determining the polarization of the valence quarks, the gluons, and the sea. In planning a new generation of spin-asymmetry experiments, it is useful to have available models for the spin transfer densities which encompass a reasonable range of variation. These models will allow experimenters to assess the precision required to extract the necessary information. For this purpose we have constructed simple model distributions which represent different theoretical scenarios. We have used existing leptoproduction data plus some simple assumptions to constrain the distribution functions.

Definitions

At this point it is useful to establish our conventions and to introduce the specific definitions we will use for the spin-weighted quark and gluon distributions.

We will initially adopt a process-independent set of definitions for the spin-weighted quark and gluon distributions. The factorization prescription will be specified by initially demanding that the distributions evolve in Q^2 with the lowest-order Altarelli-Parisi kernels.[14] In general, the connection of these process-independent distributions to the distributions "measured" in a specific hard-scattering process will involve a correction factor calculable in perturbative QCD. The choice of a factorization scale, the scale at which the distributions will be used in the calculation of various inclusive processes, will be briefly considered when we present our phenomenological projections.[15]

The language of this paper will be specific to the case of longitudinally polarized protons. We will not attempt to address the question of transverse polarization which—in QCD—raises a number of unanswered questions.

We assume that a proton is polarized along its direction of motion and we neglect the transverse momentum of the quarks and gluons so that their momenta are aligned with the proton's spin. We also neglect quark mass parameters. With this approximation we can identify helicity and the longitudinal component of spin in a straightforward way. From the structure of the Altarelli-Parisi equations, and for the purpose of comparing different processes, it is convenient to allow a distinction between the "valence" and the "sea" quarks of the proton. For the up- and down-quark number densities, we will therefore write

$$u(x,Q^2)=u_v(x,Q^2)+u_s(x,Q^2)\ ,$$
$$d(x,Q^2)=d_v(x,Q^2)+d_s(x,Q^2)\ . \tag{1.1}$$

The valence distributions are the flavor-nonsinglet components of the quark distributions in the proton and are normalized so that

$$\int_0^1 dx\, u_v(x,Q^2)=2\ ,\qquad \int_0^1 dx\, d_v(x,Q^2)=1\ . \tag{1.2}$$

All other flavor components will be included in the sea. In general, these distributions will be denoted by their flavor quantum numbers: $\bar{u}(x,Q^2)$, $\bar{d}(x,Q^2)$, $s(x,Q^2)$, $\bar{s}(x,Q^2)$, etc. When we want to write an equation valid for all flavor quantum numbers, we will use the distribution $q^i(x,Q^2)$. For a proton with its spin aligned in the $+z$ direction (along its momentum), we will parametrize the quark spin-weighted densities:

$$\begin{aligned} q^i_{+/+}(x,Q^2)&=\cos^2[\tfrac{1}{2}\theta_i(x,Q^2)]q^i(x,Q^2)\ ,\\ q^i_{-/+}(x,Q^2)&=\sin^2[\tfrac{1}{2}\theta_i(x,Q^2)]q^i(x,Q^2)\ ,\\ i&=u_v,d_v,u_s,d_s,\bar{u},\bar{d},s,\bar{s},\dots\ . \end{aligned} \tag{1.3}$$

From the parity invariance of the strong interactions we have

$$\begin{aligned} q^i_{-/-}(x,Q^2)&=q^i_{+/+}(x,Q^2)\ ,\\ q^i_{+/-}(x,Q^2)&=q^i_{-/+}(x,Q^2)\ . \end{aligned} \tag{1.4}$$

With these definitions, the difference

$$\begin{aligned} \Delta q^i(x,Q^2)&=q^i_{+/+}(x,Q^2)-q^i_{-/+}(x,Q^2)\\ &=q^i_{-/-}(x,Q^2)-q^i_{+/-}(x,Q^2)\\ &=\cos\theta_i(x,Q^2)q^i(x,Q^2) \end{aligned} \tag{1.5}$$

appears in the calculation of spin-weighted observables for various hard-scattering processes involving polarized protons. The longitudinal polarization or spin-transfer density for a given flavor of quark is given by the angle $\theta_i(x,Q^2)$:

$$\cos\theta_i(x,Q^2)=\frac{\Delta q^i(x,Q^2)}{q^i(x,Q^2)}\ . \tag{1.6}$$

For the valence distributions $u_v(x,Q^2)$ and $d_v(x,Q^2)$ we will also define another, more model-dependent angle. Taking a simple SU(6) picture of the valence-quark structure, we will assume that, as $x\to 1$ (see the discussion in Appendix A),

$$\begin{aligned} \lim_{x\to 1}\Delta u_v(x,Q^2)&\sim u_v(x,Q^2)-\tfrac{2}{3}d_v(x,Q^2)\ ,\\ \lim_{x\to 1}\Delta d_v(x,Q^2)&\sim -\tfrac{1}{3}d_v(x,Q^2)\ . \end{aligned} \tag{1.7}$$

We will therefore define a valence spin-dilution angle Θ_D which measures the deviation of the observed valence spin-weighted distributions from this SU(6) ideal:

$$\begin{aligned} \Delta u_v(x,Q^2)&=\cos\Theta_D(x,Q^2)[u_v(x,Q^2)-\tfrac{2}{3}d_v(x,Q^2)]\ ,\\ \Delta d_v(x,Q^2)&=-\tfrac{1}{3}\cos\Theta_D(x,Q^2)d_v(x,Q^2)\ . \end{aligned} \tag{1.8}$$

These spin-dilution terms, which approach 1 at large x in our models, were first introduced in this form by Carlitz and Kaur[16] who hypothesized a specific form for them. We will use the Carlitz-Kaur ansatz as a reference point in our construction of new models. In Sec. II we will present an alternative approach to the valence spin-dilution angle which emphasizes its connection to a possible nonperturbative polarization of the sea and the glue.

For the quarks and antiquarks in the sea, the specification of spin-weighted distributions involves new issues concerning the choice of a factorization prescription which need explicit discussion. The general form of factorization in QCD requires that one specify a factorization prescription which separates the "hard" perturbative process from the "soft" hadronic dynamics. Because of the chiral properties of QCD with massless quarks, there exist two distinct approaches to the problem. In analogy to the definition of valence-quark distributions, one could define for sea quarks and antiquarks, distributions $\Delta q_c^s(x,Q^2)$ and $\Delta\bar{q}_c(x,Q^2)$ (suppressing the flavor index i) which obey the lowest-order Altarelli-Parisi equations. With this abstract definition, the net helicity associated with each flavor of quark or antiquark in the sea, would not evolve with Q^2. However, there are potential problems with this factorization scheme.[17–19] A more physical definition of these distributions which we will label $\Delta q_5^s(x,Q^2)$ and $\Delta\bar{q}_5^s(x,Q^2)$ involves the observable axial-vector current. Because the axial-vector current in QCD receives a contribution from polarized gluons due to the "anomalous" current,

$$K_\mu^5=\frac{\alpha_s}{2\pi}\epsilon_{\mu\nu\rho\sigma}\mathrm{tr}[A^\nu(F^{\rho\sigma}-\tfrac{2}{3}A^\rho A^\sigma)]\ , \tag{1.9}$$

the relationship between these two possible definitions, can be written (for each flavor)

$$\Delta q_5^s(x,Q^2)=\Delta q_c^s(x,Q^2)-\frac{\alpha_s(Q^2)}{4\pi}\hat{\gamma}(x)\otimes\Delta G(x,Q^2)\ , \tag{1.10a}$$

$$\Delta\bar{q}_5(x,Q^2)=\Delta\bar{q}_c(x,Q^2)-\frac{\alpha_s(Q^2)}{4\pi}\hat{\gamma}(x)\otimes\Delta G(x,Q^2)\ , \tag{1.10b}$$

where $\hat{\gamma}(x)$ is convention dependent and a choice of renormalization prescription for α_s must be made. We will discuss the specific choice of $\hat{\gamma}(x)$ in Sec. II. At this point we want to emphasize that either type of prescription choice is possible but that physical observables are more directly related to Δq_5^s or $\Delta\bar{q}_5$ than to their c counterparts.

As indicated above, we must also consider gluon distributions. In this paper, the unpolarized gluon distribution in a proton will be denoted $G(x,Q^2)$. For gluons in a proton with helicity aligned along the $+z$ direction, we must allow for the possibility of "scalar" vector-gluon polarization which corresponds to a vector particle with spin projection $s_z=0$. The longitudinal spin-weighted distributions can then be parametrized in terms of two angles:

$$G_{+/+}(x,Q^2)=\left[\left[\frac{1+\cos\Phi_L}{2}\right]^2\cos^2\theta_g+\left[\frac{1-\cos\Phi_L}{2}\right]^2\sin^2\theta_g\right]G(x,Q^2)\ ,$$

$$G_{0/+}(x,Q^2)=\tfrac{1}{2}\sin^2\Phi_L G(x,Q^2)\ , \tag{1.11}$$

$$G_{-/+}(x,Q^2)=\left[\left[\frac{1+\cos\Phi_L}{2}\right]^2\sin^2\theta_g+\left[\frac{1-\cos\Phi_L}{2}\right]^2\cos^2\theta_g\right]G(x,Q^2)\ .$$

For massless, "on-shell" gluons traveling parallel to the direction of parent proton $\Phi_L \rightarrow 0$, so that with our approximations the scalar polarization density vanishes. In this paper, we will not discuss experimental observables sensitive to the scalar polarization density of the gluons but will retain the full complexity of the notation in (1.11) in order to be compatible with more general treatments of vector polarization. From the parity invariance of the strong interactions,

$$G_{-/-}(x,Q^2)=G_{+/+}(x,Q^2)\ ,$$
$$G_{0/-}(x,Q^2)=G_{0/+}(x,Q^2)\ , \tag{1.12}$$
$$G_{+/-}(x,Q^2)=G_{-/+}(x,Q^2)\ .$$

The difference

$$\begin{aligned}\Delta G(x,Q^2)&=G_{+/+}(x,Q^2)-G_{-/+}(x,Q^2)\\&=G_{-/-}(x,Q^2)-G_{+/-}(x,Q^2)\\&=\cos\Phi_L(x,Q^2)\cos2\theta_g(x,Q^2)G(x,Q^2)\end{aligned} \tag{1.13}$$

appears in the calculation of spin-related observables involving polarized protons in the same manner as the $\Delta q_i(x,Q^2)$. As we shall see, this distribution also occurs in the evolution equations for $\Delta q_i(x,Q^2)$. The ratio

$$\frac{\Delta G(x,Q^2)}{G(x,Q^2)}=\cos\Phi_L(x,Q^2)\cos2\theta_g(x,Q^2) \tag{1.14}$$

will be called the gluon polarization or gluon spin-transfer distribution.

Equations (1.10a) and (1.10b) above show that the gluon spin-transfer density can affect the relationship between the "observable" quark and antiquark spin densities $\Delta q_5^s(x,Q^2)$ and $\Delta\bar{q}_5(x,Q^2)$ and the abstract densities $\Delta q_c^s(x,Q^2)$ and $\Delta\bar{q}_c(x,Q^2)$ which have simple chiral properties in the limit $m_q \rightarrow 0$. Although the contribution of the polarized gluons is formally of higher order in perturbation theory, the QCD evolution equations suggest that $(4\pi)^{-1}\alpha_s(Q^2)\Delta G(x,Q^2)$ need not be small at large Q^2 (Refs. 1, 10, and 13). The suggestion has been made[13] that a large contribution from polarized gluons in (1.10a) and (1.10b) would help reconcile the EMC data with certain theoretical ideas for proton structure based on the quark model. It is therefore important to attempt to measure $\Delta G(x,Q^2)$ in some process.

The remainder of this paper is organized as follows. In Sec. II we will discuss some simple dynamical assumptions that will guide us in constructing the polarized parton distributions. Section III will begin with a discussion of the phenomenological and theoretical constraints on the spin-weighted parton structure functions. We will then present a range of models for the spin structure of the proton, and compare them to the data on electroproduction. Finally in Sec. IV we shall discuss some experiments which can help to complete our knowledge of the spin-weighted distributions.

II. SIMPLE EXPECTATIONS

For unpolarized distributions, there presently exists a large body of phenomenological knowledge concerning the various parton densities. This knowledge has been compiled into various sets of models. The range of difference between the different parametrizations of the unpolarized distributions is quite small compared to the uncertainties in the spin-weighted distributions discussed here. For our purposes, therefore, it is convenient to pick one set of unpolarized distributions and stick with them. We will primarily formulate our models in terms of spin-transfer densities, (1.6) and (1.14), or the valence spin-dilution angle (1.8). The spin-transfer densities thus obtained can be used with other sets of unpolarized distributions with little or no modification.

The usual application of the QCD-parton model assumes that the appropriate distributions are measured in one set of processes. Predictions are then made for other processes by using factorization. For the purpose of estimating the impact of possible experiments, it is convenient to have model distributions which represent an informed guess concerning the allowable range of distributions. We will, in general, call a class of possible models which embody a certain physics assumption a *scenario*. In the next section we will introduce models which represent an old-fashioned scenario with small sea polarization for quark distributions and models which represent an EMC-motivated scenario in which the polarization of the sea is strongly negative. In each case, we will consider a large range of possible behavior for the spin-weighted gluon distribution.

The two basic ideas which will help us in constructing models are the following.

(1) There is a strong connection between the spin of the valence quarks and the spin of the proton.

(2) There are spin-dependent forces between the constituents in the proton which influence the shape of the spin-weighted distributions.

The idea that there is a strong connection between the spin of the valence quarks and the spin of the proton is supported by the theoretical predictions[20] of the quark-gluon model of hadrons and by the existing experimental data[5,6] on the spin-spin asymmetry in deep-inelastic leptoproduction. For the low-lying baryons, the magnetic moments can be written[21]

$$\mu_B=\sum_i (e_i/m_i)g_i s_{z_i}\ ,$$

where the sum is over the valence, constituent quarks. A successful description of the baryons within the framework of the "naive" constituent-quark model suggests strongly that, in some approximation, the spin of the proton is associated with its valence quarks.[22]

The observation that there are significant spin-dependent forces between the constituents in the hadrons is supported by the success of the hadron mass formula[23]

$$M_h=\sum_i m_i+\sum_{i>j} \mathbf{s}_i\cdot\mathbf{s}_j v_{ij}\ , \tag{2.1}$$

where, again, the summation is over valence quarks. The m_i are effective-mass parameters and v_{ij} represents an effective hyperfine interaction. Such spin-dependent forces also have been found in simulations of QCD on a lattice.[24]

A. Spin-dependent forces and spin dilution

We shall now examine the influence that such soft, nonperturbative, two-body forces can have on the spin-weighted quark and gluon distributions. In this subsection we shall, unless otherwise stated, suppress flavor labels in order to simplify notation. We intend to consider the consequences of a general two-parton process of the type shown in Fig. 1. For two-body interactions involving only valence quarks, we assume the flavor and spin dependence of such interactions can be taken into account by specifying an approximate SU(6) wave function for the valence quarks of the proton. Before allowing for the same type of spin-dependent forces involving the other constituents, the spin-weighted valence-quark distributions are assumed to be related to the spin-averaged distributions as given in Eq. (1.7). If we now turn on adiabatically such spin-dependent forces we can observe two related effects: (1) the dilution of the valence-quark distributions from the original SU(6) limit, and (2) the polarization of the gluons and the $q\bar{q}$ pairs in the sea.

We believe that such effects are inevitable consequences of spin-dependent forces and that they must be taken into account in constructing realistic models. In order to obtain a feel for the manner in which spin can be transferred from the valence quarks to other constituents we can formulate a specific model calculation. We will first concentrate on the interaction of valence quarks and gluons. In the absence of two-body forces, we label the quark and gluon densities of definite spin by $q_s^0(x)$ and $G_h^0(x)$, respectively. The inclusion of the two-body spin force of Fig. 1 modifies these distributions to $q_s^1(x)$ and $G_h^1(x)$. We assume that long-distance, coherent effects are responsible for the spin rearrangement and, in the diagram of Fig. 1, we will neglect the momentum transfer between the interacting partons. We will also require that these two-body forces not change the spin-averaged distributions. Taking account of the interaction of quarks and gluons as shown in Fig. 2(a), we therefore have

$$\begin{aligned} q_s^1(x)\simeq q_s^0(x)&+\sum_{s_1\neq s,h_1}\int dx_1dx_2\delta(x_1+x_2-x-x')q_{s_1}^0(x_1)\\ &\quad\times G_{h_1}^0(x_2)R_{s_1,h_1;s}(x_1,x_2;x)\\ &-\sum_{h_1,s_1'\neq s}\int dx_1'dx_2\delta(x+x_2-x_1'-x')q_s^0(x)\\ &\quad\times G_{h_1}^0(x_2)R_{s,h_1;s_1'}(x,x_2;x_1')\ , \end{aligned}\tag{2.2}$$

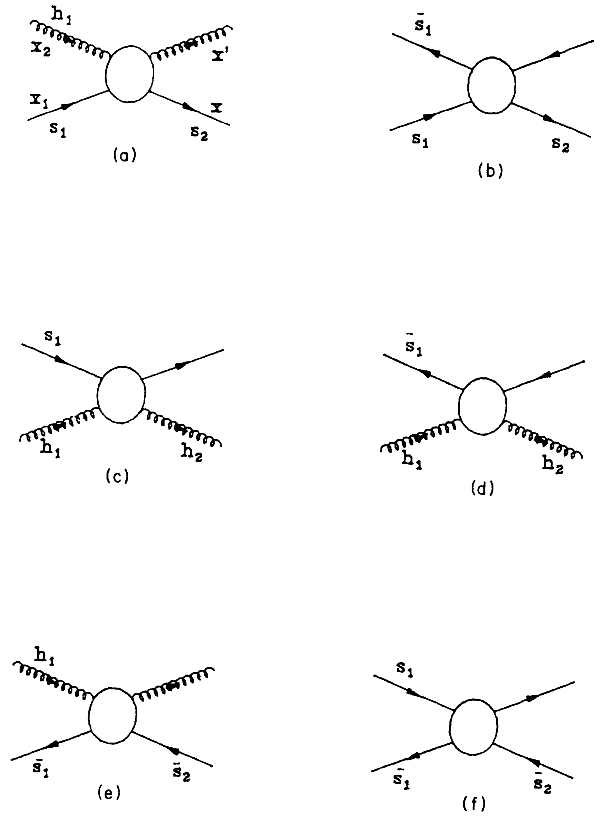

FIG. 1. The scattering diagrams for the flavor-independent processes which contribute to the spin-weighted parton distributions. (a) Quark-gluon spin-flip processes characterized by helicities s_1,s_2,h and momenta x_1,x_2,x,x'. This process contributes to the R terms in Eqs. (2.3)–(2.5). (b) Spin-dependent quark-antiquark process contributing to the parameter Q in Eqs. (2.12) and (2.13). (c) and (d) Gluon-quark and quark-gluon processes which contribute to the spin-flip terms P in Eqs. (2.8) and (2.9). (e) and (f) Antiquark spin-flip interactions which contribute to the $\bar{Q}$ terms of Eqs. (2.15) and (2.16).

where the summation of s_1 or s_1' is over $+\frac{1}{2}$ and $-\frac{1}{2}$, and that of $h_1=+1,-1$. The function $R_{s_1,h_1;s}(x_1,x_2,x)$ represents the interaction between the two participating partons due to the two-body spin force.

Equation (2.2) can be interpreted as follows: the change of the valence-quark density of spin s due to the effective spin force between quarks and gluons is equal to the sum of a "gain" term and a "loss" term. The "gain" term includes all possibilities to have a quark of spin s_1 to turn into a quark of spin s through the effective quark-

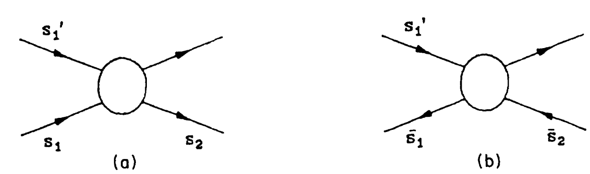

FIG. 2. The scattering diagrams which contribute to the flavor-dependent interactions which determine the O^{ij} terms in Eqs. (2.12)–(2.15).

gluon spin force, while the "loss" term includes all possibilities to have a quark of spin s to turn into a quark of spin not equal to s. To simplify our analysis we can use the assumption of the long-distance nature of the nonperturbative interaction to approximate Eq. (2.2) as

$$q_s^1(x) \simeq q_s^0(x) + \sum_{s_1 \neq s, h_1} \int dx_2 q_{s_1}^0(x) G_{h_1}^0(x_2) \times R_{s_1,h_1;s}(x,x_2;x) - \sum_{h_1, s_1' \neq s} \int dx_2 q_s^0(x) G_{h_1}^0(x_2) R_{s,h_1;s}(x,x_2;x) . \tag{2.3}$$

Although it is assumed that the momentum transfer between two interacting partons is small, the function R can still depend on x and x_2 through the flux factor. Because of parity conservation, we have only two independent spin-flip interaction functions:

$$R_{+1/2,+1;-1/2}(x,x_2;x) = R_{-1/2,-1;+1/2}(x,x_2;x) , \tag{2.4}$$
$$R_{+1/2,-1;-1/2}(x,x_2;x) = R_{-1/2,+1;+1/2}(x,x_2;x) .$$

By defining

$$R = R_{+1/2,+1;-1/2}(x,x_2;x) + R_{+1/2,-1;-1/2}(x,x_2;x) , \tag{2.5}$$
$$\Delta R = R_{+1/2,+1;-1/2}(x,x_2;x) - R_{+1/2,-1;-1/2}(x,x_2;x) ,$$

we can rearrange terms to write

$$\Delta q^1(x) \simeq \Delta q^0(x) - \int dx_2 [\Delta q^0(x) G^0(x_2) R(x,x_2;x) + q^0(x) \Delta G^0(x_2) \Delta R(x,x_2;x)] . \tag{2.6}$$

Note that we also have

$$q^1(x) = q^0(x) , \tag{2.7}$$

so that we have satisfied our requirement that the spin-averaged parton distributions be unchanged by these long-range interactions.

The interaction specified in Fig. 1 can also transfer spin from the valence quarks to the gluons. To emphasize that the final gluon helicity and quark helicity are not necessarily constrained, we will define $P_{h_1,s_1;h_1'}(x,x_2;x)$ to parametrize the dependence of the interaction on gluon helicities. Using the parity invariance of the strong interactions, we can then define

$$P(x,x_2;x) = P_{+1,+1/2;-1}(x,x_2;x) + P_{+1,-1/2;-1}(x,x_2;x), \tag{2.8}$$
$$\Delta P(x,x_2;x) = P_{+1,+1/2;-1}(x,x_2;x) - P_{+1,-1/2;-1}(x,x_2;x) ,$$

and write in analogy to (2.6) an estimate for the change in the gluon asymmetry

$$\Delta G^1(x) \simeq \Delta G^0(x) - \sum_f \int dx_2 [\Delta G^0(x) q_f^0(x_2) P(x,x_2;x) + G^0(x) \Delta q_f^0(x_2) \Delta P(x,x_2;x)] , \tag{2.9}$$

where f runs over quark flavors.

Note that if we now assume

$$\Delta q_v^0(x) = \Delta q_v^{\mathrm{SU}(6)}(x) , \quad \Delta G^0(x) = 0 , \tag{2.10}$$

we obtain

$$\Delta q_v^1(x) \simeq \Delta q_v^{\mathrm{SU}(6)}(x) - \int dx_2 [\Delta q_v^{\mathrm{SU}(6)}(x) G^0(x_2) \times R(x,x_2;x)] , \tag{2.11}$$
$$\Delta G^1(x) \simeq -\int dx_2 G^0(x) \sum_f \Delta q_{v_f}^0(x_2) \Delta P(x,x_2;x) ,$$

which demonstrates both of the features mentioned above: dilution of the spin carried by valence quarks and transfer of spin to the gluons.

To complete the discussion, we must also incorporate two-body interactions involving sea quarks and antiquarks. These interact with the valence quarks, the gluons, and with each other. The total number of combinations for two-body spin-dependent interactions in a proton can become large. Using notation similar to (2.6) and (2.9), we denote the two combinations of quark-antiquark interactions by $Q(x,x_2;x)$ and $\Delta Q(x,x_2;x)$. Because of identical particle effects, the interaction involving quarks is necessarily flavor dependent. We denote the flavor dependence by keeping labels on $O^{ij}(x,x_2;x)$ and $\Delta O^{ij}(x,x_2;x)$. Accounting for all possible two-body interactions gives

$$\Delta q_{v_i}^1(x) \simeq \Delta q_{v_i}^0(x) - \int dx_2 \Big\{ \sum_j [\Delta q_{v_i}^0(x) q_j^0(x_2) O^{ij}(x,x_2;x) + q_{v_i}^0(x) \Delta q_j^0(x_2) \Delta O^{ij}(x,x_2;x)] + \Delta q_{v_i}^0(x) \bar{q}_j(x_2) Q(x,x_2;x) + q_{v_i}^0(x) \Delta \bar{q}_j(x) \Delta Q(x,x_2;x) + \Delta q_{v_i}^0(x) G^0(x_2) R(x,x_2;x) + q_{v_i}^0(x) \Delta G^0(x_2) \Delta R(x,x_2;x) \Big\} . \tag{2.12}$$

The polarization of the quarks in the sea is then given by

$$\Delta q_i^1(x) \simeq \Delta q_i^0(x) - \int dx_2 \left[\sum_j [\Delta q_i^0(x) q_j^0(x_2) O^{ij}(x,x_2;x) + q_i^0(x) \Delta q_j^0(x_2) \Delta O^{ij}(x,x_2;x)] + \Delta q_i^0(x) G^0(x_2) R(x,x_2;x) + q_i^0(x) \Delta G^0(x_2) \Delta R(x,x_2;x) + \Delta q_i^0(x) \sum_j \bar{q}_j^0(x_2) Q(x,x_2;x) + q_i^0(x) \sum_j \Delta \bar{q}_j^0(x_2) \Delta Q(x,x_2;x) \right] . \tag{2.13}$$

The impact on the gluon polarization is given by

$$\Delta G^1(x) \simeq \Delta G^0(x) - \int dx_2 \left[\Delta G^0(x) \sum_f [q^0(x_2) + \bar{q}^0(x_2)] P(x,x_2;x) + G^0(x) \sum_f [\Delta q^0(x_2) + \Delta \bar{q}^0(x_2)] \Delta P(x,x_2;x) \right] . \tag{2.14}$$

Finally, the polarization of the antiquarks can be written

$$\Delta \bar{q}_i^1(x) \simeq \Delta \bar{q}_i^0(x) - \int dx_2 \left[\sum_j [\Delta \bar{q}_i^0(x) \bar{q}_j^0(x_2) O^{ij}(x,x_2;x) + \bar{q}_i^0(x) \Delta \bar{q}_j^0(x_2) \Delta O^{ij}(x,x_2;x)] + \Delta \bar{q}_i^0(x) G^0(x_2) R(x,x_2;x) + \bar{q}_i^0(x) \Delta G^0(x_2) \Delta R(x,x_2;x) + \Delta \bar{q}_i^0(x) \sum_j q_j^0(x_2) Q(x,x_2;x) + \bar{q}_i^0(x) \sum_j \Delta q_j^0(x_2) \Delta Q(x,x_2;x) \right] . \tag{2.15}$$

These equations can be simplified considerably by making the assumption [as in (2.10)] that the initial polarization of the sea quark and antiquark distribution vanishes:

$$\Delta q_i^0(x) = \Delta \bar{q}_i^0(x) = 0 . \tag{2.16}$$

At this point, we will make the further assumption that the interaction kernels, $R(x,x_2;x), Q(x,x_2;x)$ and $O^{ij}(x,x_2;x)$ are sharply peaked at $x_2 \simeq x$ (low subenergy) so that we can approximate the integrations in (2.15) by an algebraic factor. We can then write

$$\begin{aligned} \Delta q_{v_i}^1 &\simeq \Delta q_{v_i}^{SU(6)}(x) \left[1 - x R G^0(x) - \sum_j (x Q \bar{q}_j + x O^{ij} q_j) \right] , \\ \Delta G^1(x) &\simeq -x \Delta P G^0(x) \sum_i \Delta q_{v_i}^{SU(6)} , \\ \Delta \bar{q}_i^1(x) &\simeq -x \Delta Q \bar{q}_i^0(x) \sum_j \Delta q_{v_j}^{SU(6)} , \\ \Delta q_i^1(x) &\simeq -x \Delta O^{ij} q_i^0(x) \sum_j \Delta q_{v_j}^{SU(6)} . \end{aligned} \tag{2.17}$$

These simple equations can now be used to provide a rough estimate for the polarization densities.

It is worthwhile to try to estimate the sign of the terms in Eq. (2.17). For sea quarks and antiquarks, the assumption that the spin-dependent forces enhance the probability that a sea antiquark will combine with a valence quark to form a pseudoscalar ($q\bar{q}$) meson implies[9] $\Delta \bar{q}^1(x) \leq 0$. For the sea quarks, the enhancement of the $S=0$ diquark configuration also leads to $\Delta q_{sea}^1(x) \leq 0$. These signs are consistent with baryon and meson spectroscopy and with the spin-dependent forces observed in lattice simulations. We have no good phenomenological information about the sign of ΔP. One consistent assumption is to take $\Delta P \simeq 0$ so that there is *no* gluon spin density generated at this heuristic nonperturbative level. However we shall see in the next subsection how, at the perturbative level, the bremsstrahlung from the valence quarks will generate a positive ΔG.

The structure of the spin-dilution implied by (2.17) is quite close to that of the Carlitz-Kaur model.[16] The assumption of valence dominance which leads to (2.17) is suggestive of the picture of constituent quarks are assumed to consist of a valence-quark "core" surrounded by gluons and $q\bar{q}$ pairs in the sea. The connection between constituent quarks and current or "parton-level" quarks has been recently discussed within the context of the EMC data in Refs. 25 and 26.

The more general structure of (2.12)–(2.15) is important to keep in mind if more complicated spin-dependent forces are considered. For example, it is interesting to consider putting in a nontrivial initial gluon polarization $\Delta G^0(x)$ to explore its impact on the other distributions.

B. Q^2 evolution and the $J_z = \frac{1}{2}$ sum rule

The nonperturbative dynamical processes described in the preceding section leave the momenta of the interacting partons unaltered. We must consider them as providing start-up distributions at low values of momentum transfer. To account for the evolution of the polarized distributions with increasing Q^2 we must turn to perturbation theory and consider the Altarelli-Parisi[14] equations for the spin-dependent distributions. If we define

$$t = \frac{2}{\beta_0} \ln \left[\frac{\alpha_s(Q_0^2)}{\alpha_s(Q^2)} \right]$$

with

$$\beta_0 = 11 - 2N_f$$

the Altarelli-Parisi equations take the form

$$\begin{aligned} \frac{d}{dt}[\Delta q_v^i(x,t)] &= \Delta P_{qq}(x) \otimes \Delta q_v^i(x,t) , \\ \frac{d}{dt}[\Delta q_c^i(x,t)] &= \Delta P_{qq}(x) \otimes \Delta q_c^i(x,t) + \Delta P_{qG}(x) \otimes \Delta G(x,t) , \\ \frac{d}{dt}[\Delta G(x,t)] &= \sum_i \Delta P_{Gq}(x) \otimes \Delta q_c^i(x,t) + \Delta P_{GG}(x) \otimes \Delta G(x,t) , \end{aligned} \tag{2.18}$$

with

$$P(x)\otimes Q(x,t)\equiv\int_x^1\frac{dz}{z}P(z)Q(x/z;t)\ .$$

The leading-order probability kernels $\Delta P_{ij}(x)$ are given by

$$\begin{aligned}\Delta P_{qq}(z)&=\tfrac{4}{3}\left[\frac{1+z^2}{1-z}\right]_+ ,\\ \Delta P_{qG}(z)&=\tfrac{1}{2}(2z-1)\ ,\\ \Delta P_{Gq}(z)&=\tfrac{4}{3}(2-z)\ ,\\ \Delta P_{GG}(z)&=3\left[\left[\frac{1+z^4}{1-z}\right]_+ +(3-3z+z^2+z^3)\right.\\ &\qquad\left.-\tfrac{7}{12}\delta(1-z)\right]\ .\end{aligned}\tag{2.19}$$

The evolution equations (2.18) and (2.19) and their behavior have been discussed by Einhorn and Soffer in Ref. 3. Other discussions of the implications of these evolution equations can be found in Ref. 4. The small-x behavior of these equations is discussed in Appendix B. We can see from these equations that even if there is no gluon polarization generated from the nonperturbative mechanisms discussed above, the strong positive polarization of $\Delta u_v(x,Q_0^2)$ observed at large x will generate a positive $\Delta G(x,Q^2)$ since $\Delta P_{Gq}(x)$ is everywhere positive. Moreover, the positive $\Delta G(x,Q^2)$ will tend to give a positive polarization to the large-x tail of the sea distributions. However, since the first moments of $\Delta P_{qq}(x)$ and $\Delta P_{qG}(x)$ vanish, the first moment for each flavor

$$\langle\Delta q_i\rangle=\int_0^1 dx\ \Delta q_i(x,t)\tag{2.20}$$

will be constant in t.

It is interesting to consider the impact of these results on angular momentum conservation. For a proton with momentum and spin in the z direction, O(2) invariance gives the conservation of J_z. Writing $J_z=\frac{1}{2}$ in terms of the integrals of the parton densities and angular momentum gives the sum rule

$$\sum_i\tfrac{1}{2}\langle\Delta q_v^i\rangle+\tfrac{1}{2}\langle\Delta q_s^i+\Delta\bar q^{\,i}\rangle_c+\langle\Delta G\rangle+\langle L_z\rangle=\tfrac{1}{2}\ .\tag{2.21}$$

Taking the t derivative of this sum rule, we find

$$\frac{d}{dt}\langle\Delta G\rangle=-\frac{d}{dt}\langle L_z\rangle\ .\tag{2.22}$$

Since the first moments of ΔP_{Gq} and ΔP_{GG} are positive, the net spin associated with gluons will grow with increasing t and will be balanced by orbital angular momentum. This was first discussed by Babcock *et al.*[1] and has been elaborated upon by many others.[27,10]

In the discussion above we have used the c or "chiral" distributions for the q and $\bar q$ in the sea. To consider the evolution of the physical 5 distribution, we must also include a contribution

$$\frac{d}{dt}(\Delta q_5^i-\Delta q_c^i)(x,t)=\frac{d}{dt}\left[-\frac{e^{-\beta_0 t/2}}{4\pi}\hat\gamma(x)\otimes\Delta G(x,t)\right]\ .\tag{2.23}$$

Consistent with Berger and Qiu,[28] we choose

$$\hat\gamma(x)=-(1-2x)(2\ln x-\tfrac{3}{2})\tag{2.24}$$

to specify the relationship between the 5 definition and the c definition of the sea distribution and will use (2.19) and (2.23) to evolve the 5 distributions. Note that the first moments of the 5 distributions are *not* t independent:

$$\frac{d}{dt}\langle\Delta q_5^i\rangle=\frac{-1}{4\pi}\left[e^{-\beta_0 t/2}\sum_i\langle\Delta q_i\rangle\right]\ .\tag{2.25}$$

As Jaffe[7] has pointed out, this equation is responsible for the flavor-singlet moment of the structure function being multiplicatively renormalized. To this order in α_s, the net-spin difference associated with the alternative definitions

$$\Gamma(t)=\sum_i\langle\Delta q_5^i-\Delta q_c^i(t)\rangle=\frac{N_f\alpha_s(t)}{2\pi}\langle\Delta G(t)\rangle\tag{2.26}$$

goes asymptotically to a constant. We have to measure $\Delta G(x,Q^2)$ for a range of x at large Q^2 in order to find out the magnitude of this constant and judge whether it is phenomenologically significant. For some applications it may be necessary to go beyond the "leading-order" Altarelli-Parisi equations discussed here in order to get a consistent picture of the impact of polarized glue.

The small-x behavior of the valence spin-dilution angle $\cos\Theta_D$ can be investigated by using the nonsinglet Altarelli-Parisi equation in the form[3]

$$\frac{d}{dt}(\cos\Theta_D)=\frac{1}{2\pi\beta_0}\int_x^1\frac{dy}{y}P_{qq}\left[\frac{x}{y}\right]\frac{q(y)}{q(x)}\times\left[\frac{\Delta q(y)}{q(y)}-\frac{\Delta q(x)}{q(x)}\right]\ .\tag{2.27}$$

We will assume the small-x behavior of the unpolarized $[q(x)]$ and polarized $[\Delta q(x)]$ quark densities to be governed by Regge behavior. Thus, $q(x)\sim x^{-\alpha_\rho}$ and $\Delta q(x)\sim x^{-\alpha_{A_1}}$. Equation (2.27) can therefore be written as

$$\frac{d}{dt}(\cos\Theta_D)=\frac{4x^{\alpha_\rho}}{27\pi}\int_x^1 dy\left[\frac{1+(x^2/y^2)}{y-x}\right]\times(y^{\alpha_{A_1}}-y^{-\alpha_\rho}x^{\alpha_\rho-\alpha_{A_1}})\ .\tag{2.28}$$

If we take $\alpha_\rho\simeq\frac{1}{2}$ and $\alpha_{A_1}\simeq0$, we can evaluate the integral explicitly:

$$\frac{d}{dt}(\cos\Theta_D)\underset{x\to0}{\sim}x^{1/2}\ln\left[\frac{1}{x}\right]\sim\cos\theta_D\ln\left[\frac{1}{x}\right]\ .\tag{2.29}$$

This is an expected result, since a higher-Q^2 probe reveals the small-x behavior of the constituents. It is also compatible with the moment treatment of Einhorn and Soffer in Ref. 3.

A similar analysis can be done for the gluon distribution. If we assume $\Delta G=xG$ and $\Delta S\simeq0$ at some Q_0^2, then

$$\frac{\partial}{\partial t}\left[\frac{\Delta G}{xG}\right]=\frac{(xG)\frac{\partial \Delta G}{\partial t}-\Delta G\frac{\partial (xG)}{\partial t}}{(xG)^2}$$

$$=\frac{1}{\Delta G}[(\Delta P_{GG}-xP_{GG})\otimes\Delta G] \ . \tag{2.30}$$

Using the usual forms for ΔP_{GG} and P_{GG}, we get

$$\frac{\partial}{\partial t}\left[\frac{\Delta G}{xG}\right]=\frac{6}{\Delta G}\{[1-x^2+x^3-\tfrac{25}{8}\delta(1-x)]\otimes\Delta G\} \ . \tag{2.31}$$

The convolution in (2.31) brings in an incomplete beta function $B_x(\alpha,\beta+1)$. At small x,

$$B_x(\alpha,\beta+1)\simeq x^\alpha\left[1-\frac{\beta x}{\Gamma(1+\alpha)}\right] ,$$

where $\Gamma(1+\alpha)$ is the Γ function. Using this expansion with $\alpha=-\frac{1}{2}$ and $\beta=6$ for the polarized gluon distribution, we get the correction

$$\frac{\partial}{\partial t}\left[\frac{\Delta G}{xG}\right]\simeq -12.75+33x \quad (<0 \quad \text{for } x<0.38) \ . \tag{2.32}$$

For further discussion of the small-x behavior we refer the reader to Appendix B.

III. MODELS FOR THE SPIN-WEIGHTED DISTRIBUTIONS

We have studied some of the dynamical assumptions that can help us formulate models of the spin structure of the proton. In this section we shall examine some of the theoretical and phenomenological constraints that have to be satisfied, and present some explicit models for the spin-weighted parton distributions. We do not intend in this paper to give an exhaustive parametrization of the various possibilities, but rather to illustrate a range of allowed behavior for these basic functions. We will address the question of completing the specification of these distributions using new experimental tests in Sec. IV.

The basic sources of information for fixing the parameters in our models are data on the longitudinal spin-spin asymmetry in deep-inelastic lepton-proton scattering and the sum rules involving the polarized parton distributions. In QCD, the structure function $g_1^p(x,Q^2)$ is given by

$$g_1^p(x,Q^2)=\tfrac{1}{2}\sum_i e_i^2\Delta q_i(x,Q^2)\otimes\left[\mathbb{1}-\frac{\alpha_s}{\pi}\tau_i(x)+\cdots\right]+\cdots , \tag{3.1}$$

where $\tau_i(x)$ is a calculable correction factor, which depends, in general, on the factorization prescription used. In our prescriptions, the first moment of $\tau_i(x)$ is one. In one scenario, we use the integral of g_1^p to constrain our parameters. In the other, it is used as an experimental comparison to the theoretical model which limits the polarization of the strange sea. As discussed previously, we distinguish between valence and sea contributions. In analyzing the QCD corrections in Eq. (3.1) it can be seen that the sea components of $g_1^p(x,Q^2)$ receive a contribution from the anomaly discussed earlier. Since g_1^p is measured by experiment, the parton density expansion in (3.1) is most conveniently given in terms of the Δq_5^i distributions, so that the gluon contribution is absorbed in the sea. For numerical work, we will use (3.1) with $\tau_i^5(x)\simeq 1$. Defining the first moments of the parton densities as in Sec. II, the polarized gluon distribution relates the measured spin densities to the chiral densities (satisfying the lowest-order evolution equations) by

$$\sum_i(\langle\Delta q_5^i\rangle-\langle\Delta q_c^i\rangle)=\frac{N_f\alpha_s(Q^2)}{2\pi}\langle\Delta G(Q^2)\rangle\equiv\Gamma(Q^2) , \tag{3.2}$$

which may or may not be large.

The first of three sum rules we will use to fix some of the model parameters is the Bjorken sum rule:[29]

$$A_3=\int_0^1 dx[\Delta u_v(x,Q^2)-\Delta d_v(x,Q^2)]_c$$
$$=1.258\pm0.004 \ . \tag{3.3}$$

The value of A_3 is determined from neutron β decay. Since the anomaly terms cancel in the difference, the distributions used can be either the measured or the chiral ones. For now, we will use the chiral distributions. The second sum rule we use involves an SU(3) rotation of the Bjorken sum rule which involves hyperon decay and relates the valence-quark integrals to the strange quarks in the sea. A traditional SU(3) analysis of hyperon decays determines two empirical constants $D=0.795\pm0.009$ and $F=0.466\pm0.009$ (Ref. 30), and gives

$$A_8=\int_0^1 dx(\Delta u_v+\Delta d_v+\Delta u_s+\Delta d_s$$
$$+\Delta\bar{u}+\Delta\bar{d}-2\Delta s-2\Delta\bar{s})_c$$
$$=3F-D\simeq 0.60\pm0.02 \ . \tag{3.4}$$

The anomaly terms also cancel from this integral, so that the sum rule in this form restricts the chiral distributions. The third sum rule represents the total spin carried by all quarks and antiquarks and is just the matrix element of the singlet current A_0, given in terms of the chiral distributions by

$$(A_0)_c=\int_0^1 dx(\Delta u_v+\Delta d_v+\Delta u_s+\Delta d_s$$
$$+\Delta\bar{u}+\Delta\bar{d}+\Delta s+\Delta\bar{s})_c \ . \tag{3.5}$$

Since the anomaly terms do not cancel in this sum rule, there is a difference between the chiral $(A_0)_c$ and the value of A_0 determined from the data: namely,

$$(A_0)_5=(A_0)_c-\Gamma(Q^2) , \tag{3.6}$$

with $\Gamma(Q^2)$ given by Eq. (3.2). To compare the theoretical models with data, we can express A_0 in terms of A_3, A_8, and the measured value of the integral of xg_1^p as follows:

$$(A_0)_c=9\left[1+\frac{\alpha_s}{\pi}\right]\int_0^1 dx\, g_1^p(x)-\tfrac{1}{4}A_8-\tfrac{3}{4}A_3+\Gamma \ . \tag{3.6a}$$

From (3.5) we can relate this to the fraction of the proton's helicity carried by quarks

$$(A_0)_c \equiv f_v + f_s \equiv f_v + \tilde{f}_s + \Gamma \ . \tag{3.7}$$

Here, f_v and f_s are the fractions of the proton spin carried by the valence and sea quarks, respectively, given by the chiral distributions. Only f_s is modified by the anomaly as $f_s = \tilde{f}_s + \Gamma$. In the rest of this section, all polarized distributions given in the models will be assumed to be the chiral ones, unless otherwise noted.

For calculations of our valence-quark distributions, we start with the simple SU(6) wave functions described in Appendix A [Eqs. (A12) and (A13)]. We will use the *unpolarized* parton distributions given by Gluck, Hoffmann, and Reya,[31] evaluated at $Q_0^2 = 10.7$ GeV^2:

$$xu_v(x,Q_0^2) = 1.39x^{0.419}(1-x^{1.94})^{3.40} \tag{3.8}$$

and

$$xd_v(x,Q_0^2) = 0.65x^{0.361}(1-x^{1.94})^{5.11} \ .$$

We can then evolve the polarized distributions via the Altarelli-Parisi equations.[32] Defining the evolution parameter t to be

$$t \equiv \frac{2}{\beta_0} \ln \left[\frac{\ln(Q^2/\Lambda^2)}{\ln(Q_0^2/\Lambda^2)} \right]$$

with $\Lambda = 0.2$ GeV, the total polarized valence-quark distribution can be parametrized as

$$x\,\Delta u_v(x,t) + x\,\Delta d_v(x,t) = 2x^{1.05(1-3t)}(1-x)^{3.1(1+1.3t)} \times [1+2.7(1-5t)x] \ . \tag{3.9}$$

The other available parametrizations of the quark and antiquark distributions differ by only a few percent in the range 5 GeV$^2 < Q^2 < 500$ GeV^2. Hence, our models for $\cos\Theta_D$ can be used with these distributions without modification.

We shall consider two forms for the spin-dilution angle. The first is that proposed by Carlitz and Kaur[16] (CK) using statistical arguments, and takes the form

$$\cos\Theta_D = [1 + H_0(Q^2)(1-x)^2/\sqrt{x}\,]^{-1} \quad \text{(CK model)} \ . \tag{3.10}$$

The second is derived from the spin-flip (SF) model described in Sec. II, with $\cos\Theta_D$ given by

$$\cos\Theta_D = [1 - RxG^0(x)] \ . \tag{3.11}$$

At low x, $xG^0(x)$ becomes large. However many of the dynamical assumptions underlying the SF model lose their validity at very small x; in particular there is substantial shadowing,[33] and multiparton processes becomes important. To account for these effects, we replace Eq. (3.11) by

$$\cos\Theta_D = [1 + R(Q^2)xG(x,Q^2)]^{-1} \quad \text{(SF model)} \ , \tag{3.12}$$

which is well behaved at small x, and may be viewed in the spirit of "resumming" the multiparton interactions. For the unpolarized gluon distribution occurring in the spin-dilution factor of Eq. (3.12), there is a wide range of uncertainty, especially at small x. Since the spin-flip model dilution factor contains the gluon distribution, which is large at small x, we can more closely match the data by adjusting the power of x in $G(x,Q^2)$. This power is subject to constraints in the unpolarized data used to determine the gluon distribution at the appropriate Q_0^2. We choose the unpolarized gluon distribution to be parametrized as

$$G(x,Q^2) = A_g x^{-\alpha_g}(1+\gamma_g x)(1-x)^{\beta_g} \ . \tag{3.13}$$

Then, at small x, our spin-dilution factor has the form

$$\cos\Theta_D = (1 + RA_g x^{1-\alpha_g})^{-1} \simeq \frac{1}{RA_g} x^{\alpha_g - 1} \ . \tag{3.14}$$

The Carlitz-Kaur form for $\cos\Theta_D$ is numerically equivalent to choosing $\alpha_g = 1.5$ when using the EMC data to fix the sea parameters. If this value is decreased, as has been suggested by other models,[34] the theoretical values may match the data very well at all x. See Figs. 3 and 4.

It should be noted that in both models the spin-dilution angle is flavor independent, and thus can be determined by the Bjorken sum rule

$$A_3 = \int_0^1 dx[u_v(x,Q^2) - \tfrac{1}{3}d_v(x,Q^2)]\cos\Theta_D(x,Q^2)$$
$$= 1.258 \pm 0.004 \ . \tag{3.15}$$

In particular, evaluating the integral at $Q^2 = 10.7$ GeV^2, we obtain $H_0 = 0.10$ for the CK model, and $R = 0.18$ for the SF model. For both models, the fraction of the proton spin carried by the valence quarks, f_v, is about 78%. It is interesting to note that this result is consistent to within ten percent of the naive SU(2) flavor constraint on the ratio of the proton and neutron magnetic moments, given by

$$\frac{\mu_p}{\mu_n} = \frac{2\langle \Delta u_v \rangle - \langle \Delta d_v \rangle}{2\langle \Delta d_v \rangle - \langle \Delta u_v \rangle} = -\frac{3}{2} \ . \tag{3.16}$$

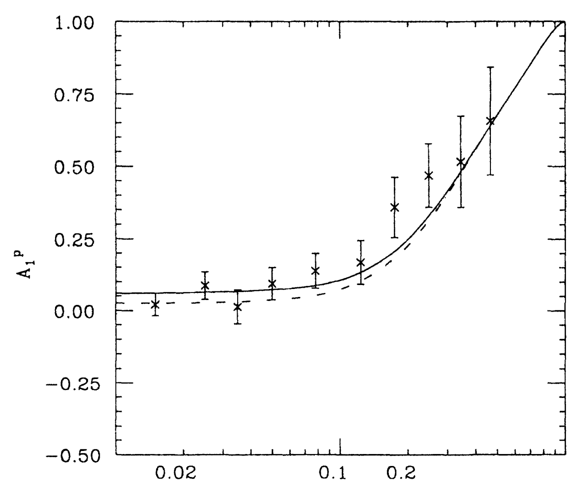

FIG. 3. The proton asymmetry A_1^p as a function of Bjorken x using the parton distributions motivated by the EMC data. The solid line corresponds to $\alpha_g = 1.5$ and the dot-dash line to $\alpha_g = 1.2$ in the valence spin-dilution factor of Eq. (3.12).

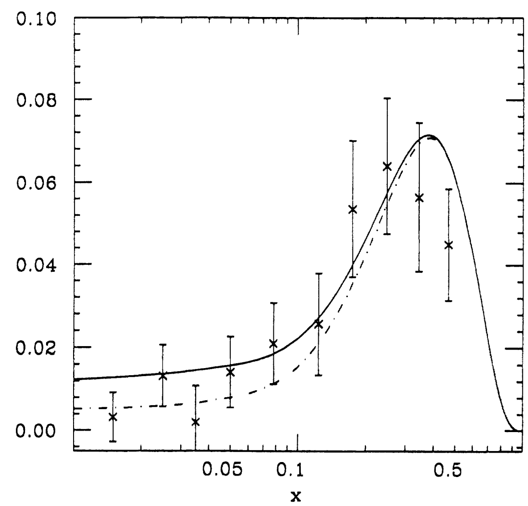

FIG. 4. The structure function xg_1^p is shown as a function of x using the EMC-motivated parton distributions. The α_g factors are the same as those in Fig. 3.

Having determined a suitable form for the polarized valence distributions, we now consider the polarized gluon distribution. Traditionally, this distribution was thought to be small at accessible values of Q^2, limited by the gluon's momentum. However, recent attention to the γ_5 anomaly[13] has allowed speculation that the polarized gluon distribution may be significantly larger and that large positive values of $\Delta G(x,Q^2)$ are needed to fit the data. Possible models for $\cos\Phi_L\cos 2\theta_g$ fall into three basic categories. The first is an extreme version, which assumes $\cos\Phi_L\cos 2\theta_g=0$ at some value of Q_0^2. In this model one can then generate the polarization of the gluons using the Altarelli-Parisi equations. This model of the gluon polarization is consistent with the Skyrme approach to the proton structure. As we pointed out in Ref. 10, it is possible to avoid the growth of $\langle \Delta G(Q^2)\rangle$ and $\langle L_z(Q^2)\rangle$ using this starting point. The Altarelli-Parisi equations still generate a positive $\Delta G(x,Q^2)$ at large x.

A second starting point assumes $\cos\Phi_L\cos 2\theta_g \simeq x$ at $Q_0^2\simeq 10.7\ \text{GeV}^2$. With this assumption, the net spin carried by gluons is approximately equal to the fraction of momentum carried by the gluons: $\langle \Delta G(Q_0^2)\rangle \simeq \int_0^1 dx\, xG(x,Q^2)\simeq 0.50$. Depending on the form for the $\Delta q_i(x,Q^2)$, this starting point is relatively stable under the Altarelli-Parisi evolution equations. This model can be compatible with either the EMC-motivated quark distributions or with theoretically motivated distributions. In this model, $\Gamma(Q^2)$, the difference between the process-independent spin fraction carried by the sea f_s and the measured spin fraction $\tilde{f}_s$, is about 0.06.

Finally, one can assume that the spin-spin forces discussed in Sec. II lead to a saturation of the gluon polarization and $\cos 2\Phi\cos\theta_g\simeq 1$ at large x. This dramatic possibility is suggested by the discrepancy of the EMC data with certain theoretical estimates on the amount of sea polarization allowed. It is compatible with what little we know about nonperturbative spin-spin forces. One explicit parametrization is

$$\begin{aligned}&\cos 2\Phi\cos\theta_g=\frac{x}{x_0},\quad x<x_0\ ,\\&\cos\theta_g\simeq 1,\quad x>x_0\ ,\end{aligned}\tag{3.17}$$

with the further evolution of the structure function given by the Altarelli-Parisi equations. Note that the second starting point discussed above corresponds to $x_0=1$ in this model. If we choose x_0 at the other extreme, say at the smallest value of x in the EMC data, then $x_0\simeq 0.04$ and $\langle \Delta G(x)\rangle\simeq 5.0$, which implies that $\Gamma(Q^2)\simeq 0.65$. Thus, if we use this model for the gluon polarization there is considerable difference between the measured and calculated values of the spin fraction of the proton carried by the sea.

In order to complete our specification of the spin-weighted parton distributions, we now turn to the sea distributions. As discussed in Sec. II, there exist good theoretical reasons to believe *ab initio* that the quarks and antiquarks in the sea are polarized. As we will see, the baryon decays and the EMC data suggest strongly that the sea pairs are polarized in a direction opposite to the proton's spin. However, the experimental errors and theoretical ambiguities in the interpretation of the data leave a good deal of uncertainty concerning the magnitude of polarization.

The polarized sea distribution consists of the chiral parton distributions:

$$\begin{aligned}\Delta S(x)=&\Delta u_s(x)+\Delta\bar{u}(x)+\Delta d_s(x)\\&+\Delta\bar{d}(x)+\Delta s(x)+\Delta\bar{s}(x)\ .\end{aligned}\tag{3.18}$$

In order to construct our models for the process-independent distributions, we make the following assumptions relating the various components of quarks in the sea. Since the spin-spin forces are different in $q\bar{q}$ and qq channels, we make the distinction that

$$\Delta\bar{q}(x,Q_0^2)=c(x)\Delta q_s(x,Q_0^2)\ ,\tag{3.19}$$

at some Q_0^2 for each flavor. The parameter $c(x)$ represents the enhancement of antiquarks over quarks in the sea. As a first approximation, we let $c(x)$ be constant and flavor independent. We have implicitly assumed that the quark and antiquark distributions have the same x dependence. Additionally, we assume isospin invariance of the sea with respect to the up and down quarks, but a broken-flavor-SU(3) sea so that

$$\Delta u_s=\Delta d_s=[1+\epsilon(x)]\Delta s\ .\tag{3.20}$$

The symmetry-breaking parameter ϵ indicates the difficulty in polarizing the strange quarks due to their larger masses. The x dependence of ϵ is most likely related to the quark masses, but as a first approximation, we assume a mild mass dependence, so that ϵ is almost constant over the important kinematic range. Although the sea quark and antiquark distributions are distinct, we will require the same relations between flavors for the antiquarks as in Eq. (3.20): namely,

$$\Delta\bar{u}=\Delta\bar{d}=[1+\bar{\epsilon}(x)]\Delta\bar{s}\ . \tag{3.21}$$

Since we have assumed that c is flavor independent, it is reasonable to conclude that $\epsilon=\bar{\epsilon}$.

With these assumptions we can write all of the sea components in terms of $\Delta\bar{u}$, which is the dominant distribution of the sea within the proton. The values of $\Delta\bar{u}$ and its integral over x give an accurate indication of the contributions of the sea polarization to the proton spin. Then,

$$\Delta S=\left[\left[\frac{1}{c}+1\right]\left[\frac{3+2\epsilon}{1+\epsilon}\right]\right]\Delta\bar{u}$$

and

$$\Delta\bar{u}=c\,\Delta u_s=\Delta\bar{d}=c\,\Delta d_s=(1+\bar{\epsilon})\Delta\bar{s}=c(1+\epsilon)\Delta s\ . \tag{3.22}$$

Now we can rewrite the A_8 sum rule in the form

$$A_8=\langle\Delta u_v\rangle_c+\langle\Delta d_v\rangle_c+\left[\frac{2\epsilon}{1+\epsilon}\right]\left[1+\frac{1}{c}\right]\langle\Delta\bar{u}\rangle_c\ . \tag{3.23a}$$

Using an SU(3)-flavor analysis of data on hyperon decays gives a value

$$A_8=0.60\pm0.02 \tag{3.23b}$$

since the Bjorken sum rule plus our assumption concerning the isospin independence of $\cos\Theta_D$ fixes

$$\langle\Delta u_v\rangle_c+\langle\Delta d_v\rangle_c=0.78\pm0.01\ . \tag{3.23c}$$

The A_8 sum rule requires that the total spin carried by a sea of $q\bar{q}$ pairs, f_s, be negative, but allows a considerable range in the magnitude of that contribution, due to the experimental error and the theoretical uncertainty in ϵ.

This estimate is *independent* of the EMC data. Any anomalous contribution from the gluons cancels in A_8 so we have an estimate for

$$\frac{2\epsilon}{1+\epsilon}\langle\Delta u+\Delta\bar{u}\rangle_c=-0.18\pm0.01 \tag{3.24}$$

directly from symmetry arguments applied to baryon decays.

If we accept SU(3) flavor as a good symmetry for baryon decays we can use the parton model expression for the integral of g_1^p and the A_8 sum rule to set reasonable limits on the polarized gluon distribution. The latest analysis of the EMC data[35] gives a value for the integral:

$$\int_0^1 dx\, g_1^p(x)=0.126\pm0.010(\text{stat})\pm0.015(\text{sys})\ . \tag{3.25}$$

(Note that this differs from earlier numbers.) Combining statistical and systematic errors and using $\alpha_s=0.27$ we can then write the integral with the QCD correction as

$$\begin{aligned}\left[1-\frac{\alpha_s}{\pi}\right]^{-1}2\int_0^1 dx\, g_1^p(x)&=0.272\pm0.037\\ &=\tfrac{4}{9}\langle\Delta u_v\rangle+\tfrac{1}{9}\langle\Delta d_v\rangle+\tfrac{4}{9}\langle\Delta u_s+\Delta\bar{u}\rangle_5+\tfrac{1}{9}\langle\Delta d_s+\Delta\bar{d}+\Delta s+\Delta\bar{s}\rangle_5\\ &=\tfrac{4}{9}\langle\Delta u_v\rangle+\tfrac{1}{9}\langle\Delta d_v\rangle+\frac{6+5\epsilon}{9}\langle\Delta s+\Delta\bar{s}\rangle_c-\tfrac{2}{9}\Gamma\ .\end{aligned} \tag{3.26}$$

The value of $\Gamma(Q^2)$ obtained from this exercise depends on the amount of SU(3) breaking in the sea polarization. Using (3.15) and (3.22) we can write this in the form

$$-1.39\pm0.33=\left[5+\frac{1}{1+\epsilon}\right]\langle\Delta u_s+\Delta\bar{u}\rangle_c-2\Gamma\ .$$

From (3.24), we have, for $\epsilon=1$,

$$\Gamma(Q^2)|_{\epsilon=1}=0.202\pm0.180$$

and (using $\alpha_s=0.27$)

$$\langle\Delta G\rangle=(1.57\pm1.39)\ .$$

We will not use these arguments to constrain the value of $\langle\Delta G\rangle$ in our models but will keep the flexibility mentioned above.

In the two major scenarios which follow, the first uses the EMC data to fix the sea parameters via the A_8 and A_0 sum rules and the second uses theoretical arguments and the A_8 sum rule to restrict the size of the polarized sea. The first scenario we consider assumes a small $\langle\Delta G\rangle$ and uses the EMC experimental determination of $\int_0^1 dx\, g_1^p(x,Q^2)$ to fit the parameters. After choosing a suitable value for the parameter c, we find $(A_0)_c$ from Eq. (3.6a) and then ϵ using Eqs. (3.4) and (3.5). The A_8 sum rule will then determine $\langle\Delta\bar{u}\rangle$, with the other sea parameters to follow. Using $\alpha_s\simeq0.27$, this fixes the value of the spin fractions $f_v+\tilde{f}_s$ to be 0.14 ± 0.18, which is consistent with the conjecture that the net quark contribution to the proton spin is small. From the A_8 sum rule, Eq. (3.4), we find that $\epsilon=0.6\pm0.4$ and the total spin carried by each of the quark flavors, including valence and sea, is

$$\begin{aligned}&\langle\Delta u_v\rangle+\langle\Delta u_s+\Delta\bar{u}\rangle=1.02+(-0.24)=0.78\ ,\\ &\langle\Delta d_v\rangle+\langle\Delta d_s+\Delta\bar{d}\rangle=-0.25+(-0.24)=-0.49\ ,\\ &\langle\Delta s+\Delta\bar{s}\rangle=-0.15\ .\end{aligned} \tag{3.27}$$

The total contribution from all flavors of sea quarks in this scenario is calculated to be -0.63 ± 0.60.

If we assume that the polarized gluon distribution is large, say $\langle\Delta G\rangle\simeq5$, then when the above results are translated into parton model language, we find that

$$f_v+f_s=f_v+\tilde{f}_s+N_f\frac{\alpha_s}{2\pi}\langle\Delta G\rangle\simeq0.79\pm0.25\ , \tag{3.28}$$

consistent with the naive quark model. A theoretical concern with this scenario is that the spin carried by the sea-quark pairs is much larger than the corresponding momentum. This is easily seen by comparing the integrals over x of the unpolarized and polarized sea. Using the EMC data, $\int_0^1 dx\, \Delta S = -0.81$, while $\int_0^1 dx\, xS \simeq 0.15$ for typical models of the unpolarized sea.[31]

To resolve this discrepancy, we introduce a theoretically motivated model in which the sea quarks obtain their spin through a localized interaction with the valence quarks. In this valence-dominated model (VDM), the spin-transfer density of the sea antiquarks is limited by $\Delta\bar{s}(x) \simeq -\frac{1}{3}x\bar{s}(x)$. For simplicity, we assume that the polarized strange quarks are similarly limited by $c\,\Delta s(x) \simeq -\frac{1}{3}xs(x)$. Then, for the process-independent sea distributions,

$$\frac{\Delta S}{S} = \frac{(3+2\epsilon)(c\,\Delta s + \Delta\bar{s})}{5(s+\bar{s})} = -\frac{3+2\epsilon}{15}x \ . \tag{3.29}$$

Here, we have used the conjecture that the unpolarized strange quarks comprise about 20% of the unpolarized sea.[9] Then the total spin carried by the sea is related to the total momentum of the sea by

$$f_s \equiv \int_0^1 dx\, \Delta S = -\frac{3+2\epsilon}{15}\int_0^1 dx\, xS \ . \tag{3.30}$$

Typical values of the last integral range from 0.10 to 0.15 (Ref. 20). Using the larger value for this integral and the A_8 and A_0 sum rules, the values of the sea parameters become

$$\begin{aligned} &\epsilon = 12 \pm 1.8 \ , \\ &f_v + \tilde{f}_s = 0.56 \pm 0.07 - N_f \frac{\alpha_s(Q^2)}{2\pi}\langle \Delta G(Q^2)\rangle \ , \\ &\int_0^1 dx\, g_1^p = 0.170 \pm 0.008 - (0.1)N_f \frac{\alpha_s(Q^2)}{2\pi}\langle \Delta G(Q^2)\rangle \ . \end{aligned} \tag{3.31}$$

For a small polarized gluon distribution, the integral of g_1^p falls outside the error bars of the EMC value. The large value of ϵ implies that the strange sea is much harder to polarize, possibly due to mass differences of the quark flavors. We compare the VDM-based functions A_1^p and g_1^p (versus x) to data in Figs. 5 and 6. The fraction of spin carried by all quarks in this model is about one-half of the total spin of the proton, which is significantly greater than that implied by the model which directly fits the EMC data. These conclusions are similar to other theoretical models.[7] Preparata and Soffer[36] have analyzed the EMC data and have made some reasonable assumptions concerning the small-x behavior of the polarized distributions. Their analysis leads to the bound

$$\langle \Delta s + \Delta\bar{s}\rangle \leq 0.072 \pm 0.030 \ . \tag{3.32}$$

Our valence-dominated model includes a smaller negatively polarized sea contribution consistent with the above analysis and indicates a very large symmetry-breaking effect ($\epsilon \simeq 12.0$). Here, the spin carried by the quark flavors is

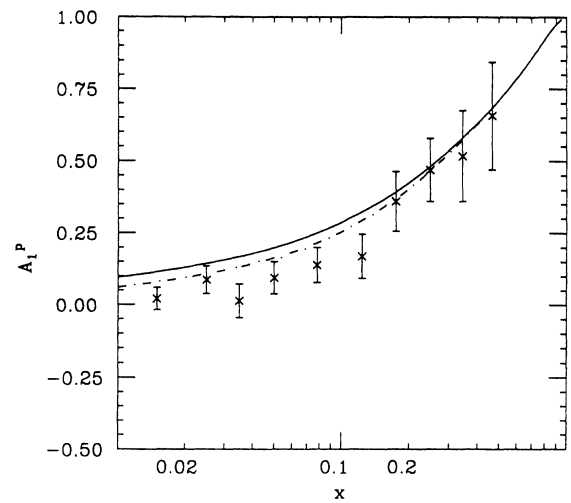

FIG. 5. The proton asymmetry A_1^p plotted as a function of x using the parton distributions obtained from the valence-dominated model (VDM) discussed in the text. The α_g factors are the same as in Fig. 3.

$$\begin{aligned} &\langle \Delta u_v \rangle + \langle \Delta u_s + \Delta\bar{u}\rangle = 1.02 + (-0.10) = 0.92 \ , \\ &\langle \Delta d_v \rangle + \langle \Delta d_s + \Delta\bar{d}\rangle = -0.25 + (-0.10) = -0.35 \ , \\ &\langle \Delta s + \Delta\bar{s}\rangle = -0.01 \ , \end{aligned} \tag{3.33}$$

indicating that the quarks carry a larger fraction of the proton's spin than in the EMC-motivated model. We note that introducing a large polarized gluon distribution on the order of $\langle \Delta G\rangle = 3.5$ in this model yields a value for the integral of g_1^p in agreement with the EMC experimental value. This is consistent with the limits on $\langle \Delta G\rangle$ given by Eq. (3.26).

The EMC-motivated model has a significantly larger negatively polarized sea and less SU(3)-flavor breaking (small ϵ) than the valence-dominated model (VDM). The larger value of $\langle \Delta s + \Delta\bar{s}\rangle$ has been connected to the

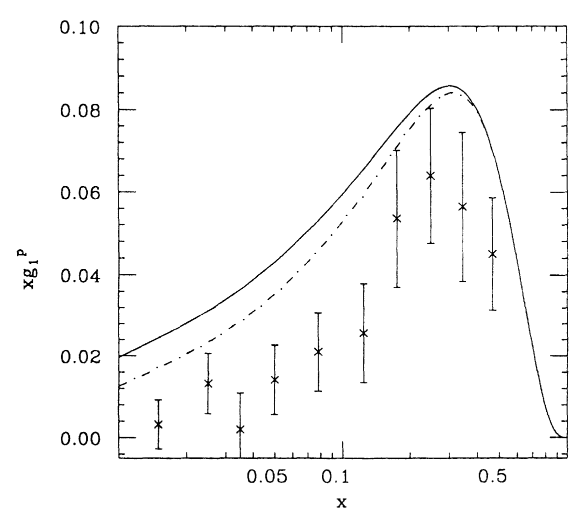

FIG. 6. The structure function xg_1^p shown as a function of x calculated from the VDM distributions, as in Fig. 5.

large value of the σ term in πN scattering, which also suggests a large amount of strangeness in the proton. It has also been suggested that the weak neutral-current data $\nu p \to \nu p$ and $\bar{\nu} p \to \bar{\nu} p$ support this result by providing an estimate $\langle \Delta s + \Delta \bar{s} \rangle = -0.15 \pm 0.09$. On the other extreme, the VDM with a smaller polarized gluon contribution is more consistent with a Skyrme model of the proton. Only a comprehensive experimental program in polarized beams can help to distinguish the accuracy of these theoretical models.

IV. A PROSPECTIVE EXPERIMENTAL PROGRAM

We have discussed a range of possible models for the spin-weighted parton distributions. As we have seen, the valence-quark distributions can be determined with some confidence from existing data on the longitudinal asymmetry in deep-inelastic lepton-proton scattering. Additional experimental information is needed in order to determine the spin-weighted gluon and sea distributions. Only when these distributions have been measured can we proceed to formulate tests of QCD using spin-related observables. In this section, we will discuss a specific set of experiments in terms of the models introduced in Sec. III. This will enable us to estimate the sensitivity of the distributions to new experimental data.

A complete set of measurements of this type will require high-energy beams of polarized protons. There have been important technical developments along this line which mean that hard-scattering processes with polarized protons can be measured in kinematic regimes where QCD perturbation theory can be valid.[2,3] Further progress should vastly improve the ability to do measurements of this type. For example, recent progress in the development of Siberian snakes may allow the acceleration of high-energy polarized proton beams and the design of polarized colliding beam facilities.[37] For this paper, however, we will concentrate on measurements which can be done with existing facilities. We shall consider in turn experiments which are primarily sensitive to valence-quark distributions, to the antiquark distribution, and, finally to the gluon distribution.

The interest generated by the measurements of the EMC group has served to emphasize the need for further precision measurements of the lepton-scattering asymmetry. Measurements at small x can serve to reduce the uncertainties in the extrapolation of the integral of $g_1^p(x,Q^2)$ mentioned by Close and Roberts.[8] A natural extension of the work on the proton involves a measurement of A_1^{en}. A set of measurements of A_1^{en} would allow an experimental test of the Bjorken sum rule. Using Crewther's relation,[38] the perturbative corrections to this sum rule are known[39] and the sum can be written in the modified minimal-subtraction ($\overline{\text{MS}}$) prescription

$$\left[1+\frac{\alpha_s}{\pi}(Q^2)+c_2\left[\frac{\alpha_s(Q^2)}{\pi}\right]^2+\cdots\right] \times \int dx[g_1^p(x,Q^2)-g_1^n(x,Q^2)]=g_A \,, \quad (4.1)$$

where the coefficients on the left-hand side of (4.1) are the same as those in $R_{e^+e^-}$. All of the models we consider treat the Bjorken sum rule as inviolate but we recognize that it is crucial to test it experimentally. In particular, Gianelli, Nitti, Sforza, and Preparata[40] and Preparata, Ratcliffe, and Soffer[41] have examined the possibility that our basic understanding of QCD may be flawed in such a way to allow a violation of the Bjorken sum rule.

In addition to allowing a test of the Bjorken sum rule, *en* data provide significant additional information concerning nucleon structure. Because of charge factors, $g_1^n(x,Q^2)$ will be more sensitive to the polarization of the sea than is $g_1^p(x,Q^2)$. Predictions for A_1^{en} using both the EMC- and VDM-motivated models are shown in Fig. 7. In Fig. 8 we show the possibility for A_1^{ed} using the EMC and VDM models, with the assumptions that the deuteron is an incoherent superposition of proton and neutron, and that the "contamination" from a small component of D wave in the deuteron wave function can be ignored.

Semi-inclusive processes involving the associated production of new flavors offer an alternate mechanism for probing the sea. For example, many analyses of the EMC data suggest that a substantial (negative) component of the spin proton can be ascribed to the strange sea. It is thus tempting to look for other quantities that may be sensitive to the spin-weighted distributions of heavy quarks. A promising candidate is the semi-inclusive electroproduction of strange and charmed quarks. We expect the ratio of charmed to strange quarks in the proton sea at a scale Q^2 to be given by

$$\frac{d\sigma(c+\bar{c})}{d\sigma(s+\bar{s})} \simeq \frac{m_\phi^2+Q^2}{m_\psi^2+Q^2}\frac{m_\phi^2}{m_\psi^2}\frac{\frac{4}{9}}{\frac{1}{9}} \simeq 0.20 \,, \quad (4.2)$$

at $Q^2 \simeq m_\psi^2$. Thus in this picture the charmed sea comprises about 4% of the total sea. It is presumably more difficult to polarize a charmed quark than a strange quark. Nevertheless the polarized charmed sea could also be a few percent of the polarized sea, and it may be instructive to measure the asymmetry

$$A_c \equiv \frac{\Delta\sigma_L(\mu p \to c\bar{c})}{\sigma_L(\mu p \to c\bar{c})} \,, \quad (4.3)$$

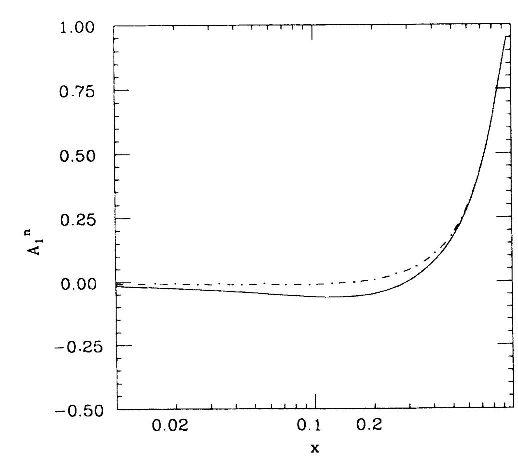

FIG. 7. A prediction for the neutron asymmetry A_1^n from the parton distributions motivated by the EMC data (solid line) and the VDM model (dot-dash line).

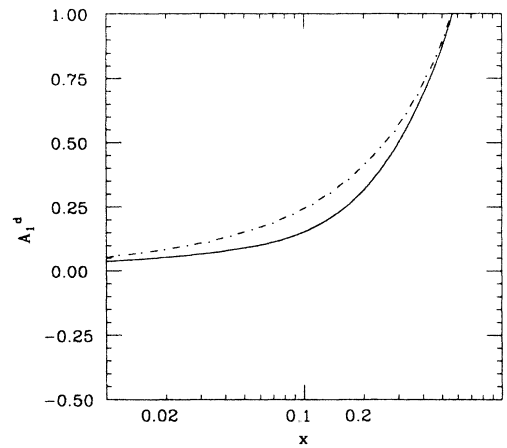

FIG. 8. A prediction for the deuteron asymmetry A_1^d using the same distributions as those in Fig. 7.

with a polarized muon beam on a polarized proton target. The sign and size of this asymmetry could give insight into the flavor composition of the polarized sea.

The Drell-Yan process allows the most direct probe of the polarization of the antiquarks in the sea (see Fig. 9).[42] After integrating over transverse momentum, the spin-spin asymmetry for the production of lepton pairs in pp collisions can be given, in lowest order, by

$$\frac{a_{LL}d\sigma}{dM^2dx_F}=\frac{4\pi\alpha^2}{3M^2}\frac{1}{3}\sum_q e_q^2\left[x_F^2+\frac{4M^2}{s}\right]^{-1/2}\times[\Delta q(x_a)\Delta\bar{q}(x_b)+\Delta\bar{q}(x_a)\Delta q(x_b)]\,, \tag{4.4}$$

where

$$x_a=\frac{1}{2}\left[\left[x_F^2+4\frac{M^2}{s}\right]^{1/2}+x_F\right],\qquad x_b=\frac{1}{2}\left[\left[x_F^2+4\frac{M^2}{s}\right]^{1/2}-x_F\right]. \tag{4.5}$$

To the extent that the quark spin transfer density is dominated by the valence contribution this gives a straightforward measure of the amount of polarization of the antiquarks. Figures 10 and 11 show our model estimates for the Drell-Yan asymmetry. The dominant contribution in (4.4) involves the distribution $\Delta\bar{u}(x,Q^2)$. In terms of our model parameters, these types of measurements will therefore allow us to constrain the SU(3)-breaking parameter ϵ as well as the parameter c which distinguishes the polarization of antiquarks and that of sea quarks.

The production of jets and hadrons at large transverse momentum using polarized beams and targets presents a wide range of experimental opportunities. The fundamental asymmetries for the $2\rightarrow 2$ processes are known from perturbation theory[1] and can be used to determine the polarized distributions.[2]

It is very interesting to explore experimental observables which are sensitive to the gluon polarization. It has been known for a long time[1,3] that QCD perturbation theory leads to strong polarization of the gluons, $\cos\Phi_L\cos2\theta_g\rightarrow 1$, at large Q^2. The deep-inelastic lepton scattering data constrain the gluon polarization only if one accepts speculative theoretical arguments. It is therefore important to measure $\Delta G(x,Q^2)$ as directly as possible by looking for hard-scattering processes where gluons are involved.

We shall consider here the process $a_{LL}d\sigma(pp\rightarrow\text{jet}+X)$ with polarized beam and target and the integrated hard-scattering process[4] $\Delta\sigma_L^{\text{jet}}(pp;p_0,\sqrt{s}\,)$. At large c.m. ener-

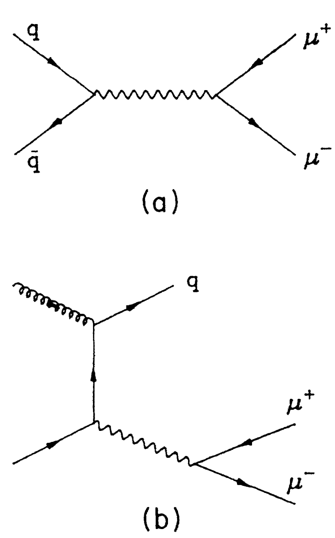

FIG. 9. The Feynman diagrams are shown for lepton pair production: (a) $q\bar{q}$ diagrams and (b) quark-gluon diagrams.

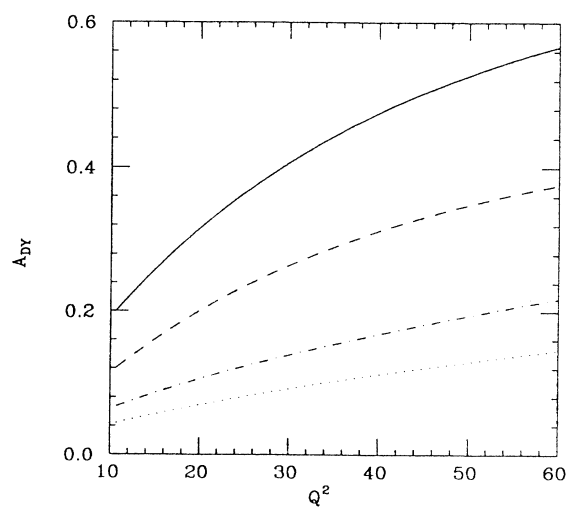

FIG. 10. The Drell-Yan asymmetry A_{LL}^{DY} for $\sqrt{s}=14$ GeV and $x_F=0$. The asymmetries with EMC sea distributions are represented by solid ($c=3$) and dashed ($c=1$) curves, while the VDM-motivated asymmetries are represented by dot-dash ($c=3$) and dotted ($c=1$) curves, respectively.

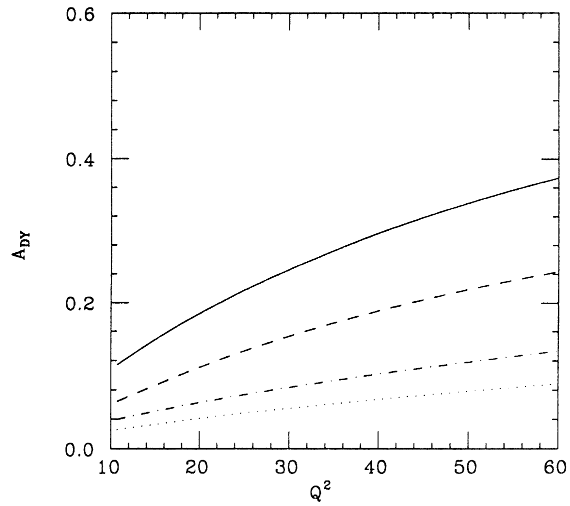

FIG. 11. Identical to Fig. 10, except $\sqrt{s}=20$ GeV.

gies, jet production is dominated by gluon-gluon scattering processes. It might be expected, therefore, that a measurement of jet production with polarized beam and target would be sensitive to the gluon polarization. Our explicit calculations show that this expectation is reasonable.

In Fig. 12, we show estimates for the polarized jet cross section $\Delta\sigma_L^{\rm jet}(pp;p_0,\sqrt{s}\,)$ at $\sqrt{s}=14$ GeV. We use both the large and small polarized gluon distributions discussed in Sec. III and use the EMC and VDM models for the polarization of the sea. The predictions are very sensitive to the gluon polarization and so a measurement of $\Delta\sigma_L^{\rm jet}$ should provide important early clues to the magnitude of $\Delta G(x,Q_0^2)$.

In order to disentangle the contribution of the gluon-gluon subprocess from those involving sea quarks and antiquarks, it is important to consider different subenergies and jet momenta. Should the polarized sea be large and

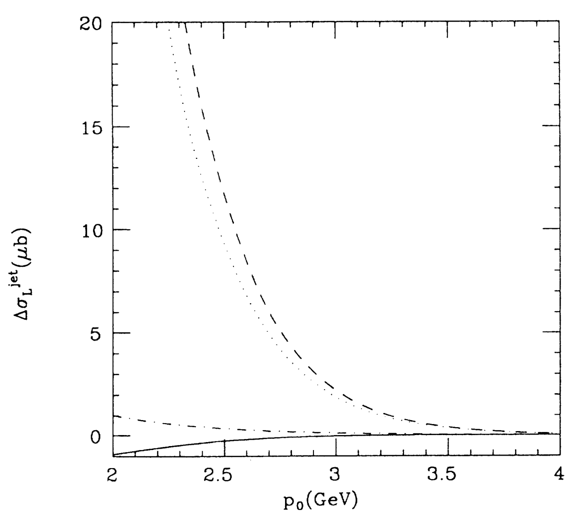

FIG. 12. The polarized jet cross section $\Delta\sigma_L^{\rm jet}$ as a function of momentum cutoff for distinguishing a jet, p_0 with $\sqrt{s}=14$ GeV. Key: solid curve=EMC sea, small ΔG; dot-dash curve=VDM sea, small ΔG; dotted curve=EMC sea, large ΔG ($\langle \Delta G\rangle \simeq 5$); dashed curve=VDM sea, large ΔG.

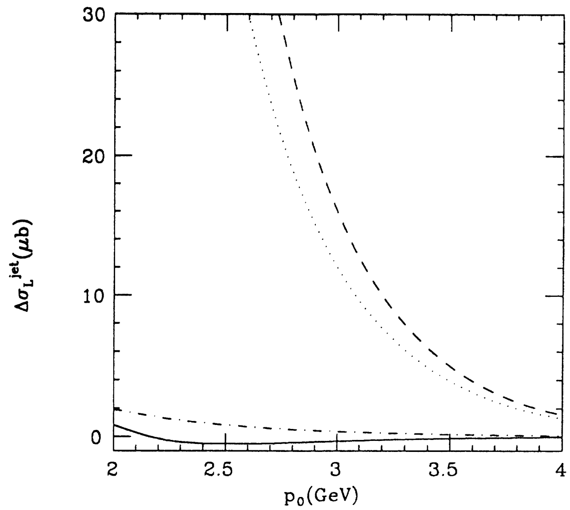

FIG. 13. Identical to Fig. 12, except $\sqrt{s}=20$ GeV.

negative, as suggested by the EMC data, the positive contribution to the jet cross section would be decreased. However, a large polarized gluon distribution would make the positive contribution significant. Figure 13 shows the same set of curves at $\sqrt{s}=20$ GeV. The predictions made by the curves in Figs. 12 and 13 can be tested with the polarized proton beam at Fermilab. With the appropriate calorimeters, it may be possible to measure the total cross section $\Delta\sigma_L$ as a function of momentum cutoff, p_0, and extract the jet cross section $\Delta\sigma_L^{\rm jet}$ by an appropriate assumption about the Regge behavior of the soft part. By boosting the beam energy and measuring the cross section at $\sqrt{s}=20$ GeV, the differences in model predictions are enhanced (Fig. 13) and the relative size of the gluon polarization could be confirmed.

Perhaps the cleanest method for determining the gluon polarization is through direct photon production from a polarized beam on a polarized target. The process receives contributions from diagrams in Fig. 14 and are ex-

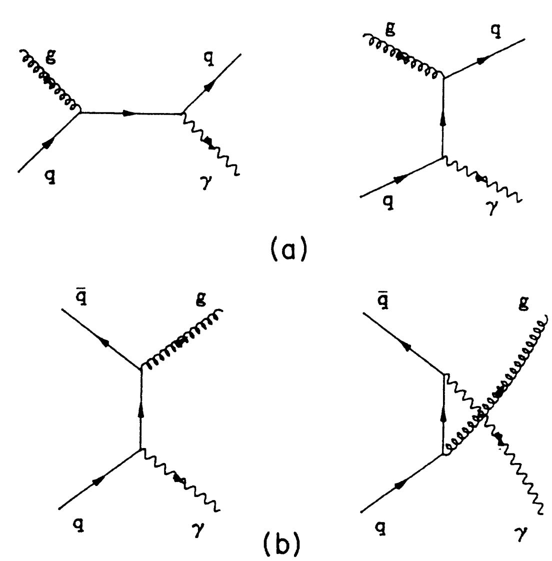

FIG. 14. The scattering diagrams contributing to direct photon production are shown: (a) diagrams with a quark in the final state and (b) diagrams with a gluon in the final state.

pected to be dominated by the quark-gluon diagrams. This type of measurement would be interesting. The discussion of Berger and Qiu[28] shows the importance of measurements in the small-x region where the polarized gluon distribution would be sufficiently large. At this point, it seems that the immediate aim should be to do a variety of experiments to get an indication as to the size of the polarized gluon distribution and thus, the anomaly term. With this knowledge, we may begin to understand the constituent contributions to the spin of the proton.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy, Division of High Energy Physics, Contract No. W-31-109-ENG-38.

APPENDIX A: VALENCE-QUARK DISTRIBUTIONS FOR THE BARYON OCTET AND DIQUARKS

To elucidate the spin and flavor content of the baryon octet, it is frequently convenient to use a language based on the decomposition of the three quarks in the proton into a "bachelor" or "spinster" quark and two quarks bound together to form a diquark. From this point of view, the probability density as a function of x for a quark can be decomposed

$$P_{q/B}(x)=P^B_{q/B}(x)+P_{D/p}(x)\otimes P_{q/D}(x)\ , \tag{A1}$$

where $P_{q/B}(x)$ is the probability density of the bachelor quark and

$$\begin{aligned} P_{D/p}(x)\otimes P_{q/D}(x) &= \int_0^1 dy\, dz\, P_{D/p}(y)P_{q/D}(z)\delta(x-yz) \\ &= \int_0^1 \frac{dy}{y} P_{D/p}(y)P_{q/D}\left[\frac{x}{y}\right]\ , \end{aligned} \tag{A2}$$

gives the density of a quark within the diquark. To take account of the spin and flavor content of the diquark, we will use the notation $D_{s,s_z;I,I_z}$. We can decompose the valence-quark proton wave function in terms of the direct product of a quark and a diquark:

$$\begin{aligned} |p;(+)\rangle = C_0 f_0(x_i)|D_{0,0;0,0}\rangle\otimes|u(+)\rangle + C_1 f_1(x_i)\Bigg[&\frac{2}{3}|D_{1,+1;1,+1}\rangle\otimes|d(-)\rangle-\frac{\sqrt{2}}{3}|D_{1,0;1+1}\rangle\otimes|d(+)\rangle \\ &-\frac{\sqrt{2}}{3}|D_{1,+1;1,0}\rangle\otimes|u(-)\rangle+\frac{1}{3}|D_{1,0;1,0}\rangle\otimes|u(+)\rangle\Bigg]\ , \end{aligned} \tag{A3}$$

where $C_0^2+C_1^2=1$ while $f_0(x_1,x_2,x_3)$ and $f_1(x_1,x_2,x_3)$ are functions of the momentum fractions. Using the fact that all the momentum dependence is contained in f_0 and f_1 both the quark distributions and the spin-weighted quark distributions for quarks in the diquark are easy to read off from the quantum numbers of the diquarks:

$$\begin{aligned} &P_{q/D(0,0;00)}:\ u(x)=q_0(x),\quad d(x)=q_0(x)\ , \\ &\qquad \Delta u(x)=0,\quad \Delta d(x)=0\ ; \\ &P_{q/D(1,+1;1,+1)}:\ u(x)=2q_1(x),\quad d(x)=0\ , \\ &\qquad \Delta u(x)=2q_1(x),\quad \Delta d(x)=0\ ; \\ &P_{q/D(1,+1;1,0)}:\ u(x)=q_1(x),\quad d(x)=q_1(x)\ , \\ &\qquad \Delta u(x)=q_1(x),\quad \Delta d(x)=q_1(x)\ ; \\ &P_{q/D(1,0;1,1)}:\ u(x)=2q_1(x),\quad d(x)=0\ , \\ &\qquad \Delta u(x)=0,\quad \Delta d(x)=0\ ; \\ &P_{q/D(1,0;1,0)}:\ u(x)=q_1(x),\quad d(x)=q_1(x)\ , \\ &\qquad \Delta u(x)=0,\quad \Delta d(x)=0\ ; \end{aligned} \tag{A4}$$

where $q_0(x)$ and $q_1(x)$ are two distinct quark distributions normalized to unity. From (A1), (A3), and (A4) the valence-quark distributions can be written

$$\begin{aligned} u_v(x) &= C_0^2[D_0(x)\otimes q_0(x)+q_0^B(x)] \\ &\quad + C_1^2[\tfrac{5}{3}D_1(x)\otimes q_1(x)+\tfrac{1}{3}q_1^B(x)]\ , \\ d_v(x) &= C_0^2[D_0(x)\otimes q_0(x)] \\ &\quad + C_1^2[\tfrac{1}{3}D_1(x)\otimes q_1(x)+\tfrac{2}{3}q_1^B(x)]\ , \end{aligned} \tag{A5}$$

where the "bachelor" distributions are also normalized to unity. Note that for any mixture of C_0 and C_1 we have

$$\int_0^1 dx\, u_v(x)=2,\quad \int_0^1 dx\, d_v(x)=1\ . \tag{A6}$$

In the same way, it is also possible to write the spin-weighted distributions from (A3) as

$$\begin{aligned} \Delta u_v(x) &= C_0^2[q_0^B(x)]+C_1^2[\tfrac{10}{9}D_1(x)\otimes q_1(x)-\tfrac{1}{9}q_1^B(x)]\ , \\ \Delta d_v(x) &= C_1^2[\tfrac{2}{9}D_1(x)\otimes q_1(x)-\tfrac{2}{9}q_1^B(x)]\ . \end{aligned} \tag{A7}$$

The assumption of a quark-diquark decomposition in (A3) which appears to be quite general, in fact, leads directly to the constraints

$$\int_0^1 dx\, \Delta u_v(x)=1,\quad \int_0^1 dx\, \Delta d_v(x)=0\ , \tag{A8}$$

independently of the "dynamics" represented by the shapes of $D_0(x)$, $D_1(x)$; $g_0(x)$, $g_1(x)$; $q_0^B(x)$; or the mixture C_0, C_1. This string implication of the diquark picture for spin-weighted quark distributions was pointed out by Babcock *et al.*[1] We can make the further assumption of "weak correlation"

$$D_1(x)\otimes q_1(x)=D_0(x)\otimes q_0(x)=q(x)\ ,$$
$$q_1^B(x)=q_0^B(x)=q(x)\ , \tag{A9}$$

which leads to

$$u_v(x)=2q(x),\quad d_v(x)=q(x)\ ,$$
$$\Delta u_v(x)=q(x),\quad \Delta d_v(x)=0\ , \tag{A10}$$

which is the explicit representation of the quark-diquark "model" discussed in Ref. 1.

An alternate assumption is also informative. If we assume $D_i(x)\otimes q_i(x)\ll q_B(x)$ except at small x where the whole picture might break down for other reasons, we get

$$u_v(x)=C_0^2 q_0^B(x)+\frac{C_1^2}{3}q_1^B(x)\ ,$$
$$d_v(x)=\frac{2C_1^2}{3}q_1^B(x)\ ,$$
$$\Delta u_v(x)=C_0^2 q_0^B(x)-9C_1^2 q_1^B(x)\ , \tag{A11}$$
$$\Delta d_v(x)=-\frac{2C_1^2}{9}q_1^B(x)\ .$$

In this limit, the spin-weighted distributions can be expressed in terms of $u_v(x)$ and $d_v(x)$,

$$\Delta u_v(x)=u_v(x)-\tfrac{2}{3}d_v(x)\ ,$$
$$\Delta d_v(x)=-\tfrac{1}{3}d_v(x)\ , \tag{A12}$$

which comprise the starting point for the valence-quark distributions discussed in the text.

In our numerical work in Sec. III we follow Carlitz and Kaur and modify the ratios in (A12),

$$\Delta u_v(x,Q^2)=\cos\Theta_D(x,Q^2)[u_v(x,Q^2)-\tfrac{2}{3}d_v(x,Q^2)]\ ,$$
$$\Delta d_v(x,Q^2)=-\tfrac{1}{3}\cos\Theta_D(x,Q^2)d_v(x,Q^2)\ , \tag{A13}$$

by an x-dependent, Q^2-dependent dilution factor. More details on the numerical form used can be found there.

APPENDIX B: POLARIZED VALENCE-QUARK DISTRIBUTIONS AT SMALL x

The small-x behavior of the polarized valence-quark distributions is modified with increasing Q^2 according to the Altarelli-Parisi equations. To investigate this behavior further it is easiest to work with the moments of the quark distributions where the Mellin moment $\tilde{f}(n)$ of a function $f(x)$ is defined by

$$\tilde{f}(n)=\int_0^1 dx\, x^{n-1}f(x)\ . \tag{B1}$$

The inverse Mellin transform is then

$$f(x)=\frac{1}{2\pi i}\int_c dn\,\tilde{f}(n)x^{-n}\ , \tag{B2}$$

where c is a contour running parallel to the imaginary axis to the right of all the singularities of $\tilde{f}$. It is the rightmost singularity in $\tilde{f}(n)$ that governs the leading small-x behavior of $f(x)$.

The nonsinglet polarized quark distributions evolve according to

$$\frac{\partial}{\partial t}\Delta Q_v(x,t)=\int_x^1 dz\,\Delta P_{qq}\left[\frac{x}{z}\right]\Delta Q_v(z,t)\ , \tag{B3}$$

where $\Delta Q_v(x,t)=x\Delta q_v(x,t)$. The corresponding equation for the moments is

$$\frac{\partial}{\partial t}\Delta\tilde{q}_v(n,t)=\Delta\tilde{P}_{qq}(n)\Delta\tilde{q}_v(n,t)\ , \tag{B4}$$

with

$$\Delta\tilde{P}_{qq}(n)=C_2(R)\left[\frac{3}{2}-2\gamma_2+\frac{1}{n}-\frac{1}{n+1}-2\psi(n+1)\right]. \tag{B5}$$

Equation (B4) has the solution

$$\Delta\tilde{q}_v(n,t)=\exp[\Delta\tilde{P}_{qq}(n)(t-t_0)]\Delta\tilde{q}_v(n,t_0)\ . \tag{B6}$$

We will assume the following simple form for the starting polarized valence-quark distributions

$$\Delta q_v(x,t_0)=Ax^{\alpha}(1-x)^{\beta}\ . \tag{B7}$$

The Mellin transform is

$$\Delta\tilde{q}_v(n,t_0)=AB(n+\alpha,1+\beta)\ , \tag{B8}$$

which has its rightmost pole at $n=-\alpha$. Since we expect $\alpha>0$, $\Delta\tilde{q}_v(n,t_0)$ has all its poles in the left half plane. Thus the leading contribution to $\Delta q_v(x,t)$ at small x is controlled by the pole in $\Delta\tilde{P}_{qq}(n)$ at $n=0$. We can quantitatively investigate this contribution using the method of steepest descent.

We begin by inverting Eq. (B6) and writing the solution in the form

$$\Delta q(x,t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}dn\,\Delta\tilde{q}_v(n,t_0)\exp\left\{\ln\frac{1}{x}\left[n+(t-t_0)\Delta\tilde{P}_{qq}(x)\left[\ln\frac{1}{x}\right]^{-1}\right]\right\}. \tag{B9}$$

The stationary point n_0 of the exponent occurs at

$$n_0=\left[\frac{tc_2(R)}{\ln(1/x)}\right]^{1/2}\left[1+O\left[\ln\frac{1}{x}\right]\right] \tag{B10}$$

and approximating the contribution to the integral about $n=n_0$ by a Gaussian yields

$$\Delta q_v(x,t) \underset{x \to 0}{\sim} A\beta(\alpha,1+\beta)\left[4\pi\left[\frac{\ln(1/s)}{\Delta t\, C_2(R)}\right]^{1/2}\right]^{-1/2} \exp\left[2\left[\ln\frac{1}{x}(t-t_0)C_2(R)\right]^{1/2}\right] . \tag{B11}$$

The form of (B11) is similar to that obtained by Einhorn and Soffer.[3] However, here the coefficient of the exponent is suppressed by powers of $\ln(1/x)$, and is negligible over the range of x and Q^2 appropriate to the data.

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