Flavor-mixing structure functions in the Nambu–Jona-Lasinio model

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The distribution of the strange quarks in the constituent light (up or down) quark, as seen in deep-inelastic scattering, is calculated for first nonvanishing order in the generalized Nambu–Jona-Lasinio model. Using a previous fit to determine the parameters of the model, the result is compared to experimental bounds on the momentum carried by strange quarks and antiquarks in the nucleon. No violation of experimental bounds is obtained.

The strangeness content of the nucleon has been the subject of vigorous investigation in the last few years.[1–11] There is a long-standing problem of incompatibility of the value of the pion-nucleon σ term $\Sigma_{\pi N}$ extracted from measured scattering amplitudes and its value obtained from lowest-order expansion in the strange-quark mass, neglecting the scalar bilinear $\bar{s}s$ matrix element compared to the matrix element of the light quarks $\bar{u}u+\bar{d}d$ (Ref. 12). The assumption of the nucleon made from up and down quarks with negligible strange-quark admixture can be given up, with some interesting consequences.[13–15] One can also question the validity of the first-order expansion in m_s. Calculations using different models indeed suggest that treating the term $m_s\bar{s}s$ as a first-order perturbation may not be justified[2,3,11] and that the strangeness content of the nucleon is suppressed for values of m_s/m_u suggested by current algebra. While it is rather difficult to extract from experiments the nucleon matrix elements of the zero-spin dimension-three operator $\bar{q}q$ (Ref. 13), another source on the strangeness content of the nucleon is deep-inelastic scattering. The twist-two operator matrix elements (relevant for deep-inelastic scattering) cannot in general be related to those of $\bar{q}q$; hence it is, in principle, possible to have small former and large latter ones. However, both quantities can be computed in specific models and in that way one can hope that exploiting the stringent experimental limits on the momentum of the proton carried by strange quarks and antiquarks, it may be possible to put limits on the $\bar{s}s$ matrix element as well.[16]

The above reasoning is the motivation for the calculation of the strange-quark structure function in the physical (dressed) up quark in the Nambu–Jona-Lasinio (NJL) model[17] which will be presented here. Since the latter is not confining one cannot calculate the proton structure functions, but only the admixture of strange-quark–antiquark pairs present in the up or down quarks due to multiquark interactions mixing flavor. The NJL model, modified to take into account the flavor and color degrees of freedom as well as the breaking of the axial U(1) symmetry, has been recently extensively studied.[18] It has been shown that a very good fit to the quark vacuum condensates and constituent masses and pseudoscalar-meson masses and decay constants is obtained in the Hartree-Fock approximation by fitting the three parameters G, K, and Λ (see below for their definition) of the model, while taking for the current quark masses (in the isospin symmetry limit) $m_u=m_d=6$ MeV, $m_s=150-200$ MeV. The model also predicts a value for the $\bar{s}s$ matrix element closer to a very large value predicted by the Skyrme model,[1] then to a very small value obtained in a hybrid chiral bag model calculation.[2] Thus, it is of interest to examine if the model is in accordance with experimental limits on the strange sea as seen in deep-inelastic scattering.

In Ref. 18 the procedure for calculating the strange-quark distribution in the physical up (or down) quark has been outlined. We follow the notation and use the results of that work. The interaction Lagrangian is given by

$$L_{\rm int}=L_{4f}+L_{6f}\ , \tag{1}$$

$$L_{4f}=G\sum_{a=0}^{8}[(\bar{q}\lambda^a q)^2-(\bar{q}\gamma_5\lambda^a q)^2]\ , \tag{2}$$

$$L_{6f}=-K\{\det[\bar{q}(1+\gamma_5)q]+\det[\bar{q}(1-\gamma_5)q]\}\ , \tag{3}$$

with λ^a being the SU(3)-flavor generators, and the determinant is to be taken in the flavor space. An implicit summation over colors is understood. The above interactions are nonrenormalizable; hence, a cutoff procedure has to be implemented. In Ref. 18 a Lorentz-invariant cutoff on the fermion propagator in Euclidean space was used, restricting the integration over loop momenta to the region $k_E^2<\Lambda^2$. The same procedure cannot be used for the calculation of structure functions since the imaginary parts of the relevant forward-scattering amplitudes become negative, violating unitarity. The imaginary parts of the relevant diagrams to the lowest nonvanishing order in the couplings G and K (see Fig. 4 of Ref. 18) are finite even without a cutoff, but they do not approach zero when the virtual parton mass approaches infinity, thus violating an assumption of the covariant parton model developed by Landshoff, Polkinghorne, and Short,[19] making that approach not applicable. To overcome this difficulty we introduce exponential damping factors in the virtual parton mass and center-of-mass energy by hand. A heuristic justification for these damping factors will be given below from exponentially falling form factors for the four-quark and six-quark interaction vertices, in Euclidean space. That corresponds to replacing the delta-function interactions in coordinate space by exponentially falling ones, i.e., smearing them.

The result of Ref. 18, based on the covariant parton model[19] is that the strange-quark distribution in the physical up quark is given by

$$f_{s/U}=\frac{1}{16\pi^2}\int_{s_{\rm th}}^{\infty}ds\int_{-\infty}^{\mu^2_{\rm max}}d\mu^2[w_1(s,\mu^2)+xw_2(s,\mu^2)]\ , \quad (4)$$

where x is the Bjorken scaling variable $x=-q^2/2p\cdot q$, $w_j=(1/2\pi)\mathrm{Im}t_j$, and t_j $(j=1,2)$ are determined by the decomposition of the forward-scattering amplitude of a strange antiquark on an up quark (for details, see Ref. 18). s is the center-of-mass energy $s=(p-k+q)^2$ and $\mu^2=(k-q)^2$. The limits of integrations in (4) are $s_{\rm th}=(M_u+M_s)^2$, $\mu^2_{\rm max}=xM_u^2-xs/(1-x)$, where M_q denotes the constituent quark mass.

The lowest-order diagrams having nonzero imaginary parts are shown in Fig. 4 of Ref. 18. Since the coupling is not small, there is no justification for stopping at the lowest order. The result will be changed by higher-order terms, but, as discussed in Ref. 18, working to lowest-order giving physical insight should lead to results of interest. An assumption of the general covariant parton model[19] is that the scattering amplitude $t(p,k-q)$ approaches zero "sufficiently fast" when the virtual parton mass $(k-q)^2$ approaches infinity. Also, in order that the integral in expression (4) be finite the amplitude must go to zero when the center-of-mass energy s goes to infinity. To assure this we shall put in by hand exponential damping factors in the imaginary parts of the pertinent diagrams. In a more fundamental theory the four- and six-fermion couplings would not be pointlike; they would soften at high-momentum transfers. To mimic that, preserving the symmetry between the legs (in the chiral limit), we introduce, in the Euclidean space, the following exponential damping factor in the n-fermion vertex:

$$F^{(n)}(p_1,\ldots,p_n)=\exp\left[-\frac{P}{2^{n-2}\Lambda^2}\right]\ , \quad (5)$$

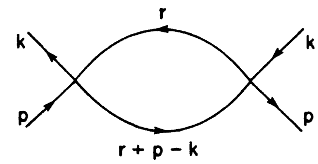

FIG. 1. The basic form of the diagram describing forward quark-antiquark scattering.

$$P=\sum_{i=1}^{n-1}p_i^2+\sum_{i>j}^{n-1}(p_i+p_j)^2+\sum_{i>j>k}^{n-1}(p_i+p_j+p_k)^2+\cdots+(p_1+p_2+\cdots+p_{n-1})^2 \quad (6)$$

(all the momenta are oriented toward the vertex). The above form (5) assures that the form factor goes to zero when any linear combination of momenta $p_1,\ldots,p_n$ is large. Using this approach for the Schwinger-Dyson equation in the Hartree-Fock approximation, one sees that it corresponds closely to the proper-time regularization[18,20] and reproduces the covariant Euclidean cutoff results of Ref. 18 to first order in M_q^2/Λ^2. Of course, the quark masses due to spontaneous chiral-symmetry breakdown will be momentum dependent, having the form $M_i(p^2)=A_i\exp(-p^2/\Lambda^2)+m_i$, m_i being the current mass. The form of $M_i(p^2)$ follows by inspection of the gap equation.

Using (5) and (6) in the diagram of the basic form shown in Fig. 1 gives

$$\exp\left[-\frac{p^2+k^2}{\Lambda^2}\right]\int\frac{d^4r\,N\exp\left[-\dfrac{r^2+(r+a)^2}{\Lambda^2}\right]}{(r^2+M^2)[(r+a)^2+M^2]}\ , \quad (7)$$

N being the numerator and $a=(p-k)$. The exact form of N is not of interest here. The above integral at large a^2/Λ^2 behaves like $\exp(-a^2/2\Lambda^2)$. Technical difficulties related to carrying out explicitly the integration in (7) and rotating back to Minkowski space do not allow one to proceed directly. Still, for an estimate it should suffice to take the general dependence of the diagram on the virtual quark mass k^2 and the center-of-mass energy $(p-k)^2$, as suggested by expression (7), in the Minkowski space. Note that p^2 will be fixed at M_q^2 and since $M_q^2\ll\Lambda^2$ that factor in (7) can be neglected. Next, we calculate the imaginary parts of the diagrams in Fig. 4 of Ref. 18 using the simplicity of Cutkosky's rules[21] for point couplings and then include the factor $\exp[-\Lambda^{-2}(|\mu^2|+s/2)]$. While mathematically not exact, this approach reflects the general physical behavior of the amplitude and should be regarded as an approximation for the present estimate of the flavor mixing as seen in deep-inelastic scattering. After straightforward algebra, one gets

$$f_{s/U}=\int_{s_{\rm th}}^{\infty}\frac{ds}{(2\pi)^4}\left[1+\frac{M_s^2-M_u^2}{s}\right]\left[1-\frac{4sM_s^2}{(s+M_s^2-M_u^2)^2}\right]^{1/2}\exp\left[-\frac{s}{2\Lambda^2}\right]\int_{-\infty}^{\mu^2_{\rm max}}d\mu^2\frac{\exp(\mu^2/\Lambda^2)}{(M_s^2-\mu^2)^2}(A^G+A^{KG}+A^K)\ . \quad (8)$$

The contributions proportional to G^2, GK, and K^2 are (for N_c colors)

$$A^{G}=N_cG^2\{(s-M_u^2-M_s^2)[xM_u^2-(1-x)\mu^2-xs+M_s^2]-4xM_u^2M_s^2\}\ , \tag{9a}$$

$$A^{KG}=4KG(2N_c+1)(N_c+1)M_uM_sS_d^{\Lambda}(0)[(1+x)M_s^2+2xM_u^2-(1-x)\mu^2-2xs]\ , \tag{9b}$$

$$\begin{aligned}A^{K}=4K^2(N_c+1)^2S_d^{\Lambda}(0)^2\Bigg[&[M_s^2+xM_u^2-(1-x)\mu^2-xs]\left[(N_c+1)(s-M_u^2-M_s^2)+N_c\frac{(s-M_u^2-M_s^2)^2-4M_u^2M_s^2}{6s}\right]\\&+N_c[(1-x)M_s^2+xM_u^2-\mu^2-xs](s-\mu^2+M_u^2)\\&\times\frac{s(s+M_u^2+M_s^2)-2(M_s^2-M_u^2)^2}{6s^2}-4(2N_c+1)xM_u^2M_s^2\Bigg]\ ,\end{aligned} \tag{9c}$$

where the one propagator loop is regularized as a consequence of the damping factors (5) introduced into the vertices:

$$\begin{aligned}S_d^{\Lambda}(0)&\equiv\int\frac{d^4q}{(2\pi)^4}\frac{M_d\exp(-q^2/\Lambda^2)}{q^2+M_d^2}\\&=\frac{M_d\Lambda^2}{16\pi^2}[1-Z\exp(Z)\mathrm{Ei}(Z)]\ ,\end{aligned} \tag{10}$$

with $Z\equiv M_d^2/\Lambda^2$, and $\mathrm{Ei}(Z)\equiv\int_Z^\infty(dx/x)e^{-x}\ (Z>0)$.

For the values of the parameters of the model we use the results of the extensive study of Ref. 18, where a very good fit to the quark constituent masses, vacuum scalar bilinear condensates, and the meson masses and decay constants has been obtained, with the following values of parameters: $G=3.43$ GeV^{-2}, $K=46.3$ GeV^{-5}, $\Lambda=1$ GeV. Since we use an exponentially falling cutoff function, we make a correction for that by keeping the coupling constants G and K, but adjusting Λ in such a way as to reproduce the solution of the gap equation of Ref. 18 in the chiral limit. This gives $\Lambda\approx1.4$ GeV. If one would instead try to reproduce the quark condensates, a somewhat smaller value of Λ would be needed. For a more exact treatment one should repeat the analysis of Ref. 18 with damping factors given by (5) and (6), introduced into four- and six-fermion vertices. That seems possible, but is out of the scope of this work.[22] In Fig. 2 the distribution of the strange quark (or antiquark) in the constituent up (or down) quark is given as a function of x, for $\Lambda=1.2$, 1.4, and 1.5 GeV. The dependence on Λ is very strong, as one can anticipate by using the dimensionless couplings $G\Lambda^2$ and $K\Lambda^5$. The distributions calculated in this model are Q^2 independent and one should interpret them as some "intrinsic" distributions, corresponding to some low scale of the order of Λ^2 (Ref. 18). To compare the results with experiments we should evolve them according to perturbative QCD to experimental values of $Q^2\gg\Lambda^2$. Since we do not know the valence distributions we cannot do that. However, we are mainly interested in the predicted fraction of the momentum carried by strange quarks in the constituent up quark, given by the integrals $\int_0^1dx\,xf_{s/U}(x)$ (see Fig. 2). To compare these numbers with experimental bounds we scale the latter from large Q^2 (for which we take 15 GeV^2) to low $Q^2=1$ GeV^2. At large Q^2 we take the experimental upper limit on the fraction of momentum carried by strange quarks to be 1.5% (Ref. 23), together with up and down antiquarks carrying four times as much momentum[23] and the gluons carrying one-half of the nucleon momentum. A simple calculation using the anomalous dimension matrix[24] then shows that strange quarks at $Q^2=1$ GeV^2 carry only 0.8% of the nucleon momentum. One can see that this experimental limit is not violated by the results of our calculation, even though the momentum carried by gluons has been neglected. Looking at the distributions of Fig. 2 one notices that they fall off very quickly with increasing x and are practically nonzero only in the region $x\leq0.2$, in accordance with the limited experimental information available.[23] On the other hand, $f_{s/U}(x)$ in the considered model approaches a finite limit as $x\to0$ (although increasing sharply), contrary to general expectations that it should diverge. Of course, in a more realistic model the low-x behavior of $f_{s/U}$ would be modified by hard-gluon exchange.[25] While the results shown in Fig. 2 correspond to the barely broken phase (of the chiral symmetry), which was found to be phenomenologically successful,[18] we also performed a calculation for the firmly broken phase. The latter does not give a good fit to scalar bilinear vacuum condensates and predicts a

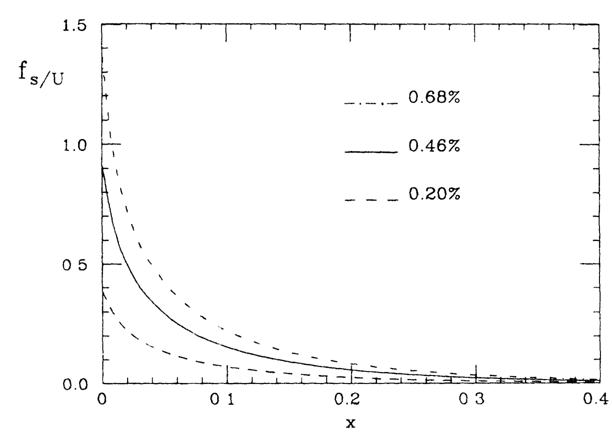

FIG. 2. The strange-quark distribution in the constituent up quark, $f_{s/U}(x)$, as a function of the Bjorken-x variable. The dashed line corresponds to $\Lambda=1.2$ GeV, the solid line to $\Lambda=1.4$ GeV and the dotted-dashed line to $\Lambda=1.5$ GeV. The values of the other parameters are taken from Ref. 15: $G=3.43$ GeV^{-2}, $K=46.3$ GeV^{-5}, $M_s=368$ MeV, $M_u=314$ MeV. The numbers next to each line are the integrals of $xf_{s/U}(x)$, corresponding to the fraction of the momentum carried by strange quarks.

small pion-nucleon Σ term, but the results for the flavor-mixing structure functions are not significantly different from the case of the barely broken phase. Also, we analyzed the case when the chiral symmetry was not explicitly broken by nonzero current quark masses. Again, we did not find significant changes in our results. However, one must not forget that we used the constituent quark masses in our calculation, a reflection of the fact that we were calculating the "intrinsic" distribution, effectively corresponding to some low Q^2 of the order of Λ^2.

In conclusion, the calculation of the strange-quark structure function for a constituent light quark, to first nontrivial order in the coupling constants in the generalized Nambu–Jona-Lasinio model, has been performed. An estimate, using the values of the parameters of the model from a previous analysis, shows that the stringent experimental limits on the magnitude of the strange sea as seen in deep-inelastic scattering are not violated.

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