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The decay $b \to sgg$ in the standard model and a nonsoft bremsstrahlung extension of the low-energy theorem

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An extension to the low-energy theorem for a flavor-changing two-gluon decay is discussed. It is shown that in cases where external momenta are much smaller than M_W but not small compared with the mass of the decaying particle, contributions with the lowest powers of external momenta are only proportional to the corresponding one-gluon decay dipole-moment form factor. In particular, contributions from the color charge-radius form factor must cancel by gauge invariance. As an illustration, an explicit calculation for the decay $b \to sgg$ is carried out in the standard model. The result of the calculation agrees with our theorem. We estimate that the branching ratio $B(b \to sgg)$ is roughly $\lesssim 10^{-4}$ and hence completely negligible in the study of nonleptonic inclusive charmless B decays. This differs from other estimates.

I. INTRODUCTION

During the last few years considerable attention has focused on the study of B-meson rare decays. That such decays occur so rarely in nature implies that they can only be induced either by some unknown new physics or by highly suppressed radiative loop corrections. A careful study of these processes may, therefore, provide insight into the structure of theoretical models, and hence is important in the formulation of a more fundamental theory.

Recently, a study on the decay $b \to s\gamma\gamma$ has been carried out.[1] It turns out that the photon spin orientations in this decay are sensitive to the value of the top-quark mass m_t. A precise measurement on the photon spin structures may thus provide useful limits on m_t. Another interesting feature observed in this study is that in the limit to be discussed below, an extension to Low's low-energy theorem[2] exists. With such an extension, the lowest-dimensional contributions can be simply calculated in terms of the decay $b \to s\gamma$ and external bremsstrahlungs. Note that the momenta of the photons are not small compared with the b-quark mass. Thus an extension is nontrivial.

In this paper we wish to extend our earlier work to cover the case of a flavor-changing two-gluon decay such as $b \to sgg$ (with the two final-state gluons on shell). Previous studies[3] seem to suggest that this decay gives dominant contributions to the nonleptonic inclusive charmless B decay. Furthermore, such a domination, if true, would probably remain unchanged[4] under QCD radiative corrections. The expectation that the decay $b \to sgg$ might dominate arises from an observation that the self-interaction graph [Fig. 1(d)], with the $b \to sg$ vertex given by its color charge-radius form factor, appears to have a large log factor. Calculations[5] based on the zero-external-momentum approximation indicate that the log factor of a flavor-changing charge-radius form factor is $\ln(m_i^2/M_W^2)$ with m_i given by the mass of an internal participating quark. In the case of B_s decay, one may therefore expect that the log factor is $\ln(m_c^2/m_t^2)$. As a result, for m_t heavier or not much smaller than M_W, an enhancement of $\sim(10)$ (and thus $\sim 10^2$ for the decay rate) would be anticipated. The enhancement would be even more significant if $m_t \ll M_W$, for in that case a strong Glashow-Iliopoulos-Maiani[6] (GIM) suppression is applicable to some of the other terms.

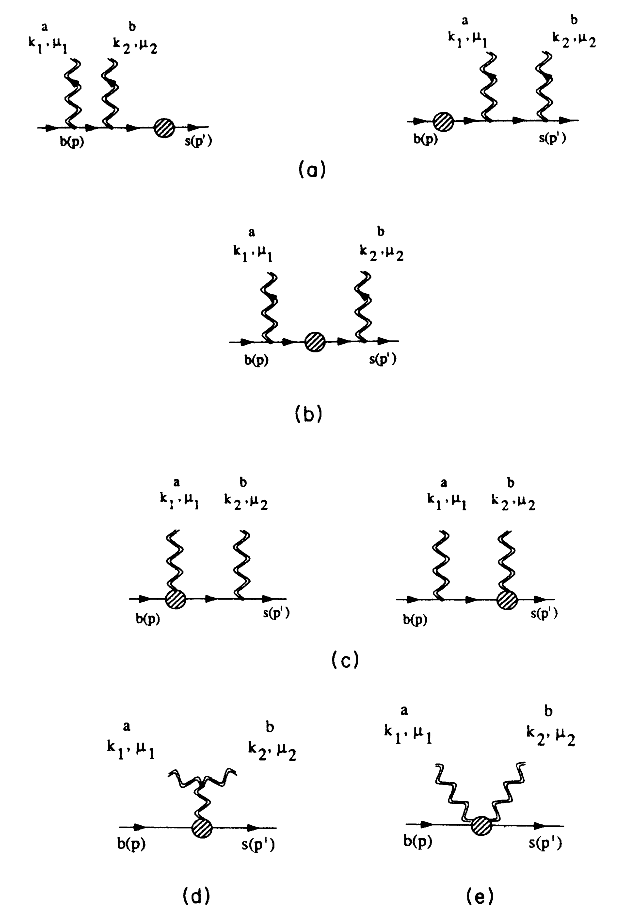

FIG. 1. Feynman graphs contributing to the decay $b \to sgg$. The double wiggling lines represent gluons. For simplicity, graphs with the interchange $1 \leftrightarrow 2$ are not shown explicitly here.

According to the classification given by our previous studies of $b \to s\gamma\gamma$, although leading terms from the color charge-radius form factors also have the lowest dimensionality with respect to external momenta and fields, in the case of a two-photon decay, the lowest-dimensional terms which survive after cancellations are only proportional to the dipole-moment form factor. This proportionality is a consequence of an extension to Low's theorem originated from gauge invariance. Therefore, similar features should also be expected for the decay $b \to sgg$. Although there are many similarities between $b \to s\gamma\gamma$ and $b \to sgg$, the situation nevertheless has to be carefully reexamined due to the non-Abelian nature of QCD.

It turns out that for a two-gluon decay an extension to the low-energy theorem also exists. We shall extract this theorem in the next section. In particular, we find that the color charge-radius term must cancel. It then follows that the final result should not have a large log factor. As a result, the branching ratio of the decay should be about $\ln(m_c^2/m_t^2) \sim 10^2$ times smaller than the previous expectations, and hence completely negligible in the study of the inclusive B_s decay. This part of the discussions is given in Sec. IV which will be followed by a conclusion in Sec. V.

The rest of the paper, which covers Sec. III and two appendixes, is given to an explicit calculation of the decay $b \to sgg$ in the standard model (without QCD corrections). The cancellations of the color charge-radius form factor and other "off-shell" effects are shown indeed to take place. The result of the calculation completely agrees with our theorem. By explicit calculations, we also show that when external momenta are not negligible, which is the case in the study of B decay, the dependence of the charge-radius form factor on the internal quark masses actually takes a different form from that quite commonly quoted in the literature. Its physical implications are also briefly examined.

II. THE LOW-ENERGY THEOREM AND FLAVOR-CHANGING TWO-GLUON DECAY

It is useful to begin with some considerations on the constraints imposed by gauge invariance on the decay amplitude. To do so, we follow the idea of Low (Ref. 2).

According to Low's theorem (Ref. 2), leading terms of a soft bremsstrahlung-scattering cross section can be calculated exactly in terms of the cross section of the hard scattering (without the photon emission) and an external photon bremsstrahlung (without the scattering). Two important conditions are required for the theorem to hold. First, the energy emitted by the photon must be much smaller than the energy scale for producing the scattering, and second, the photon energy must be much smaller than the external momenta of the participating particles. The first condition permits the separation of radiation from the scattering, whereas the second one amounts to allowing for a momentum expansion to single out the leading terms. The usefulness of Low's theorem is that, thanks to gauge invariance, potentially complicated calculations of internal bremsstralungs are now no longer necessary.

It is straightforward to extend Low's theorem to places where a scattering is replaced by a decay process provided the two necessary conditions are satisfied. For the case in point, one can think of the decay $b \to s\gamma\gamma$ as a decay $b \to s\gamma$ followed by a bremsstrahlung (both internal and external). The first condition required for Low's theorem to hold is clearly satisfied, because the energy scale responsible for the flavor-changing decay $b \to s\gamma$ is of the order M_W, which is much higher than the photon energy. However, the second condition, which requires the photon energy be much smaller than m_b and m_s, cannot be satisfied. Still, it is found (Ref. 1) that in the lowest-dimensional terms, the photons can be regarded as if they were soft and thus an extension to Low's theorem follows. The price for relaxing the second condition is that these terms do not necessarily dominate the process.

Now, we consider the decay $b \to sgg$ without QCD loops. Following Low (Ref. 2), we write the transition amplitude $T^{\alpha\beta}_{\mu_1\mu_2}$ as

$$T^{\alpha\beta}_{\mu_1\mu_2}(b(P) \to s(P')g(k_1)g(k_2)) = g_s^2(T^{\alpha\beta}_{1,\mu_1\mu_2} + T^{\alpha\beta}_{2,\mu_1\mu_2}) \tag{1}$$

with

$$T^{\alpha\beta}_{1,\mu_1\mu_2} = \gamma_{\mu_2}\frac{1}{\not P' + \not k_2 - m_s}\Gamma_{\mu_1}(P,k_1)t^\beta t^\alpha + \Gamma_{\mu_1}(P-k_2,k_1)\frac{1}{\not P - \not k_2 - m_b}\gamma_{\mu_2}t^\alpha t^\beta + \begin{bmatrix} 1\leftrightarrow 2 \\ \alpha \leftrightarrow \beta \end{bmatrix}$$
$$+\Gamma^\mu(P,k_1+k_2)\frac{1}{(k_1+k_2)^2}[2(k_{2,\mu_1}g_{\mu_2\mu} - k_{1,\mu_2}g_{\mu_1\mu}) + g_{\mu_1\mu_2}(k_1-k_2)_\mu][t^\beta, t^\alpha] \tag{2}$$

corresponding to contributions from external bremsstrahlung diagrams. The last term arises from the self-interaction diagram [Fig. 1(d)] and the others from Fig. 1(c). In Eq. (2), t^α is an $SU(3)_C$ generator, and $\Gamma_\mu(P,k)$ represents the $b \to sg$ vertex. Contributions from the rest of the diagrams [Figs. 1(a), 1(b), 1(e)] are represented by $T^{\alpha\beta}_{2,\mu_1\mu_2}$, which are to be determined by gauge invariance

$$k_1^{\mu_1}T^{\alpha\beta}_{\mu_1\mu_2} = 0, \quad k_2^{\mu_2}T^{\alpha\beta}_{\mu_1\mu_2} = 0 . \tag{3}$$

The most general form of $\Gamma_\mu(P,k)$ can be written as follows. Consider the Ward-Takahashi identity for the renormalized quantities

$$k^\mu\Gamma_\mu(P,k) = \Sigma(P-k) - \Sigma(P) . \tag{4}$$

Here Σ is the on-shell renormalized $b \to s$ self-energy. It can always be written as $\Sigma(P)=(\not P - m_s)\tilde{\Sigma}(P)(\not P - m_b)$. The explicit expression of $\tilde{\Sigma}$ is model dependent. However, for our purpose, one only needs to know that $\tilde{\Sigma}$ is finite when operating on the external fields b and s. In other words, $\tilde{\Sigma}$ does not have Dirac projections in its denominator. This result is a consequence of the on-shell conditions

$$\bar{s}\Sigma(P^2=m_s^2)=0, \quad \Sigma(P^2=m_b^2)b=0 \tag{5}$$

required by our renormalization procedure. It then follows from Eq. (4) that

$$\Gamma_\mu(P,k)=i\sigma_{\mu\nu}k^\nu\frac{mF}{M^2}+(\not k k_\mu - k^2\gamma_\mu)L\frac{H}{M^2} +[(\not P-\not k-m_s)O_\mu(P,k) +Q_\mu(P,k)(\not P-m_b)]\frac{I}{M^2} . \tag{6}$$

Here M, which is typically of the order of M_W or higher, represents the energy scale responsible for the flavor-changing transition. m is a mass that flips helicities. O_μ and Q_μ are two combinations of operators. They have the dimension of mass and satisfy

$$k^\mu O_\mu(P,k)\neq 0 , \tag{7}$$

$$k^\mu Q_\mu(P,k)\neq 0 . \tag{8}$$

That Eqs. (7) and (8) must be true is a consequence that the relation, $k^\mu\Gamma_\mu(P,k)=0$, holds only if the external quark fields are on shell. Finally, F, H, and I are dimensionless functions of P^2/M_W^2,$(P-k)^2/M_W^2$, and other internal participating particle masses. In Eq. (6), the first term is the usual dipole-moment term and the second one is the charge radius. The last term gives rise to off-shell effects. It vanishes in $b \to sg$ when both b and s are on shell. The charge-radius term vanishes also if the gluon emitted from Γ is real.

It is useful to classify the relevant operators by their dimensionality with respect to external fields (b,s,g) and momenta. Such a classification is possible provided $m_b \ll M$ (the first condition of Low's theorem). Now, we can write F, H, and I as series expansions of external momenta with respect to M. Their first terms in these series, denoted by F_0, H_0, and I_0 hereafter, are external momenta independent, and hence have the lowest dimensionality. From Eqs. (2) and (6), we have explicitly

$$\begin{aligned}
T^{\alpha\beta}_{1,\mu_1\mu_2} = & \left[\gamma_{\mu_2}\frac{1}{\not P'+\not k_2-m_s}i\sigma_{\mu_1\nu}k_1^\nu t^\beta t^\alpha + i\sigma_{\mu_1\nu}k_1^\nu\frac{1}{\not P-\not k_2-m_b}\gamma_{\mu_2}t^\alpha t^\beta\right]\frac{mF_0}{M^2} \\
& +[\gamma_{\mu_2}O_{\mu_1}(P,k_1)t^\beta t^\alpha + Q_{\mu_1}(P-k_2,k_1)\gamma_{\mu_2}t^\alpha t^\beta]\frac{I_0}{M^2}+\begin{bmatrix}1\leftrightarrow 2\\ \alpha\leftrightarrow\beta\end{bmatrix} \\
& +\frac{i}{k_1\cdot k_2}[(\sigma_{\mu_2\nu}k_{2,\mu_1}-\sigma_{\mu_1,\nu}k_{1,\mu_2})(k_1+k_2)^\nu+\sigma_{\mu\nu}k_1^\mu k_2^\nu g_{\mu_1\mu_2}][t^\beta,t^\alpha]\frac{mF_0}{M^2} \\
& +[2(k_{2,\mu_1}\gamma_{\mu_2}-k_{1,\mu_2}\gamma_{\mu_1})+g_{\mu_1\mu_2}(\not k_1-\not k_2)][t^\beta,t^\alpha]\frac{H_0}{M^2}+(\text{higher-dimensional terms}) .
\end{aligned} \tag{9}$$

Here all the external fields in the decay $b \to sgg$ are on their mass shell and higher-dimensional terms represent the rest in the series expansion. Now, the calculations of $k_1^{\mu_1}T^{\alpha\beta}_{1,\mu_1\mu_2}$ and $k_2^{\mu_2}T^{\alpha\beta}_{1,\mu_1\mu_2}$ are straightforward. We find that the most general solution to Eq. (3) that satisfies the constraints given by Eqs. (7) and (8) is

$$\begin{aligned}
T^{\alpha\beta}_{2,\mu_1\mu_2} = & i\sigma_{\mu_1\mu_2}[t^\beta,t^\alpha]\frac{mF_0}{M^2}-[2(k_{2,\mu_1}\gamma_{\mu_2}-k_{1,\mu_2}\gamma_{\mu_1})+g_{\mu_1\mu_2}(\not k_1-\not k_2)][t^\beta,t^\alpha]\frac{H_0}{M^2} \\
& -\left[\gamma_{\mu_2}O_{\mu_1}(P,k_1)t^\beta t^\alpha+Q_{\mu_1}(P-k_2,k_1)\gamma_{\mu_2}t^\alpha t^\beta+\begin{bmatrix}1\leftrightarrow 2\\ \alpha\leftrightarrow\beta\end{bmatrix}\right]\frac{I_0}{M^2}+(\text{higher-dimensional terms}) .
\end{aligned} \tag{10}$$

Adding $T^{\alpha\beta}_{2,\mu_1\mu_2}$ to $T^{\beta}_{1,\mu_1\mu_2}$, we find that the charge-radius term as well as the other off-shell terms are canceled out, resulting in our theorem

$$T^{\alpha\beta}_{\mu_1\mu_2}(b(P)\to s(P')g_1(k_1)g_2(k_2))=T_{S,\mu_1\mu_2}S^{\beta\alpha}+T_{A,\mu_1\mu_2}A^{\beta\alpha}+(\text{higher-dimensional terms}) , \tag{11}$$

where $S^{\beta\alpha}=\{t^\beta,t^\alpha\}/2$, $A^{\beta\alpha}=[t^\beta,t^\alpha]/2$, and

$$\begin{aligned}
T^{\alpha\beta}_{S,\mu_1\mu_2} = ig_s^2\frac{mF_0}{M^2}\Bigg\{ & \left[\frac{P'_{\mu_2}}{P'\cdot k_2}-\frac{P_{\mu_2}}{P\cdot k_2}\right]\sigma_{\mu_1\nu}k_1^\nu+\left[\frac{P'_{\mu_1}}{P'\cdot k_1}-\frac{P_{\mu_1}}{P\cdot k_1}\right]\sigma_{\mu_2\nu}k_2^\nu \\
& -\frac{i}{2}\left[\left[\frac{1}{P'\cdot k_2}-\frac{1}{P\cdot k_1}\right]\sigma_{\mu_2\mu}\sigma_{\mu_1\nu}k_2^\mu k_1^\nu+\left[\frac{1}{P'\cdot k_1}-\frac{1}{P\cdot k_2}\right]\sigma_{\mu_1\mu}\sigma_{\mu_2\nu}k_1^\mu k_2^\nu\right]\Bigg\} ,
\end{aligned} \tag{12}$$

and

$$T^{\alpha\beta}_{A,\mu_1\mu_2}=ig_s^2\frac{mF_0}{M^2}\left\{\left[\frac{P'_{\mu_2}}{P'\cdot k_2}-\frac{P_{\mu_2}}{P\cdot k_2}\right]\sigma_{\mu_1\nu}k_1^\nu-\left[\frac{P'_{\mu_1}}{P'\cdot k_1}-\frac{P_{\mu_1}}{P\cdot k_1}\right]\sigma_{\mu_2\nu}k_2^\nu\right.$$
$$+2\left[\frac{P_{\mu_2}}{P\cdot k_2}-\frac{k_{1,\mu_2}}{k_1\cdot k_2}\right]\sigma_{\mu_1\nu}k_1^\nu-2\left[\frac{P_{\mu_1}}{P\cdot k_1}-\frac{k_{2,\mu_1}}{k_1,k_2}\right]\sigma_{\mu_2\nu}k_2^\nu$$
$$\left.-\frac{i}{2}\left[\left[\frac{1}{P'\cdot k_2}-\frac{1}{P\cdot k_1}\right]\sigma_{\mu_2\mu}\sigma_{\mu_1\nu}k_2^\mu k_1^\nu-\left[\frac{1}{P'\cdot k_1}-\frac{1}{p\cdot k_2}\right]\sigma_{\mu_1\mu}\sigma_{\mu_2\nu}k_1^\mu k_2^\nu\right]-i\frac{\delta_{\mu_1\mu_2}}{k_1\cdot k_2}\right\}. \tag{13}$$

In Eq. (13), the quantity $\delta_{\mu_1\mu_2}$ is given by

$$\delta_{\mu_1\mu_2}=k_{2,\mu_1}[\gamma_{\mu_2},\not k_1]+k_{1,\mu_2}[\not k_2,\gamma_{\mu_1}]+g_{\mu_1\mu_2}[\not k_1,\not k_2]+(k_1\cdot k_2)[\gamma_{\mu_1},\gamma_{\mu_2}]\ . \tag{14}$$

Both T_S and T_A are manifestly gauge invariant. T_S is symmetric under the interchange $1\leftrightarrow 2$. This part is precisely the same as for $b\to s\gamma\gamma$. It can simply be obtained by calculating the effective tree diagrams [Fig. 1(c)] with the $b\to sg$ vertex given by its dipole-moment form factor. The T_A term is antisymmetric under the interchange $1\leftrightarrow 2$ and therefore exists only in theories with non-Abelian nature such as QCD. That T_S and T_A ought to take the forms shown in Eqs. (12) and (13) is, of course, not surprising. We know that in momentum space, a gauge-invariant two-gluon state $G^a_{\mu\nu}G^{b,\mu\nu}$ or $\tilde G^a_{\mu\nu}G^{b,\mu\nu}$ must be bilinear in (external) momenta; here, $G^a_{\mu\nu}$ is the gluon field tensor with its dual $\tilde G^a_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{a,\alpha\beta}$. To construct a gauge-invariant term that has the lowest dimension with respect to external momenta and fields, one has to introduce terms with external momenta in the denominator.

Our theorem has two interesting implications. First, a flavor-changing two-gluon decay amplitude should not have log factor. Second, the lowest-dimensional terms can always be represented in forms given by Eqs. (12) and (13) with only the dipole moment to be determined by the specific decay process and theoretical model. As far as the lowest-dimensional terms are concerned, potentially complicated calculations can now be simplified to a much simpler calculation of the corresponding dipole-moment form factor.

Finally, it should be pointed out that F_0 may depend on the internal participating quark masses, which we have so far ignored for simplicity, in a very complicated way. Thus, there is no reason in general to expect that the lowest-dimensional terms given by Eqs. (12) and (13) are the dominant terms. To assess their importance, complete model calculations are called for.

III. $b\to sgg$ IN THE STANDARD MODEL

As an illustration, we now consider the decay $b\to sgg$ in the standard model. The calculation of $b\to sgg$ is very similar to that of $b\to s\gamma\gamma$ (Ref. 1). In the case involving gluons, however, one has fewer Feynman diagrams because gluons do not couple to W. The complete answer to the amplitude of $b\to sgg$ can be written as

$$T^{\alpha\beta}_{\mu_1\mu_2}=\sum_{j=u,c,t}V_{bj}V^*_{sj}T^{\alpha\beta}_{j,\mu_1\mu_2}\ , \tag{15}$$

where V is the Kobayashi-Maskawa (KM) matrix and T_j ($j=u,c,t$) represents contributions from an internal u-, c-, or t-quark exchange.

For $T^{\alpha\beta}_{t,\mu_1\mu_2}$, at one-loop level, all the external momenta are much smaller than the internal particle masses. Consequently, the usual zero-external-momentum approximation can be applied. At this point, it is however important to realize that for $T^{\alpha\beta}_{c,\mu_1\mu_2}$ and particularly for $T^{\alpha\beta}_{u,\mu_1\mu_2}$ such a convenient approximation is no longer applicable, because now the internal quark masses are much smaller than the external momenta, which are typically of the order of m_b. Thus, these graphs have to be treated separately. To the leading order, we find that the only effect is on the color charge-radius form factor R. For instance (for details see Appendixes A and B), in the Feynman–'t Hooft gauge, the graph with a t quark and W yields

$$R_W(m_t)=\frac{-g^2g_s}{16\pi^2}\int_0^1 dx\frac{(1-x)(2+5x-x^2)}{6[m_t^2+x(M_W^2-m_t^2)]}\ , \tag{16}$$

whereas the graph with a c or u quark and W gives

$$R_W(m_{u,c})=\frac{-g^2g_s}{16\pi^2M_W^2}\left[2\int_0^1 dx\, x(x-1)\ln\frac{m_{u,c}^2+x(x-1)k^2}{M_W^2}+\frac{1}{18}\right], \tag{17}$$

where k is the momentum transfer. Equation (17) holds for arbitrary $m^2,k^2\ll M_W^2$. It is readily seen that both equations give the same result with a log factor $\ln(m_i^2/M_W^2)$ ($i=u,c,t$) in the limit $m_t\ll M_W$ and $k^2=0$. However, for $k^2\gtrsim m_{u,c}^2$, which is the case for B_s decay, the charge-radius form factor is not simply given by such a log factor. This important fact has so far been overlooked in the literature.[7] Its consequences are currently under our investigation.

Now, we turn back to $T^{\alpha\beta}_{\mu_1\mu_2}$. By a unitarity condition, $\sum_j V_{bj}V^*_{sj}=0$, Eq. (15) can be conveniently rewritten as

$$T^{\alpha\beta}_{\mu_1\mu_2}=V_{bu}V^*_{su}(T^{\alpha\beta}_{u,\mu_1\mu_2}-T^{\alpha\beta}_{c,\mu_1\mu_2})+V_{bt}V^*_{st}(T^{\alpha\beta}_{t,\mu_1\mu_2}-T^{\alpha\beta}_{c,\mu_1\mu_2}) . \tag{18}$$

To a good approximation, the first term in Eq. (18) can be ignored because of the small value of $V_{bu}V^*_{su}$. As we already mentioned, contributions to $T^{\alpha\beta}_{\mu_1\mu_2}$ can be classified in terms of their dimensionality with respect to the external momenta and fields. To the order of $O(1/M_W^2)$, we find (details can be found in Appendixes A and B) that the lowest-dimensional contributions are given by Eqs. (12) and (13). The dipole-moment form factor $D\equiv mF_0/M^2$ is

$$D(m_c)=\frac{g^2g_s}{16\pi^2}(m_bR+m_sL)\int_0^1 dx\frac{x(1-x^2)}{2[m_c^2+x(M_W^2-m_c^2)]} \tag{19}$$

for $T^{\alpha\beta}_{c,\mu_1\mu_2}$ and

$$D(m_t^2)=\frac{g^2g_s}{16\pi^2}(m_bR+m_sL)\left[\int_0^1 dx\frac{x(1-x^2)}{2[m_t^2+x(M_W^2-m_t^2)]}+\frac{m_t^2}{2M_W^2}\int_0^1 dx\frac{(1-x)^2(2-x)}{2[m_t^2+x(M_W^2-m_t^2)]}\right] \tag{20}$$

for $T^{\alpha\beta}_{t,\mu_1\mu_2}$.

Higher-dimensional terms, which are generated from the one-particle-irreducible (1PI) graphs, will also contribute to the decay. For a heavy top, i.e., $m_t\gtrsim M_W$, the only term of $O(1/M_W^2)$ comes from the diagram with an internal c quark and W. The result of the calculation is

$$\frac{4\sqrt{2}}{\pi}G_F\alpha_s[iR_{\mu_1\mu_2\sigma}S^{\beta\alpha}I_1+R'_{\mu_1\mu_2\sigma}A^{\beta\alpha}(I_1-2I_2)]\gamma^\sigma L , \tag{21}$$

where

$$R_{\mu_1\mu_2\sigma}=k_{1,\mu_2}\epsilon_{\mu_1\nu\rho\sigma}k_1^\nu k_2^\rho-k_{2,\mu_1}\epsilon_{\mu_2\nu\rho\sigma}k_1^\nu k_2^\rho +k_1\cdot k_2\epsilon_{\mu_1\mu_2\nu\sigma}(k_1-k_2)^\nu \tag{22}$$

is the third-quark tensor, which is symmetric under the interchange $1\leftrightarrow2$, suggested originally by Rosenberg[8] and Adler.[9] $R'_{\mu_1\mu_2\sigma}$ is given by

$$R'_{\mu_1\mu_2\sigma}=(k_{1,\mu_2}k_{2,\mu_1}-g_{\mu_1\mu_2}k_1\cdot k_2)(k_2-k_1)_\sigma , \tag{23}$$

which is antisymmetric under the interchange $1\leftrightarrow2$. The two integrals in Eq. (21) are

$$I_1=\int_0^1 dx\int_0^{1-x}dy\frac{xy}{m_c^2-2xyk_1\cdot k_2} ,\qquad I_2=\int_0^1 dx\int_0^{1-x}dy\frac{x^2y}{m_c^2-2xyk_1\cdot k_2} . \tag{24}$$

The first term is precisely what one would get for a two-photon decay.[10] The final result is a sum of Eqs. (12), (13), and (21) according to Eq. (18).

IV. FURTHER DISCUSSIONS

In this section we will give a simple (but fairly reliable) estimate on the order of magnitude of our result. In a future publication we shall present a more complete treatment (including all the relevant processes in the inclusive B_s decay) and give more details of the computation. We now consider the limit $m_t\gg M_W$. To determine the lowest-dimensional contributions, we have to compute $D(m_t^2)-D(m_c^2)$. For a heavy top, we have, from Eqs. (19) and (20),

$$D(m_t)-D(m_c)=\frac{1}{4}\left[\frac{g_sG_F}{4\sqrt{2}\pi^2}\right](m_bR+m_sL) . \tag{25}$$

This should be compared with the color charge-radius form factors $R(m_t)-R(m_c)$, which enters into the estimate of Ref. 3, obtained from Eq. (16) or (17) by assuming, incorrectly, $k^2=0$. The squared ratio of these two factors is 450. As a result, the branching ratio given in Ref. 3 should approximately be scaled down by this amount. This yields

$$B(b\to sgg)\sim3\times10^{-5} . \tag{26}$$

One can see by inspection that the order of magnitude of the higher-dimensional terms [Eq. (21)] for a heavy top is roughly the same as that of the lowest-dimensional terms. In view of the various uncertainties involved in such a simple scaling, we expect that the branching ratio should at least be about 100 times less than that given by the previous estimates (Ref. 3).

It is instructive to compare Eq. (26) with the branching ratio of $B_s\to\gamma\gamma$ which is of the order $B(B_s\to\gamma\gamma)\sim10^{-7}$ (Ref. 1). Taking $\alpha_s(m_b)=0.235$, one would expect from

$$\frac{B(b\to sgg)}{B(B_s\to\gamma\gamma)}\sim\frac{\alpha_s^2}{\alpha^2}\sim10^3 \tag{27}$$

that $B(b\to sgg)\sim10^3\times B(B_s\to\gamma\gamma)\sim10^{-4}$, which agrees qualitatively with Eq. (26).

In comparison with the one-gluon decay $b\to sg$, the amplitude of the two-gluon decay is suppressed by a factor of α_s/π. For $m_t\gtrsim M_W$, the two-gluon decay channel is therefore negligible. It is certainly much smaller than that of $b\to sq\bar{q}$. Hence, the two-gluon decay channel is completely negligible in the study of nonleptonic inclusive charmless decays.

Finally, we should point out that, unlike in $b\to sg$, it is rather complicated to include QCD corrections to the decay $b\to sgg$. First, all the operators involved [see Eqs. (12), (13), and (21)] are highly nonlocal. Second, the complete set for these operators are rather large. Although the lowest-dimensional terms are related to $b\to sg$, it is not clear whether the results with QCD corrections for $b\to sg$ can be used for $b\to sgg$. This is because, for the two-gluon decay, one of the quark lines connecting to the

$b \to sg$ vertex is necessarily off shell, whereas the result in existing literature (Ref. 4) for $b \to sg$ is obtained by assuming both lines on shell. In addition, the gluon-gluon vertices and the quark propagators should also be QCD corrected. Because of all these complications, we will not discuss QCD corrections in this paper. It is nonetheless fairly reasonable to believe that our result gives the right order of magnitude.

V. CONCLUSION

We have shown that, for a flavor-changing two-gluon decay, there is an extension [see Eqs. (12) and (13)] to the low-energy theorem. Such an extension suggests that the lowest-dimensional terms are only proportional to the dipole-moment form factor. As an illustration, we calculated the decay $b \to sgg$ explicitly in the standard model. Without QCD corrections, we find that the result of the calculation completely agrees with our theorem. Off-shell effects, particularly the color charge-radius form factor, actually do not make any contribution in leading order. As a result, the branching ratio is found below the order of 10^{-4}, which is approximately about 2 orders of magnitude smaller than the previous suggestions. Since our theorem arises from gauge invariance, as long as the first condition (the gluon energies are smaller than the energy scale M_W in $b \to sg$) for the low-energy theorem is satisfied, similar features should therefore be expected in all theoretical models for any flavor-changing two-gluon decays.

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APPENDIX A: HEAVY-QUARK CONTRIBUTIONS

In this appendix we present some details of the calculation involving t-quark-exchange diagrams. The calculation is carried out in the Feynman–'t Hooft gauge. There are basically two classes of diagrams: one with a W exchange and the other with a charged unphysical Higgs boson. The results presented below hold for $m_t \gg m_b$.

1. Self-energy

We begin with self-energy diagrams [Figs. 1(a) and 1(b)]. The $b \to s$ self-energy due to a W and a t-quark exchange [Fig. 2(a)], $\Sigma_W^0(P)$, is given by[11]

$$\begin{aligned}\Sigma_W^0(P) &= (-i)\left[\frac{-ig}{\sqrt{2}}\right]^2 (\mu^2)^{\epsilon/2} \int \frac{d^nK}{(2\pi)^n} \gamma_\rho L \frac{i}{\not P + \not K - m_t} \gamma^\rho L \frac{-i}{K^2 - M_W^2} \\ &= \frac{-g^2}{16\pi^2} \not P L \int_0^1 dx\, x \left[-\frac{2}{\epsilon} + \ln \frac{B(P^2)}{4\pi\mu^2} - \Gamma' + 1 \right],\end{aligned} \tag{A1}$$

where a dimensional regularization scheme has been used with $\epsilon = 4 - n$ and

$$B(P^2) = \Delta(m_t^2) + P^2 x(x-1), \tag{A2a}$$

where

$$\Delta(m^2) = m^2 + x(M_W^2 - m^2). \tag{A2b}$$

For simplicity, we have taken out the KM matrix elements factor. It is, of course, understood that the final result should be a sum of all terms multiplied by this factor. Since $P^2 \ll M_W^2$, m_t^2, $1/B(P^2)$ can be written as a series expansion of $1/\Delta(m_t^2)$:

$$\frac{1}{B(P^2)} = \frac{1}{\Delta(m_t^2)} \left[1 - \frac{P^2 x(x-1)}{\Delta(m_t^2)} + \cdots \right]. \tag{A3}$$

To calculate terms linear in external momenta in the decay $b \to sgg$ [higher-dimensional terms arising from these graphs are of the order of $O(1/M_W^4)$ or $O(1/M_W^2 m_t^2)$], we need to compute Σ up to terms with three powers of external momenta. To this order, the counterterm of $\Sigma_W^0(P^2)$ required by on-shell subtraction of the theory is[11]

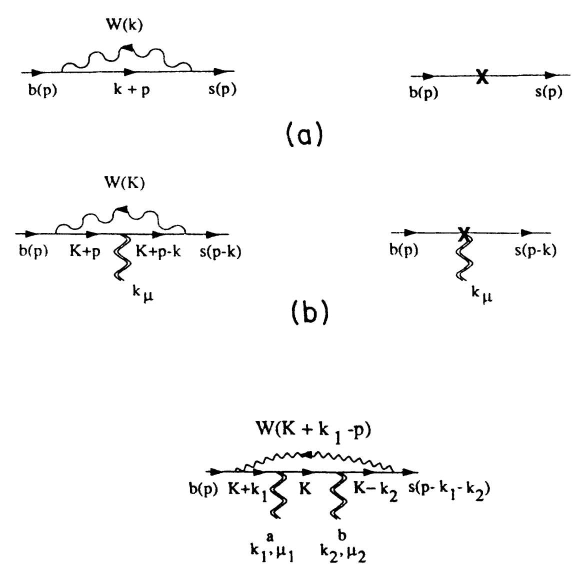

FIG. 2. Feynman graphs due to W exchange contributing to (a) the self-energy and its counterterm, (b) the one-gluon vertex and its counterterm, and (c) the decay $b \to sgg$ directly.

$$\Sigma_W^c(P) = \frac{-g^2}{16\pi^2}(C_1 \not{P} L + C_2 \not{P} R + C_3 L + C_4 R) , \quad \text{(A4)}$$

where

$$\begin{aligned} C_1 &= (-P^2 + m_b^2 + m_s^2) f_1(m_t^2) \\ &\quad - \int_0^1 dx\, x \left[-\frac{2}{\epsilon} + \ln \frac{B(P^2)}{4\pi\mu^2} - \Gamma' + 1 \right] , \\ C_2 &= m_b m_s f_1(m_t^2), \quad C_3 = -m_b^2 m_s f_1(m_t^2) , \\ C_4 &= -m_b m_s^2 f_1(m_t^2) , \end{aligned} \quad \text{(A5)}$$

and $f_1(m_t^2)$ is given by (A9) (see below). Adding $\Sigma_W^0(P)$ to $\Sigma_W^c(P)$ we obtain the renormalized self-energy

$$\Sigma_{\text{ren},W}(P) = \Lambda_s(P)(\not{P} R + m_b L + m_s R)\Lambda_b(P) \times \left[\frac{g^2}{16\pi^2} \right] f_1(m_t^2) , \quad \text{(A6)}$$

where

$$\Lambda_{b,s}(P) = \not{P} - m_{b,s} \quad \text{(A6a)}$$

is the Dirac projection. Clearly, $\Sigma_{\text{ren},W}$ satisfies the on-shell conditions [Eq. (5)].

Similarly, we find that the renormalized self-energy due to an unphysical Higgs boson and t-quark exchange [Fig. 3(a)] is

$$\begin{aligned} \Sigma_{\text{ren},H}(P) &= \frac{m_t^2}{2M_W^2} \Sigma_{\text{ren},W}(P) \\ &\quad + \frac{g^2 m_t^2}{32\pi^2 M_W^2} \Lambda_s(P)(m_b L + m_s R)\Lambda_b(P) \\ &\quad \times \int_0^1 dx \frac{x(x-1)}{\Delta(m_t^2)} . \end{aligned} \quad \text{(A7)}$$

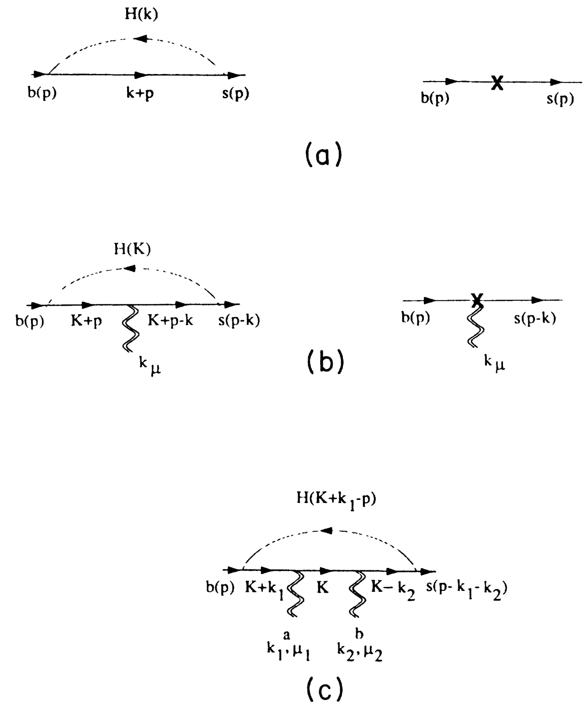

FIG. 3. Feynman graphs due to the unphysical-Higgs-boson exchange contributing to (a) the self-energy and its counterterm, (b) the one-gluon vertex and its counterterm, and (c) the decay $b \to sgg$ directly.

We now calculate the self-energy contributions to $b \to sgg$. Evidently, Fig. 1(a) will not make any contribution (assuming the external b and s quark are on shell). For Fig. 1(b), we have, from Eqs. (A6) and (A7),

$$T(1(\text{b}))_{W,\mu_1\mu_2}^{\alpha\beta} = \frac{g^2 g_s^2}{16\pi^2}[\gamma_{\mu_2}(\not{P} - \not{k}_1)\gamma_{\mu_1} L + \gamma_{\mu_2}\gamma_{\mu_1}(m_b R + m_s L)] t^\beta t^\alpha f_1(m_t^2) + \left[\begin{matrix} 1 \leftrightarrow 2 \\ \alpha \leftrightarrow \beta \end{matrix} \right] , \quad \text{(A8)}$$

where

$$f_1(m^2) = \int_0^1 dx \frac{x^2(1-x)}{\Delta(m^2)} , \quad \text{(A9)}$$

and

$$T(1(\text{b}))_{H,\mu_1\mu_2}^{\alpha\beta} = \frac{g^2 g_s^2}{16\pi^2} \left[\frac{m_t^2}{2M_W^2} \right] [\gamma_{\mu_2}\gamma_{\mu_1}(m_b R + m_s L)] t^\beta t^\alpha \int_0^1 dx \frac{x(x-1)}{\Delta(m_t^2)} + \left[\begin{matrix} 1 \leftrightarrow 2 \\ \alpha \leftrightarrow \beta \end{matrix} \right] + \left[\frac{m_t^2}{2M_W^2} \right] T(1(\text{b}))_{W,\mu_1\mu_2}^{\alpha\beta} . \quad \text{(A10)}$$

2. Flavor-changing one-gluon vertex

We now consider graphs with a $b \to sg$ vertex. The flavor-changing one-gluon vertex generated by a W exchange [Fig. 2(b)] is

$$\begin{aligned} \Gamma_{W,0}^\mu(P,k) &= (-i) \left[\frac{-ig}{\sqrt{2}} \right]^2 i g_s (\mu^2)^{\epsilon/2} \int \frac{d^n K}{(2\pi)^n} \gamma_\rho L \frac{i}{\not{P} + \not{K} - \not{k} - m_t} \gamma^\mu \frac{i}{\not{P} + \not{K} - m_t} \gamma^\rho L \frac{-i}{K^2 - M_W^2} \\ &= \frac{-g^2 g_s}{16\pi^2} \int_0^1 dx_1 \int_0^{1-x_1} dx_2 \left[\frac{-(\not{P} x_1 + \not{k} x_2)\gamma^\mu [\not{P} x_1 + \not{k}(x_2 - 1)] - \gamma^\mu m_t^2}{Y} + \gamma^\mu \left[-\frac{2}{\epsilon} + 2 - \Gamma' + \ln \frac{Y}{4\pi\mu^2} \right] \right] L , \end{aligned}$$

(A11)

where

$$Y=m_t^2+x_1(M_W^2-m_t^2)+P^2x_1(x_1-1)+k^2x_2(x_2-1)+2P\cdot kx_1x_2\ . \tag{A12}$$

Again, one can write Y as a series expansion of $m_t^2+x_1(M_W^2-m_t^2)$ (the usual zero-external-momentum expansion). Here we have dropped the color matrix t^α in (A11) for simplicity. The counterterm of (A12) is[11]

$$\Gamma^\mu_{W,c}(P,k)=\frac{-g^2g_s}{16\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\left[\left[\frac{m_t^2}{Y_0}+\frac{2}{\epsilon}-2+\Gamma'-\ln\frac{Y_0}{4\pi\mu^2}\right]\gamma^\mu L+x_1^2\frac{m_bm_s}{Y_0}\gamma^\mu R\right]\ , \tag{A13}$$

where

$$Y_0=m_t^2+x_1(M_W^2-m_t^2)+m_b^2x_1(x_1-1)+(m_b^2-m_s^2)x_1x_2\ . \tag{A14}$$

Adding (A13) to (A11) we obtain

$$\Gamma_{W,\mathrm{ren}}(P,k)_\mu=\frac{-g^2g_s}{16\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\left\{\left[\ln\frac{Y}{Y_0}+m_t^2\left[\frac{1}{Y_0}-\frac{1}{Y}\right]\right]\gamma_\mu L\right.$$
$$\left.+x_1^2\frac{m_bm_s}{Y_0}\gamma_\mu R-\frac{(x_1\not P+x_2\not k)\gamma_\mu[x_1\not P+(x_2-1)\not k]}{Y}L\right\}\ . \tag{A15}$$

It is easy to verify that $\Gamma_{W,\mathrm{ren}}(P,k)_\mu$ satisfies the Takahashi-Ward identity. In particular, when all the external fields are on shell one finds $\Gamma_{W,\mathrm{ren}}(P,k)_\mu k^\mu=0$.

To calculate terms linear in external momenta in the decay $b\to sgg$ [terms with higher powers of external momenta are of the order $O(1/M_W^4)$ or $O(1/M_W^2m_t^2)$], we need to compute $\Gamma_{W,\mathrm{ren}}(P,k)_\mu$ up to terms quadratic in external momenta. To this order, one finds easily

$$\int_0^1 dx_1\int_0^{1-x_1}dx_2\left[\ln\frac{Y}{Y_0}+m_t^2\left[\frac{1}{Y_0}-\frac{1}{Y}\right]\right]=-\int_0^1\frac{dx}{\Delta(m_t^2)}\left[[\Lambda_b^2(P)+\Lambda_s^2(P-k)]x^2(1-x)+k^2\frac{(1-x)^2(1+2x)}{6}\right]\ , \tag{A16}$$

where $\Lambda_{b,s}^2(P)=P^2-m_{b,s}^2$. The only algebraic point needed to be notified is

$$\int_0^1 dx\frac{1}{\Delta(m^2)}\left[1+\frac{m^2}{\Delta(m^2)}\right](1-x)^3=\int_0^1 dx\frac{(1-x)^2(1+2x)}{\Delta(m^2)}\ . \tag{A17}$$

This identity can be verified by an explicit calculation. The rest of the terms of (A15) lead to

$$\frac{-g^2g_s}{16\pi^2}\int_0^1\frac{dx}{\Delta(m_t^2)}\left[-i\sigma_{\mu\nu}k^\nu(m_bR+m_sL)\frac{x(1-x^2)}{2}-\Lambda_s(P-k)\left[\gamma_\mu m_bRx^2(1-x)+(k_\mu-\gamma_\mu\not k)L\frac{x(1-x^2)}{2}\right]\right.$$
$$-\left[\gamma_\mu m_sRx^2(1-x)-(k_\mu-\not k\gamma_\mu)R\frac{x(1-x^2)}{2}\right]\Lambda_b(P)$$
$$\left.+(\not kk_\mu-k^2\gamma_\mu)L\frac{(1-x)(1+4x+x^2)}{6}+\not kk_\mu L\frac{(1-x)^2(1+2x)}{6}-\Lambda_s(P-k)\gamma_\mu R\Lambda_b(P)x^2(1-x)\right]\ . \tag{18}$$

Adding them together, one obtains the general one-gluon vertex

$$\Gamma_{W,\mathrm{ren}}(P,k)^\alpha_\mu=\frac{-g^2g_s}{16\pi^2}\int_0^1\frac{dx}{\Delta(m_t^2)}\left[-i\sigma_{\mu\nu}k^\nu(m_bR+m_sL)\frac{x(1-x^2)}{2}+(\not kk_\mu-k^2\gamma_\mu)L\frac{(1-x)(2+5x-x^2)}{6}\right.$$
$$-\Lambda_s(P-k)\left[[(\not P-\not k)R+m_sR+m_bL]\gamma_\mu x^2(1-x)+(k_\mu-\gamma_\mu\not k)L\frac{x(1-x^2)}{2}\right]$$
$$-\left[\gamma_\mu(\not PR+m_sR+m_bL)x^2(1-x)-(k_\mu-\not k\gamma_\mu)R\frac{x(1-x^2)}{2}\right]\Lambda_b(P)$$
$$\left.-\Lambda_s(P-k)\gamma_\mu R\Lambda_b(P)x^2(1-x)\right]t^\alpha\ . \tag{A19}$$

In reaching (A19), the following relations turn out to be useful:

$$\begin{aligned}
&\not{k}\gamma_\mu \not{P} L = i\sigma_{\mu\nu}k^\nu m_b R + \not{k}\gamma_\mu R\Lambda_b(P) + k_\mu m_b R \ , \\
&\not{P}\gamma_\mu \not{k} L = -i\sigma_{\mu\nu}k^\nu m_s L + \Lambda_s(P-k)\gamma_\mu \not{k} L + (2\not{k}k_\mu - k^2\gamma_\mu)L + k_\mu m_s L \ , \\
&\not{k} L = m_b R - m_s L - \Lambda_s(P-k)L + R\Lambda_b(P) \ , \\
&\not{P}\gamma_\mu \not{P} L = i\sigma_{\mu\nu}k^\nu m_b R + \Lambda_s(P-k)\gamma_\mu m_b R + (\not{k}+m_s)\gamma_\mu R\Lambda_b(P) + k_\mu m_b R + m_b m_s \gamma_\mu R + \Lambda_s(P-k)\gamma_\mu R\Lambda_b(P) \ .
\end{aligned} \tag{A19a}$$

Clearly, $\Gamma_{W,\mathrm{ren}}$ satisfies, as it should, the Takahashi-Ward identity [Eq. (4)]. Again, in (A19) we have factorized out the KM matrix elements. From (A19) we see that the general one-gluon vertex actually has more than just the dipole-moment and charge-radius form factors. When both b and s quark are on shell, $\Gamma_{W,\mathrm{ren}}$ reduces to the more familiar form with only the dipole-moment and the charge-radius terms. More specifically, the dipole-moment form factor (from W exchange) is

$$D_W(m_t^2) = \frac{g^2 g_s}{16\pi^2}(m_b R + m_s L)\int_0^1 dx \frac{x(1-x^2)}{2\Delta(m_t^2)} \tag{A20a}$$

and the corresponding color charge-radius form factor is given by

$$R_W(m_t^2) = \frac{-g^2 g_s}{16\pi^2}\int_0^1 dx \frac{(1-x)(2+5x-x^2)}{6\Delta(m_t^2)} \ . \tag{A20b}$$

The last term in (A19) will not contribute to $b \to sgg$ if the external b and s quark are on shell. When all the external fields are on shell, $\Gamma_{W,\mathrm{ren}}$ becomes the usual $b \to sg$ dipole-moment form factor. In what follows we will call contributions from the rest of the terms in (A19) as "off-shell" effects.

Similarly, we find that the renormalized flavor-changing one-gluon vertex from the unphysical Higgs diagram [Fig. 3(b)] is

$$\Gamma_{H,\mathrm{ren}}(P,k)_\mu = \frac{m_t^2}{2M_W^2}[\Gamma_{W,\mathrm{ren}}(P,k)_\mu + \Gamma'(P,k)_\mu] \ , \tag{A21}$$

where

$$\begin{aligned}
\Gamma'(P,k)_\mu = \frac{g^2 g_s}{16\pi^2}\int_0^1 \frac{dx}{\Delta(m_t^2)} \{ & i\sigma_{\mu\nu}k^\nu(m_b R + m_s L)(1-x)(1-2x) \\
& -\Lambda_s(P-k)[\gamma_\mu(m_b R + m_s L) - (\gamma_\mu \not{k} - k_\mu)L]x(1-x) \\
& -[(m_b R + m_s L)\gamma_\mu - (k_\mu - \not{k}\gamma_\mu)R]\Lambda_b(P)x(1-x) + (\not{k}k_\mu - k^2\gamma_\mu)Lx(1-x)\} \ .
\end{aligned} \tag{A22}$$

We are now in the position to compute the external bremsstrahlung diagrams [Figs. 1(c) and 1(d)]. First, we consider the off-shell effects (terms not from the dipole-moment form factor). For Fig. 1(c), we find

$$T(1(\mathrm{c}))^{\alpha\beta}_{W,\mu_1\mu_2} = \frac{-g^2 g_s^2}{16\pi^2}\{[\gamma_{\mu_2}(\not{P}-\not{k}_1)\gamma_{\mu_1}L + \gamma_{\mu_2}\gamma_{\mu_1}(m_b R + m_s L)]2f_1(m_t^2) + \gamma_{\mu_2}(\not{k}_1 - \not{k}_2)\gamma_{\mu_1}Lf_2(m_t^2)\}t^\beta t^\alpha + \begin{bmatrix} 1\leftrightarrow 2 \\ \alpha\leftrightarrow\beta \end{bmatrix} \ , \tag{A23}$$

where

$$f_2(m^2) = \frac{1}{2}\int_0^1 dx \frac{x(1-x^2)}{\Delta(m^2)} \ .$$

This piece is generated from the third and the fourth terms in $\Gamma_{W,\mathrm{ren}}$. Similarly, for the unphysical-Higgs-boson diagram we have

$$\begin{aligned}
T((1\mathrm{c}))^{\alpha\beta}_{H,\mu_1\mu_2} = & \frac{g^2 g_s^2}{16\pi^2}\frac{m_t^2}{2M_W^2}[2\gamma_{\mu_2}\gamma_{\mu_1}(m_b R + m_s L) + \gamma_{\mu_2}(\not{k}_1 - \not{k}_2)\gamma_{\mu_1}]t^\beta t^\alpha \int_0^1 dx \frac{x(1-x)}{\Delta(m_t^2)} \\
& + \begin{bmatrix} 1\leftrightarrow 2 \\ \alpha\leftrightarrow\beta \end{bmatrix} + \frac{m_t^2}{2M_W^2}T(1(\mathrm{c}))^{\alpha\beta}_{W,\mu_1\mu_2} \ .
\end{aligned} \tag{A24}$$

The self-interaction graph [Fig. 1(d)] also generates an off-shell contribution from the color charge-radius form factor. For the W-exchange diagram we find

$$T(1(\mathrm{d}))^{\alpha\beta}_{W,\mu_1\mu_2} = g_s[2(k_{2,\mu_1}\gamma_{\mu_2} - k_{1,\mu_2}\gamma_{\mu_1}) + g_{\mu_1\mu_2}(\not{k}_1 - \not{k}_2)]LA^{\beta\alpha}2R_W(m_t^2) \ , \tag{A25}$$

where $A^{\beta\alpha}$ is defined in Sec. II, and $R_W(m_t^2)$ is given by Eq. (16) in the text. This term is totally antisymmetric under the interchange $1\leftrightarrow 2$ and thus exists only in theories with a non-Abelian nature such as QCD. Also, for the unphysical-Higgs-boson diagram, we have

$$T(1(\mathrm{d}))^{\alpha\beta}_{H,\mu_1\mu_2}=\frac{g^2g_s^2}{16\pi^2}\frac{m_t^2}{2M_W^2}[2(k_{2,\mu_1}\gamma_{\mu_2}-k_{1\mu_2}\gamma_{\mu_1})+g_{\mu_1\mu_2}(\not{k}_1-\not{k}_2)]LA^{\beta\alpha}\int_0^1 dx\frac{2x(1-x)}{\Delta(m_t^2)}+\frac{m_t^2}{2M_W^2}T(1(\mathrm{d}))^{\alpha\beta}_{W,\mu_1\mu_2}\ . \tag{A26}$$

3. The one-particle-irreducible graph contribution

The one-particle-irreducible (1PI) Feynman diagram that generates $T(1(\mathrm{e}))^{ab}_{W,\mu_1\mu_2}$ is depicted explicitly in Fig. 2(c) which gives

$$T(1(\mathrm{e}))^{\alpha\beta}_{W,\mu_1\mu_2}=i\left[\frac{-ig}{\sqrt{2}}\right]^2 g_s^2\int\frac{d^4K}{(2\pi)^2}\gamma_\rho L\frac{i}{\not{K}-\not{k}_2-m_t}\gamma_{\mu_2}\frac{i}{\not{K}-m_t}\gamma_{\mu_1}\frac{i}{\not{K}+\not{k}_1-m_t}\gamma^\rho L$$
$$\times\frac{-i}{(K-P+k_1)^2-M_W^2}t^\beta t^\alpha+\begin{bmatrix}a\leftrightarrow b\\ 1\leftrightarrow 2\end{bmatrix}\ . \tag{A27}$$

Terms linear in external momenta in (A27) are given by [terms with higher powers of external momenta are of the order of $O(1/M_W^2m_t^2)$ or $O(1/M_W^4)$ and thus negligible]

$$T(1(\mathrm{e}))^{\alpha\beta}_{W,\mu_1\mu_2}=\frac{g^2g_s^2}{16\pi^2}t^\beta t^\alpha\int\frac{dx_1dx_2dx_3}{\Delta(m_t^2)}\left[(R_1+R_2)+\frac{m_t^2}{\Delta(m_t^2)}(R_1+R_3)\right]\ . \tag{A28}$$

Again, $\Delta(m_t^2)$ is defined in (A2b) with the integration parameter x given by x_1 and

$$R_1=\gamma_{\mu_1}\gamma_{\mu_2}[x_1\not{P}-(x_1+x_2)\not{k}_1-(1-x_3)\not{k}_2]L+[x_1\not{P}+(1-x_1-x_2)\not{k}_1+x_3\not{k}_2]\gamma_{\mu_1}\gamma_{\mu_2}L\ ,$$
$$R_2=\gamma_{\mu_2}[x_1\not{P}-(x_1+x_2)\not{k}_1+x_3\not{k}_2]\gamma_{\mu_1}L,\quad R_3=\gamma_{\mu_1}[x_1\not{P}-(x_1+x_2)\not{k}_1+x_3\not{k}_2]\gamma_{\mu_2}L\ . \tag{A29}$$

After integrating over x_2 and x_3, and then using (A17) as well as the identities

$$\int_0^1\frac{dx}{\Delta(m^2)}\left[1+\frac{m^2}{\Delta(m^2)}\right](1-x)^2(1+2x)=\int_0^1\frac{dx}{\Delta(m^2)}(1-x)(1+x+4x^2)\ ,$$
$$\int_0^1\frac{dx}{\Delta(m^2)}\left[1+\frac{m^2}{\Delta(m^2)}\right](1-x)^2(2+x)=2\int_0^1\frac{dx}{\Delta(m^2)}(1-x)(1+x+x^2)\ , \tag{A30}$$

one can show

$$T(1(\mathrm{e}))^{\alpha\beta}_{W,\mu_1\mu_2}=\frac{g^2g_s^2}{16\pi^2}\{[\gamma_{\mu_1}(\not{P}-\not{k}_2)\gamma_{\mu_2}L+\gamma_{\mu_1}\gamma_{\mu_2}(m_bR+m_sL)]f_1(m_t^2)$$
$$+\gamma_{\mu_1}(\not{k}_2-\not{k}_1)\gamma_{\mu_2}Lf_2(m_t^2)+(k_{2,\mu_1}\gamma_{\mu_2}-k_{1,\mu_2}\gamma_{\mu_1})Lf_3(m_t^2)$$
$$+[\gamma_{\mu_2}(\not{P}-\not{k}_2)\gamma_{\mu_1}-\gamma_{\mu_1}(\not{P}-\not{k}_1)\gamma_{\mu_2}]Lf_4(m_t^2)$$
$$+[\gamma_{\mu_2}(\not{k}_2-\not{k}_1)\gamma_{\mu_1}-\gamma_{\mu_1}(\not{k}_1-\not{k}_2)\gamma_{\mu_2}]Lf_5(m_t^2)\}t^\beta t^\alpha+\begin{bmatrix}1\leftrightarrow 2\\ \alpha\leftrightarrow\beta\end{bmatrix}\ , \tag{A31}$$

where f_1 and f_2 are defined in (A9) and (A24), respectively, and

$$f_3(m^2)=\frac{1}{3}\int_0^1 dx\frac{(1-x)(1+x+4x)}{\Delta(m^2)},\quad f_4(m^2)=\frac{1}{2}\int_0^1 dx\frac{x(1-x)^2}{\Delta(m^2)}\ ,$$
$$f_5(m^2)=\frac{1}{6}\int_0^1 dx\frac{(1-x)^2(1+2x)}{\Delta(m^2)}\ . \tag{A32}$$

Following the same procedures, we find that for the unphysical-Higgs-boson diagram [Fig. 2(c)], we have

$$T(1(\mathrm{e}))^{\alpha\beta}_{H,\mu_1\mu_2}=\frac{g^2g_s^2m_t^2}{32\pi^2M_W^2}\int_0^1\frac{dx}{\Delta(m_t^2)}\left[[\gamma_{\mu_2},\gamma_{\mu_1}](m_bR+m_sL)\frac{(1-x)(2x-1)}{2}+g_{\mu_1\mu_2}(\not k_1-\not k_2)Lx(1-x)\right.$$
$$\left.+[\gamma_{\mu_1}(\not k_1-\not k_2)\gamma_{\mu_2}L-\gamma_{\mu_2}\gamma_{\mu_1}(m_bR+m_sL)]x(1-x)\right]t^\beta t^\alpha$$
$$+\left[\begin{matrix}1\leftrightarrow2\\ \alpha\leftrightarrow\beta\end{matrix}\right]+\frac{m_t^2}{2M_W^2}T(1(\mathrm{e}))^{\alpha\beta}_{W,\mu_1\mu_2}\ . \tag{A33}$$

A direct sum of $T(1(\mathrm{b}))$ to $T(1(\mathrm{e}))$ (all the off-shell effects) appears to be rather complicated at first. It can be greatly simplified, however, by using the relations

$$(\gamma_{\mu_1}\not P\gamma_{\mu_2}-\gamma_{\mu_2}\not P\gamma_{\mu_1})L=\tfrac{1}{2}[\gamma_{\mu_2},\gamma_{\mu_1}](m_bR+m_sL)+\tfrac{1}{2}(\not k_1+\not k_2)[\gamma_{\mu_2},\gamma_{\mu_1}]L\ ,$$
$$(\gamma_{\mu_1}\not k_2\gamma_{\mu_2}-\gamma_{\mu_2}\not k_1\gamma_{\mu_1})=2(k_{2,\mu_1}\gamma_{\mu_2}-k_{1,\mu_2}\gamma_{\mu_1})+g_{\mu_1\mu_2}(\not k_1-\not k_2)+\tfrac{1}{2}(\not k_1+\not k_2)[\gamma_{\mu_2},\gamma_{\mu_1}]\ , \tag{A34}$$
$$(\gamma_{\mu_1}\not k_1\gamma_{\mu_2}-\gamma_{\mu_2}\not k_2\gamma_{\mu_1})=-g_{\mu_1\mu_2}(\not k_1-\not k_2)+\tfrac{1}{2}(\not k_1+\not k_2)[\gamma_{\mu_2},\gamma_{\mu_1}]\ .$$

The final result takes a simple form

$$T(1(\mathrm{a})+1(\mathrm{b})+1(\mathrm{c})+1(\mathrm{d})+1(\mathrm{e}))^{\alpha\beta}_{W(H),\mu_1\mu_2}=g_sD_{W(H)}(m_t^2)[\gamma_{\mu_1},\gamma_{\mu_2}]A^{\beta\alpha}\ . \tag{A35}$$

Here $D_W(m_t^2)$ is the dipole-moment form factor given by the W-exchange diagram [see (A20a)], and $D_H(m_t^2)$ is generated from the unphysical-Higgs-boson exchange. One can read off $D_H(m_t^2)$ from (A21) and (A22):

$$D_H(m_t^2)=\left[\frac{m_t^2}{2M_W^2}\right]\left[D_W(m_t^2)+\frac{g^2g_s}{16\pi^2}(m_bR+m_sL)\int_0^1dx\frac{(1-x)(1-2x)}{\Delta(m_t^2)}\right]\ . \tag{A36}$$

From (A35) we see that the sum of all the off-shell effects, together with contributions from the 1PI graph, is totally antisymmetric under the interchange $1\leftrightarrow2$. The sum would have been zero if the underlying group were Abelian. Such an interesting feature was, indeed, discovered (Ref. 1) in the study of $b\to s\gamma\gamma$. Here a nonzero sum is required by gauge invariance.

Finally, we proceed to compute the on-shell effects. For a heavy top, $m_t\gtrsim M_W$, higher-dimensional terms arising from the 1PI graph are negligible. This leaves only those terms from (A35) and Figs. 1(c) and Fig. 1(d) with the $b\to sg$ vertex given by its dipole moment. The result of the calculation is given by Eqs. (12) and (13) in the text with

$$\frac{mF_0}{M^2}=D_W(m_t^2)+D_H(m_t^2)\ . \tag{A37}$$

Together with other terms from the self-interaction graph [Fig. 1(d)], the residue of the off-shell effects (A35) leads to a gauge-invariant tensor $\delta_{\mu_1\mu_2}$ defined in Eq. (14).

APPENDIX B: LIGHT-QUARK CONTRIBUTIONS

In this appendix we present some details of the calculation involving c-quark-exchange diagrams. The calculation is again carried out in the Feynman–'t Hooft gauge. Since $m_c\ll M_W$, diagrams with an unphysical-Higgs-boson exchange can now be ignored. With a replacement of $m_c\to m_u$, our results presented below are equally valid for the u-quark-exchange diagrams.

1. Summary of the results

The lowest-dimensional terms are still given by Eqs. (12) and (13) with the dipole-moment form factor given by Eq. (20a) (replacing m_t by m_c). Cancellations which occurred for the heavy-quark contributions also take place for the light-quark contributions. Now, the higher-dimensional terms must be included. These are given by Eq. (21).

The renormalized $b\to s$ self-energy due to a c-quark exchange is

$$\Sigma_{\mathrm{ren},W}(P)=\Lambda_s(P)(\not PR+m_bL+m_sR)\Lambda_b(P)\times\left[\frac{-g^2}{16\pi^2}\right]\int_0^1dx\frac{x^2(x-1)}{\Delta(m_c^2)}\ , \tag{B1}$$

which can be obtained directly from (A6) by replacing m_t by m_c. That such a replacement can be applied is because the expansion (A3) is always valid for arbitrary small masses [see Eqs. (A2a) and (A2b)].

To order $O(1/M_W^2)$, the flavor-changing one-gluon vertex is given by

$$\Gamma_{W,\mathrm{ren}}(P,k)^\alpha_\mu=\frac{-g^2g_s}{16\pi^2M_W^2}\left[-i\sigma_{\mu\nu}k^\nu(m_bR+m_sL)\tfrac{1}{3}+(\not kk_\mu-k^2\gamma_\mu)L\left[2\int_0^1dx\,x(x-1)\ln\frac{m_c^2+x(x-1)k^2}{M_W^2}+\frac{1}{18}\right]\right.$$
$$-\Lambda_s(P-k)\{[(\not P-\not k)R+m_sR+m_bL]\gamma_\mu+2(k_\mu-\gamma_\mu\not k)L\}\tfrac{1}{6}$$
$$\left.-[\gamma_\mu(\not PR+m_sR+m_bL)-2(k_\mu-\not k\gamma_\mu)R]\Lambda_b(P)\tfrac{1}{6}-\Lambda_s(P-k)\gamma_\mu R\Lambda_b(P)\tfrac{1}{6}\right]t^\alpha\ . \tag{B2}$$

Except for the color charge-radius form factor, the rest of the terms in (B2) can be obtained from (A19) by replacing m_t by m_c. Thus, to the leading order, the only difference between that from a heavy quark and a light quark happens to the charge-radius form factor.

The lowest-dimensional flavor-changing two-gluon vertex (from the 1PI graph) is

$$T(1(\mathrm{e}))^{\alpha\beta}_{W,\mu_1\mu_2}=\frac{g^2g_s^2}{16\pi^2M_W^2}\left[[\gamma_{\mu_1}(\not P-\not k_2)\gamma_{\mu_2}L+\gamma_{\mu_1}\gamma_{\mu_2}(m_bR+m_sL)]\tfrac{1}{6}+\gamma_{\mu_1}(\not k_2-\not k_1)\gamma_{\mu_2}L\tfrac{1}{3}\right.$$
$$+(k_{2,\mu_1}\gamma_{\mu_2}-k_{1,\mu_2}\gamma_{\mu_1})L\left[2\int_0^1 dx\,x(x-1)\ln\frac{m_c^2+x(x-1)k^2}{M_W^2}+\frac{1}{18}\right]$$
$$+[\gamma_{\mu_2}(\not P-\not k_2)\gamma_{\mu_1}-\gamma_{\mu_1}(\not P-\not k_1)\gamma_{\mu_2}]L\tfrac{1}{6}+[\gamma_{\mu_2}(\not k_2-\not k_1)\gamma_{\mu_1}-\gamma_{\mu_1}(\not k_1-\not k_2)\gamma_{\mu_2}]L$$
$$\left.\times\left[\int_0^1 dx\,x(x-1)\ln\frac{m_c^2+x(x-1)k^2}{M_W^2}-\frac{5}{36}\right]\right]t^\beta t^\alpha+\begin{bmatrix}1\leftrightarrow 2\\ \alpha\leftrightarrow\beta\end{bmatrix}, \tag{B3}$$

where $k=(k_1+k_2)$. The structure of (B3) is the same as that of (A31). Except for the third and the last terms, which are antisymmetric under the interchange $1\leftrightarrow 2$, the others can be obtained directly from (A31) by replacing m_t by m_c. The higher-dimensional terms are shown in Eq. (21) of the text.

2. Flavor-changing one-gluon vertex

For the $b\to sg$ induced by an internal c quark, equations given by (A11) to (A15) still hold after we replace m_t by m_c. However, for a light quark, the external-momentum expansion used in Appendix A is no longer applicable. Take the function Y defined originally in (A12) as an example:

$$Y=m_c^2+x_1(M_W^2-m_c^2)+P^2x_1(x_1-1)$$
$$+k^2x_2(x_2-1)+2P\cdot kx_1x_2\ . \tag{B4}$$

It is easy to see that, for values of $k^2\gg m_c^2$, there is a region in the Feynman parameter space (x_1,x_2) in which

$$m_c^2+x_1(M_W^2-m_c^2)\ll P^2x_1(x_1-1)+k^2x_2(x_2-1)$$
$$+2P\cdot kx_1x_2\ . \tag{B5}$$

This happens in places where x_2 is not close to zero and $x_1\to 0$. Because of this, a new approximation method had to be introduced. Of course, such a complication disappears if the radiation from the vertex is real ($k^2=0$), for in that case the external-momentum expansion is always valid for arbitrary small masses.

Our method can be illustrated as follows. Considering the integral that enters into the determination of the flavor-changing one-gluon vertex

$$\int\frac{d^nK}{(2\pi)^n}\gamma_\rho L\frac{i}{\not K-\not k-m_c}\gamma^\mu\frac{i}{\not K-m_c}\gamma_\rho L\frac{-i}{(K-P)^2-M_W^2}\ .$$

It contains three denominators. Instead of introducing Feynman parameters to combine them together at one time, we take two steps. First, we combine the two terms with light-quark masses:

$$\frac{1}{[(K-k)^2-m_c^2](K^2-m_c^2)[(K-P)^2-M_W^2]}$$
$$=\frac{1}{(K-P)^2-M_W^2}$$
$$\times\int_0^1 dx\frac{1}{\{(K-xk)^2-[m_c^2+x(x-1)k^2]\}^2}\ . \tag{B6}$$

Next, we shift the momentum $K\to K+xk$ and then introduce another parameter y to complete the combination.

Following these procedures, the renormalized one-gluon vertex can be written as

$$\Gamma_{\mathrm{ren}}(P,k)^\mu=\frac{-g^2g_s}{16\pi^2}\int_0^1 dx\int_0^1 dy(1-y)\left\{\left[\ln\frac{X}{X_0}+m_c^2\left[\frac{1}{X_0}-\frac{1}{X}\right]\right]\gamma^\mu L+y^2\frac{m_bm_s}{X_0}\gamma^\mu R\right.$$
$$\left.-\frac{[x\not k-y(x\not k-\not P)]\gamma^\mu[(x-1)\not k-y(x\not k-\not P)]}{X}L\right\}, \tag{B7}$$

with

$$X=\mu^2+y(M_W^2-\mu^2)+y(y-1)(xk-P)^2,\quad \mu^2\equiv m_c^2+x(x-1)k^2\ , \tag{B8}$$

and

$$X_0=m_c^2+y(M_W^2-m_c^2)+y(y-1)[(1-x)m_b^2+xm_s^2]\ . \tag{B9}$$

Now, it is easy to see that expansions in terms of $m_c^2+y(M_W^2-m_c^2)$ in (B9) and $\mu^2+y(M_W^2-\mu^2)$ in (B8) can be introduced. To order $O(1/M_W^2)$, the integrations over the parameter y and x are easy to carry out. The result of the calculation is given in Eq. (B2).

3. The 1PI graph

For the 1PI diagram [Fig. 2(c)], we first introduce Feynman parameters x_1 and x_2 to combine the three denominators containing m_c. The remaining term, the W propagator, is then combined with this resultant form by another parameter y. We find

$$T(1(e))^{\alpha\beta}_{W,\mu_1\mu_2}=\frac{-g^2g_s^2}{16\pi^2}\int_0^1 dx_1\int_0^{1-x_1}dx_2\int_0^1 dy(1-y)^3\frac{1}{M^2}$$
$$\times\left\{[\gamma_{\mu_1}\gamma_{\mu_2}(\not{Q}+\not{k}_2)+(\not{Q}-\not{k}_1)\gamma_{\mu_1}\gamma_{\mu_2}+\gamma_{\mu_2}\not{Q}\gamma_{\mu_1}]L\right.$$
$$+\frac{m_c^2}{M^2}[\gamma_{\mu_1}\gamma_{\mu_2}(\not{Q}+\not{k}_2)+(\not{Q}-\not{k}_1)\gamma_{\mu_1}\gamma_{\mu_2}+\gamma_{\mu_1}\not{Q}\gamma_{\mu_2}]L$$
$$\left.-\frac{1}{2M^2}\gamma_\rho(\not{Q}+\not{k}_2)\gamma_{\mu_2}\not{Q}\gamma_{\mu_1}(\not{Q}-\not{k}_1)\gamma^\rho L\right\}t^\beta t^\alpha+\left\{\begin{matrix}1\leftrightarrow 2\\ \alpha\leftrightarrow\beta\end{matrix}\right\}, \tag{B10}$$

where

$$Q=y[(1-x_1)k_1+x_2k_2-P]+x_1k_1-x_2k_2\ , \tag{B11}$$

and

$$M^2=\mu^2+y(M_W^2-\mu^2)+y(y-1)[(1-x_1)k_1+x_2k_2-P]^2\ . \tag{B12}$$

Here $\mu^2=m_c^2-x_1x_2k^2$ and $k=k_1+k_2$. Again, one finds from (B12) that an expansion in terms of $\mu^2+y(M_W^2-\mu^2)$ is valid in the whole Feynman parameter space. After integrating over the Feynman parameters, one obtains the results given in (B3) and Eq. (21).

It becomes clear that for the study of B physics, one should treat heavy-quark and light-quark contributions differently. In particular, studies involving "penguin diagrams," where the charge form factor (either electromagnetic or gluonic) plays an important role, should be carefully reexamined. It appears that so far such a precaution has not been taken properly. On the other hand, since the value of m_b is not too far away from that of m_c, we do not expect that this will change the order of magnitude of the results given by the existing studies. Taking the decay $b\to s\bar{q}q$ (through penguin diagrams) as an example, we find that the correction could be of the order of $\sim 20\%$, depending on the choice of m_t.

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