Naturally light Majorana neutrinos with no neutrinoless double-β decay

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It is argued that neutrinoless double-β decay [$(\beta\beta)_{0\nu}$] does not depend on the physical Dirac or Majorana mass of the electron neutrino. A new scenario is proposed where the physical neutrino can naturally be a light Majorana particle, without any $(\beta\beta)_{0\nu}$. Supersymmetric grand unified theories are constructed incorporating this scenario.

In 1937, Racah[1] pointed out that if the neutrino emitted by a neutron is a Majorana particle, then it can stimulate the decay of a second neutron. Furry[2] then pointed out that this neutrino can be a virtual one thus inventing the process of neutrinoless double-β decay [$(\beta\beta)_{0\nu}$]. We now expect that the $(\beta\beta)_{0\nu}$ experiment will give us the physical Majorana mass of the neutrino. Since a Dirac particle can be thought of as two Majorana particles with opposite CP properties, their contributions to $(\beta\beta)_{0\nu}$ cancel.[3-6] Thus we also expect that $(\beta\beta)_{0\nu}$ experiments will allow us to distinguish between a Dirac and a Majorana particle.

In this paper, from a general analysis of the neutrino mass matrix, we argue that the $(\beta\beta)_{0\nu}$ amplitude does not depend on the physical Dirac or Majorana mass of the electron neutrino. We discuss the situation in which ν_e is a Majorana neutrino (and may even be the only light one) and yet there is no $(\beta\beta)_{0\nu}$. Conversely we also know of situations wherein there is $(\beta\beta)_{0\nu}$ in spite of the ν_e being a Dirac (to be precise, a pseudo-Dirac) particle. We then proceed to construct certain supersymmetric grand unified theories that naturally have a light Majorana ν_e with no $(\beta\beta)_{0\nu}$ or on the contrary a massless ν_e with considerable $(\beta\beta)_{0\nu}$. Thus the experimental signature or otherwise of $(\beta\beta)_{0\nu}$ will not allow us to infer about the physical mass of the neutrinos.

The mass term for the neutrinos, in general, is of the form

$$\mathcal{L}_m = \bar{\nu}^c_R M \nu_L \ , \tag{1}$$

where M is a symmetric matrix. We shall, for simplicity, assume that there is no CP violation in the leptonic sector thus constraining M to be real. Going over to the mass basis χ_L $(=U\nu_L)$ one has $\mathcal{L}_m = \sum_i m_i \bar{\chi}^c_{Ri} \chi_{Li}$, where U is an orthogonal matrix such that $U^T M U = \mathrm{diag}(m_i)$. We shall allow m_i to take either sign but restrict the ν's to be CP positive. This is equivalent to requiring m_i to be positive and let the neutrinos behave in opposite fashion under CP. The terms m_i here denote the Majorana masses of the neutrinos and give rise to a nonzero value of the propagator $\langle \chi^T(x_1)\chi(x_2)\rangle$, a correlator identically zero for a Dirac particle. It can be seen easily that if two of the mass eigenvalues are degenerate and of opposite signs, then the corresponding eigenvectors can be combined to give a Dirac particle.

A mass matrix of the form in (1) in general induces $(\beta\beta)_{0\nu}$. It has been shown that the amplitude for this event goes as

$$A((\beta\beta)_{0\nu}) \propto \langle m \rangle = \sum_{k=1}^{n} (U_{ek})^2 m_k F(m_k, N) \ , \tag{2}$$

where

$$F(m_i, N) = \langle e^{-m_i r}/r \rangle \langle 1/r \rangle^{-1}$$

the average being done over the nucleus N in question. For neutrinos lighter than a few MeV the suppression factor F is nearly one and then one has

$$\langle m \rangle \approx \sum_{k=1}^{n} (U_{ek})^2 m_k = M_{11} \tag{3}$$

the last equality following from the definition of U. Thus for light neutrinos, the $(\beta\beta)_{0\nu}$ level depends only on M_{11}. (Although this result was obtained by Wolfenstein[3] in 1981, its significance was not quite appreciated.) It is quite independent of whether ν_e is massless or, if massive, it is of the Dirac or Majorana type.

However if one or more of the ν species are too heavy to be kinematically produced inside the nucleus, then the effective mass $\langle m \rangle$ gets modified to

$$\langle m \rangle = \sum_{\text{light } \nu} (U_{ek})^2 m_k F(m_k, N) \simeq \sum_{\text{light } \nu} (U_{ek})^2 m_k \ . \tag{4}$$

The last approximate equality follows under the assumption that the neutrinos are either too heavy to be of kinematic importance or quite light; i.e., their masses do not lie in the MeV region. In this case M_{11} is no longer a measure of $(\beta\beta)_{0\nu}$.

To consider a concrete case, assume the mass matrix to be of the form

$$M = \begin{bmatrix} M_1 & M_2 \\ M_2^T & M_3 \end{bmatrix} , \tag{5}$$

where $M_{1,3}$ are real symmetric matrices and one has a hierarchy $M_1 \ll M_2 \ll M_3$ such that $M_1 = O(M_2 M_3^{-1} M_2^T)$. M can then be approximately

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block diagonalized by an orthogonal matrix

$$V=\begin{bmatrix} 1-\frac{1}{2}\rho^T\rho & \rho^T \\ -\rho & 1-\frac{1}{2}\rho\rho^T \end{bmatrix}, \tag{6}$$

where $\rho=M_3^{-1}M_2^T$. Then one has

$$M_{BD}\equiv V^T M V=\begin{bmatrix} \tilde{m} & 0 \\ 0 & \tilde{M} \end{bmatrix}+O(\rho^2 M_1), \tag{7}$$

where

$$\tilde{m}=M_1-M_2\rho \quad \text{and} \quad \tilde{M}=M_3+\tfrac{1}{2}(\rho M_2+M_2^T\rho^T). \tag{8}$$

Let $K_{1,3}$ be orthogonal matrices such that $K=\text{diag}(K_1,K_3)$ diagonalizes M_{BD}. Then

$$U\equiv VK=\begin{bmatrix} (1-\frac{1}{2}\rho^T\rho)K_1 & \rho^T K_3 \\ -\rho K_1 & (1-\frac{1}{2}\rho\rho^T)K_3 \end{bmatrix} \tag{9}$$

diagonalizes the mass matrix M. If we assume the eigenvalues of $\tilde{M}$ to be be very large, then the effective mass $\langle m\rangle$ for $(\beta\beta)_{0\nu}$ is given by

$$\langle m\rangle=\sum_{k\in\tilde{m}}(U_{ek})^2 m_k=\tilde{m}_{11}. \tag{10}$$

Thus if we block diagonalize the mass matrix M into two blocks $\tilde{m}$ and $\tilde{M}$, such that the eigenvalues of $\tilde{M}$ are much larger than 1 MeV while those of $\tilde{m}$ are not, then the $(\beta\beta)_{0\nu}$ amplitude depends only upon the element $(\tilde{m})_{11}$ and not on the actual eigenvalues of $\tilde{m}$. We are here working in the basis $(\nu_e\nu_a\nu_b\cdots)$ where $\nu_a,\nu_b,\cdots$ can either be of different generations or sterile.

Let us now consider a few special cases. If

$$M=\begin{bmatrix} 0 & m \\ m & m' \end{bmatrix},$$

where $m'\gg 1$ MeV $\gg m$, then we get a light Majorana neutrino of mass m^2/m'. In this case $M_{11}=0$, yet we do get a nonzero contribution to $(\beta\beta)_{0\nu}$. This is the well-known seesaw mechanism.[7] On the other hand, if in the same mass matrix we have $m,m'\lesssim 1$ MeV, then M itself describes low-energy ν phenomena and we do not have to take recourse to constructing $\tilde{m}$. In this case though we have two Majorana particles of masses $(m'\pm\sqrt{m'^2+4m^2})/2$ yet $A[(\beta\beta)_{0\nu}]=0$ as $M_{11}=0$.

We have demonstrated two situations. In both the cases $M_{11}=0$, but while $(\beta\beta)_{0\nu}$ is present in one it is absent in the other. In either case the physical neutrino is a Majorana particle. Let us now consider the case when it is a Dirac particle, at least at the tree level. Since our analysis does not depend on the radiative corrections, we shall not talk about loop effects. Consider the mass matrix

$$M=\begin{bmatrix} m' & m \\ m & -m' \end{bmatrix}.$$

This corresponds to two Majorana particles of equal masses $\sqrt{m^2+m'^2}$ with opposite CP properties, and hence they combine to give a two-helicity-state Dirac neutrino.[8] Both the eigenvalues being equal we can ignore the factor F and write $\langle m\rangle=m'$. Now we can have two scenarios $m'=0$ or $m'\neq 0$. Each will predict a Dirac neutrino but in the first there is no $(\beta\beta)_{0\nu}$ whereas in the second it does appear.

Although in the simplest example cited above, if the physical neutrino is a Dirac particle, then the contribution to $(\beta\beta)_{0\nu}$ is given by $\langle m\rangle=M_{11}$, this is not the case in general. Consider, for example,

$$M=\begin{bmatrix} 0 & 0 & 0 & m \\ 0 & 0 & m & 0 \\ 0 & m & m' & 0 \\ m & 0 & 0 & -m' \end{bmatrix},$$

where $m'\gg m$. This mass matrix predicts one light Dirac neutrino of mass m^2/m' and also gives nonzero $(\beta\beta)_{0\nu}$ $(\langle m\rangle=m^2/m')$ although $M_{11}=0$.

The question we attempt to answer next is regarding the naturalness of the above arguments. We have demonstrated many scenarios which, in principle, can exist. But if we cannot get them naturally from any realistic theory, then it does not make much sense.

Models[9,10] were constructed to predict light Dirac neutrinos naturally, which give no $(\beta\beta)_{0\nu}$. The most popular version is to start with three additional sterile neutrinos per generation. Then using some symmetry of the theory one gets a mass matrix in the $(\nu_e\nu_a\nu_b\nu_c)$ basis of the form[9]

$$M=\begin{bmatrix} 0 & 0 & A & 0 \\ 0 & 0 & B & C \\ A & B & 0 & 0 \\ 0 & C & 0 & 0 \end{bmatrix},$$

where $B\gg A,C$. This predicts a light Dirac neutrino of mass $AC/B\sim$ a few eV, so that this can explain the ITEP result[11] of $m_\nu\sim 20$ eV, as well as the absence of $(\beta\beta)_{0\nu}$ (Ref. 12). It is believed that this is the only natural solution of the ITEP result and the absence of $(\beta\beta)_{0\nu}$.

We shall proceed in a similar fashion to demonstrate a scenario where we have a light Majorana neutrino with $m_\nu\sim 20$ eV but no $(\beta\beta)_{0\nu}$. The model can also accommodate a 17-keV Majorana ν with a small mixing with ν_e (as seen by Simpson[13]). The numbers are not very special to the model. What we would like to emphasize is that one can obtain light Majorana neutrinos from realistic grand unified theories (GUT's) naturally which do not admit $(\beta\beta)_{0\nu}$. We also start with three sterile neutrinos along with ν_e and seek to get in the $(\nu_e\nu_a\nu_b\nu_c)$ basis a mass matrix of the form

$$M=\begin{bmatrix} \alpha & 0 & 0 & a \\ 0 & 0 & k & 0 \\ 0 & k & 0 & G \\ a & 0 & G & B \end{bmatrix}, \tag{11}$$

where $G\gg k$, $B\gg a\gg\alpha$. In fact α can even be zero. This mass matrix be block diagonalized to

$M_{BD}=\mathrm{diag}(\tilde{m},\tilde{M})$ with

$$\tilde{m}=\begin{bmatrix}\alpha & -ak/G\\ -ak/G & k^2B/G^2\end{bmatrix} \quad \text{and } \tilde{M}\approx\begin{bmatrix}0 & G\\ G & B\end{bmatrix}. \quad (12)$$

Then if $aG \ll kB$, we have four Majorana neutrinos with masses

$$m_1=\alpha-a^2/B,\quad m_2=k^2B/G,\quad m_{3,4}=B\pm G/2\ . \quad (13)$$

With a suitable choice for the five parameters appearing in M we can obtain two light neutrinos and two superheavy ones. The ν-less double-β decay is proportional to $(\tilde{m})_{11}=\alpha$ as two of the neutrinos are too heavy to be kinematically produced at the ordinary decay energies.

The most interesting aspect of this exercise is the relation between α and m_1. As has been mentioned earlier, α is a parameter in the mass matrix much smaller than the others. In the explicit models to be considered later, it turns out to be of the order of a^2/B or smaller. Thus we have three possibilities.

(a) α of the same order as m_1. This gives the usual picture of $(\beta\beta)_{0\nu}$ being proportional to the Majorana mass of ν_e.

(b) $\alpha\approx 0$. Then we get a light Majorana ν without appreciable $(\beta\beta)_{0\nu}$.

(c) $\alpha\approx a^2/B$. This leads to a very small Majorana mass but a rather large amount of $(\beta\beta)_{0\nu}$.

We now proceed to construct a model based on a supersymmetric SO(10) grand unified theory in which the hierarchy of the parameters that we require appears naturally. We do not aim to construct a complete gauge theory; rather we give an illustration of how we can get light Majorana neutrinos, with no $(\beta\beta)_{0\nu}$. This model is on the same footing as those which predict light Dirac neutrinos. In particular, we require one U(1) global symmetry to get the required form of the mass matrix as compared to the three U(1) symmetries required for a light Dirac neutrino in a similar model.

We shall focus our attention on a single family assuming intergeneration mixing to be small. We start a SO(10) model with two fermion singlet superfields S and S' in addition to the usual 16-plet matter superfield ψ. The symmetry-breaking chain being considered is

$$\begin{aligned}&\mathrm{SO}(10)\xrightarrow{M_{\mathrm{GUT}}}\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{U}(1)_{B-L}\\ &\xrightarrow{M_{LR}}\mathrm{SU}(3)_c\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\\ &\xrightarrow{M_W}\mathrm{SU}(3)_c\times\mathrm{U}(1)_Q\ .\end{aligned}$$

To give masses to the fermion fields as well as to prompt the last two stages of the above symmetry-breaking chain, we introduce four Higgs superfields $\chi(\overline{16})$, $\Phi(10)$, $\Delta(126)$, and $\sigma(1)$ where the numbers in parentheses denote their transformation properties under SO(10). The most general Yukawa coupling allowed then is

$$\begin{aligned}\mathcal{L}_y=&\bar{\psi}^c\psi(f_1\Phi+f_2\Delta)+\bar{\psi}^c\chi(f_3S+f_4S')\\ &+(f_5SS'+f_6SS+f_7S'S')\sigma\ .\end{aligned}$$

We impose an additional U(1) global symmetry, the nontrivial transformations under it being

$$\chi\to e^{i\theta}\chi,\quad \sigma\to e^{-i\theta}\sigma,\quad S\to e^{-i\theta}S,\quad S'\to e^{2i\theta}S'\ .$$

This eliminates the Yukawa couplings given by f_4, f_6, and f_7 as well the bare mass terms for S' and S. Then in the basis $(\nu\ S\ S'\ \nu^c)$, the mass matrix reads as in Eq. (11), and with a particular hierarchy of vacuum expectation values[14]: $G=f_3\langle\chi\rangle\sim 10^9$ GeV, $k=f_5\langle\sigma\rangle\sim 10$ TeV, $B=f_2\langle\Delta\rangle\sim 10$ TeV, $a=f_1\langle\Phi\rangle\sim 10$ MeV, and $\alpha\sim 10$ eV.

While the value of G is self-evident the others need some explanation. The scales of k and B, proportional to M_{SUSY} appear naturally in a certain class of supersymmetric models where the corresponding scalars remain massless at the tree level, only to gain mass through radiative corrections. a reflects the electroweak-breaking scale, assuming a value comparable to the light-quark masses. A small value of α (proportional to $\langle\Phi\rangle^2/\langle\Delta\rangle$) is generated due to the features of potential minimization in a left-right-symmetric model. (The details of the model will be discussed elsewhere.[15])

An SU(5) analog of this model can easily be constructed using three singlet fermions (S_1, S_2, and S_3) apart from the usual $10(\chi)$ and $\bar{5}(\psi)$ superfields. The Higgs sector is enlarged to accommodate two singlets (σ_1 and σ_2) and a 5-plet $\tilde{\Phi}$ along with the usual 24-plet Σ and the 5-plet Φ. Then the imposition of a U(1) symmetry,

$$\begin{aligned}&S_1\to e^{i\theta}S_1,\quad S_2\to e^{3i\theta}S_2,\quad S_3\to e^{2i\theta}S_3\ ,\\ &\sigma_1\to e^{-4i\theta}\sigma_1,\quad \sigma_2\to e^{-5i\theta}\sigma_2,\quad \tilde{\Phi}\to e^{-2i\theta}\tilde{\Phi}\ ,\end{aligned}$$

will give us the Yukawa coupling

$$\begin{aligned}\mathcal{L}_y=&(f_1\psi\chi+f_2\chi\chi)\Phi+g_1\psi S_3\tilde{\Phi}+g_4S_2S_3\sigma_2\\ &+(g_2S_1S_2+g_3S_3S_3)\sigma_1\ .\end{aligned}$$

This gives a neutrino mass matrix of the form of Eq. (11) with $G=g_4\langle\sigma_2\rangle$, $k=g_2\langle\sigma_1\rangle$, $B=g_3\langle\sigma_1\rangle$, and $a=g_1\langle\tilde{\Phi}\rangle$ and $a\ll k$, $B\ll G$. It is to be noted that in this case $\alpha=0$. With a suitable choice (depending on the details of the model concerned) of a, k, B, and G we obtain naturally light Majorana neutrinos with no $(\beta\beta)_{0\nu}$.

As an aside we point out that in these scenarios with a careful, but not too unnatural, choice of the Yukawa couplings one could simultaneously accommodate a 25-eV Majorana neutrino with very low $(\beta\beta)_{0\nu}$ rate along with a 17-keV neutrino with a small mixing. Also the cosmological constraint on the masses of stable light neutrinos would not pose much of a problem as the keV mass neutrino could be made to decay through Majorons.[16] (The exact details shall be considered in a future work.)

In summary we argue that the widely held belief that the neutrinoless double-β decay experiments would give us the Majorana mass of the physical electron neutrino is tenable if and only if there is just a single species of ultralight neutrino per generation (as, for example, in a minimal extension of the standard model with the inclusion of right-handed neutrino singlets) and if the interfamily mixing is nonexistent. But in a generic grand unified theory, where there is more than one type of neutrino, it fails to go through. While the absence of $(\beta\beta)_{0\nu}$ cannot say anything about the mass matrix except that $\tilde{m}_{11}=0$, its presence only confirms the presence of a lepton-violating mass term but does not distinguish between a massless, a massive pseudo-Dirac or a Majorana particle. This is exploited in the construction of a supersymmetric grand unified theory in which a Majorana particle that can explain the ITEP results while not succumbing to the $(\beta\beta)_{0\nu}$ constraints is naturally generated.

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