PHYSICAL REVIEW D VOLUME 41, NUMBER 11 1 JUNE 1990

Diquarks in the Nambu–Jona-Lasinio model

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(Received 15 December 1989)

We investigate the possibility of binding locally two quarks in the Nambu–Jona-Lasinio model. For a range of model parameters consistent with vacuum parameters the scalar, isoscalar, and color-antitriplet diquark is found to bind with a mass of about the constituent-quark mass. The relevance of this light diquark for baryons in the Nambu–Jona-Lasinio model is discussed.

In QCD, hadrons are believed to be either color-singlet bound states of quarks, which are baryons, or of quarks and antiquarks, which are mesons. The binding in these states is due to strong gluonic interactions. To form a baryon, one requires a color singlet of three quarks, in which two quarks are always a color $\overline{3}$ state and the remaining quark is in a color 3 state. Hadrons may possibly be viewed as color-singlet quark-diquark bound states provided that diquarks are light.

The concept of diquarks was introduced by Gell-Mann[1] in his seminal paper on the quark model and by Ida and Kobayashi[2] and Lichtenberg and co-workers[3] in the context of constituent-quark models. It was later argued that the presence of diquark substructure in the nucleon could account for observed scaling violations in lepton-nucleon scattering experiments, where structure functions would be smeared by the diquark form factor.[4] These experiments suggested that the scalar, isoscalar diquark is the lightest and that its radius is small ($\approx 1\ \mathrm{GeV}^{-1}$). Diquarks also prove useful in string-model analyses of baryon yields in e^+e^- collisions.[5,6] In addition, a large amount of data from pp collisions[7,8] favor quark-diquark models of nucleons over models with three independent quarks. It has been suggested that quark matter just above the deconfinement phase transition exists primarily in a configuration of diquark clusters[9] and in Ref. 10 the effect of a diquarklike scalar field within a hadron bag was examined. Values of the mass of the lightest diquark in the literature include 225 MeV (Ref. 5), 321 MeV (Ref. 3), $\approx$420 MeV (Ref. 6), and a lower bound of 400 MeV in Ref. 8, obtained from a string-model analysis of inclusive production of large-p_T protons.

Diquark masses have been calculated in various models with nonlocal interactions.[3,11,12] In general, the nonlocality necessitates the use of nontrivial form factors. In this paper we shall investigate the possibility of diquark formation in the Nambu–Jona-Lasinio model using local interactions. Since diquarks are thought to be two interacting quarks with a high degree of collectivity, this approach is perhaps appropriate. Our analysis will parallel the discussion on the formation of collective quark-antiquark excitations in the above-mentioned model.

Consider the following (massless) version of the Nambu–Jona-Lasinio model:

$$\mathcal{L}=\bar{\psi}i\partial\!\!\!/\psi+g_1[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\tau_a\psi)^2]-g_2\left[\bar{\psi}\gamma_\mu\frac{\lambda_a}{2}\psi\right]^2 , \tag{1}$$

where ψ refers to the quark field in the color 3 representation, τ_a are the generators of $SU_f(2)$, and λ_a the generators of $SU_c(3)$. Other flavor-singlet and chiral-invariant combinations of the same or higher-dimensional order are also possible but in the long-wavelength limit the higher-dimensional operators are suppressed.

The effective interactions occurring in Eq. (1) can be thought of as generated from gluonic interactions by coarse graining.[13] Similar interaction Lagrangians can be derived from an instanton-based description of the QCD vacuum.[14] In these descriptions the resulting multifermion interactions are usually nonlocal. We have retained the scalar and vector combinations of lower dimensions to mimic the scalar character of confining forces (long-range fluctuations) and the vector nature of one-gluon exchange (short-range fluctuations). Since the coupling constants g_1 and g_2 are dimensionful the model is not renormalizable. A four-dimensional cutoff Λ will be understood throughout.

As was first pointed out by Nambu and Jona-Lasinio,[15] the vacuum spontaneously breaks chiral symmetry for a critical value of the coupling. The quarks thereby acquire a nonvanishing constituent mass M. In the Hartree-Fock approximation (Euclidean space),

$$M=-(a_1g_1+a_2g_2)\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\frac{1}{i q\!\!\!/-M} \tag{2}$$

with $a_1=2N_fN_c+1=13$ and $a_2=\frac{8}{3}$. Using the covariant cutoff Λ we have

$$1=(a_1g_1+a_2g_2)\frac{\Lambda^2}{4\pi^2}f\left[\frac{M}{\Lambda}\right] , \tag{3}$$

where $f(M/\Lambda)$ is a dimensionless function. Solutions to Eq. (3) exist for $(a_1g_1+a_2g_2)\geq 4\pi^2/\Lambda^2$.

We use the Bethe-Salpeter equation as shown in Fig. 1(a) to describe pseudoscalar quark-antiquark bound states. Denoting the pion vertex by Γ_b^π, the contribution of the vector interaction in Eq. (1) reads

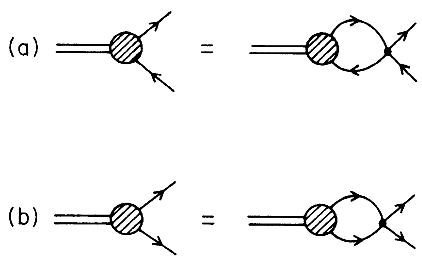

FIG. 1. (a) Bethe-Salpeter equation for mesons, Eq. (4); (b) Bethe-Salpeter equation for diquarks, Eq. (7).

$$\Gamma_b^\pi = 2g_2 \int \frac{d^4q}{(2\pi)^4}\gamma_\mu \frac{\lambda_a}{2} \frac{1}{i\left(\not{q}+\frac{P}{2}\right)-M} \times \Gamma_b^\pi \frac{1}{i\left(\not{q}-\frac{P}{2}\right)-M}\gamma^\mu \frac{\lambda_a}{2} \,. \tag{4}$$

The above is purely an exchange process. The contribution of the scalar interaction is analogous to this except for additional direct terms. The pion vertex corresponds to the singlet combination $\Gamma_b^\pi = \tau_b\gamma_5 1_c$. Writing $P=(iM_\pi,0)$ we find

$$1=(a_1g_1+a_2g_2)\frac{\Lambda^2}{4\pi^2} I\left[\frac{M}{\Lambda},\frac{M_\pi}{\Lambda}\right] \tag{5}$$

for a specific routing of the momenta, where I is a dimensionless function satisfying $I(M/\Lambda,0)=f(M/\Lambda)$. Equation (5) leads to a hierarchy of pseudoscalar poles with a massless pion $M_\pi=0$. The pion vertex in general satisfies the chiral Ward identity so that $\Gamma_b^\pi=\tau_b\gamma_5 M/f_\pi$ where f_π is the pion decay constant. It is straightforward to show that Eq. (1) also yields a scalar mass at threshold with $M_\sigma=2M$.

Diquarks are quark-quark bound states. Since the quarks are originally in the color-triplet representation they can be combined into either an antitriplet color state or a color-sextet state with various flavor contents. Following the color decomposition $3\otimes 3=\bar{3}\oplus 6$ the antitriplet states are antisymmetric while the sextet states are symmetric. The only local, color-antitriplet Lorentz scalars are

$$\psi_a^T(x)C\gamma_5\tau_2\psi_b(x)\epsilon_{abc} \,, \tag{6}$$

where C is the charge-conjugation matrix $C\gamma^{\mu^T}C^{-1} = -\gamma^\mu$ and the superscript T stands for transposition. Other flavor combinations vanish due to the anticommuting nature of the quark fields and the fact that $C^T=-C$:

$$\begin{aligned}\psi_a^T(x)C\gamma_5\psi_b(x)\epsilon_{abc} &= \psi_a^T(x)C\gamma_5\tau_1\psi_b(x)\epsilon_{abc}\\ &= \psi_a^T(x)C\gamma_5\tau_3\psi_b(x)\epsilon_{abc}\\ &= 0 \,.\end{aligned}$$

Note that the scalar diquark (6) is invariant under $SU_L(2)\times SU_R(2)$. In a *nonlocal* description of diquarks, the combinations $\psi_a^T(x_1)C\gamma_5\psi_b(x_2)\epsilon_{abc}$, etc., do not vanish. However, since they are antisymmetric under the interchange of x_1 and x_2, we conclude that contributions from diquarks with symmetric flavor structure should not be included in the action of Ref. 16.

The diquark mass follows from the Bethe-Salpeter equation of Fig. 1(b). Note that in this case the total contribution is through exchange effects. The contribution of the vector interaction to the diquark vertex function Γ^D reads

$$\Gamma^D = 2g_2 \int \frac{d^4q}{(2\pi)^4}\gamma_\mu^T \frac{\lambda_a^T}{2} \frac{1}{i\left(\not{q}+\frac{P}{2}\right)^T-M} \times \Gamma^D \frac{1}{-i\left(\not{q}-\frac{P}{2}\right)-M}\gamma^\mu \frac{\lambda_a}{2} \,. \tag{7}$$

The spin-flavor structure of the color $\bar{3}$ scalar diquarks is $\Gamma^D = C\gamma_5\tau_2\Gamma^A$ where Γ^A is an antisymmetric color matrix. Proceeding as with the mesons, and using the color and flavor identities $\lambda^a\lambda^a=\frac{16}{3}$, $\lambda^a\Gamma^A\lambda^{aT}=-\frac{8}{3}\Gamma^A$, and $\tau_a\tau_2\tau_a^T=-3$ we obtain

$$1=(b_1g_1+b_2g_2)\frac{\Lambda^2}{4\pi^2} I\left[\frac{M}{\Lambda},\frac{M_D}{\Lambda}\right] \tag{8}$$

with $b_i=2$ and $b_2=\frac{4}{3}$. Here I is the function occurring in Eq. (5). Note that the interaction is attractive in the antitriplet state.

The constituent mass M and cutoff Λ are chosen so that the vacuum quark condensate $\langle\bar{\psi}\psi\rangle \approx(-250\text{ MeV})^3$ and the pion decay constant $f_\pi\approx 93$ MeV. The quark condensate is given by

$$\langle\bar{\psi}\psi\rangle = -N_f N_c \frac{\Lambda^2}{4\pi^2} M f\left[\frac{M}{\Lambda}\right] \,. \tag{9}$$

The pion decay constant follows from the normalization of the axial-vector current between one pion state and the

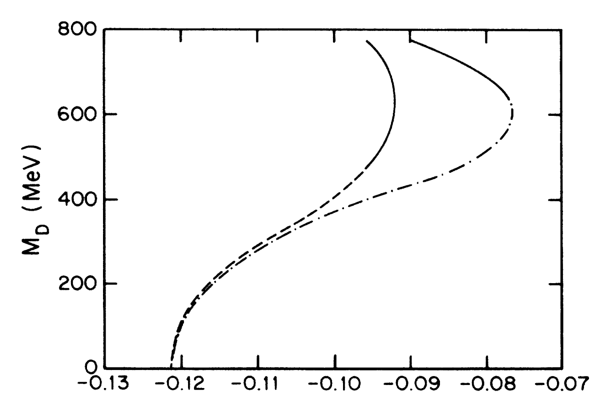

FIG. 2. Diquark mass as a function of g_1/g_2. The dashed-dotted curve gives the result for $\Lambda=800$ MeV and the dashed one for $\Lambda=1$ GeV. The solid curves show the region in which there is no bound state.

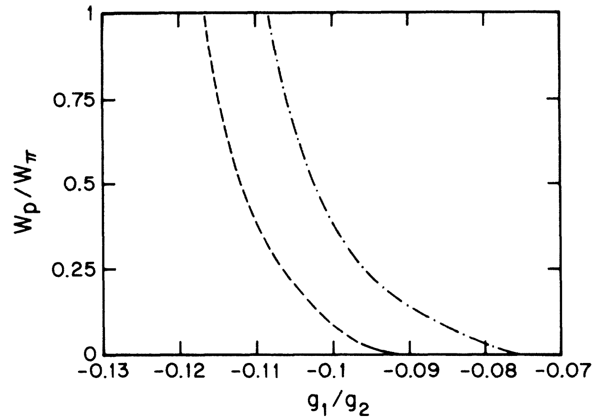

FIG. 3. Ratio of the proton-to-pion fragmentation function following string breaking as discussed in the text for the two cutoffs of Fig. 2. The string tension is $\sigma=0.18\ \mathrm{GeV}^{-2}$.

vacuum:

$$if_\pi M_\pi=-\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}\left[\gamma_0\gamma_5\frac{\tau_3}{2}\frac{1}{i\left(\not{q}+\frac{P_\pi}{2}\right)-M}\times\tau_3\gamma_5\frac{M}{f_\pi}\frac{1}{i\left(\not{q}-\frac{P_\pi}{2}\right)-M}\right]. \tag{10}$$

If we were to switch off *one* of the *two* couplings, the remaining coupling would be determined by Eq. (3). In this case, we find no bound-state solutions to Eq. (8). Therefore, if we insist on reproducing the vacuum parameters, two couplings are required. It is possible, however, that since diquarks are to be found within baryons the "vacuum parameters" might be altered. As a result diquark couplings are independent of meson couplings. This alternative will not be pursued here. Rather, we will investigate the meson and diquark formation using the same vacuum parameters and nonvanishing couplings.

We have set f_π equal to 93 MeV throughout and found the value of M after choosing the cutoff Λ. The value of $\langle\bar{\psi}\psi\rangle$ follows from Eq. (9) and the value of $G=a_1g_1+a_2g_2$ from Eq. (3). For $\Lambda=800$ MeV we have $M=317$ MeV, $\langle\bar{\psi}\psi\rangle=(-277\ \mathrm{MeV})^3$, and $G=8.98\times10^{-5}\ \mathrm{MeV}^{-2}$. For $\Lambda=1$ GeV; $M=241$ MeV, $\langle\bar{\psi}\psi\rangle=(-3.12\ \mathrm{MeV})^3$, and $G=4.75\times10^{-5}\ \mathrm{MeV}^{-2}$. Using Eq. (3) to eliminate the cutoff in Eq. (8) allows one to study the diquark mass M_D as an implicit function of g_1/g_2 as shown in Fig. 2 for the above cutoffs. The upper parts of the curves (solid) lie in the continuum.

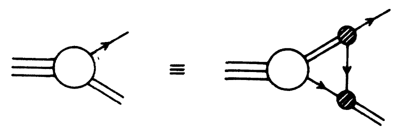

FIG. 4. Bethe-Salpeter equation for a nucleon as a quark-diquark bound state. The diquark- (quark-quark) vertex function is $C\gamma_5\tau_2\Gamma^A$.

Note that for $g_1/g_2=-\frac{4}{33}$ the diquark is massless. Using the first set of parameters, a mass of $M_D=400$ MeV is given by $g_1\Lambda^2=-3.87$ and $g_2\Lambda^2=40.4$ and a mass of $M_D=500$ MeV by $g_1\Lambda^2=-2.88$ and $g_2\Lambda^2=35.6$. With these parameter sets we find that the interaction is repulsive in the octet and sextet.

In a string-model analysis of the inclusive production of high-p_T protons the fragmentation function decreases exponentially with the diquark mass squared. Assuming that inclusive proton production follows from string breaking, the ratio of the fragmentation functions for inclusive protons to pions reads

$$\frac{W_p}{W_\pi}\approx\tfrac{1}{2}\exp\left[-\frac{\pi}{\sigma}(M_D^2-M^2)\right], \tag{11}$$

where $\sigma=(2\pi\alpha)^{-1}$ is the string tension.[12] For $\sigma=0.18\ \mathrm{GeV}^{-2}$, Fig. 3 shows the behavior of (11) as a function of g_1/g_2 in the range of couplings considered here and for the two cutoffs discussed above. The *pp* data for inclusive proton production favor a ratio of about 0.03 in the kinematical range $x>0.5$ (Ref. 8). This curve should be viewed as an "empirical" constraint on the relative coupling strengths and the choice of the bare cutoff in this schematic model.

To summarize, we note that in the Nambu–Jona-Lasinio model the attractive character of the scalar interaction in the vacuum causes the quarks to condense and the pion to be massless. The same interaction induces an attraction between two quarks but contributes only exchange terms, which are weaker by the ratio 2:13. However, this interaction does not discriminate between colors. It is also attractive for the antitriplet and sextet quark-quark configurations and the octet quark-antiquark configurations. Vector-type interactions, suggestive of one-gluon exchange, are therefore needed. It would be interesting to see whether the light scalar diquarks can bind with the constituent quarks to form color-singlet baryons in the Nambu–Jona-Lasinio model. To do this one needs to investigate the Bethe-Salpeter equations for a quark-diquark bound state as shown in Fig. 4.

This work was supported in part by the U.S. Department of Energy under Grant No. DE-FG02-88ER40388.

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