Evolution of nonspherical bubbles

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We study the evolution of nonspherical bubbles which arise in the theory of a real scalar field that has nondegenerate ground states. A calculational framework to study radial perturbations to the evolution of spherical shells of domain wall is presented. We focus on the (classical) evolution of true-vacuum bubbles which nucleate in a sea of false vacuum in a first-order phase transition, e.g., as in old or extended inflation (we work in the zero-temperature, zero-gravity limit). As the bubbles evolve, perturbations with nonzero initial velocity and low angular wave numbers grow initially and then freeze out; perturbations with high wave number oscillate about the unperturbed state and eventually decay. Perturbations with finite initial amplitude and zero initial velocity quickly decay for all wave numbers. Applications of our work to closed-surface domain walls (which have recently been invoked to explain large-scale structures in cosmology) are also discussed.

I. INTRODUCTION

Potentials with nondegenerate minima in field theory give rise to interesting behavior. Domain walls can separate regions of true vacuum (the stable, lowest-energy state) from regions of false vacuum (a state of higher energy, a local minimum). Spherical shells of domain wall have been studied in many circumstances. In particular, such bubbles can arise at early epochs as the Universe passes through a first-order phase transition. For example, in the original version of the inflationary Universe[1] now known as "old inflation," the Universe was in a false vacuum at early times; then bubbles of true vacuum began to nucleate in the sea of false vacuum. These bubbles expanded and the hope was that they would coalesce and ultimately convert the entire Universe to true vacuum. However, previous work has shown that such percolation does not occur for the case of perfectly spherical bubbles—the "graceful exit" problem of old inflation.[2] In an entirely different context, closed-surface domain walls may also arise as topological defects which form, e.g., during a late-time phase transition; these walls may serve as seeds for the growth of large-scale structure in cosmology.[3]

Previous work has focused on perfectly spherical bubbles. In this paper, we study the evolution of nonspherical bubbles; in particular, we consider the growth of radial perturbations to spherical walls. Our motivation was to investigate whether these perturbations could grow rapidly enough so that the bubbles would become highly nonspherical and evolve in novel ways. Our work may also have application to detonation waves and the evolution of nonspherical bubbles in other contexts.

In this paper, we will focus on a simple but illustrative example—a quantum field theory of a single scalar field. The Lagrangian of the field is

$$\mathcal{L}=\tfrac{1}{2}\partial_\mu\Phi\partial^\mu\Phi-V(\Phi)\ , \tag{1.1}$$

where $V(\Phi)$ is an asymmetric potential with metastable minimum Φ_+ and absolute minimum Φ_- (see Fig. 1). The energy difference between the vacua is ϵ. We consider spherical bubbles of true vacuum (Φ_-) expanding into a false-vacuum (Φ_+) background.

For definiteness, we take the potential to have the form

$$V(\Phi)=\tfrac{1}{8}\lambda(\Phi^2-\eta^2)^2+\frac{\epsilon}{2\eta}(\Phi-\eta)\ , \tag{1.2}$$

where $\eta^2\equiv\mu^2/\lambda$. To leading order, the metastable minimum is given by $\Phi_+=+\eta$ and the absolute minimum by $\Phi_-=-\eta$. A bubble wall separating a region of true vacuum from a region of false vacuum can be characterized by a surface tension σ and by the energy density ϵ of the false vacuum swept up by the bubble. For the above potential, for a static, plane symmetric domain wall, the surface tension is equal to the surface energy density and is given by[4]

$$\sigma=\tfrac{1}{3}\sqrt{\lambda}\eta^3 \tag{1.3}$$

and the wall thickness is $O(1/\sqrt{\lambda}\eta)$. The stress-energy tensor for a wall of arbitrary shape as measured by a local observer moving with the wall is similar to that of a plane-symmetric wall.

In this paper, we will ignore the effects of gravity and work in flat spacetime. We will also take the "thin-wall" limit and assume that the thickness of the bubble wall is

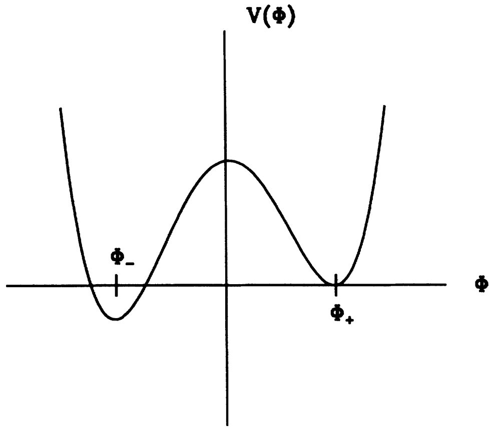

FIG. 1. Potential energy density as a function of field strength for a scalar field theory with nondegenerate ground states.

much smaller than any dynamical scale in the problem. For a perfectly spherical bubble wall of radius $R \gg 1/\sqrt{\lambda}\eta$, the Lagrangian takes the form[5]

$$L=-4\pi\sigma R^2(1-\dot{R}^2)^{1/2}+\frac{4\pi}{3}R^3\epsilon \quad (1.4a)$$

and the corresponding Hamiltonian takes the form[5]

$$H=p_R\dot{R}-L=4\pi\sigma R^2(1-\dot{R}^2)^{-1/2}-\frac{4\pi}{3}R^3\epsilon\ . \quad (1.4b)$$

The first term in the Hamiltonian is simply the Lorentz boost of the energy of a spherical wall with surface density σ. The second term is a potential-energy term which measures how much of the false-vacuum energy has been swept out by the bubble as false vacuum is converted to true vacuum. In the thin-wall limit, we consider only bubbles and perturbations much larger than the thickness of the wall. Hence, the behavior of the Φ field within the wall can be ignored.

Our analysis is relevant to the study of bubble nucleation in first-order phase transitions. For the potential of Fig. 1, Coleman[6] has calculated the rate for quantum tunneling of a universe in a metastable vacuum Φ_+ to a true vacuum Φ_-. He has shown that the solution to the Euclidean equations of motion with the lowest (classical) action has O(4) symmetry. By analytic continuation, he then argues that the most likely bubble to emerge via barrier penetration is perfectly spherically symmetric. Once this spherical true-vacuum bubbles has formed, it expands into the sea of false vacuum. The (classical) motion of the bubble wall is given by $R^2=t^2+R_C^2$, where $R_C=3\sigma/\epsilon$ is the initial radius (i.e., the radius at the time of formation). The wall rapidly asymptotically approaches the speed of light. In this paper, we study radial perturbations (presumably induced by quantum fluctuations) about this spherical solution.

We hope that our work will be relevant to old inflation[1] and to extended inflation,[7] although we consider zero-temperature bubbles and ignore the effects of gravity. Old inflation has been shown to fail[2] because the interiors of expanding spherical bubbles of true vacuum fail to thermalize. Hence, this model does not produce a universe such as our own. The latent heat of the phase transition (ϵ) is entirely converted into the kinetic energy of the bubble wall rather than thermalizing the interior of the bubble; in addition, separate bubbles fail to intersect and release wall energy into thermalization of the bubble interior, i.e., percolation does not occur. Our original motivation was to determine whether perturbations of the spherical bubbles could modify these conclusions. Instability to these perturbations might have led to the growth of radial structures such as those occurring in the formation of dendrites (e.g., snowflakes, see Ref. 8) although the mechanism is markedly different.

Our work can be applied to the study of instabilities in relativistic detonation waves in the zero-temperature limit. Although the theory of relativistic shock waves is well established,[9] relativistic detonation waves have been studied only recently.[10] Detonation waves are characterized by a surface of discontinuity which separates regions of different "chemical" composition, i.e., the surface separates a region of "unburnt" fluid from a region of "burnt" fluid. As the detonation wave propagates through the unburnt region, the stored chemical (or in our case vacuum) energy gets converted (through "burning") into kinetic energy of the wave front (or the fluid behind the wave front). In the present case of a bubble wall separating two nondegenerate vacua, the stored "chemical" energy of the unburnt fluid corresponds to the energy difference ϵ between the two vacua. As the bubble wall propagates, the potential energy of the false vacuum gets "burned" and *all* of this energy gets channelled into further acceleration of the wall. Since we are considering both sides of the bubble wall to be in a state of pure vacuum (i.e., no particles), we are essentially considering the zero-temperature limit of a detonation wave. Hence, our results can be applied to any detonation wave in which the $T\to 0$ limit is valid.

Our analysis is also relevant to the expansion and collapse of spherical topological domain walls (in the case of two nondegenerate minima of the potential). Stable domain walls which have a mass per unit area greater than 4×10^{-3} g/cm^2 [$(10\ \mathrm{MeV})^3$ in units where $\hbar=c=1$] are a cosmological disaster.[11,12] However, unstable (but perhaps long-lived) domain walls could have been present and perhaps important in the early Universe.[13] Recently Hill, Schramm, and Fry[3] have revived interest in cosmological domain walls with their suggestion of a late ($z\sim 10$) phase transition that gives rise to domain walls. Their model attempts to explain formation of large-scale structure with perturbations that are nonlinear at the time of the phase transition but would leave the microwave background unaffected at the level of $\Delta T/T\sim 10^{-6}$. Their model consists of a sine-Gordon potential with degenerate minima. Gelmini, Gleiser, and Kolb[14] consider the astrophysics of a classical field theory with nondegenerate minima. If spherical domain walls arise in this case, their evolution can be described by our calculations.

In this paper, we study radial perturbations to spherical walls. If these perturbations grew rapidly enough relative to the overall expansion of the bubble, one could end up with highly nonspherical bubbles. Here we give a heuristic argument for instability to such perturbations. If a section of the bubble has a slightly larger radius than average $R' > R$, then the energy in that section due to false-vacuum conversion $\propto \epsilon R'^3$ is slightly higher than for the rest of the bubble [see Eq. (1.4)]. This excess energy can increase the kinetic energy in that section of the wall and cause the section to grow faster, sweep out more energy, grow still faster, and so forth. Thus, one might expect the wall to be unstable to radial perturbations. However, because the entire bubble asymptotically approaches the speed of light,[5,6] these perturbations should freeze out at some finite value. Whether or not bubbles are formed of dramatically nonspherical shape and significantly different subsequent evolution depends on how large these perturbations grow before they freeze out. Interesting modifications to the zeroth-order behavior could result if the perturbations quickly reach nonlinearity, before the asymptotic solution is approached. For example, one might speculate that sections of the bubble wall could intersect one another and pinch off.

Our results show that the qualitative behavior outlined above does indeed occur for perturbations with low angular wave number; namely, these perturbations grow and then freeze out. However, in the case of inflationary bubbles, the perturbations remain small enough that nonlinearity is never reached. For perturbations with higher angular wave number, we find an interesting oscillatory behavior. Thus one might speculate that a significant fraction of the energy density in the walls is released due to the oscillations, e.g., in the form of particle production. In this manner, the future evolution of the bubbles could be drastically affected. However, as shown in the results, the amplitude of the oscillations is small and the oscillations damp before the radius has expanded by a factor of 10. Thus, in the case of inflation, the energy density in the oscillations is only a small fraction of the total energy in the wall and cannot drastically affect the expansion of the wall.

The paper is organized as follows. In Sec. II, we develop the mathematical formalism necessary to study the classical evolution of spherical bubble walls and of radial perturbations to these bubbles. We then consider the evolution of nucleating bubbles in phase transitions (Sec. III). In Sec. IV we present a summary and discussion of our conclusions.

II. CLASSICAL EVOLUTION OF BUBBLES

In this section, we derive the general equations of motion for a nearly spherical wall. A bubble wall of negligible thickness can be described as a $(2+1)$-dimensional hypersurface. The motion of such a wall can then be parametrized by the equations $x^\mu = x^\mu(\zeta^a)$ $(a=0,1,2)$ where x^μ are the usual spacetime coordinates and ζ^a are the coordinates for the $(2+1)$-dimensional hypersurface swept out by the wall. (Throughout, Greek letter indices will refer to the usual spacetime coordinates and Roman letter indices will refer to hypersurface coordinates.) We use the spherical coordinates, $x^\mu = (t, r, \theta, \varphi)$, and choose $\zeta^a = (\tau, \alpha, \beta)$ with τ being the timelike coordinate. The metric on the three-surface is given by

$$g^{(3)}_{ab} = g_{\mu\nu} x^\mu_{,a} x^\nu_{,b} \ , \tag{2.1}$$

where $x^\mu_{,a} \equiv \partial x^\mu / \partial \zeta^a$. The Lagrangian for the surface is (see, e.g., the review of Vilenkin[4])

$$L = -\sigma [\det(-g^{(3)})]^{1/2} - U \ , \tag{2.2}$$

where σ is the mass per unit area of the surface as given by Eq. (1.3) and where U is the potential energy due to the nondegeneracy of the vacua. The equations of motion for the wall are

$$\frac{\partial}{\partial \zeta^a} \frac{\delta L}{\delta x^\mu_{,a}} - \frac{\delta L}{\delta x^\mu} = 0 \ . \tag{2.3}$$

We study perturbations from the spherical case by expanding in terms of spherical harmonics, $Y_{lm}(\alpha,\beta)$. The equations that describe the wall are

$$\begin{gathered} t = \tau, \quad \theta = \alpha, \quad \varphi = \beta \ , \\ r = R(\tau) + \sum_{l,m} \Delta_{lm}(\tau) Y_{lm}(\alpha,\beta) \ . \end{gathered} \tag{2.4}$$

Thus, the quantity Δ_{lm}/R measures the size of the departure from sphericity. In the analysis that follows, we will consider only first-order perturbations; i.e., we assume that $\Delta_{lm}/R \ll 1$.

The Lagrangian for the surface described by Eq. (2.3) is given by

$$\begin{aligned} L = -\sigma [& (x^\mu_{,0} x_{\mu,\alpha})^2 (x^\mu_{,\beta} x_{\mu,\beta}) + (x^\mu_{,0} x_{\mu,\beta})^2 (x^\mu_{,\alpha} x_{\mu,\alpha}) \\ & - (x^\mu_{,0} x_{\mu,0})(x^\mu_{,\alpha} x_{\mu,\alpha})(x^\mu_{,\beta} x_{\mu,\beta})]^{1/2} + \tfrac{1}{3}\epsilon r^3 \sin\alpha \ . \end{aligned} \tag{2.5a}$$

Notice that the determinant given in Eq. (2.1) has six terms whereas we have only included three in Eq. (2.5a); the extra terms are all fourth order (in Δ_{lm}/R) and do not contribute to the zeroth- or first-order equations of motion. The last term is written so that $\epsilon > 0$ corresponds to the case in which a bubble of true vacuum is expanding into a background of false vacuum. The Lagrangian (2.5a) for the bubble surface can be evaluated to first order in Δ_{lm}/R as

$$-\frac{1}{\sigma} L = r^2 (1 - \dot{r}^2)^{1/2} \sin\alpha - \tfrac{1}{3} r^3 \frac{\epsilon}{\sigma} \sin\alpha \ , \tag{2.5b}$$

where r is the (local) radius of the bubble and is defined in Eq. (2.4) above. The Lagrangian for the entire spherical shell [see Eq. (1.4)] can be obtained by integrating Eq. (2.5b) over the surface of the sphere.

From the above Lagrangian we now derive the equations of motion. Since we wish to consider perturbations only in the radial direction, it is sufficient to consider only the $\mu = r$ equation. Taking the derivative of Eq. (2.5a), we find, to first order in Δ_{lm}/R,

$$-\frac{1}{\sigma}\frac{\delta L}{\delta x^r}=2R\,\sin\alpha(1-\dot{R}^2)^{1/2}\times\left[1+\sum_{l,m}Y_{lm}\left[\frac{\Delta_{lm}}{R}-\frac{\dot{R}\dot{\Delta}_{lm}}{1-\dot{R}^2}\right]\right]-\frac{\epsilon}{\sigma}\sin\alpha R^2\left[1+\frac{2}{R}\sum_{l,m}Y_{lm}\Delta_{lm}\right], \quad (2.6a)$$

$$-\frac{1}{\sigma}\frac{\delta L}{\delta x^r_{,\alpha}}=\sin\alpha(1-\dot{R}^2)^{-1/2}\sum_{l,m}\Delta_{lm}\frac{\partial Y_{lm}}{\partial\alpha}, \quad (2.6b)$$

$$-\frac{1}{\sigma}\frac{\delta L}{\delta x^r_{,\beta}}=\frac{1}{\sin\alpha}(1-\dot{R}^2)^{-1/2}\sum_{l,m}\Delta_{lm}\frac{\partial Y_{lm}}{\partial\beta}, \quad (2.6c)$$

$$-\frac{1}{\sigma}\frac{\delta L}{\delta x^r_{,\tau}}=-R^2\dot{R}\,\sin\alpha(1-\dot{R}^2)^{-1/2}\times\left[1+\sum_{l,m}Y_{lm}\left[\frac{2\Delta_{lm}}{R}+\frac{\dot{\Delta}_{lm}}{\dot{R}(1-\dot{R}^2)}\right]\right], \quad (2.6d)$$

where we have denoted derivatives of R with respect to τ by $\dot{R}$ (see also Ref. 15). The above expressions can now be substituted into the equation of motion (2.3). The zeroth-order equation has the form

$$\ddot{R}=\frac{\epsilon}{\sigma}(1-\dot{R}^2)^{3/2}-\frac{2}{R}(1-\dot{R}^2), \quad (2.7)$$

which can be integrated to give

$$(1-\dot{R}^2)^{1/2}=\frac{R^2}{\tilde{\epsilon}R^3/3+\mathcal{C}}, \quad (2.8)$$

where we have defined an inverse length scale $\tilde{\epsilon}\equiv\epsilon/\sigma$ and where $\mathcal{C}$ is a constant of integration to be determined by the initial conditions. To obtain the full zeroth-order solution $R(\tau)$, Eq. (2.8) can be solved for $\dot{R}$ and then integrated again; however, for finding the behavior of first-order perturbations, it is sufficient to simply use Eqs. (2.7) and (2.8) to express $\ddot{R}$ and $\dot{R}$ in terms of the bubble radius R. In general, to completely specify the zeroth-order solution, two constants must be specified. If we take one of these constants to be the initial radius R_0 of the bubble, then the second constant will be $\mathcal{C}$, which can be written in terms of the initial velocity of the bubble. For example, if $\dot{R}=0$ initially, then $\mathcal{C}$ is given by $\mathcal{C}=R_0^2(1-\tilde{\epsilon}R_0/3)$. For this paper, we will take $\dot{R}(\tau=0)=0$ for the zeroth-order solution; this choice is not especially restrictive because any nonzero initial velocity can be treated as an $l=0$ perturbation.

From the form of Eqs. (2.7) and (2.8), the zeroth-order solution has the following properties. In the limits for both $R\rightarrow 0$ (collapse) and $R\rightarrow\infty$ (expansion), the wall speed $|\dot{R}|\rightarrow 1$, i.e., the bubble wall always asymptotically approaches the speed of light. If we choose $\dot{R}=0$ initially, the initial radius R_0 determines whether the bubble will either expand or collapse. For $R_0>R_S\equiv 2/\tilde{\epsilon}$, the acceleration will always be positive [Eq. (2.7)] and the bubble will expand; for $R_0<R_S$, the bubble will collapse. Thus, there exists a critical initial radius R_S for which the surface tension is exactly balanced by the potential energy term and the bubble can remain static. However, it can be shown that the static solution is unstable [perturbations ξ about the static solution have the form $\xi=\exp(\pm\tau/\tau_C)$] and the bubble will either start to expand or collapse on a characteristic time scale given by $\tau_C=R_S/\sqrt{2}$, which is of order the light crossing time for the bubble.

The equation of motion for first-order perturbation can be written

$$\ddot{\Delta}_{lm}+\dot{R}\left[3\tilde{\epsilon}(1-\dot{R}^2)^{1/2}-\frac{4}{R}\right]\dot{\Delta}_{lm}-(1-\dot{R}^2)(2-J)\frac{\Delta_{lm}}{R^2}=0, \quad (2.9)$$

where $J\equiv l(l+1)$ is the angular eigenvalue associated with the spherical harmonics. Since the time coordinate does not appear in the equation of motion, we can eliminate the time dependence of Δ_{lm} in favor of the coordinate R; i.e., we can measure time by the position of the bubble radius for the unperturbed case. Furthermore, we will suppress the subscripts of the function Δ for the remainder of this paper. If we then use

$$\dot{\Delta}=\dot{R}\frac{d\Delta}{dR}, \quad (2.10a)$$

$$\ddot{\Delta}=\ddot{R}\frac{d\Delta}{dR}+\dot{R}^2\frac{d^2\Delta}{dR^2}, \quad (2.10b)$$

the perturbation equation becomes

$$\dot{R}^2\frac{d^2\Delta}{dR^2}+\left[\ddot{R}+\dot{R}^2\left[3\tilde{\epsilon}(1-\dot{R}^2)^{1/2}-\frac{4}{R}\right]\right]\frac{d\Delta}{dR}-(1-\dot{R}^2)(2-J)\frac{\Delta}{R^2}=0. \quad (2.11)$$

III. EVOLUTION OF NUCLEATION BUBBLES

In this section we find the solutions to Eqs. (2.7) and (2.11) for the case of zero-temperature nucleation bubbles considered by Coleman. For the initial conditions $R(t=0)=R_C=3\sigma/\epsilon$ (the radius at which the bubble materializes), and $\dot{R}(t=0)=0$, the solution to the zeroth order [Eq. (2.7)] is

$$R^2=t^2+R_C^2, \quad (3.1)$$

which corresponds to growing bubbles that asymptotically approach the speed of light, in agreement with Coleman. Making the substitution $x=R/R_C$ and using Eq. (2.11), we can rewrite the first-order equation as

$$x^2(x^2-1)\Delta''+x(5x^2-4)\Delta'+(J-2)\Delta=0, \quad (3.2)$$

where the primes denote derivatives with respect to x. Notice that the equation of motion is singular at $x=1$. Hence, we must find a series solution in the neighborhood of the point $x=1$. As required, the equation of motion has two linearly independent solutions, which can be written in the form

$$\Delta(\xi)=\Delta_a(1+a_1\xi+a_2\xi^2+\cdots), \quad (3.3a)$$

$$\Delta(\xi)=\Delta_b\xi^{1/2}(1+b_1\xi+b_2\xi^2+\cdots), \quad (3.3b)$$

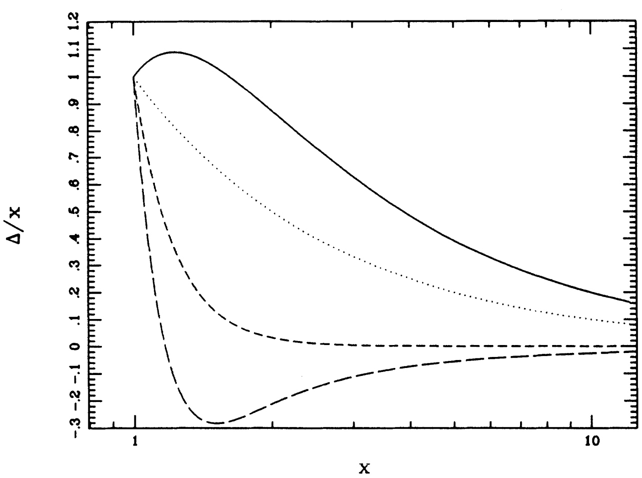

FIG. 2. Relative displacement Δ/x as a function of x for perturbations with finite initial amplitude and zero initial velocity. The various curves are for angular wave numbers of $l=0$ (solid line), $l=1$ (dots), $l=2$ (short dashes), and $l=3$ (long dashes).

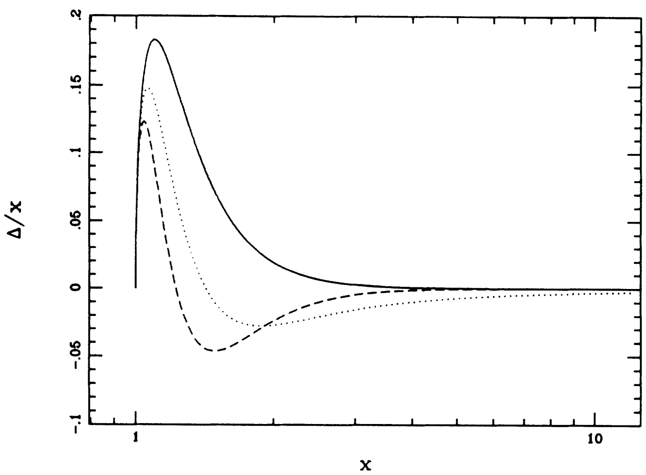

FIG. 4. Relative displacement Δ/x as a function of x for perturbations with zero initial amplitude and finite initial velocity. The various curves are for angular wave numbers of $l=3$ (solid line), ($l=4$) (dots), and $l=5$ (short dashes).

where $\xi \equiv x-1$. Initially, $\xi=0$, so the first solution (3.3a) corresponds to a perturbation with a finite initial amplitude Δ_a and the second solution (3.3b) corresponds to a perturbation with zero initial amplitude. Recall that the initial velocity of the perturbation is given by

$$\frac{d\Delta}{d\tau}=\frac{\dot{R}}{R_C}\frac{d\Delta}{d\xi}\,, \tag{3.4a}$$

and that $\dot{R}=0$ initially. For the first solution (3.3a), $d\Delta/d\xi$ is finite at $\xi=0$ and hence the initial velocity of the perturbation is zero. For the second solution (3.3b), $d\Delta/d\xi$ is infinite at $\xi=0$ and the initial velocity of the perturbation is finite, i.e.,

$$\frac{d\Delta}{d\tau}\rightarrow\frac{\sqrt{2}}{2}\frac{\Delta_b}{R_C}\,. \tag{3.4b}$$

Thus, the length scale Δ_a determines the initial amplitude for the first type of perturbation (3.3a) and the length scale Δ_b determines the initial velocity for the second type of perturbation (3.3b).

The equation of motion (3.2) for radial perturbations has been solved for a variety of initial conditions using a fourth-order Runge-Kutta routine. For perturbations with finite initial amplitude and zero initial velocity [which have the form (3.3a) near $x=1$], we find decaying solutions as shown in Fig. 2 for an initial length scale $\Delta_a=1$. The $l=1$ perturbation exhibits the behavior $\Delta=$const, $\Delta/x\sim x^{-1}$. For all $l>1$, the perturbations begin to oscillate but Δ decays rapidly to zero. For perturbations with zero initial amplitude and finite initial velocity [which have the form (3.3b) near $x=1$], we find growing solutions. A series of these finite velocity perturbations is shown in Figs. 3–5 for a variety of spherical harmonics, $l=0$–5, and 20. The figures are plotted for a length scale $\Delta_b=1$, which corresponds to an initial velocity of $c/\sqrt{2}$ [see Eq. (3.4b) above]. The growing solutions peak at $\Delta/R_C=0.3$ (e.g., for $l=2$) and freeze out

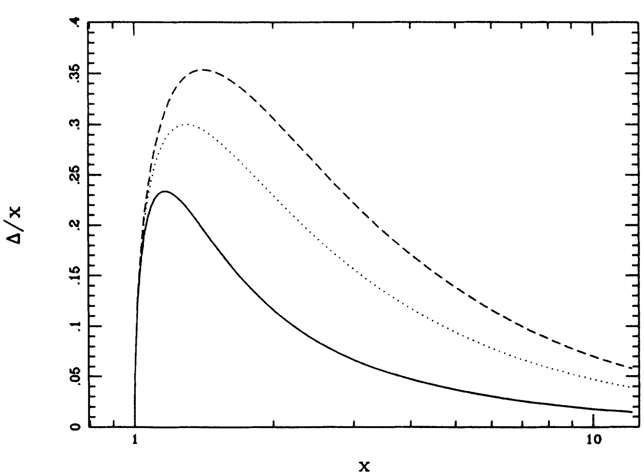

FIG. 3. Relative displacement Δ/x as a function of x for perturbations with zero initial amplitude and finite initial velocity. The various curves are for angular wave numbers of $l=0$ (short dashes), $l=1$ (dots), and $l=2$ (solid line).

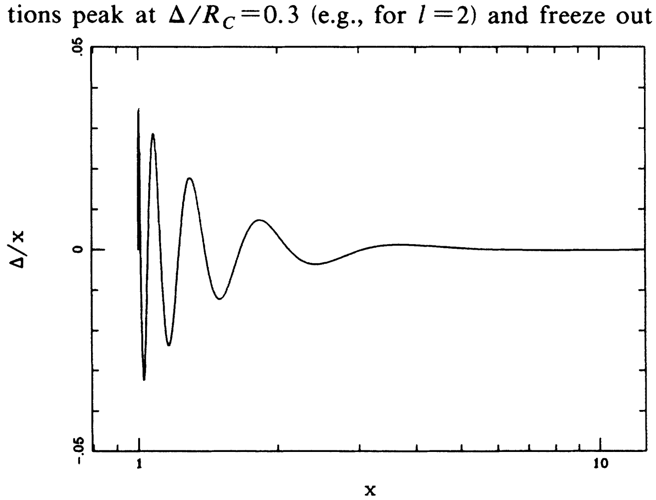

FIG. 5. Relative displacement Δ/x as a function of x for an $l=20$ perturbation with zero initial amplitude and finite initial velocity.

after that, i.e., as $R\rightarrow\infty,\Delta\rightarrow$const and hence $\Delta/x\sim x^{-1}$. This behavior of initial growth and eventual freezing out can be seen in power-law solutions for the short-time and long-time behavior as well.

Perturbations with higher angular wave number ($l>5$) exhibit oscillatory behavior as shown, e.g., in Fig. 5. The amplitude of the oscillations is small and after a few wavelengths the oscillations are damped. For increasing l, the solutions exhibit more oscillations with ever decreasing amplitude. The energy in the oscillations is a small fraction of the total energy in the bubble wall. Furthermore, the oscillating perturbations decay before the radius of the bubble has expanded by a factor of 10. In inflationary scenarios, the radius must expand by a factor of 10^{28}; thus, these oscillations are probably insignificant.

In summary, we find that finite velocity perturbations with low l grow and subsequently freeze out before they achieve nonlinearity (in the case of nucleation bubbles considered here). We find that finite velocity perturbations with high l have short-lived oscillations of small amplitude and that finite amplitude perturbations decay for all l. Thus our calculations (if we extrapolate from the zero gravity and zero-temperature limit) indicate that inflationary bubbles are well described by the zeroth-order solution for bubble evolution.

IV. DISCUSSION

In this paper, we have studied the growth of radial perturbations to spherical bubbles. We have focused on the case of expanding bubbles of true vacuum nucleated at a phase transition in the early Universe. However, the formulation presented here can be used to describe many similar problems. In the cases we considered, all the energy swept up by a growing bubble is converted into the kinetic energy of the wall. Generalization to the case where only some fraction is in the wall and the rest goes, e.g., to particle creation would not be difficult; one would have a new effective ϵ (some fraction of the total ϵ) which could also be a function of time, e.g., a function of the radius of the bubble.

Our work is also relevant to spherical topological domain walls that may arise at late-time phase transitions in models where the potential has nondegenerate minima. The evolution of perturbations to spherical bubbles would then be described by our formalism. We have considered the case of no friction between the walls and the particles in the Universe. In a version of the model of Hill and Ross,[16] the coupling of the Φ field (here a schizon) to neutrinos Ψ has the form $\Phi\bar{\Psi}\Psi/f$, where we assume $f\sim10^{15}$ GeV (GUT scale). Thus the coupling and the friction are very small. It might be interesting to consider structure formation with small $f\sim$weak scale [as suggested by Chris Hill (private communication)]. In this case the neutrino on different sides of the wall has a different mass (for a potential with nondegenerate minima)[12,17] and friction would serve to slow down the walls. Trapped neutrinos might even halt the collapse and lead to the formation of nontopological solitons.[18]

Bubble walls such as those arising in the model of Hill, Schramm, and Fry[3] can lose energy through particle radiation. The shape of a wall during its collapse determines how quickly a wall will lose energy due to particle radiation, a process that occurs for collapsing walls once their finite thickness is taken into account.[19] However, in this paper, we have assumed that no particle radiation takes place. For the model of Ref. 3, the thickness of the wall $\delta\sim m_\Phi^{-1}$. Hence in the thin-wall limit, we are looking at oscillations whose wavelength exceeds the Compton wavelength of schizons (Φ particles where Φ is the field responsible for the wall) and neutrinos (the only particles to which the schizon couples). The time scale for gravitational radiation is longer than a Hubble time and can be ignored. Hence, the assumption of no particle radiation is consistent with the thin-wall limit considered here.

In this paper, we have ignored gravity and worked in a flat spacetime metric and focused on expanding spherical shells. For the case of collapsing spherical walls, we will include the effects of gravity in a future paper.

To summarize, we have presented a calculational framework to study radial perturbations to the evolution of spherical shells of domain walls. We focused on expanding bubbles of true vacuum such as those arising at phase transitions in the early Universe. We found that perturbations with finite initial amplitude decay. Perturbations with an initial velocity and low angular wave number l grow and subsequently freeze out; perturbations with an initial velocity and high l exhibit oscillatory behavior which eventually damps out. Hence, (in the flat space time limit) inflationary bubbles are well described by the zeroth-order solution for bubble evolution. Our analysis may also be relevant to cosmological domain walls, detonation waves, and the evolution of nonspherical bubbles in other contexts.

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