# Multiple production of supersymmetric Higgs bosons in $Z^0$ decays

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Multi-Higgs-boson production in $Z^0$ decays is discussed in the context of low-energy supergravity models. For Higgs-boson masses lighter than about 20 GeV, $Z^0 \to H_2^0 H_2^0 H_3^0$ naturally has a branching ratio in the range $10^{-4}$–$10^{-6}$. $Z^0 \to H_2^0 H_2^0 l\bar{l}$, where $l$ is a neutral or charged lepton, has a branching ratio in the same range if $m_{H_2} < 10$ GeV. Detection of these processes will give information about the structure of the Higgs sector and about the $HHZZ$ and $HHH$ couplings.

Future experiments in $e^+e^-$ collisions at the $Z^0$ peak will be able to search for the Higgs boson in a large range of masses. When the CERN collider LEP reaches its planned luminosity, it can probe the standard-model (SM) Higgs boson ($H^0$) up to a mass of about 50 GeV through the process[1]

$$Z^0 \to H^0 \mu^+ \mu^- , \tag{1}$$

where $H^0$ is produced via the $HZZ$ coupling. Moreover, the radiative decay[2]

$$Z^0 \to H^0 \gamma \tag{2}$$

could also be detected at LEP, although with a much smaller rate than (1). [Only for a rather heavy $H^0$ ($m_H \gtrsim 60$ GeV) does process (2) become dominant over (1).]

Recently, it has been pointed out[3] that the process

$$Z^0 \to H^0 H^0 f\bar{f} \tag{3}$$

has a rate within the reach of LEP, if the Higgs-boson mass is less than about 10 GeV. The detection of this decay is mainly interesting because, being sensitive to the $HHZZ$ and $HHH$ couplings, it represents a crucial test of the Higgs sector. In view of this intriguing possibility, it is important to compare the SM results for process (3) with the predictions coming from new theoretical models. This paper will study the processes analogous to (3) in the minimal low-energy supergravity model, compare the results with the SM predictions, and briefly discuss their sensitivity to the $HHZZ$ and $HHH$ couplings.

It is worth mentioning that all processes considered in this paper can also occur in a SM scenario with two Higgs doublets. However, the strong constraints imposed by supersymmetry, which allow one to relate the masses of the various Higgs bosons and the different production processes, are no longer valid.

Low-energy supergravity is one of the best-motivated extensions of the SM. The introduction of supersymmetry at the Fermi scale is the only known way to describe a light elementary Higgs boson consistent with the naturalness criterion.[4] Therefore, the experimental discovery of the Higgs boson would deeply strengthen the motivation for supersymmetry. In any case, the Higgs-boson search represents a crucial test for any low-energy supergravity model.

The minimal $N=1$ supergravity model requires two Higgs doublets. Then, the supersymmetric Higgs sector[5] contains two real scalars ($H_1^0, H_2^0$): a pseudoscalar ($H_3^0$) and two charged scalars ($H^\pm$) as physical states. Because of the constraints imposed by supersymmetry, the quartic Higgs-boson couplings are related to gauge couplings and the Higgs sector is fully describable in terms of only two free parameters. I choose as independent variables $m_{H_2^0}$, the mass of the lighter of the two real scalars, and $v_2/v_1$, the ratio of the vacuum expectation values of the two Higgs neutral components. The masses of the other Higgs particles can then be written as

$$\frac{m_{H_1^0}^2}{m_Z^2} = c^2 \left| \frac{1-r}{c^2-r} \right| , \tag{4}$$

$$\frac{m_{H_3^0}^2}{m_Z^2} = r \left| \frac{1-r}{c^2-r} \right| , \tag{5}$$

$$m_{H^\pm}^2 = m_{H_3^0}^2 + m_{W^\pm}^2 , \tag{6}$$

where

$$r \equiv \frac{m_{H_2^0}^2}{m_Z^2}, \quad c \equiv \cos 2\beta, \quad \tan\beta \equiv \frac{v_2}{v_1} , \tag{7}$$

$$r \le c^2 . \tag{8}$$

Since $v_2$ is the vacuum expectation value which gives mass to up-type quarks, one can choose $\pi/4 \le \beta \le \pi/2$. From Eqs. (4)–(8) one infers $m_{H_2^0} \le m_{Z^0} \le m_{H_1^0}$, $m_{H_2^0} \le m_{H_3^0}$, and $m_{H^\pm} \ge m_{W^\pm}$. Supersymmetry implies that $H_2^0$ should be necessarily lighter than the $Z^0$ boson, while $H_1^0$ and $H^\pm$, being heavy, cannot be directly produced at LEP I.

The production of supersymmetric Higgs bosons in $Z^0$ decays can occur via the process

$$Z^0 \to H_2^0 \mu^+ \mu^- , \tag{9}$$

 

analogous to (1), or via[5,6]

$$Z^0 \to H_2^0 H_3^0 \ . \tag{10}$$

(The decay $Z^0 \to H_3^0 \mu^+ \mu^-$ is highly suppressed, since the $H_3^0 ZZ$ coupling vanishes at the tree level.) The decay rate for (9) is $J_2^2$ times the rate for the SM process (1), where $J_2$ is the $H_2^0 ZZ$ coupling in units of SM Higgs-boson coupling given by

$$J_2 \equiv \left[ \frac{c^2(1-c^2)}{r^2+c^2-2rc^2} \right]^{1/2} . \tag{11}$$

(The Feynman rules for the supersymmetric Higgs sector can be found in Ref. 5.) The rate for (10) is

$$\frac{\Gamma(Z^0 \to H_2^0 H_3^0)}{\Gamma(Z^0 \to \nu_e \bar{\nu}_e)} = \frac{L_2^2}{2} \left[ \left[ 1 - \frac{m_{H_2}^2 + m_{H_3}^2}{m_Z^2} \right]^2 - \frac{4 m_{H_2}^2 m_{H_3}^2}{m_Z^4} \right]^{3/2} , \tag{12}$$

where $L_2$ is proportional to the $H_2^0 H_3^0 Z$ coupling:

$$L_2 \equiv \frac{r-c^2}{(r^2+c^2-2rc^2)^{1/2}} \ . \tag{13}$$

If $v_2/v_1 \simeq 1$, the rates for the processes (9) and (1) are nearly equal, the decay (10) is suppressed, and $H_2^0$ is undistinguishable from a SM Higgs boson. However, if $v_2/v_1$ is somewhat larger than 1, as suggested by supergravity models with radiative gauge symmetry breaking, $H_3^0$ becomes almost degenerate in mass with $H_2^0$ [see Eq. (5)] and the process (10) rapidly dominates over (9), giving a different Higgs-boson signature at LEP (see Ref. 6).

The processes analogous to (2) (Ref. 7),

$$Z^0 \to H_2^0 \gamma, \quad Z^0 \to H_3^0 \gamma \ , \tag{14}$$

could also be observed at LEP. Since these decays occur at the one-loop level, they depend on the mass spectrum of all the charged supersymmetric particles. Slight enhancements with respect to the SM are possible.

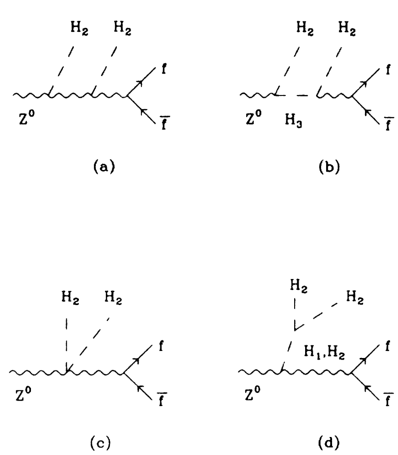

FIG. 1. Feynman diagrams for $Z^0 \to H_2^0 H_2^0 f\bar{f}$, for massless fermions $f$.

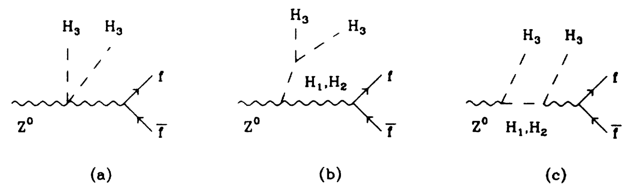

FIG. 2. Feynman diagrams for $Z^0 \to H_3^0 H_3^0 f\bar{f}$, for massless fermions $f$.

If the Higgs boson is discovered at LEP, multi-Higgs-boson production processes can provide us with interesting information about the structure of the Higgs sector. The processes analogous to (3) are now

$$Z^0 \to H_2^0 H_2^0 f\bar{f}, \quad Z^0 \to H_3^0 H_3^0 f\bar{f} \ , \tag{15}$$

which occur through the diagrams of Figs. 1 and 2, respectively. These processes are sensitive to the $HHZZ$ and $HHH$ couplings. Since we are interested in the leptonic modes [$f = e, \mu, \tau, \nu$ in Eq. (15)], which allow a better experimental identification, diagrams mediated by Yukawa interactions proportional to the small lepton masses have been neglected in Figs. 1 and 2.

The decay rates for the processes (15) are given by

$$\Gamma(Z^0 \to H_i^0 H_i^0 f\bar{f}) = \frac{32}{3} \frac{\pi^3 \alpha^3}{\sin^6\theta_W \cos^6\theta_W} \frac{C_L^2 + C_R^2}{m_Z^3} \times \int d\phi^{(4)} \frac{g^{\mu\nu} m_Z^2 - p_Z^\mu p_Z^\nu}{(2k_1 k_2 - m_Z^2)^2 + \Gamma_Z^2 m_Z^2} \times (g^{\rho\sigma} k_1 k_2 - k_1^\rho k_2^\sigma - k_2^\rho k_1^\sigma) \Lambda_{\mu\rho}^{(i)} \Lambda_{\nu\sigma}^{*(i)} , \tag{16}$$

where $H_i^0$ is one of the three kinds of Higgs boson: $H_2^0, H_3^0$, or the SM Higgs boson $H^0$. Also, $p_Z, k_1, k_2, p_1, p_2$ are, respectively, the $Z^0$, massless fermions, and Higgs bosons four-momenta, and $d\phi^{(4)}$ is the phase-space measure. $C_L, C_R$ are the left- and right-handed fermion couplings to the $Z^0$ ($C_L \equiv Q \sin^2\theta_W - T_3$, $C_R \equiv Q \sin^2\theta_W$) and

$$\Lambda_{\mu\nu}^{(i)} \equiv A^{(i)} g_{\mu\nu} + B^{(i)} \frac{p_{1\mu} p_{2\nu}}{m_Z^2} + C^{(i)} \frac{p_{2\mu} p_{1\nu}}{m_Z^2} \ . \tag{17}$$

For the supersymmetric Higgs boson $H_2^0$,

$$A = \tfrac{1}{2} + \tfrac{3}{2} \sum_{\alpha=1,2} J_\alpha K_{\alpha 2} P_{2\alpha} + J_2^2 [D(p_1) + D(p_2)] \ , \tag{18}$$

$$B = J_2^2 D(p_1), \quad C = J_2^2 D(p_2) \ , \tag{19}$$

and, for the pseudoscalar $H_3^0$,

$$A = \tfrac{1}{2} + \tfrac{3}{2} \sum_{\alpha=1,2} J_\alpha K_{\alpha 3} P_{3\alpha} \ , \tag{20}$$

$$B = \sum_{\alpha=1,2} L_\alpha^2 S_{3\alpha}(p_1), \quad C = \sum_{\alpha=1,2} L_\alpha^2 S_{3\alpha}(p_2) \ , \tag{21}$$

where

$$D(p_i) \equiv \frac{m_Z^2}{m_{H_2^0}^2 - 2 p_Z p_i + i \Gamma_Z m_Z} , \quad i = 1,2 \ , \tag{22}$$

$$P_{\beta\alpha} \equiv \frac{m_{Z^0}^2}{2p_1p_2 + 2m_{H_\beta^0}^2 - m_{H_\alpha^0}^2}\ , \quad \beta=2,3, \quad \alpha=1,2\ , \qquad (23)$$

$$S_{\beta\alpha(p_i)} \equiv \frac{m_{Z^0}^2}{m_{Z^0}^2 + m_{H_\beta^0}^2 - m_{H_\alpha^0}^2 - 2p_Z p_i}\ ,$$

$$i=1,2, \quad \alpha,\beta=1,2,3\ . \qquad (24)$$

Finally, $J_\alpha$ and $L_\alpha$, proportional, respectively, to the $Z^0Z^0H_\alpha^0$ and $Z^0H_3^0H_\alpha^0$ couplings, are specified by Eqs. (11) and (13) and by $J_1=L_2$, $L_1=-J_2$. The relevant coefficients $K_{\alpha\beta}$, proportional to the trilinear $H_\alpha^0H_\beta^0H_\beta^0$ couplings, are given by

$$K_{12} = \frac{2r(1-c^2)(r^2-c^2)-c(r-c^2)(r^2+c^2-2r)}{3(r^2+c^2-2rc^2)^{3/2}}\ ,$$

$$K_{13} = \frac{c^2(1-r)}{3(r^2+c^2-2rc^2)^{1/2}}\ ; \qquad (25)$$

$$K_{22} = \frac{cr(r^2+c^2-2r)\sqrt{1-c^2}}{(r^2+c^2-2rc^2)^{3/2}}\ ,$$

$$K_{23} = \frac{cr\sqrt{1-c^2}}{3(r^2+c^2-2rc^2)^{1/2}}\ . \qquad (26)$$

The rate for the decay chain $Z^0 \to H_3^0H_2^0$, $H_3 \to H_2^0 f\bar{f}$ from Fig. 1(b) has been computed separately. The decay rate for the standard Higgs boson $H^0$ can be recovered from Eqs. (16)–(19), after the replacement $J_2=1$, $J_1=0$, $K_{22}=m_{H^0}^2/m_{Z^0}^2$, and $m_{H_2^0}=m_{H^0}$.

Figure 3 shows $\Gamma(Z^0 \to H_2^0H_2^0 f\bar{f})$ in units of the SM decay width $\Gamma(Z^0 \to H^0H^0 f\bar{f})$, for $m_{H_2^0}=m_{H^0}$. (In the numerical calculations throughout this paper, I have taken $m_Z=91.1$ GeV, $\sin^2\theta_W=0.23$, and $\alpha=\frac{1}{128}$.) The supersymmetric process is in general suppressed. Since Fig. 1(a) gives the leading contribution, the suppression factor with respect to the SM decay is, for small Higgs-boson masses, roughly equal to $J_2^4 \simeq \sin^4 2\beta$, where $\tan\beta \equiv v_2/v_1$. Similarly to the SM case, the dependence of the total rate on the $HHZZ$ and $HHH$ couplings is rather weak, since Fig. 1(a) dominates over the others. Lacking the leading contribution, $Z^0 \to H_3^0H_3^0 f\bar{f}$ has a much smaller rate.

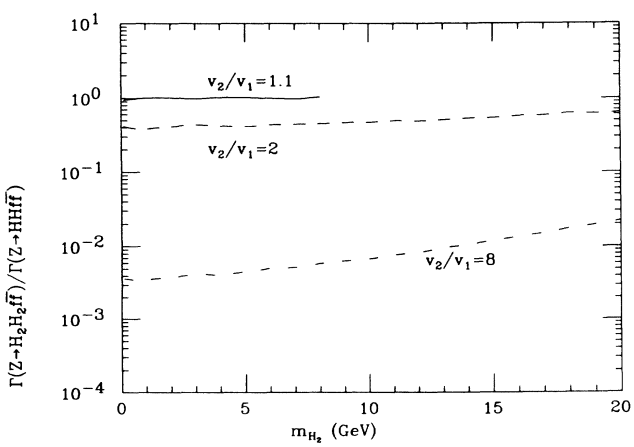


FIG. 3. $\Gamma(Z^0 \to H_2^0H_2^0 f\bar{f})$ in units of the SM process $\Gamma(Z^0 \to H^0H^0 f\bar{f})$ for $m_{H_2^0}=m_{H^0}$. In the case $v_2/v_1=1.1$, consistency requires $m_{H^0} \lesssim 9$ GeV [See Eq. (8)].

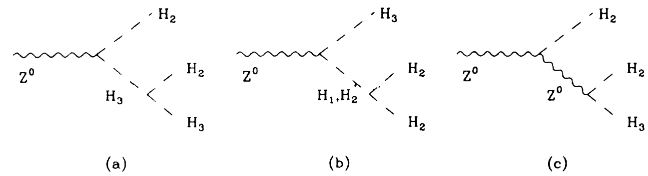


FIG. 4. Feynman diagrams for $Z^0 \to H_2^0H_2^0H_3^0$.

Unlike the SM, supersymmetry, in addition to the decays (15), may lead to triple-Higgs-boson production:

$$Z^0 \to H_2^0H_2^0H_3^0, \quad Z^0 \to H_3^0H_3^0H_3^0\ . \qquad (27)$$

The processes occur through the diagrams of Figs. 4 and 5, respectively, which include the trilinear Higgs coupling, and yield the partial decay widths:

$$\Gamma(Z^0 \to H_2^0H_2^0H_3^0) = \frac{\pi^2\alpha^2}{3\sin^4\theta_W\cos^4\theta_W m_Z^3} \int d\phi^{(3)} \left| \sum_{i=1}^{3} \mathbf{p}_i T_i \right|^2 , \qquad (28)$$

$$\Gamma(Z^0 \to H_3^0H_3^0H_3^0) = \frac{\pi^2\alpha^2}{\sin^4\theta_W\cos^4\theta_W m_Z^3} \int d\phi^{(3)} \left| \sum_{i=1}^{3} \mathbf{p}_i V_i \right|^2 . \qquad (29)$$

$d\phi^{(3)}$ is the three-body phase space and $E_i, \mathbf{p}_i$ ($i=1,2,3$) are, respectively, the energy and momentum of the Higgs bosons in the $Z^0$ rest frame. In the case of Eq. (28), the index $i=3$ refers to the pseudoscalar $H_3^0$. Also

$$T_1 = -3L_2K_{23}S_{23}(p_1) + J_2L_2\left[\frac{m_{H_3^0}^2 - m_{H_2^0}^2}{m_{Z^0}^2}D(p_1) - D(p_2)\right] , \qquad (30)$$

$$T_2 = -3L_2K_{23}S_{23}(p_2) + J_2L_2\left[\frac{m_{H_3^0}^2 - m_{H_2^0}^2}{m_{Z^0}^2}D(p_2) - D(p_1)\right] , \qquad (31)$$

$$T_3 = 3\sum_{\alpha=1,2} L_\alpha K_{\alpha 2} S_{3\alpha}(p_3) + J_2L_2[D(p_1)+D(p_2)]\ , \qquad (32)$$

$$V_i = \sum_{\alpha=1,2} L_\alpha K_{\alpha 3} S_{3\alpha}(p_i), \quad i=1,2,3\ . \qquad (33)$$

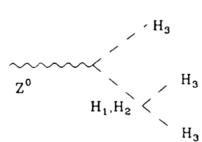


FIG. 5. Feynman diagrams for $Z^0 \to H_3^0H_3^0H_3^0$.

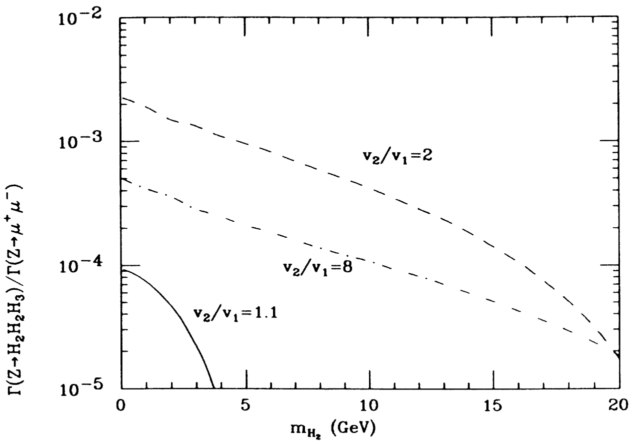


FIG. 6. $\Gamma(Z^0\to H_2^0H_2^0H_3^0)$ in units of $\Gamma(Z^0\to\mu^+\mu^-)$ as a function of $m_{H_2^0}$.

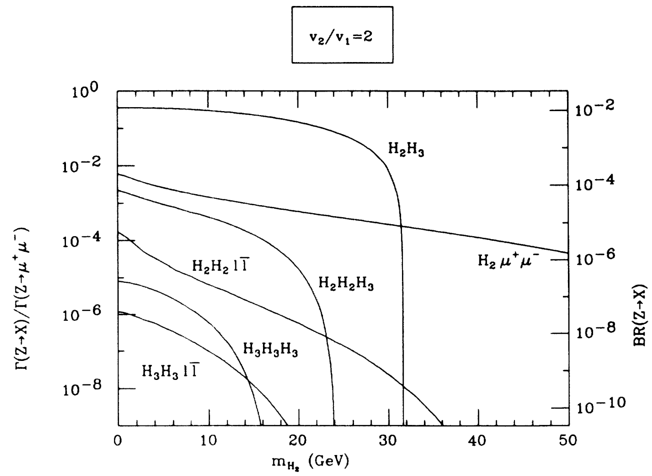


FIG. 7. The partial width of the different $Z^0$ decays into supersymmetric Higgs bosons, for $v_2/v_1=2$. The scale on the left shows the rate in units of $\Gamma(Z^0\to\mu^+\mu^-)$ and the scale on the right shows the branching ratio of the process, assuming $B(Z^0\to\mu^+\mu^-)=3.3\%$.

The decay rate for $Z^0\to H_2^0H_2^0H_3^0$ as a function of $m_{H_2}$, for three different values of $v_2/v_1$, is shown in Fig. 6. The mass of $H_3^0$ is fixed by the choice of these two parameters [see Eq. (5)]. For $m_{H_2}$ smaller than about 20 GeV, the branching ratio for triple-Higgs-boson decay of the $Z^0$ is naturally in the range $10^{-4}$–$10^{-6}$ and a sizable number of events is expected at LEP. The leading contribution to the process comes from Fig. 4(c), which is independent of the trilinear Higgs-boson coupling. This also means that the decay rate, for light Higgs bosons, is roughly proportional to $\sin^2 4\beta$, where $\tan\beta\equiv v_2/v_1$. Thus the maximum signal is expected to occur for $\beta=3\pi/8$, i.e., $v_2/v_1\simeq 2.4$. The decay $Z^0\to H_3^0H_3^0H_3^0$, lacking the leading contribution (see Fig. 5) has a much smaller rate.

Figure 7 compares the rates for the different Higgs-boson production processes from $Z^0$ decays, for the choice $v_2/v_1=2$. The rates for $Z^0\to H_i^0H_i^0f\bar{f}$ $(i=2,3)$ have been summed over all leptonic modes $(f=e,\mu,\tau,\nu)$, in order to allow an easier comparison with the SM predictions, given in Ref. 3. The rate for $Z^0\to H_i^0H_i^0\mu^+\mu^-$ is then about nine times smaller. The predictions for the processes (14) are not shown, since they depend also on the unknown supersymmetric mass parameters (see Ref. 7). If kinematically allowed, $Z^0\to H_2^0H_3^0$ dominates over $Z^0\to H_2^0\mu^+\mu^-$, which is suppressed with respect to the analogous SM process. A sizable rate for $Z^0\to H_2^0H_2^0H_3^0$ is also predicted, while $Z^0\to H_2^0H_2^0f\bar{f}$ is suppressed with respect to the SM. The rates for $Z^0\to H_3^0H_3^0f\bar{f}$ and $Z^0\to H_3^0H_3^0H_3^0$ are unobservably small.

The effect of varying the value of $v_2/v_1$ is the following. Increasing $v_2/v_1$ will further enhance $Z^0\to H_2^0H_3^0$ and suppress $Z^0\to H_2\mu^+\mu^-, H_2^0H_2^0f\bar{f}$. The rate for $Z^0\to H_2^0H_2^0H_3^0$ has a maximum for $v_2/v_1\simeq 2.4$ and then decreases for larger $v_2/v_1$. On the contrary, if $v_2/v_1$ is very close to one, $H_2^0$ is forced to be very light, and the supersymmetric Higgs-boson production processes will closely resemble those of the SM scenario. It is important to stress that, given the large expected value of the top-quark mass, $v_2/v_1\simeq 1$ is a rather unnatural solution of the conditions for supergravity-induced electroweak-symmetry breaking. Larger values of $v_2/v_1$ are much more likely to occur and then, as shown in this paper, the supersymmetric Higgs sector will have very distinctive features.

The detection of the processes discussed here relies upon the experimental ability of disentangling the signal from the QCD background. A complete calculation of the multijet background is not available at present, but the possible discovery of a light Higgs boson should stimulate further study.

In conclusion, if a light Higgs boson is discovered at LEP, detection of rare production processes can provide interesting information and constraints about the structure of nonminimal Higgs sectors. In low-energy supergravity models, multi-Higgs-boson productions in $Z^0$ decays can occur with rates substantially larger than in the SM.

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