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New quark mass matrix based on the BCS form

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A new quark mass matrix based on the BCS form is proposed. At the first stage, the desirable two mass gaps are derived and generation mixing occurs between the second and third generations. At the second stage, the first-generation quarks obtain masses and the *CP*-violating phase appears. An interesting prediction is found in the V_{cb} element of the Kobayashi-Maskawa matrix, which is favorable to the heavy top quark. The new mass matrix is consistent with the observed B_d^0-$\bar{B}_d^0$ mixing.

The generation problem of quarks and leptons is one of the most important subjects of particle physics. In particular, one has not yet understood the origin of the mass spectra of the quark-lepton system and the Kobayashi-Maskawa (KM) mixing angles.[1] In order to discover the clues to the fundamental theory beyond the standard model, it important to investigate the quark-lepton mass matrices which give the observed KM matrix elements and the quark-lepton mass hierarchies. Two familiar mass matrices were proposed in past years: one is the Fritzsch mass matrix[2] and the other is the Stech mass matrix,[3] both having been investigated intensively.[4,5]

These models have confronted recent data of the B_d^0-$\bar{B}_d^0$ mixing by the ARGUS and CLEO Collaborations.[6] Harari and Nir concluded[5] that the Stech model is inconsistent with the experimental constraints and the Fritzsch model is consistent with them only if several experimental and theoretical quantities are taken at the limits of their allowed ranges. These rather unfavorable situations for these models follow from the fact that the top quark with mass heavier than the previously expected one is called on by the observed B_d^0-$\bar{B}_d^0$ mixing[6] in the standard model.

Recently, a new approach for the quark mass matrix has been introduced by some authors.[7-10] The analogy to BCS superconductivity and the nuclear pairing force[11] leads to a generation-independent (so-called democratic) quark mass matrix, which gives a large mass gap. Kaus and Meshkov[9] obtained the quark mass matrix numerically based on the "BCS quark mass matrix," and Koide[10] proposed the mass matrix which is the BCS quark mass matrix with the diagonal correction terms. We believe that these approaches are important to take a step forward in understanding the origin of quark masses. However, their correction terms are not so transparent for us to understand these physical meanings. Instead of their mass matrices, we propose a new quark mass matrix with a simple structure based on the BCS form, and analyze it by using recent experimental quantities such as B_d^0-$\bar{B}_d^0$ mixing.[6]

If all six quark masses vanish, the Lagrangian of the standard model is invariant under the chiral-symmetry $U(3)_L \times U(3)_R$, where U(3) is the symmetry group connecting the three generations. This chiral symmetry is broken down to $U(2)_L \times U(2)_R$ by the mass matrix of the BCS form[7-10]

$$\frac{1}{3}M_0(q)\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}, \qquad (1)$$

where q denotes the up-quark sector (u) or the down-quark sector (d). This matrix is diagonalized as $\mathrm{diag}[0,0,M_0(q)]$ by using the orthogonal matrix U_0:

$$U_0=\begin{bmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0\\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{2}{\sqrt{6}}\\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}}\end{bmatrix}. \qquad (2)$$

The up-quark mass matrix for quark state $u=(u_0,c_0,t_0)$ provides two massless quarks $u_1=(u_0-c_0)/\sqrt{2}$, $c_1=(u_0+c_0-2t_0)/\sqrt{6}$, and one heavy quark $t_1=(u_0+c_0+t_0)/\sqrt{3}$. For the down-quark sector, d_1, s_1, and b_1 are defined analogously. Thus, the BCS matrix form gives a large mass gap between $M_0(q)$ and zeros, which is favorable to the quark mass hierarchies. However, two mass gaps are demanded by the observed quark mass hierarchies such as m (first generation) $\ll m$ (second generation) $\ll m$ (third generation). One of the simplest ways to get two mass gaps is to modify the BCS form in (1) as

$$M(q)=\frac{1}{3}M_0(q)\begin{bmatrix}1+\epsilon_q & 1+\epsilon_q & 1\\ 1+\epsilon_q & 1+\epsilon_q & 1\\ 1 & 1 & 1\end{bmatrix}, \qquad (3)$$

where ϵ_q is a real correction factor and to be $\ll 1$. Mass matrix elements between the first two generations and the third generation are slightly different. Any exchanges of columns and rows in (3) do not change following our results. In this stage chiral-symmetry $U(3)_L \times U(3)_R$ is violated and is reduced to the subsymmetry $U(1)_L \times U(1)_R$ acting on the (u_1,d_1) system. This correction term was introduced in a different way from ours by

Harari, Haut, and Weyers[8] who gave however a light top-quark mass unfortunately. The mass matrix in (3) is no longer diagonalized by $U_0 M(q) U_0^T$ as shown in the following:

$$U_0 M(q) U_0^T = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -B_q & \sqrt{2}B_q \\ 0 & \sqrt{2}B_q & A_q \end{bmatrix} , \tag{4}$$

where $A_q = (1+4\epsilon_q/9)M_0(q)$ and $B_q = -2(\epsilon_q/9)M_0(q)$. In order to diagonalize $M(q)$, we introduce an orthogonal matrix U_{q2}, which is

$$U_{q2} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_{q2} & -\sin\theta_{q2} \\ 0 & \sin\theta_{q2} & \cos\theta_{q2} \end{bmatrix} . \tag{5}$$

Diagonalizing $M(q)$ by $U_{q2}U_0 M(q)(U_{q2}U_0)^T$, we obtain diag$(0, z_{q2}, z_{q3})$, where

$$\begin{aligned} &z_{q2} + z_{q3} = A_q - B_q , \quad z_{q2} z_{q3} = -A_q B_q - 2B_q^2 , \\ &\tan 2\theta_{q2} = \frac{2\sqrt{2}B_q}{A_q + B_q} . \end{aligned} \tag{6}$$

Then, the parameters A_q and B_q are expressed in terms of the eigenvalues z_{q2} and z_{q3} as

$$\begin{aligned} A_q &= z_{q2} + z_{q3} + B_q , \\ B_q &= -\tfrac{1}{6}(z_{q2} + z_{q3}) \\ &\quad \times \{ 1 - [1 - 12 z_{q2} z_{q3} (z_{q2} + z_{q3})^{-2}]^{1/2} \} . \end{aligned} \tag{7}$$

The approximate values of z_{q2} and z_{q3} are given as

$$\begin{aligned} z_{q2} &\approx -B_q \approx \tfrac{2}{9}\epsilon_q M_0(q) , \\ z_{q3} &\approx A_q \approx (1 + \tfrac{4}{9}\epsilon_q) M_0(q) , \end{aligned} \tag{8}$$

where we have chosen the solution of $z_{q2} \ll z_{q3}$ because of the quark mass hierarchies. Thus, we find that $M(q)$ in (3) shares a nice feature to give the quark mass hierarchies in three generations. Moreover, we remark that the generation mixing between the second and third generations occurs at this stage due to U_{q2}.

At this stage, the first generation quarks are massless and do not mix with the other generation quarks. Now, we assume that the masses of the first-generation quarks follow from the other origin, which breaks residual $U(1)_L \times U(1)_R$ chiral symmetry. We may consider the Yukawa coupling of the Higgs particle in the first generation sector. At the next stage, we give simple correction factors on the base of diag$(0, z_{q2}, z_{q3})$ as

$$M^{\text{phys}}(q) = \begin{bmatrix} 0 & C_q \exp(i\alpha_q) & 0 \\ C_q \exp(i\beta_q) & z_{q2} & 0 \\ 0 & 0 & z_{q3} \end{bmatrix} , \tag{9}$$

where C_q is a real number and α_q, β_q are phases. The correction is given at off-diagonal elements in order to derive the Cabibbo angle in terms of the quark masses.[2] Of course, we may have other possible choice of the correction terms to give mass of the first-generation quark and the Cabibbo angle. But, Eq. (9) is the simplest form for our purpose and so we investigate only this form in this paper.

Since $M^{\text{phys}}(q)$ is a complex matrix, we get the *CP*-violating phase in the KM matrix at this stage. For our convenience, we redefine the left-handed and the right-handed quark fields to eliminate the phase factor in (9). Then, the mass matrix in (9) is brought to the real 3×3 matrix as

$$M_R^{\text{phys}}(q) = \begin{bmatrix} 0 & C_q & 0 \\ C_q & z_{q2} & 0 \\ 0 & 0 & z_{q3} \end{bmatrix} , \tag{10}$$

but the phase matrix appears in deriving the Kobayashi-Maskawa (KM) matrix such as

$$P_0 = \begin{bmatrix} \exp(i\sigma) & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} , \tag{11}$$

with the definition $\sigma = \alpha_d - \alpha_u$.

The mass matrix $M_R^{\text{phys}}(q)$ in (10) is easily diagonalized by $U_{q1} M_R^{\text{phys}}(R) U_{q2}^T$, where

$$U_{q1} = \begin{bmatrix} \cos\theta_{q1} & -\sin\theta_{q1} & 0 \\ \sin\theta_{q1} & \cos\theta_{q1} & 0 \\ 0 & 0 & 1 \end{bmatrix} . \tag{12}$$

In terms of the eigenvalues diag$(-m_{q1}, m_{q2}, m_{q3})$, we get

$$\begin{aligned} &\tan 2\theta_{q1} = \frac{2C_q}{z_{q2}} , \quad C_q = \sqrt{m_{q1} m_{q2}} , \\ &z_{q2} = -m_{q1} + m_{q2} , \quad z_{q3} = m_{q3} . \end{aligned} \tag{13}$$

From Eqs. (2), (5), (11), and (12), we obtain the KM matrix as

$$V_{\text{KM}} = U_{u1} P_0 U_{u2} U_0 (U_{d1} U_{d2} U_0)^T = U_{u1} P_0 U_{u2} U_{d2}^T U_{d1}^T . \tag{14}$$

It is remarked that the *CP*-violating phase follows from the phase matrix P_0.

Now, we give the concrete expressions of the nine KM matrix elements in the following:

$$V_{ud}=e^{i\sigma}\cos\theta_{d1}\cos\theta_{u1}+\cos\theta_{d2}\cos\theta_{u2}\sin\theta_{d1}\sin\theta_{u1}+\sin\theta_{d1}\sin\theta_{d2}\sin\theta_{u1}\sin\theta_{u2}\ ,$$
$$V_{us}=e^{i\sigma}\cos\theta_{u1}\sin\theta_{d1}-\cos\theta_{d1}\cos\theta_{d2}\cos\theta_{u2}\sin\theta_{u1}-\cos\theta_{d1}\sin\theta_{d2}\sin\theta_{u1}\sin\theta_{u2}\ ,$$
$$V_{ub}=\sin\theta_{u1}(\cos\theta_{d2}\sin\theta_{u2}-\cos\theta_{u2}\sin\theta_{d2})\ ,$$
$$V_{cd}=e^{i\sigma}\cos\theta_{d1}\sin\theta_{u1}-\cos\theta_{d2}\cos\theta_{u1}\cos\theta_{u2}\sin\theta_{d1}-\cos\theta_{u1}\sin\theta_{d1}\sin\theta_{d2}\sin\theta_{u2}\ ,$$
$$V_{cs}=e^{i\sigma}\sin\theta_{d1}\sin\theta_{u1}+\cos\theta_{d1}\cos\theta_{d2}\cos\theta_{u1}\cos\theta_{u2}+\cos\theta_{d1}\cos\theta_{u1}\sin\theta_{d2}\sin\theta_{u2}\ , \tag{15}$$
$$V_{cb}=-\cos\theta_{u1}(\cos\theta_{d2}\sin\theta_{u2}-\cos\theta_{u2}\sin\theta_{d2})\ ,$$
$$V_{td}=-\sin\theta_{d1}(\cos\theta_{d2}\sin\theta_{u2}-\cos\theta_{u2}\sin\theta_{d2})\ ,$$
$$V_{ts}=\cos\theta_{d1}(\cos\theta_{d2}\sin\theta_{u2}-\cos\theta_{u2}\sin\theta_{d2})\ ,$$
$$V_{tb}=\cos\theta_{d2}\cos\theta_{u2}+\sin\theta_{d2}\sin\theta_{u2}\ ,$$

with

$$\sin\theta_{q1}=\left[\frac{m_{q1}}{m_{q2}-m_{q1}}\right]^{1/2}\ ,$$
$$\sin\theta_{q2}=\left\{\frac{1}{2}\left[1-\left[\frac{1}{1+r}\right]^{1/2}\right]\right\}^{1/2}\ , \tag{16}$$
$$r=8B_q^2(A_q+B_q)^{-2}\ ,$$

A_q and B_q being given in (7).

The relative phases of the KM matrix elements in (15) are not fixed because the left-handed up- and down-quark fields can be redefined including phase factors. It is convenient to take V_{ud}, V_{us}, V_{cb}, and V_{tb} as real numbers in order to compare our prediction with the usual parametrization of the KM matrix.[12] For this purpose, the diagonal matrices with phases are multiplied on both sides of $V_{\rm KM}$ to be $P_u V_{\rm KM} P_d^\dagger$, where

$$P_u=\mathrm{diag}[\exp(-i\gamma),\exp(i\delta),\exp(i\delta)]$$

and $I_d=\mathrm{diag}[1,\exp(i\delta),\exp(i\delta)]$. The phases γ and δ are defined to make $V(1,1)$ and $V(1,2)$ real, respectively. Then, γ is approximately equal to σ and $\tan^2\delta\approx m_u m_s/(m_d m_c)$. Since σ is a free parameter, it should be determined so as to be consistent with the experimental value of $|V(1,2)|$. The simplest choice is to take $\exp(i\sigma)=i$ (or $-i$) as already discussed by other authors.[4] So we take $\sigma=90°$ in the following numerical calculations.

Now, we give the approximate KM matrix elements as follows:

$$V_{ud}\approx 1,\quad V_{us}\approx\left[\frac{m_d}{m_s}+\frac{m_u}{m_c}\right]^{1/2}\ ,$$
$$V_{ub}\approx\left[2\frac{m_u}{m_c}\right]^{1/2}\left[\frac{m_s}{m_b}-\frac{m_c}{m_t}\right]\exp[-i(\sigma+\delta-180°)]\ ,$$
$$V_{cd}\approx-\left[\frac{m_d}{m_s}+\frac{m_u}{m_c}\right]^{1/2},\quad V_{cs}\approx 1\ ,$$
$$V_{cb}\approx\sqrt{2}\left[\frac{m_s}{m_b}-\frac{m_c}{m_t}\right]\ , \tag{17}$$
$$V_{td}\approx\left[2\frac{m_d}{m_s}\right]^{1/2}\left[\frac{m_s}{m_b}-\frac{m_c}{m_t}\right]\exp(i\delta)\ ,$$
$$V_{ts}\approx-\sqrt{2}\left[\frac{m_s}{m_b}-\frac{m_c}{m_t}\right],\quad V_{tb}\approx 1\ .$$

One of the remarkable features in our KM matrix is found in V_{cb}, which depends on the difference between the m_s/m_b ratio and the m_c/m_t one. Remember that

$$V_{cb}\approx\left[\frac{m_s}{m_b}\right]^{1/2}-\left[\frac{m_c}{m_t}\right]^{1/2}$$

and

$$V_{cb}\approx\left[\frac{m_s}{m_b}-\frac{m_c}{m_t}\right]^{1/2}$$

in the Fritzsch model[2] and Stech model,[3] respectively. Our V_{cb} is favorable to the heavy top quark. Since the ratio m_s/m_b is roughly 1/30, the ratio m_c/m_t is expected to be much smaller than 1/30 by the experimental value $|V_{cb}|=0.045\pm0.08$ (Ref. 13).

Another remarkable feature is found in the $|V_{ub}|/|V_{cb}|$ ratio. This ratio is equal to $\sqrt{m_u/m_c}$, which is independent of the top-quark mass. Furthermore, we predict a CP-violating phase such as $\arg(V_{ub})=180°-\gamma-\delta\approx 90°-\delta\approx 75°$.

Let us show the numerical results. We calculate the $V_{\rm KM}$ in (15) without any approximations, using the central values of the following quark masses at the energy scale of 1 GeV as[14]

$$m_u=5.1\pm1.5\ \text{MeV},\quad m_d=8.9\pm2.6\ \text{MeV}\ ,$$
$$m_c=1.35\pm0.05\ \text{GeV}\ , \tag{18}$$
$$m_s=175\pm55\ \text{MeV},\quad m_b=5.3\pm0.1\ \text{GeV}\ .$$

The top-quark mass at the 1 GeV scale is a free parameter. The physical top-quark mass is obtained by the QCD evolution formula, where we take the QCD parameter Λ as $\Lambda=0.1$ GeV.

We show in Fig. 1 the predicted numerical value of $|V_{cb}|$ versus the physical top-quark mass $m_t^{\rm phys}$. The variation range of $m_t^{\rm phys}$ is taken to be 41 GeV (Ref. 15)–200 GeV (Ref. 16) from the experimental constraints. The present experimental limit $0.037<|V_{cb}|<0.053$ (Ref. 13) is also shown in Fig. 1, from which the top-quark mass is suggested to be larger

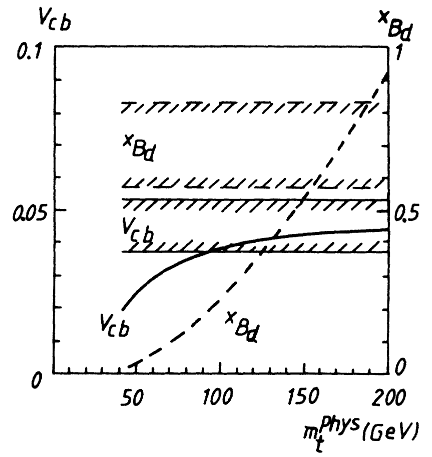

FIG. 1. The predictions of $|V_{cb}|$ and x_{Bd} vs the physical top-quark mass in the case of $m_s=175$ MeV. The solid line denotes $|V_{cb}|$ by the scale of the left-side vertical axis and the dashed line denotes x_{Bd} by the scale of the right-side vertical axis.

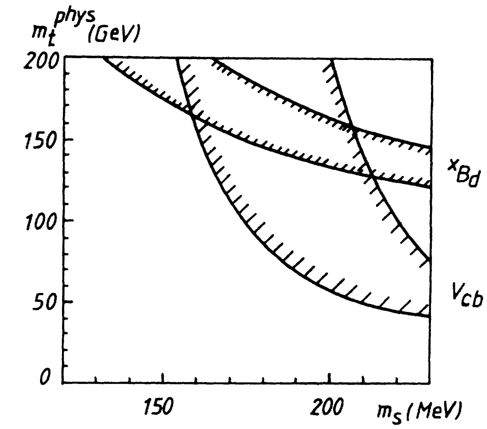

FIG. 2. The allowed regions of m_s and $m_t^{\rm phys}$ for $|V_{cb}|$ and x_{Bd}.

than 90 GeV. Since the predicted value of $|V_{cb}|$ depends on the s-quark mass considerably, we show the allowed region in the $m_s-m_t^{\rm phys}$ plane for the experimental region of $|V_{cb}|$ in Fig. 2.

The $|V_{ub}|/|V_{cb}|$ ratio is predicted to be $\sqrt{m_u/m_c}=0.0615$, which is independent of m_s and $m_t^{\rm phys}$, and so this prediction is crucial for our mass matrix. The CP-violating phase depends on the value of m_s. In the parametrization $V_{ub}=|V_{ub}|\exp(i\phi)$, we predict $\phi=77.28°$, 74.76°, 72.68° for $m_s=120$, 175, 230 MeV, respectively.

Another physical quantity crucially depending on the top-quark mass is the B_d^0-$\bar{B}_d^0$ mixing. In the standard model, the mixing parameter $x_{Bd}=\Delta m_{Bd}/\Gamma_{Bd}$ is given as[17]

$$x_{Bd}=1.49\times10^5\ \mathrm{GeV}^{-2}B_{Bd}f_{Bd}^2\eta_{tt}E(y_t)|V_{td}^*V_{tb}|^2\ , \qquad (19)$$

where we have taken $\tau_{Bd}=1/\Gamma_{Bd}=(1.18\pm0.14)\times10^{-12}$ sec, $m_W=82$ GeV, and $m_B=5.275$ GeV (Ref. 13). The parameter η_{tt} is the QCD correction factor, which is taken to be 0.85 (Ref. 18) in our analysis, and $E(y_t)$ is the loop-integral function given by Inami and Lim.[19] Although the $B_{Bd}f_{Bd}^2$ value is somewhat ambiguous for the present, we use the typical value $B_{Bd}f_{Bd}^2=(0.16\ \mathrm{GeV})^2$ (Ref. 17).

The predicted value of x_{Bd} vs $m_t^{\rm phys}$ is shown in the case of $m_s=175$ MeV in Fig. 1, where the experimental value $x_{Bd}=0.70\pm0.13$ (Ref. 6) is also shown. From Fig. 1, the top-quark mass is suggested to be 150–190 GeV. This value is consistent with the allowed region of $m_t^{\rm phys}$ obtained from the analysis of V_{cb}. We also show the allowed region in the $m_s-m_t^{\rm phys}$ plane for the experimental value of x_{Bd} in Fig. 2. Combining two regions for V_{cb} and x_{Bd} in the $m_s-m_t^{\rm phys}$ plane, we get the allowed region of $m_t^{\rm phys}=130$–200 GeV and $m_s=155-210$ MeV.

Now, we briefly comment on the ϵ'/ϵ ratio in the neutral-K-meson system since it somewhat depends on the top-quark mass. The hadronic matrix element associated with the $K\to\pi\pi$ decay depends on the hadronic dynamics at the long distance. If we adopt the systematic analyses by $1/N$ expansion given by Buras and Gerard,[20] we obtain $\epsilon'/\epsilon=(1.2-1.5)\times10^{-3}$ in the case of $m_t^{\rm phys}=130-200$ Gev with $m_s=175$ MeV. This predicted value is somewhat smaller than the experimental value $\epsilon'/\epsilon=(3.3\pm1.1)\times10^{-3}$ by the NA31 Collaboration at CERN,[21] but if we take account of the recent data $\epsilon'/\epsilon=(-0.5\pm1.5)\times10^{-3}$ by the E731 Collaboration at Fermilab,[22] our prediction is consistent. At any rate, since the theoretical prediction of ϵ'/ϵ depends on the long-distance dynamics of hadrons, the experimental value of ϵ'/ϵ does not give a severe constraint to our mass matrix for the present.

We note the QCD parameter Λ dependence of our result. In our calculation, we have taken $\Lambda=0.1$ GeV to connect $m_t(1\ \mathrm{GeV})$ with $m_t^{\rm phys}$. Our numerical predictions change only 4% and 7% for V_{cb} and x_{BD}, respectively, in the range of $\Lambda=0.5-0.2$ GeV.

We have proposed a new quark mass matrix based on the BCS form. At the first stage, the desirable two mass gaps are derived and generation mixing occurs between the second and third generations. At this stage, no CP-violating phase appears. At the second stage, the first-generation quarks obtain masses and the CP-violating phase appears. The phase in the KM matrix follows from the difference between the two phases of the down-quark mass matrix element and the up-quark one associated with the first generation. This difference σ is expected to be $\pm90°$. We may have possible other choices for the mass matrix element at the second stage. The most prominent prediction is found in the V_{cb} element, which is favorable to the heavy top quark. The top-quark mass is predicted to be 130–200 GeV. It is important for our model to compare the predicted $|V_{ub}|/|V_{cb}|$ ratio with the experimental one. To derive our mass matrix on the basis of the same dynamical mechanism will be the interesting work.

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