Domain-wall number in various invisible-axion models

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We study the vacuum structures for various invisible-axion models. We illustrate that the assignment of $U(1)_{PQ}$ charge for the Higgs-singlet field plays a very important role in determining the domain-wall number N_{DW}. Especially, we find that the existence of trilinear terms in the Higgs potential would reduce N_{DW} by a factor 2 compared with the usual quartic-type one. This factor is important in constructing domain-wall-free models. Several possibilities of constructing such models are pointed out.

It has been pointed out by Sikivie[1] that many invisible-axion models have a spontaneously broken discrete $Z(N_{DW})$ symmetry and hence N_{DW} degenerate disconnected vacua, which lead to an unwanted domain-wall structure in the early Universe.[2] He showed that in the Dine-Fischler-Srednicki[3] (DFS) invisible-axion model which contains two doublet ϕ_i $(i=1,2)$ and one singlet χ Higgs fields, the number N_{DW}, called the domain-wall number, is equal to $2N_g$: i.e.,

$$N_{DW}=2N_g \ , \tag{1}$$

where N_g stands for the number of quark generations. To avoid the domain-wall problem, one needs to augment the theory so that a unique vacuum,[4] i.e., $N_{DW}=1$, emerges from it.

In this paper we will show that, in the DFS model, N_{DW} depends on the charge of Peccei-Quinn $U(1)_{PQ}$ symmetry[5] for the Higgs-singlet field χ. An alternative PQ assignment of χ which results in the trilinear terms such as $\phi_1^\dagger\phi_2\chi$ in the Higgs potential could reduce N_{DW} in Eq. (1) by a factor 2, i.e.,

$$N_{DW}=N_g \ . \tag{2}$$

With these alternative terms in the DFS Higgs potential, we will examine the vacuum structures in more general invisible-axion models.

To find out the domain-wall number in the theory, the first task is to determine the unbroken discrete symmetry Z_N due to the $SU(3)_C$-color instanton effects and then to identify the spontaneously broken discrete symmetry by studying the Z_N transformations of the Higgs-like particles in the vacuum.

As an illustration, we review the vacuum structure of the N_f-flavor QCD model which has a global $U(N_f)_L\times U(N_f)_R$ symmetry.[1,6] The quantum numbers of the model can be written as

$$\begin{array}{lcccccccc} & SU(3)_C\times & SU(N_f)_L\times & SU(N_f)_R\times & [U(1)_L\times & U(1)_R= & U(1)_{V=L+R}\times & U(1)_{A=L-R}] & \\ q_L^i & 3 & N_f & 1 & 1 & 0 & 1 & 1 & , \\ q_L^{ci} & \bar{3} & 1 & \bar{N}_f & 0 & -1 & -1 & 1 & \end{array} \tag{3}$$

where the quark fields are denied to be left handed. According to 't Hooft,[7] because of $SU(3)_C$-color instanton effects the axial chiral $U(1)_A$ symmetry breaks down to $Z_{N=2N_f}$: i.e.,

$$U(1)_A \overset{\text{instanton}}{\rightarrow} Z_{2N_f} \ , \tag{4}$$

where $N=2N_f$ is determined by the formula

$$N=\left|\sum_i^{N_g} M_i Q_i\right| \ , \tag{5}$$

where M_i and Q_i are the number of fermion zero modes under $SU(3)_C$ and the $U(1)_A$ charge for the ith quark flavor, respectively. Under Z_{2N_f}, the quark fields have the following transformations:

$$(q_L,q_L^c)^i \rightarrow e^{i\pi k/N_f}(q_L,q_L^c)^i \tag{6}$$

for $i,k=1,2,\ldots,N_f$. In QCD, the order parameters that break the chiral symmetries are the quarks condensates. For the bilinear quark condensate $q_L^i q_L^{ci}$ one has

$$\langle q_L^i q_L^{ci}\rangle_k=\mu_i^3 e^{i2\pi k/N_f} \ . \tag{7}$$

Therefore, Z_{2N_f} is spontaneously broken down to Z_2: i.e.,

$$Z_{2N_f}\rightarrow Z_{2N_f}/Z_{N_f}=Z_2 \ . \tag{8}$$

However, since Z_{N_f} is embedded in the symmetry of

$SU(N_f)_L \times SU(N_f)_R$ which is spontaneously broken down to $SU(N_f)_{L+R}$, the vacua are not topologically distinct.[8] One notes that $q_L^i q_L^{ci}$ are the lowest-dimensional $SU(3)_C$ gauge-invariant Higgs-like terms.

We now study the general invisible-axion model with two ϕ_i $(i=1,2)$ and one χ Higgs field under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{PQ}$ symmetry.[9] The Higgs and quark fields have the $U(1)_{PQ}$ transformations

$$\begin{aligned} &\phi_1 \to e^{-ih_1\alpha}\phi_1, \quad \phi_2 \to e^{ih_2\alpha}\phi_2 , \\ &\chi \to e^{-i\chi\alpha}\chi , \\ &q_L^i \to e^{iq_i\alpha} q_L^i , \\ &u_R^i \to e^{-iu_i\alpha} u_R^i, \quad d_R^i \to e^{-id_i\alpha} d_R^i , \end{aligned} \tag{9}$$

where

$$\phi_k = \begin{bmatrix} \phi_k^+ \\ \phi_k^0 \end{bmatrix} \quad (k=1,2)$$

are Higgs doublets and $i=1,\ldots,N_g$ are generation indices. There are two possible PQ assignments for χ:

$$\chi^{(\mathrm{I})} = \frac{h_1+h_2}{2} \tag{10}$$

and

$$\chi^{(\mathrm{II})} = h_1+h_2 , \tag{11}$$

which correspond to the Higgs potentials

$$\begin{aligned} V^{(\mathrm{I})} = &\sum_{i=1}^{3} m_i^2 \phi_i^\dagger \phi_i + \sum_{i,j}^{3} a_{ij} (\phi_i^\dagger\phi_i)(\phi_j^\dagger\phi_j) \\ &+ b_{12}(\phi_1^\dagger\phi_2)(\phi_2^\dagger\phi_1) + (c\phi_1^\dagger\phi_2\chi^2 + \text{ H.c.}) \\ \equiv &V_0 + (c\phi_1^\dagger\phi_2\chi^2 + \text{ H.c.}) \end{aligned} \tag{12}$$

and

$$V^{(\mathrm{II})} = V_0 + (c\phi_1^\dagger\phi_2\chi + \text{ H.c.}) , \tag{13}$$

respectively. The Higgs potential $V^{(\mathrm{I})}$ in (12) is precisely the one in the original DFS model.

With the PQ charges in (9), the $U(1)_{PQ}$ is broken down to Z_N by color instanton effects with

$$\begin{aligned} N &= \left| \sum_q M_q Q_{PQ}^q \right| \\ &= \left| \sum_{i=1}^{N_q} (2q_i + u_i + d_i) \right| , \end{aligned} \tag{14}$$

according to the anomaly of $[SU(3)_C]^2 U(1)_{PQ}$, where M_q and Q_{PQ}^q are the number of $SU(3)_C$ fermion zero modes and the PQ charge for each quark, respectively. Under Z_N, the Higgs and fermion fields have transformation

$$\begin{aligned} &\phi_1 \to e^{-i2\pi k h_1/N}\phi_1, \quad \phi_2 \to e^{i2\pi k h_2/N}\phi_2 , \\ &\chi^{(\mathrm{I,II})} \to e^{-i2\pi k \chi^{(\mathrm{I,II})}}/N\chi , \end{aligned} \tag{15}$$

where $\chi^{(\mathrm{I})} = (h_1+h_2)/2$ and $\chi^{(\mathrm{II})} = h_1+h_2$ and

$$\begin{aligned} &q_L^i \to e^{i2\pi k q_i/N} q_L^i, \quad u_R^i \to e^{-i2\pi k u_i/N} u_R^i , \\ &d_R^i \to e^{-i2\pi k d_i/N} d_R^i . \end{aligned} \tag{16}$$

To examine the domain-wall structure, one has to consider the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge-invariant terms among which the lowest-dimensional ones are $\phi_1^\dagger\phi_2$, χ, etc.[10] From (15), one obtains that

$$\begin{aligned} &\langle \chi^{(\mathrm{I})} \rangle \to e^{-i2\pi k/[2N/(h_1+h_2)]} \langle \chi^{(\mathrm{I})} \rangle , \\ &\langle \chi^{(\mathrm{II})} \rangle \to e^{-i2\pi k/[N/(h_1+h_2)]} \langle \chi^{(\mathrm{II})} \rangle , \\ &\langle \phi_1^\dagger\phi_2 \rangle \to e^{i2\pi k/[N/(h_1+h_2)]} \langle \phi_1^\dagger\phi_2 \rangle , \ldots . \end{aligned} \tag{17}$$

Therefore, one concludes that

$$N_{\mathrm{DW}}^{(\mathrm{I})} = \frac{2N}{|h_1+h_2|} = 2 \left| \frac{\sum_{i=1}^{N_g} (2q_i+u_i+d_i)}{h_1+h_2} \right| \tag{18}$$

for case (I) and

$$N_{\mathrm{DW}}^{(\mathrm{II})} = \frac{N_{\mathrm{DW}}^{(\mathrm{I})}}{2} = \left| \frac{\sum_{i=1}^{N_g} (2q_i+u_i+d_i)}{h_1+h_2} \right| \tag{19}$$

for case (II).

Let us illustrate the domain-wall number of various invisible-axion models. For the DFS model, the $U(1)_{PQ}$ charges for quarks are independent of generation and the Yukawa couplings are constructed such that ϕ_1 couples to up-type quarks and ϕ_2 couples to down-type quarks, which can be written as

$$h_{ij}^u \bar{q}_L^i \tilde{\phi}_1 u_R^j \quad \text{and} \quad h_{ij}^d \bar{q}_L^i \phi_2 d_R^j . \tag{20}$$

We derive that

$$\begin{aligned} \sum_{i=1}^{N_g} (2q_i+u_i+d_i) &= N_g(2q_i+u_i+d_i) \\ &= N_g(h_1+h_2) . \end{aligned} \tag{21}$$

From Eqs. (18), (19), and (21), we find Eqs. (1) and (2) for cases (I) and (II), respectively.

We now study the class of variant invisible-axion (VIA) models[11] proposed by us recently to see whether a class of models could indeed realize the $N_{\mathrm{DW}}=1$ property. The models modify the variant weak-scale axion models[12] by adding a Higgs-singlet field as in the DFS model. We use the same definitions given in (9)–(13) to discuss a typical model in the Ref. 11. Without loss of generality, we choose $h_1=h_2=1$ and $N_g=3$. The PQ charges of the quarks in the model are

$$\begin{aligned} &(q_1,q_2,q_3) = (0,0,0) , \\ &(u_1,u_2,u_3) = (1,-1,-1) , \\ &(d_1,d_2,d_3) = (1,1,1) . \end{aligned} \tag{22}$$

From Eqs. (18), (19), and (22), we obtain that

$$N_{\rm DW}^{(\rm I)}=2 \tag{23}$$

and

$$N_{\rm DW}^{(\rm II)}=1\ . \tag{24}$$

Therefore, $N_{\rm DW}=1$ can be achieved in VIA models if the case (II) assignment of PQ charge for χ in (11) is used. This justifies the conclusions given by us in Ref. 11.

For an interesting extension of VIA models, we point out that it is possible to construct $N_{\rm DW}=1$ models by imposing a global horizontal family U(1) symmetry as the PQ symmetry under which the quark transformations are

$$F_L^i=(q,u^c,d^c)_L^i \to e^{if_i\alpha}F_L^i\ . \tag{25}$$

We shall call such class of models as horizontal invisible-axion (HIA) models. For example, the PQ charges for the Higgs and quark fields,

$$\begin{aligned} &h_1=h_2=1,\ \ \chi=2\ ,\\ &(f_1,f_2,f_3)=(\tfrac{3}{2},-\tfrac{1}{2},-\tfrac{1}{2})\ , \end{aligned} \tag{26}$$

could result in $N_{\rm DW}=1$ for the model. The detailed discussions for avoiding the domain-wall problem with a horizontal PQ symmetry will be given elsewhere.[13]

In summary, we have discussed the vacuum structures for various invisible-axion models. We showed that the domain-wall number in the original DFS model can be reduced by a factor 2 by using an alternative PQ assignment for the Higgs singlet. Although it does not help to solve the domain-wall problem for the DFS model itself, it opens up new avenues to look for $N_{\rm DW}=1$ invisible-axion models which we demonstrated can, indeed, be achieved for the VIA as well as HIA types of models.

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