Higgs bosons and sleptons in an alternative left-right model

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The phenomenological structure of the combined Higgs-boson–slepton sector of the alternative left-right-supersymmetric model introduced by Ma is explored. Constraints upon and relations between Higgs-boson and slepton masses are derived and a tightly constrained mass spectrum is found. In general, one neutral Higgs boson is never heavier than 98 GeV, one neutral Higgs boson is always nearly degenerate in mass with the extra neutral gauge boson Z_2^0, and the charged Higgs boson can in principle be as light as 22 GeV. Further constraints require large ratios of Higgs vacuum expectation values, strongly favor the W_R mass above $\sim$423 GeV, predict one Higgs-boson mass to be always very close to 98 GeV, and masses of the other Higgs bosons and the sleptons to be bounded from above and preferably not much above the Z mass. In addition, the possibility of detecting light Higgs bosons at CERN LEP and the SLAC Linear Collider is briefly discussed.

I. INTRODUCTION

Despite its great experimental success, the standard model is generally expected to be part of a more fundamental theory. One of its most interesting left-right extensions is due to Ma[1] and has been motivated by the superstring-based E_6 theory in which each generation of matter fields belongs to the **27** representation of E_6. Ma noticed that if the low-energy gauge group to which E_6 is broken is $SU_c(3)\times SU_L(2)\times SU_R(2)\times U_Y(1)$ then an alternative assignment of quantum numbers for fermion fields is possible, which yields different phenomenological predictions. In the new left-right model, (u,d) is an $SU_L(2)$ quark doublet, while (u^c,h^c) forms an $SU_R(2)$ doublet of color fermions. The new down-type quark h, present in the **27** of E_6, also carries a nonzero lepton quantum number. Among $B=0$ states, ordinary leptons ν_e and e, along with new $L=0$ states, both charged E, and neutral ν_E, N_E^c, and n, form an $SU_L(2)$ doublet (ν_E,E), an $SU_R(2)$ doublet (e^c,n), and a bidoublet (E^c,N_E^c,ν_e,e) under both $SU_L(2)$ and $SU_R(2)$. A conserved discrete symmetry can be defined and identified with the usual supersymmetric R parity, $R=(-1)^{2j+3B+L}$, where j, B, and L denote spin, baryon, and lepton quantum numbers, respectively. All the usual quarks and leptons carry positive R parity, while the exotic fermions are assigned negative R parity, with opposite assignments given to their bosonic partners. A new right charged current W_R couples u^c to h^c; thus, it must have negative R parity as well as a nonzero lepton number. For these reasons the W_R does not mix with the W_L and escapes constraints from K_L-K_S mixing; thus, it can in principle, be much lighter than in conventional left-right models. (For recent reviews of this model, see Refs. 2 and 3.)

Most studies[4–8] focused mainly on the various properties of the extra gauge bosons and exotic fermions. The Higgs sector of the model was outlined by Babu, He, and Ma.[9] In this paper, I extend their work by exploring the phenomenological structure of the Higgs potential. Actually, because in this model both leptonic and nonleptonic states are combined into the same multiplets, the Higgs potential contains also the slepton part of the full scalar potential. Therefore, unlike the authors of Ref. 9, I explicitly distinguish between physical Higgs bosons and scalar leptons. I demonstrate a tightly constrained Higgs-boson–slepton mass spectrum and find severe constraints on the parameters in the model. I also derive several mass relations and approximate formulas which prove useful in understanding the pattern of the mass spectrum. In general, at least one Higgs boson is predicted to be always within a hundred GeV mass range, with other scalars (both leptonic and Higgs) possibly also being light. One neutral Higgs boson is always nearly degenerate with the Z_2^0. I find that the Higgs self-coupling λ, defined below, can take values in the very restricted range between ~ 0.44 and ~ 0.53, and therefore always remains in the perturbative region. Next, I apply additional constraints which require large ratios of vacuum expectation values (VEV's) and show that they practically exclude W_R lighter than approximately 423 GeV (although somewhat lower values are also in principle possible), and predict a very definite pattern of the Higgs-boson and the slepton mass spectra. The mass of one neutral scalar is always very near 98 GeV; other scalars can be as light as several GeV and should not be heavier than several hundred GeV, depending on the mass of the W_R. Over a significant region of the parameter space, the sneutrino can be very light. Finally, I comment on the prospects for searching for a light neutral Higgs bosons in e^+e^- collisions at CERN LEP and the SLAC Linear Collider (SLC).

The paper has the following structure. In Sec. II, I introduce the Higgs, slepton, and gauge bosons, and express their masses in terms of conveniently chosen parameters. I also derive and discuss several mass sum rules. In Sec. III, I examine various constraints on the model and explore the highly constrained and interrelated Higgs-boson and slepton mass spectra. Section IV is devoted to analyzing the prospect for an experimental search of the lightest Higgs-boson states. Conclusions are summarized in Sec. V.

II. GAUGE-BOSON, HIGGS-BOSON, AND SLEPTON MASSES

A. Mass Formulas

The Higgs bosons and sleptons in this model belong to a bidoublet, two doublets, and a singlet under the gauge symmetry group $G=SU_L(2)\times SU_R(2)\times U_Y(1)$. More generally, there are three families of Higgs fields, each belonging to the **27** of E_6. As usual, we will assume that a rotation has been made to a basis in which neutral Higgs bosons of only the third family acquire vacuum expectation values; hence, the third family notation will be used from now on.

The $SU_L(2)$ doublet transforms as $H_L=(2,1,-1)$:

$$H_L=\begin{bmatrix} H_L^0 \\ H_L^- \end{bmatrix}=\begin{bmatrix} \tilde{\nu}_E \\ \tilde{E} \end{bmatrix} . \tag{1}$$

The $SU_R(2)$ doublet $H_R=(1,2,1)$ is specified by

$$H_R=\begin{bmatrix} H_R^+ \\ H_R^0 \end{bmatrix}=\begin{bmatrix} \tilde{\tau}^c \\ \tilde{n} \end{bmatrix} . \tag{2}$$

The bidoublet

$$\Phi=\begin{bmatrix} \phi_{11}^+ & \phi_{12}^0 \\ \phi_{21}^0 & \phi_{22}^- \end{bmatrix}=\begin{bmatrix} \tilde{E}^c & \tilde{N}_E^c \\ \tilde{\nu}_\tau & \tilde{\tau} \end{bmatrix} \tag{3}$$

transforms under the gauge group $SU_L(2)\times SU_R(2)\times U_Y(1)$ as $\Phi=(2,2,0)$; i.e., it consists of two left doublets $(\tilde{E}^c,\tilde{N}_E^c)$ and $(\tilde{\nu}_\tau,\tilde{\tau})$ as well as two right doublets $(\tilde{E}^c,\tilde{\nu}_\tau)$ and $(\tilde{N}_E^c,\tilde{E})$. The lepton fields, $\tilde{\tau}$, $\tilde{\tau}^c$, and $\tilde{\nu}_\tau$, have negative R-parity assignments, with positive R parity assigned to the Higgs-boson states. There exists also a Higgs singlet (with respect to the whole low-energy gauge group G) which in the simplest scenario considered here does not couple to the other Higgs bosons and will be of no interest to us. It will thus be neglected in further analysis.

In order to avoid breaking R parity and lepton number, the real components of only neutral Higgs fields are allowed to acquire nonzero vacuum expectation values

$$\langle \phi_{12}^0\rangle=v,\quad \langle H_L^0\rangle=v_L,\quad \langle H_R^0\rangle=v_R , \tag{4}$$

which, by a suitable rotation of the Higgs fields, are chosen to be real and positive. Define also

$$\tan\theta_L\equiv\frac{v}{v_L} . \tag{5}$$

The ordinary charged gauge boson W_L^- receives its mass from a combination of H_L^- and ϕ_{11}^-, while the new charged boson of negative R parity W_R^- becomes massive by absorbing a combination of H_R^- and ϕ_{22}^-. Because of their different R parity and lepton-number assignments, $W_L^\pm$ and $W_R^\pm$ do not mix. Their masses are given by[9]

$$m_{W_{L,R}^\pm}^2=\tfrac{1}{2}g_{L,R}^2(v^2+v_{L,R}^2) , \tag{6}$$

where $g_{L,R}$ are the $SU_{L,R}(2)$ gauge couplings. Since W_R does not have to be much heavier than W_L, it is reasonable to assume that $g_L=g_R$. These couplings are related to the electric charge $e=\sqrt{4\pi\alpha}=\sqrt{4\pi/128}$ via $g_L=g_R=e/\sqrt{x}$ where $x\equiv\sin^2\theta_W=0.23$. The $U_Y(1)$ gauge coupling constant g' is given by $g'=e/\sqrt{1-2x}$. In the neutral gauge sector, aside from the photon, there are two massive neutral gauge bosons Z_1^0 and Z_2^0 with $m_{Z_1^0}<m_{Z_2^0}$, both of positive R parity. They acquire mass from a combination of the imaginary components of ϕ_{12}^0, H_L^0, and H_R^0. The Z_1^0-Z_2^0 mass matrix can be found in Ref. 9. For $m_{W_R}\gg m_{W_L}$, in a first approximation one has[9]

$$m_{Z_1^0}^2=\frac{m_{W_L}^2}{1-x}\left[1-\left[\frac{1}{1-x}+\frac{v_L^2}{v^2+v_L^2}\right]^2\frac{m_{W_L}^2}{m_{W_R}^2}\right] +O(1/v_R^4) \tag{7}$$

and

$$m_{Z_2^0}^2=\frac{1-x}{1-2x}m_{W_R}^2+O(v_R^0) . \tag{8}$$

The effect of the Z_1^0-Z_2^0 mixing on the Z_1^0 mass is typically quite small.[9]

The Higgs potential of only the third generation will be considered, since only the Higgs bosons of that family acquire VEV's. It arises from a term $W=\lambda\Phi H_L H_R$ in the superpotential, and has been derived in Ref. 9:

$$\begin{aligned} V=\lambda^2[&|\phi_{12}H_{R_2}-\phi_{22}H_{R_1}|^2+|\phi_{11}H_{R_2}-\phi_{21}H_{R_1}|^2+|\phi_{21}H_{L_2}-\phi_{22}H_{L_1}|^2+|\phi_{11}H_{L_2}-\phi_{12}H_{L_1}|^2+(H_{Li}^*H_{Li})(H_{Rj}^*H_{Rj})] \\ &+\frac{e^2}{8x}(4|H_{L1}^*H_{L2}+\phi_{\alpha1}^*\phi_{\alpha2}|^2+|H_{L1}^*H_{L1}-H_{L2}^*H_{L2}+\phi_{\alpha1}^*\phi_{\alpha1}-\phi_{\alpha2}^*\phi_{\alpha2}|^2 \\ &\qquad+4|H_{R_1}^*H_{R_2}+\phi_{1\alpha}^*\phi_{2\alpha}|^2+|H_{R_1}^*H_{R_1}-H_{R_2}^*H_{R_2}+\phi_{1\alpha}^*\phi_{1\alpha}-\phi_{2\alpha}^*\phi_{2\alpha}|^2) \\ &\qquad+\frac{e^2}{8(1-2x)}(H_{Li}^*H_{Li}-H_{Ri}^*H_{Ri})^2+V_{\text{soft}} , \end{aligned} \tag{9}$$

where

$$V_{\rm soft}=m_1^2\phi_{ij}^*\phi_{ij}+m_3^2(H_{Li}^*H_{Li}+H_{Ri}^*H_{Ri})$$
$$+\lambda A(\phi_{ij}H_{L\alpha}H_{R\beta}\epsilon_{i\beta}\epsilon_{j\alpha}+\text{H.c.})\ . \tag{10}$$

Other possible terms in the superpotential or in $V_{\rm soft}$ have been shown[9] to be inconsistent with the minimization condition which places three constraints on the seven parameters in the Higgs sector. (Note that the last term in $V_{\rm soft}$ [Eq. (3.15)] in Ref. 9 is actually not soft. This has, however, no impact on the subsequent analysis since this term is next forced to vanish by the minimization condition.)

If one includes also the first two families of matter fields, the Yukawa coupling λ becomes a three-index tensor in the family space. Additional assumptions have to be made with respect to the relative size of the integer-generational couplings of the type $\lambda_{113}\ll\lambda_{223}\ll\lambda_{333}=\lambda$, with nondiagonal mixings suppressed. This will not be considered here. The fermions of the first two generations can still acquire mass from the third-generation Higgs bosons. The masses of the scalar fields come from the quadratic soft terms and can be made sufficiently large.

The physical mass spectrum resulting from the potential V has been computed in Ref. 9. First, the $R=+1$ states $\sqrt{2}\,{\rm Im}\phi_{12}^0$, $\sqrt{2}\,{\rm Im}H_L^0$, and $\sqrt{2}\,{\rm Im}H_R^0$ mix to form a physical pseudoscalar P^0 whose mass is given by

$$m_{P^0}^2=\rho\frac{v_Lv_R}{v}\left[\frac{v_Lv_R}{v}+\frac{v_Lv}{v_R}+\frac{v_Rv}{v_L}\right], \tag{11}$$

where

$$\rho\equiv\lambda^2-\frac{e^2}{4x(1-2x)} \tag{12}$$

is related to the cubic soft term coupling λA via

$$\lambda A=\frac{v_Lv_R}{v}\rho\ . \tag{13}$$

Second, ϕ_{11}^- and H_L^- combine to form a charged Higgs boson H^- of positive R parity. Its mass is given by

$$m_{H^\pm}^2=m_{W_L}^2\left[\frac{1-4x}{2(1-2x)}+\rho\frac{2x}{e^2}\left[\frac{v_R^2}{v^2}-1\right]\right]. \tag{14}$$

Finally, the real components of three neutral scalar fields, $\sqrt{2}\,{\rm Re}\phi_{12}^0$, $\sqrt{2}\,{\rm Re}H_L^0$, and $\sqrt{2}\,{\rm Re}H_R^0$, mix to give three physical neutral scalar Higgs bosons, all of positive R parity. Their mass matrix reads

$$M_S^2=\begin{bmatrix}\rho\dfrac{v_L^2v_R^2}{v^2}+B_1v^2 & -\rho\dfrac{v_Lv_R^2}{v}+B_2vv_L & -\rho\dfrac{v_L^2v_R}{v}+B_2vv_R\\ & \rho v_R^2+B_3v_L^2 & \rho v_Lv_R+B_3v_Lv_R\\ \text{symmetric} & & \rho v_L^2+B_3v_R^2\end{bmatrix}, \tag{15}$$

where

$$B_1\equiv\frac{e^2}{x}\ , \tag{16}$$

$$B_2\equiv2\lambda^2-\frac{e^2}{2x}\ , \tag{17}$$

and

$$B_3\equiv\frac{e^2(1-x)}{2x(1-2x)}\ . \tag{18}$$

The mass states will be denoted by S_i^0, $i=1,2,3$, in order of increasing mass. In general, in order to obtain the neutral-scalar masses one has to diagonalize the matrix M_S^2 numerically, as the analytic formulas for its eigenvalues are not very useful.

Turning next to sleptons, the tau sneutrino $\tilde{\nu}_\tau=\phi_{21}^0$ remains an unmixed complex neutral scalar of negative R parity. Its mass is given by

$$m_{\tilde{\nu}_\tau}^2=\frac{e^2}{2x}\left[\frac{1-4x}{2(1-2x)}(v_L^2+v_R^2)-2v^2\right]$$
$$+\rho\left[\frac{v_L^2v_R^2}{v^2}-(v_L^2+v_R^2)\right]. \tag{19}$$

The supersymmetric partner of the τ is a mass eigenstate of a mixture of ϕ_{22}^- and H_R^-. Its mass is given by

$$m_{\tilde{\tau}^\pm}^2=m_{W_R}^2\left[\frac{1-4x}{2(1-2x)}+\rho\frac{2x}{e^2}\left[\frac{v_L^2}{v^2}-1\right]\right]. \tag{20}$$

In total, there are six Higgs bosons: the pseudoscalar P^0, a pair of charged Higgs bosons $H^\pm$, and three neutral scalars S_1^0, S_2^0, and S_3^0. The negative R-parity states consist of a pair of τ bosons $\tilde{\tau}^\pm$ and the tau sneutrino $\tilde{\nu}_\tau$.

B. Mass sum rules

Several mass sum rules can be immediately derived. First, the Higgs-boson masses are related by

$$m_{S_1^0}^2+m_{S_2^0}^2+m_{S_3^0}^2=m_{P^0}^2+m_{Z_1^0}^2+m_{Z_2^0}^2\ . \tag{21}$$

This relation is a particular example of a general formula

$${\rm Tr}\mathcal{M}_{S^0}^2={\rm Tr}\mathcal{M}_{P^0}^2+{\rm Tr}\mathcal{M}_{V^0}^2\ , \tag{22}$$

where *all* scalars (S^0), pseudoscalars (P^0), and neutral vector bosons (V^0) are taken into account, which is generally true in supersymmetric models with no gauge singlets.

Next, the Higgs bosons and the sleptons separately obey

$$m_{H^\pm}^2 = m_{W_L^\pm}^2 + m_{P^0}^2 - \lambda^2(v^2+v_L^2) - \rho v_L^2 , \tag{23}$$

and

$$m_{\tilde{\tau}^\pm}^2 = m_{W_R^\pm}^2 + m_{\tilde{v}_\tau}^2 + \rho v_L^2 + \lambda^2(v_L^2 - v^2) - \frac{e^2}{2x}(v_L^2+v_R^2-2v^2) . \tag{24}$$

These sum rules resemble the relationship between the masses of the charged Higgs boson $C^\pm$ and the pseudoscalar P^0 of the minimal supersymmetric standard model:[10,2]

$$m_C^2 = m_P^2 + m_{W_L}^2 . \tag{25}$$

They are, however, violated by terms involving VEV's, similarly as in other E_6-based models.[11] One should mention here that the sum rules, Eqs. (22) and (25), are also violated even in the minimal supersymmetric standard model, once one allows for an extra singlet Higgs.[12] Notice also that Eq. (23) implies that in the considered model in principle it should be possible to have $H^\pm$ lighter than $W_L^\pm$, unlike in the minimal supersymmetric standard model.

III. CONSTRAINTS AND MASS SPECTRUM

A. Constraints

Having fixed the W_L mass at $m_{W_L}=80$ GeV, one can describe the Higgs sector of the model in terms of just three independent parameters which I choose to be the mass of the right charged vector boson, m_{W_R}, the mass of the pseudoscalar, m_{P^0}, and the ratio of vacuum expectation values, $\tan\theta_L$, defined earlier. The requirement that the squared masses of all the bosons be positive provides a rather stringent constraint on the parameters of the model.

First, it is clear from Eq. (11) that $\rho>0$, and therefore

$$\lambda > \lambda_{\min} \equiv \frac{e}{2\sqrt{x(1-2x)}} = 0.444 . \tag{26}$$

(Since the Higgs sector depends only on λ^2, the sign of λ is irrelevant and can be chosen positive.) An upper bound on λ (and ρ) can also be derived. Consider the determinant of the neutral scalar mass matrix, Eq. (15); it has a remarkably simple form

$$m_{S_1^0}^2 m_{S_2^0}^2 m_{S_3^0}^2 = \rho(B_1B_3 - B_2^2)(v_R^2 - v_L^2)^2 v^2 . \tag{27}$$

Since B_2 grows with λ, there exists a maximum value of λ below which the determinant, and $m_{S_1^0}^2$, remain positive:

$$\lambda < \lambda_{\max} \equiv \frac{e}{2\sqrt{x}} \left[1 + \left[\frac{1+2x}{1-2x} \right]^{1/2} \right]^{1/2} = 0.531 . \tag{28}$$

This bound also implies that $\rho < 0.085$. The allowed range of λ is therefore always narrow and remains in the perturbative regime solely due to the requirement that the squared masses of the Higgs bosons be positive.

Second, since in this model both the Higgs bosons and the leptons lie in the same supermultiplets, the superpotential term defined above Eq. (9) implies,[4] for the third generation,

$$\frac{m_\tau}{m_E} = \frac{v_L}{v_R} , \tag{29}$$

where E is a new exotic charged fermion whose mass has to be bigger than at least 26 GeV from KEK TRISTAN (Ref. 13). Most likely the experiments at LEP and SLC will soon improve this bound to at least 40 GeV. In any case, the bound (29) implies a large ratio of v_R to v_L, and therefore suggests that m_{W_R} should be in the TeV mass region. A way to avoid this (unpleasant possibility) is to assume a large $\tan\theta_L$. In this paper, I will focus on this second option as phenomenologically more appealing. The bound (29), when applied to first two generations implies even much greater values of $\tan\theta_L$. Because of the possible intergenerational mixings, no strict lower limit on $\tan\theta_L$ can be given but, as we shall see, this will be irrelevant since the resulting mass spectrum will be essentially insensitive to varying $\tan\theta_L$ above ~ 10. The requirement of large $\tan\theta_L$ combined with the condition $m_{\tilde{v}_\tau}^2 > 0$ restricts also the mass of the W_R:

$$m_{W_R} \gtrsim \sqrt{28} m_{W_L} = 423.3 \text{ GeV} ; \tag{30}$$

although masses somewhat below this limit are, in principle, also possible. This can be seen as follows: the first term in Eq. (19) remains positive as long as

$$\tan^2\theta_L < \frac{\alpha^2+1}{28-a^2} , \tag{31}$$

where $a \equiv m_{W_R}/m_{W_L}$. In deriving (31) the following formula has been used:

$$\left[\frac{v_R}{v_L} \right]^2 = (a^2-1)\tan^2\theta_L + a^2 , \tag{32}$$

which can be trivially derived from Eq. (6). The inequality (31) is valid only for $m_{W_R} < \sqrt{28} m_{W_L} = 423.3$ GeV; however, it disappears for larger values of m_{W_R}. The second term in Eq. (19), which is multiplied by a small parameter ρ, remains positive as long as

$$\tan^2\theta_L < \frac{1}{a^2-1}[-1+\sqrt{1+a^2(a^2-1)}] , \tag{33}$$

which is always less than one. For large $\tan\theta_L$, it becomes bigger with increasing $m_{P^0} \sim \rho$ and eventually pulls $m_{\tilde{v}_\tau}^2$ below zero, placing an upper bound on m_{P^0}, as I will discuss later. Thus, for $m_{W_R} \lesssim 423$ GeV there is always an upper bound on $\tan\theta_L$ which is incompatible with the requirement of large $\tan\theta_L$, except for a mass range of a few GeV just below 423 GeV where the bound (31) becomes very weak and disappears completely as m_{W_R} increases.

The bound (30) is not strict but is stronger than the indirect lower bound $m_{W_R} \gtrsim 390$ GeV which can be inferred[14] from the limit of about 466 GeV on $n_{Z_2^0}$. This latter bound comes[14] from applying to this model the new preliminary limits of the Collider Detector at Fermilab (CDF) on the mass of a heavy neutral vector boson. Obviously, the right charged gauge boson in this model can in principle be still much lighter than in conventional left-right models.

Finally, requirement that the Higgs potential V have a minimum at the asymmetric vacuum which is lower than the one at VEV's equal to zero is never stronger than any of the constraints listed above.

We see from Eq. (27) that a very small mass of S_1^0 corresponds to a very small value of $v \sim \tan\theta_L$ which is far below the large $\tan\theta_L$ region. It is thus reasonable to adopt a somewhat stronger lower bound of 1 GeV on all the scalar masses (in practice, this will affect only S_1^0, S_2^0, P^0, and $\tilde{\nu}_\tau$) in order to cut off the region of the parameter space allowed by unreasonably light scalars. I will first present the general structure of the Higgs-boson and slepton mass spectrum *without* imposing the condition (29) and next I will discuss the strongly favored limit of large $\tan\theta_L$.

B. Mass spectrum

I plot in Figs. 1 and 2 the upper and lower bounds on the masses of the Higgs bosons and the third-generation sleptons versus the pseudoscalar mass m_{P^0} for a given mass of W_R. The limits are obtained by varying the parameter $\tan\theta_L$ at each given value of m_{P^0}. The mass of the right charged boson W_R is chosen to be either $m_{W_R} = 0.5$ TeV, which is somewhat above the limit (30), or $m_{W_R} = 1$ TeV, in order to display the dependence on v_R. We see that increasing m_{W_R} does not change the pattern qualitatively. The dependence on $\tan\theta_L$ is presented in Fig. 3 where the masses of S_1^0, S_2^0, and $\tilde{\nu}_\tau$ are plotted versus m_{P^0} for $m_{W_R} = 0.5$ TeV and for several representative choices of $\tan\theta_L$. (One finds a similar dependence also for other choices of m_{W_R}.) In Fig. 4 the allowed ranges of several parameters in the model are presented. Notice that arbitrarily large values of $\tan\theta_L$ are in princi-

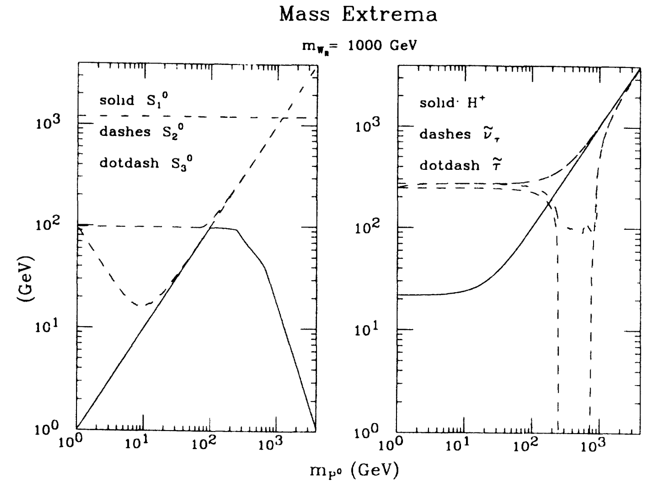

FIG. 2. Similar to Fig. 1 but for $m_{W_R} - 1$ TeV.

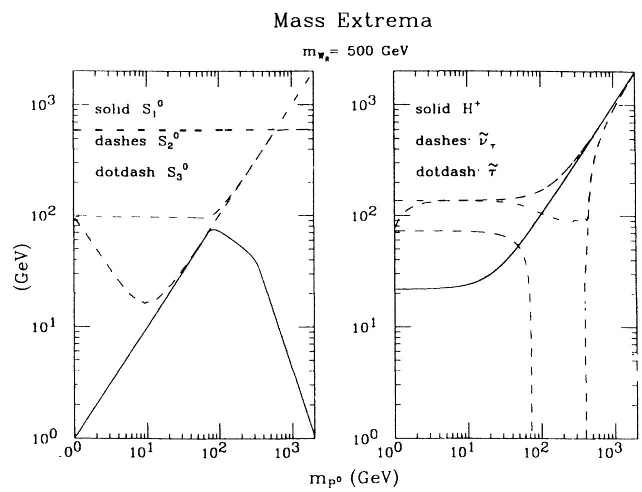

FIG. 1. Plots of upper and lower bounds for Higgs-boson and slepton masses, as functions of m_{P^0}, for $m_{W_R} = 0.5$ TeV. The allowed regions are obtained by varying $\tan\theta_L$ subject to the requirement that the scalar masses be bigger than 1 GeV. The solid curve in the left window corresponds to the *upper* bound for $m_{S_1^0}$. The upper and lower limits for $H^\pm$ practically coincide.

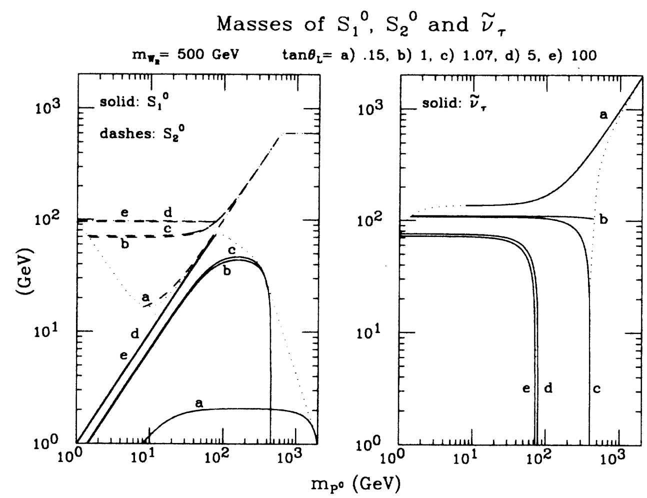

FIG. 3. For the choice of $m_{W_R} = 0.5$ TeV, I plot the masses of the two lighter neutral scalars S_1^0 and S_2^0 and the τ sneutrino $\tilde{\nu}_\tau$ vs m_{P^0} for several representative values of $\tan\theta_L$: (a) 0.15, (b) 1, (c) 1.07, (d) 5, and (e) 100. The lower and upper bounds are marked by dots. The lower bound on $m_{\tilde{\nu}_\tau}$ is in part buried under the solid curves (c) and (e). Notice that for $\tan\theta_L \geq 1.07$ the mass of $\tilde{\nu}_\tau$ may at some point vanish placing an upper bound on m_{P^0}.

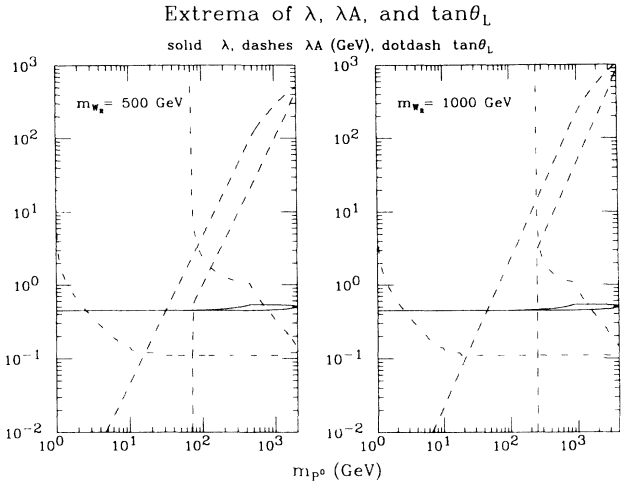

FIG. 4. Allowed ranges of λ, λA, and $\tan\theta_L$ for $m_{W_R}=0.5$ TeV and $m_{W_R}=1$ TeV. Outside of the indicated regions the condition that the square masses of all the considered bosons are positive is not satisfied.

ple allowed. However, by comparing Figs. 1 and 3 one can easily see that for larger values of $\tan\theta_L$ (above ~ 5) the masses only weakly depend on $\tan\theta_L$. In fact, increasing $\tan\theta_L$ above $\tan\theta_L \gtrsim 10$ does not cause any visible change in the presented mass spectrum.

A number of remarks can be made with regard to the Higgs-boson-mass spectrum in this model. Note first that the allowed ranges of the Higgs-boson masses form rather narrow wedges in the parameter space. This is especially true for the heaviest neutral scalar S_3^0 and for the charged Higgs boson $H^\pm$ whose mass ranges remain very constrained for all the values of the P^0 mass. There is also a remarkable asymptotic mass degeneracy among the Higgs and slepton states. Indeed, with the exception of the lightest and the heaviest neutral Higgs scalars, S_1^0 and S_3^0, all the other scalars become asymptotically degenerate with P^0 in the large mass region. The mass of the S_3^0 depends essentially only on v_R, even for rather small values of m_{W_R}. Therefore, S_3^0 always remains almost exactly degenerate with Z_2^0. There is an obvious "level crossing" between S_2^0 and S_3^0 which takes place at $m_{S_3^0} \approx m_{P^0}$. There is also a "level crossing" between S_2^0 and S_1^0 which occurs at large values of $\tan\theta_L$.

Remarkably, the mass of the lightest neutral Higgs scalar S_1^0 is in this model bounded from above, and remains always within the ~ 100 GeV range. This has already been shown analytically by the authors of Ref. 9. By studying the properties of the neutral scalar mass matrix, Eq. (15), they derived an absolute upper limit on the mass of S_1^0:

$$m_{S_1^0} < \sqrt{2} m_{W_L} \ . \tag{34}$$

For $m_{W_L}=80$ GeV this gives $m_{S_1^0} < 113$ GeV. This absolute upper bound actually corresponds[9] to very large values of $\tan\theta_L$ (more precisely to $1/\tan\theta_L = v_L/v = 0$) irrespectively of the choice of m_{W_R}, and it quickly decreases with decreasing $\tan\theta_L$. It is clear from presented graphs that the maximum of $m_{S_1^0}$ grows also with $m_{W_R} \sim v_R$. This allows us to derive an even stronger analytic upper bound on $m_{S_1^0}$. In the limit of vanishing v_L, the smallest eigenvalue of the neutral scalar mass matrix, Eq. (15), takes the form

$$m_{S_1^0}^2 \simeq 2m_{W_L}^2 - \frac{B_2^2}{B_3} v^2 \tag{35}$$

plus small corrections of $O((v/v_R)^4)$ which vanish in the large v_R limit. Notice, however, that

$$B_2 = 2\lambda^2 - \frac{e^2}{2x} > 2\lambda_{\min}^2 - \frac{e^2}{2x} = \frac{e^2}{2(1-2x)}$$

and therefore

$$m_{S_1^0}^2 \lesssim (m_{S_1^0}^{\max})^2 \equiv 2m_{W_L}^2 \left[\frac{1-3x}{(1-x)(1-2x)} \right] . \tag{36}$$

This gives

$$m_{S_1^0} \lesssim m_{S_1^0}^{\max} = 1.22 m_{W_L} = 97.7 \text{ GeV} \ , \tag{37}$$

which is stronger than the limit (34). The numerical analysis confirms this new bound on $m_{S_1^0}$. I find that for $m_{W_R}=1$ TeV the maximum of the S_1^0 mass is equal to 96.5 GeV, and is reduced to 74.8 GeV when $m_{W_R}=0.5$ TeV. In summary, depending on the actual value of $\tan\theta_L$, the lightest neutral scalar S_1^0 is likely to have a rather small mass. The value of m_{P^0}, at which this maximum is achieved, is roughly equal to $m_{S_1^0}^{\max}$ independently of the mass of W_R.

For a given value of m_{W_R}, in principle one can also place an *upper* limit on the masses of all the considered scalars. This is caused by the fact that the upper limit on the S_1^0 mass, after reaching its maximum, decreases with increasing m_{P^0} and tends asymptotically towards zero. For the adopted lower bound of 1 GeV on the Higgs-boson masses, this upper bound on m_{P^0} depends strongly on the choice of the mass of W_R, and is equal to about 1.97 TeV when $m_{W_R}=0.5$ TeV, and is shifted up to ~ 4.04 TeV for $m_{W_R}=1$ TeV. For a given choice of m_{W_R} the upper bound on m_{P^0} can be strengthened even further by considering the dependence on $\tan\theta_L$. As Fig. 3 shows, the limit becomes stronger with increasing $\tan\theta_L$. There are actually two competing effects: for smaller values of $\tan\theta_L$ the bound from the squared mass of S_1^0 becoming negative is dominant. However, as $\tan\theta_L$ increases the condition $m_{\tilde{\nu}_\tau}^2 > 0$ becomes stronger, and in the most interesting limit of large $\tan\theta_L$ gives $m_{P^0} < 72.4$ GeV for $m_{W_R}=0.5$ TeV and $m_{P^0} < 246.6$ GeV for $m_{W_R}=1$ TeV. Finally, one should mention also that if one allows for $m_{S_1^0}$ below 1 GeV then the upper limit on m_{P^0}, which is inverse proportional to $\tan\theta_L$, grows rapidly. For each finite value of $v \sim \tan\theta_L$ there is a point above which there are no more solutions. However, by decreasing v one can always evade this limit and find solutions for arbitrarily large m_{P^0}, which correspond to asymptotically small $m_{S_1^0}$. In conclusion, in this model it is possible to derive upper bounds on the masses of all the Higgs bosons and the third-generation sleptons by requiring that the mass of S_1^0 be not unreasonably small. These bounds become much stronger in the preferred region of large $\tan\theta_L$. The existence of such upper bounds is certainly not common in supersymmetric models. For example, in a recently popular superstring-inspired model with the low-energy gauge group $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)\times\mathrm{U}_{Y'}(1)$ *a priori* there are no upper bounds on the masses of neither the light nor on the heavy scalars.[11] (However, in this model one can derive such upper bounds by applying additional theoretically motivated constraints.) Similarly, in the minimal supersymmetric standard model[10] only the lightest Higgs boson has to be always lighter than the Z^0. A comprehensive summary of the properties of Higgs sectors in various interesting models can be found in Ref. 2.

For the adopted lower bound of 1 GeV on the masses of the Higgs bosons, it is also possible to put a stronger *lower* limit on m_{P^0}, and therefore on the masses of the other Higgs bosons and the sleptons, which depends on $\tan\theta_L$. This limit is stronger for smaller values of $\tan\theta_L$, as Figs. 1 and 2 indicate. One should stress, however, that this lower bound disappears for all values of $\tan\theta_L$ if one requires only that $m_{S_1^0}>0$. Indeed, the determinant (27) vanishes for $\rho=0$, which corresponds to massless P^0.

Several further remarks on the Higgs-boson-mass spectrum are also in order. The mass of the second scalar S_2^0 generally increases with m_{P^0}. In the lower regions of m_{P^0}, it may be as small as about 16 GeV. As m_{P^0} grows, it quickly increases and narrows to become degenerate with P^0 until it reaches the "level crossing" with S_3^0 where it becomes degenerate with Z_2^0.

The allowed range of the charged-scalar $H^\pm$ mass is always extremely narrow. This charged Higgs boson can be surprisingly light. Its mass can be smaller than the mass of the corresponding charged Higgs boson $C^\pm$ in the minimal supersymmetric standard model ($m_C \geq m_{W_L}$) or in an E_6 superstring-inspired model with the low-energy gauge group $\mathrm{SU}_c(3)\times\mathrm{SU}_L(2)\times\mathrm{U}_Y(1)\times\mathrm{U}_{Y'}(1)$ where m_C has been shown[11,15] to be always above ~ 54 GeV. It can be even lighter than the lightest neutral scalar S_1^0. The absolute lower limit on $m_{H^\pm}$ can be easily derived by combining Eq. (12), (14), and (26). Indeed, since v_R is always bigger than v, one necessarily has

$$m_{H^\pm} > m_{W_L}\left[\frac{1-4x}{2(1-2x)}\right]^{1/2} = 21.8\ \mathrm{GeV}\ , \qquad (38)$$

independently of the choice of m_{W_R}. This bound is approached at small m_{P^0} where ρ essentially vanishes. As m_{P^0} grows, the $H^\pm$ mass increases, and for $m_{P^0} \gtrsim 80$ GeV, it becomes practically degenerate with P^0 and asymptotically with S_2^0, $\tilde{\nu}_\tau$, and $\tilde{\tau}^\pm$. The bound (38) barely escapes the present TRISTAN limits.[13] Direct searches at LEP and SLC will soon place stronger (model-dependent) lower bounds on the masses of new charged scalar particles. Further study is needed[16] to determine precisely this limit in the presented model. However, any bound above 21.8 GeV will imply also lower bounds on the masses of S_2^0 and P^0, and in the large $\tan\theta_L$ limit, also on S_1^0, independently of the mass of W_R.

The mass pattern of the sleptons is closely related to the Higgs-boson-mass spectrum. The mass range of $\tilde{\tau}^\pm$ is quite constrained. At small m_{P^0} it narrows to about 136 GeV for $m_{W_R}=0.5$ TeV and to ~ 272 GeV for $m_{W_R}=1$ TeV. This can be easily explained in a similar fashion as in the case of $H^\pm$. Indeed, at the lower end of m_{P^0} where ρ is very small, the second term in Eq. (11) is negligible and one obtains

$$m_{\tilde{\tau}^\pm} \simeq m_{W_R}\left[\frac{1-4x}{2(1-2x)}\right]^{1/2} = 0.27 m_{W_R}\ , \qquad (39)$$

which approximately reproduces the quoted numbers. As one moves along the m_{P^0} axis, ρ grows and $m_{\tilde{\tau}^\pm}$ allows for some variations, depending on whether $\tan\theta_L$ is less or more than 1, but asymptotically increases and narrows to become degenerate with Higgs bosons.

The mass of the τ sneutrino $\tilde{\nu}_\tau$ can vary considerably. At small m_{P^0}, where ρ is negligibly small, $m_{\tilde{\nu}_\tau}$ is always bigger than zero but in the intermediate region of m_{P^0} it can be very light. At asymptotically large values of m_{P^0}, the range of allowed $\tilde{\nu}_\tau$ mass narrows rapidly and $\tilde{\nu}_\tau$ becomes degenerate with P^0. Notice that for smaller values of $\tan\theta_L$, $m^2_{\tilde{\nu}_\tau}$ is always positive, as discussed previously. The value of $\tan\theta_L$ above which the second term in Eq. (19) becomes negative is given by inequality (33) and is equal to 0.993 when $m_{W_R}=500$ GeV and 0.998 at $m_{W_R}=1$ TeV. As Fig. 3 shows, for not much larger values of $\tan\theta_L$ (>1.07 for $m_{W_R}=500$ GeV and >1.15 for $m_{W_R}=1$ TeV) and for larger values of $m_{P^0}\sim\rho$, the second term in Eq. (19) starts becoming dominant and $m^2_{\tilde{\nu}_\tau}$ is indeed driven down towards negative values. As $\tan\theta_L$ grows, $m_{\tilde{\nu}_\tau}$ becomes negative at smaller values of m_{P^0}.

C. Limit of large $\tan\theta_L$

Having explored the general structure of the Higgs-boson and slepton mass spectra, let us now focus on the important limit of large v/v_L which follows from Eq. (29) if one wants to allow for the mass of the W_R below ~ 1 TeV. As pointed out before, the considered mass spectrum is essentially insensitive to varying $\tan\theta_L$ above $\tan\theta_L \sim 10$. This leads to a great simplification of the mass pattern since, for a given value of m_{W_R}, the scalar masses form in this limit curves in the parameter space and not extended regions as before. First, note in Fig. 3 that in this limit the mass of one neutral scalar always remains very close to 97.7 GeV which is just the absolute upper bound (37) on $m_{S_1^0}$ derived earlier. The other light Higgs boson is degenerate in mass with the P^0. There is an obvious "level crossing" close to 97.7 GeV. The state degenerate with the P^0 is always predominantly $\sqrt{2}\,\mathrm{Re}H_L^0$ and the other Higgs boson is mainly $\sqrt{2}\,\mathrm{Re}\phi_{12}$. A third neutral scalar S_3^0 remains completely degenerate in mass with the Z_2^0 and consists of essentially pure $\sqrt{2}\,\mathrm{Re}H_R^0$. As mentioned before, for large $\tan\theta_L$ (in fact, for $\tan\theta_L \gtrsim 1$), the mass of $\tilde{\nu}_\tau$ becomes at some value of m_{P^0} negative, putting an upper limit on the masses of all the bosons except for the S_3^0. This limit becomes stronger with growing $\tan\theta_L$ and tends asymptotically to ~ 72.4 GeV at $m_{W_R}=0.5$ TeV and ~ 246.6 GeV at $m_{W_R}=1$ TeV. Any sizable experimental lower limit on $m_{\tilde{\nu}_\tau}$ will immediately put a stronger bound on the mass of the W_R.

The masses of the charged states are bounded both from above and from below. In this limit it is easy to give approximate expressions for the Higgs-boson masses which even in the crudest approximation of vanishing v_L reproduce the numerical results quite accurately. First, the mass of the pseudoscalar P^0 grows approximately linearly with λ:

$$m_{P^0} \simeq \sqrt{\rho}\, v_R \; . \tag{40}$$

The masses of the neutral scalars are given by

$$m_{S_1^0} \simeq m_{P^0} \; , \tag{41}$$

$$m_{S_2^0} \simeq m_{S_1^0}^{\max} \; , \tag{42}$$

where $m_{S_1^0}^{\max}$ is the absolute upper limit on the mass of the S_1^0 given in Eq. (37), and finally

$$m_{S_3^0} \simeq m_{Z_2^0} \; , \tag{43}$$

where $m_{Z_2^0}$ has been given in Eq. (8). The expression for the mass of $\tilde{\nu}_\tau$,

$$m_{\tilde{\nu}_\tau}^2 \simeq \frac{1-4x}{2(1-2x)} m_{W_R}^2 - 2m_{W_L}^2 - m_{P^0}^2 \; , \tag{44}$$

reproduces quite well both the value of $m_{\tilde{\nu}_\tau}$ at vanishing m_{P^0} (~ 72.4 GeV for $m_{W_R}=0.5$ TeV and ~ 246.6 GeV at $m_{W_R}=1$ TeV) and the upper limit on m_{P^0} (which is obviously equal to just quoted numbers, since it corresponds to vanishing $m_{\tilde{\nu}_\tau}$). The masses of the charged scalars are in the limit of vanishing v_L given by

$$m_{H^\pm}^2 \simeq \frac{1-4x}{2(1-2x)} m_{W_L}^2 + m_{P^0}^2 \tag{45}$$

and

$$m_{\tilde{\tau}^\pm}^2 \simeq \frac{1-4x}{2(1-2x)} m_{W_R}^2 - m_{P^0}^2 \; . \tag{46}$$

Given the fact that v_R is always much bigger than the other two VEV's, one might be tempted to derive approximate analytic expressions for the masses of the neutral scalar Higgs bosons in the region of nonvanishing v_L by extracting from the mass matrix M_S^2, given in Eq. (15), the "unperturbed" part containing only terms proportional to v_R and treating the remaining terms as a "small" correction. However, because ρ is typically very small, terms such as ρv_R^2 can be quite small and the formulas derived previously[5] are also applicable only in the large-$\tan\theta_L$ limit.

IV. HIGGS-BOSON PRODUCTION IN e^+e^- ACCELERATORS

In general, both S_1^0 and S_2^0 as well as $H^\pm$ can be light enough to be searched for at LEP and SLC. It is therefore worthwhile to consider prospects for their discovery. I will focus here on the limit of large $\tan\theta_L$ in which S_2^0 becomes too heavy to be even produced. On the other hand, the lightest neutral Higgs scalar S_1^0 can be discovered in the Bjorken process $Z_1 \to l^+l^-S_1$. In assessing the possibility of producing the S_1^0, it is crucial to establish what fraction of the standard-model coupling it shares. The $S_1^0Z_1^0Z_1^0$ coupling is

$$\begin{aligned} g_{S_1Z_1Z_1} &= \frac{v}{\sqrt{2}} \mathcal{S}_1(1)[g_L \mathcal{Z}_1(1) - g_R \mathcal{Z}_1(2)]^2 \\ &\quad + \frac{v_L}{\sqrt{2}} \mathcal{S}_1(2)[g_L \mathcal{Z}_1(1) - g' \mathcal{Z}_1(3)]^2 \\ &\quad + \frac{v_R}{\sqrt{2}} \mathcal{S}_1(3)[g_R \mathcal{Z}_1(2) - g' \mathcal{Z}_1(3)]^2 \; , \end{aligned} \tag{47}$$

where $\mathcal{S}_1$ denotes the eigenvector of S_1^0 in the basis $\sqrt{2}\,\mathrm{Re}\phi_{12}^0$, $\sqrt{2}\,\mathrm{Re}H_L^0$, and $\sqrt{2}\,\mathrm{Re}H_R^0$, and $\mathcal{Z}_1$ is the Z_1^0 eigenvector in the basis spanned by V_L^3, V_R^3, and V'. The following sum rule for the Higgs couplings is also obeyed

$$\begin{aligned} g_{S_1Z_1Z_1}^2 + g_{S_2Z_1Z_1}^2 + g_{S_3Z_1Z_1}^2 &= (g_{HZZ}^{\mathrm{SM}})^2 \\ &= \left[\frac{g m_Z}{\cos\theta_W} \right]^2 . \end{aligned} \tag{48}$$

Unfortunately, as mentioned before, in the large $\tan\theta_L$ limit the S_1^0 state consists mainly of $\sqrt{2}\,\mathrm{Re}H_L^0$ and the relevant coupling in the region of $m_{S_1^0} \lesssim m_{Z_1^0}/2$ is very small, independent of the mass of W_R. It is equal to $\sim 0.1 g_{HZZ}^{\mathrm{SM}}$ when S_1^0 is very light and increases only slightly ($\lesssim 0.12 g_{HZZ}^{\mathrm{SM}}$) when $m_{S_1^0} \simeq 40$ GeV which is considered as an upper limit for the standard-model (SM) Higgs-boson mass range to be explored by LEP and SLC. The whole strength is essentially "absorbed" by S_2^0 whose mass is close to 98 GeV. It is only in the region of S_1^0 approaching the "level crossing" with S_2^0 where the coupling grows significantly. Obviously, the chances of producing S_1^0 are very small, even if it is very light, and alternative processes have to be studied.[16] However, it is very likely that the region of small $m_{S_1^0}$ will be ruled out by failure to discovery pairs of H^+H^- whose coupling to Z_1^0 is not suppressed. This would eliminate the region of $m_{P^0} \sim m_{S_1^0}$ below ~ 45 GeV. The prospects for producing S_1^0 at LEP II are more promising: as the mass of S_1^0

grows and approaches 98 GeV, the S_1^0 coupling to Z_1^0 pairs becomes very close to the SM coupling.

V. CONCLUSIONS

Left-right models provide an appealing extension of the standard model. In particular, the model introduced by Ma has several features which make it distinct in this class of theories. In this paper, I have studied the combined Higgs-boson–slepton sector in this model, and found that its structure is remarkably simple and constrained. In general, one neutral Higgs boson must always remain in the ~ 100 GeV mass range, one is always nearly degenerate in mass with the Z_2^0, and all the remaining scalars are typically degenerate in mass, except for smaller masses. There exists an upper bound on all the scalar masses which may lie in the TeV region. However, in the phenomenologically required limit of large $\tan\theta_L$, all the scalars (except for the S_3^0) have significantly smaller masses, depending on the specific value of m_{W_R}. For example, if $m_{W_R}=0.5$ TeV they are always below ~ 135 GeV and typically much smaller. The mass of one neutral Higgs boson is in this limit very near 98 GeV. The charged Higgs boson can be as light as 22 GeV. Expected experimental limits from LEP and SLC will very likely produce a stronger bound on the charged-Higgs-boson mass constraining significantly the allowed parameter space. The lightest neutral Higgs scalar S_1^0 couples to Z_1^0 pairs with a significant strength only when its mass approaches its absolute upper bound of about 98 GeV, and therefore will have to wait until LEP II is built to be discovered. Clearly, in this model the properties of the light Higgs bosons are very different from the ones in the minimal supersymmetric standard model.[2] In particular, the discovery of a light neutral Higgs boson at LEP or SLC will be a very strong indication against the alternative left-right model considered in this paper. The masses of the scalar partners of τ and $\tilde{\nu}_\tau$ are closely related to the masses of the Higgs bosons. While $\tilde{\tau}$ is always heavy enough to escape even the expected bound from LEP, $\tilde{\nu}_\tau$ can be even massless. Any sizable lower limit on the mass of $m_{\tilde{\nu}_\tau}$ will in the large $\tan\theta_L$ limit place a stronger bound on the mass of the W_R.

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