BRST quantization of topological field theory

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The canonical quantization of the topological field-theory model $\int_M F\wedge B$, proposed by Horowitz, is examined through the Becchi-Rouet-Stora-Tyutin approach. With this approach we have a manifestly covariant description of a certain class of topological quantum field theories. However, if M in the present model is any orientable four-dimensional manifold, one would end up with different theories when different polarizations are chosen. Therefore a restriction $M=R^1\times\Sigma$, as pointed out by Horowitz, is required for consistency.

Topological field theories are theories that have no classical physics because they have no local dynamics. However, when quantized, the consequences are extremely rich. For instance, topological quantum Yang-Mills theory in four dimensions gives a path-integral representation of the Donaldson invariants.[1] And topological quantum Chern-Simons theory in three dimensions provides a natural framework for studying knot theory. It has also been related to $(1+1)$-dimensional conformal field theory by establishing the correspondence between conformal blocks[2] and quantum states.[3] Recently, a certain class of exactly soluble diffeomorphism-invariant theories has been suggested in Ref. 4, where the author(s) showed us a three-dimensional Chern-Simons action appearing in the solution of a four-dimensional topological quantum gauge theory, and they gave a natural description of the linking number in terms of a two-point function of a topological field theory.

As is known, for conventional constrained systems, one usually is free to solve constraints before or after quantization. But for topological field theories, the procedure of imposing constraints first and then quantizing, which has been used in Refs. 3 and 4, becomes very subtle since the reduced solution spaces usually are finite dimensional. Now we pay attention to the model proposed by Horowitz in Ref. 4.

This is a system with two fields: a Maxwell field $F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$ and a two-form axion potential $B_{\mu\nu}$, on a four-dimensional manifold M. The action reads

$$S=\int_M F\wedge B\ . \tag{1}$$

It is easy to see the theory possesses two gauge invariances:

$$(1)\quad A=d\rho \tag{2a}$$

and

$$(2)\quad B=d\xi\ , \tag{2b}$$

where d is the exterior differential operator, and $\rho(x)$ and $\xi(x)$ arbitrary zero- and one-forms on M, respectively. The variations of (1) with respect to $B_{\mu\nu}$ and A_μ give "equations of motion"

$$F_{\mu\nu}=0\quad\text{and}\quad H_{\rho\mu\nu}=3\partial_{[\rho}B_{\mu\nu]}=0\ . \tag{3}$$

It has been shown[4] that modulo the gauge transformations (2), the reduced classical solution spaces of A_μ and $B_{\mu\nu}$ are exactly the first and the second cohomology group of M, respectively. From action (1), one can see that potentials A and B are conjugate fields with each other. On the other hand, however, since the dimension of the first cohomology group is unnecessary to equal the one of the second cohomology group in a general four-manifold, the reduced solution space cannot generally be identified with the phase space. Therefore when quantized, a restriction

$$M=\Sigma\times R^1 \tag{4}$$

is required, where Σ is a three-dimensional compact manifold, on which it certainly satisfies $\dim H^1(\Sigma)=\dim H^2(\Sigma)$ (Ref. 4).

At first look, it seems that if with the Becchi-Rouet-Stora-Tyutin (BRST) quantization restriction (4) could be relaxed since coupled with auxiliary fields the potentials do not satisfy flat connection conditions (3) any more and the corresponding solution space would be infinite dimensional (see below). Therefore the phase space would be well defined on any orientable manifold M. In this paper, we shall examine this issue and we find that with the BRST quantization restriction (4) is still required otherwise the different polarization choices would probably lead to different quantum theories although BRST quantization could be formally done.

The BRST quantization will be done in the Lagrangian formalism.[8] This is because, first, topological field theories have vanishing Hamiltonians, as a result of diffeomorphism invariance. And second, we try to avoid treating Lorentz components of fields unequally so that the manifest Lorentz covariance of the theory can be maintained in each step.

At the classical level, the covariant description of the canonical formalism has been investigated and covariant symplectic structures for gauge field theories have been given in more or less different spirits.[9,10] Now we present a more practical procedure to construct a covariant symplectic structure for a relativistic theory. Suppose a theory described by Lagrangian

$\mathcal{L}=\mathcal{L}(\phi_i,\partial_\mu\phi_i)$, $i=1,2,\ldots,n$. Instead of specially treating time t, we define the conjugates of configurations ϕ_i as $\pi_{\phi i}=[\partial\mathcal{L}/\partial(\partial^\mu\phi_i)]e^\mu$, where $e^\mu=(-)^\mu dx^1\wedge dx^2\wedge\cdots\wedge dx^D$ with dx^μ missed behaves like a vector in the tangent space of M. Then all ϕ_i and $\pi_{\phi i}$ will span the phase space, which is a cotangent bundle of the configuration space ϕ_i at space-time point $x\in M$ if a symplectic structure is supplemented. To do so, we define one-forms $\delta\phi_i$ and $\delta\pi_{\phi i}$ on space $(\phi_i,\pi_{\phi i})$. Then $\Omega=\int_Y\delta\phi_i\wedge\delta\pi_{\phi i}$ defines a symplectic structure, where the integration will be performed over an arbitrary initial-value hypersurface Y of M. Such an Ω is certainly Lorentz invariant. If the theory at hand is a constrained one, Ω will in general be degenerate. A well-known example is Yang-Mills theories. In this case, one has to define Ω on either the moduli space, the original phase space modulo the transformations generated by the constraints[9] or the enlarged (super)phase space corresponding to a covariantly gauge-fixing effective theory.[10]

Now we consider the BRST canonical quantization of model (1). The point of BRST quantization is as follows. Choosing covariant gauge functions and making up a few degrees of freedom (ghosts and Lagrangian multipliers), one converts the local gauge invariances of the theory into a global BRST symmetry. After quantization one requires the quantum physical states to be annihilated by BRST charge to respect BRST invariance. Thus BRST quantization essentially is an approach with which one quantizes first and then imposes constraints.

The symmetries of (1) have been analyzed in Ref. 4. First, like any two-field theory, it is invariant under transformations

$$\delta A_\mu=V^\rho F_{\rho\mu},\quad \delta B_{\mu\nu}=V^\rho H_{\rho\mu\nu}\ , \tag{5}$$

for arbitrary vector field V^ρ on M. However, the invariance under (5) leads to no constraint. So that if two symmetries differ just by (5), they are equivalent. Next, (1) is diffeomorphism invariant. Since the Lagrangian is a top form in four dimensions, under a diffeomorphism transformation generated by an arbitrary vector field V on M via a Lie derivative, it gives

$$\mathcal{L}_V S=\int_M d[(F\wedge B)(V)]+[d(F\wedge B)](V)=0\ , \tag{6}$$

provided M is boundaryless or V^ρ vanishes on the boundary of M. The corresponding diffeomorphism transformations of A_μ and $B_{\mu\nu}$ are

$$\mathcal{L}_V A_\mu=\partial_\mu(A_\rho V^\rho)+V^\rho F_{\rho\mu}\ , \tag{7a}$$

$$\mathcal{L}_V B_{\mu\nu}=-2\partial_{[\mu}(B_{\nu]\rho}V^\rho)+V^\rho H_{\rho\mu\nu}\ . \tag{7b}$$

Finally, action (1) has the gauge invariances (2a) and (2b). Comparing (7a) and (7b) with (2a) and (2b), one immediately finds that modulo transformations (5), diffeomorphism transformations (7) completely fall in to gauge transformations. As a result, the constraints of theory (1) only relate to the gauge symmetry. In other words, only gauge transformations (2a) and (2b) need to be gauge fixed.

The BRST gauge fixing is realized by introducing proper ghosts so that a covariant nilpotent (off-shell) BRST symmetry holds. The gauge fixing for (2a) is very common: We define $\lambda c(x)=\rho(x)$; here, λ is a Grassmannian parameter and $c(x)$ anticommuting zero-form with ghost number one (usually one does not have to introduce ghosts for the Abelian gauge potential since they are decoupled; here we do so just for convenience and make it more straightforward to generalize to non-Abelian cases). We also introduce an anticommuting zero-form antighost $\bar{c}(x)$ with ghost number minus one, and a commuting zero-form Lagrange multiplier $D(x)$ with null ghost number. The BRST transformations are

$$sA_\mu=\lambda\partial_\mu c,\quad sc=0,\quad s\bar{c}=i\lambda D,\quad sD=0\ . \tag{8}$$

The ghost structure for fixing (2b) is a little more complicated because gauge parameter ζ in (2b) is a one-form. Therefore we need two generations of ghosts.[11,12] The first generation denoted by ξ and $\bar{\xi}$, being anticommuting one-forms with ghost number plus and minus one, respectively, is used to fix (2b). Since ξ and $\bar{\xi}$ behave like gauge fields, the gauge fixings for them are needed. It leads the introduction of the second generation ghosts ϕ, η, and $\bar{\phi}$, commuting zero-forms with ghost number two, zero, and minus two, respectively. Then three Lagrange multipliers G, the commuting one form with null ghost number, and E and $\bar{E}$, anticommuting zero forms with ghost number plus and minus one, respectively, also are introduced so that BRST invariance will be off shell. The BRST transformations are

$$\begin{aligned}&sB_{\mu\nu}=\lambda(\partial_\mu\xi_\nu-\partial_\nu\xi_\mu),\quad s\xi_\mu=-i\lambda\partial_\mu\phi,\quad s\phi=0\ ,\\&s\bar{\xi}_\mu=\lambda G_\mu,\quad sG_\mu=0,\quad s\eta=\lambda E\ ,\\&sE=0,\quad s\bar{\phi}=\lambda\bar{E},\quad s\bar{E}=0\ .\end{aligned} \tag{9}$$

The relations of all fields in (9) are shown as follows:

						Form number
			B			2
		ξ	G	$\bar{\xi}$		1
	ϕ	E	η	$\bar{E}$	$\bar{\phi}$	0
ghost number	2	1	0	-1	-2	

It is easy to see that BRST transformation is nilpotent, i.e., $s^2=0$ for all transformations of the fields.

To complete gauge fixing, we also need to introduce gauge functions. Taking into account of (8) and (9), we choose

$$\partial^\mu A_\mu,\quad \partial^\mu B_{\mu\nu},\quad \partial^\mu\xi_\mu,\quad \partial^\mu\bar{\xi}_\mu\ . \tag{10}$$

With all these ingredients, we can write down the effective Lagrangian with the requirement of BRST invariance:

$$S_{\text{eff}}=\int_M F\wedge B+\frac{-i}{\lambda}\int_M s[\bar{c}(\partial^\mu A_\mu-\tfrac{1}{2}\alpha D)+\bar{\xi}^\mu(\partial^\nu B_{\mu\nu}-\tfrac{1}{2}\beta G_\mu)+\eta(\partial^\mu\bar{\xi}_\mu-\tfrac{1}{2}\gamma\bar{E})+\bar{\phi}(\partial^\mu\xi_\mu+\tfrac{1}{2}\gamma E)]$$
$$=\int_M F\wedge B+\int_M(D\partial^\mu A_\mu-\tfrac{1}{2}\alpha D^2-i\partial^\mu\bar{c}\partial_\mu c)+\int(G^\mu\partial^\nu B_{\mu\nu}-\tfrac{1}{2}\beta G^\mu G_\mu+\eta\partial^\mu G_\mu+\partial^\mu\bar{\phi}\partial_\mu\phi$$
$$-\tfrac{1}{2}\bar{\xi}^{\mu\nu}\xi_{\mu\nu}-iE\partial^\mu\bar{\xi}_\mu+i\bar{E}\partial^\mu\xi_\mu+i\gamma\bar{E}E)\ , \tag{11}$$

where $\xi_{\mu\nu}=\partial_\mu\xi_\nu-\partial_\nu\xi_\mu$, and α, β, and γ are gauge parameters. All fields above are real by definition.

In covariant gauge fixing, we have unavoidably introduced a metric on M; therefore, the gauge-fixed action is no longer diffeomorphism invariant. Since a metric in general does not respect the diffeomorphism invariance, topological field theories perhaps do not accept covariant gauge fixings. However, till now no evidence has shown that the metric introduced by gauge fixing will break the general covariance of a topological quantum theory. For instance, in topological quantum Yang-Mills and topological quantum Chern-Simons theories, it seems covariant gauge fixing does not break topological invariance.[1,2,5] In the present case, as we shall see, physical quantum states as well as the constraints those states obey are metric independent. Moreover, as pointed out in Ref. 14, the energy-momentum tensor $\Theta_{\mu\nu}$ associated with the gauge-fixing Lagrangian is nonzero, but it takes the form $\Theta_{\mu\nu}=\{Q,\Upsilon_{\mu\nu}\}$ for some $\Upsilon_{\mu\nu}$, where Q is BRST charge. This means the expectation value of $\Theta_{\mu\nu}$ between physical states will vanish. However, a solid proof of general covariance anomaly-free is still lacking.

Being manifestly covariant, the effective Lagrangian (11) has an advantage for the partition function calculation as well as perturbative analysis; however, here we would like to proceed in canonical description. Extremizing action (11) gives field equations. Taking the Feynman gauge ($\gamma=1$), we find that all ghosts D and G_μ are free fields satisfying $\partial^\mu\partial_\mu\varphi=0$, $\varphi=(c,\bar{c},\xi,\bar{\xi},\phi,\eta,\bar{\phi},D,G_\mu)$ (this is not a surprise since we are treating an Abelian theory). While A_μ and $B_{\mu\nu}$, unlike the propagating fields in φ, are governed by

$$\epsilon^{\mu\nu\rho\lambda}F_{\rho\lambda}=\partial^\mu G^\nu\ , \tag{12a}$$

$$2\epsilon^{\mu\nu\rho\lambda}\partial_\nu B_{\rho\lambda}=\partial^\mu D\ . \tag{12b}$$

Equation (12) shows us that with the covariant gauge fixing unlike Eqs. (2a) and (2b), the gauge and axion potentials are no longer subjected to the zero-field-strength conditions. (A similar situation occurs in quantum topological Chern-Simons theory even with an axial gauge,[2] where, once Wilson lines are involved, a constraint such as (3) obtains extra terms, which are interpreted as having a quantum origin.) The important thing here is that the solution space is infinite dimensional, and cannot be reduced to or be identified with a finite-dimensional phase space. Thus the standard canonical approach might be used here.

To have the phase space, we identify configuration space with $\Phi=(A_\mu,c,\bar{c},\xi,\bar{\xi},\phi,\bar{\phi}$, and $G_\mu)$ at $x\in M$. And then define canonical conjugate π_Φ of Φ as

$$\pi_{A\mu}=[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\nu A_\mu)]e_\nu,\quad \pi_c=[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu c)]e_\mu\ , \tag{13}$$

and so on. Finally, equip it with a symplectic structure. Phase space $\Gamma=(\Phi,\pi_\Phi)$ defined in this way is the cotangent bundle of Φ, in which the fiber over Φ is the set of all linear maps from vectors in Φ to reals via the map $\int_Y\delta\Phi\,\pi_{\Phi\mu}e^\mu$, where we have defined the exterior derivative on Γ, noted with δ, which satisfies $\delta^2=0$ and gives one-forms $\delta\Phi$ (as well as $\delta\pi_\Phi$) in Γ. Moreover we define a canonical two-form on phase space $\Gamma,\Omega=\int_Y\delta\pi_\Phi\wedge\delta\Phi$. Precisely,

$$\Omega=\int_Y[\delta(2\epsilon^{\mu\nu\rho\lambda}B_{\rho\lambda}+\delta^{\mu\nu}D)\wedge\delta A_\nu+\delta(i\partial^\mu\bar{c})\wedge\delta c+\delta(-i\partial^\mu c)\wedge\delta\bar{c}+\delta(-B^{\mu\nu}+\delta^{\mu\nu}\eta)\wedge\delta G_\nu+\delta(i\bar{\xi}^{\mu\nu}+i\delta^{\mu\nu}E)\wedge\delta\xi_\nu$$
$$+\delta(-i\xi^{\mu\nu}+i\delta^{\mu\nu}E)\wedge\delta\bar{E}_\nu+\delta(\partial^\mu\bar{\phi})\wedge\delta\phi+\delta(\partial^\mu\phi)\wedge\delta\bar{\phi}]e_\mu\ , \tag{14}$$

where the integral is over an arbitrary initial-wave hypersurface Y. Equation (14) is obviously covariant and BRST invariant by construction. Also it is closed and nondegenerate because the local gauge symmetry has been fixed while BRST invariance is a global symmetry which brings no degeneracy into (14) (Ref. 10). So (14) does define the graded symplectic structure for the gauge-fixed theory.

From BRST transformation (8) and (9) and gauge-fixed Lagrangian (11), we have a BRST Noether current J^μ,

$$\lambda J^\mu=[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu A_\nu)]sA_\nu+[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu B_{\nu\rho})]sB_{\nu\rho}+[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu\bar{c})]s\bar{c}$$
$$+[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu\xi_\nu)]s\xi_\nu+[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu\bar{\xi}_\nu)]s\bar{\xi}_\nu+[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu\bar{\phi})]s\bar{\phi}+[\partial\mathcal{L}_{\text{eff}}/\partial(\partial_\mu\eta)]s\eta$$
$$=\lambda(2\epsilon^{\mu\nu\rho\lambda}B_{\nu\rho}\partial_\lambda c-2\epsilon^{\mu\nu\rho\lambda}A_\nu\xi_{\rho\lambda}-\xi^{\mu\nu}\partial_\nu\phi)\ , \tag{15}$$

which is conserved, $\partial_\mu J^\mu=0$, taking into account field equations. Moreover, from (15) we define a covariant BRST charge

$$Q=\int_Y J^\mu e_\mu\ . \tag{16}$$

Equation (16) generates BRST transformation (8) and (9). It is real, with ghost number one, and nilpotent, $\{Q,Q\}=0$. As usual, the Poisson brackets is defined through symplectic structure (14).

Classical "observables" of the theory are defined as real, null ghost number functions of phase space. As any observable, for instance, T, should be gauge invariant in a gauge theory; it must be BRST invariant here; i.e., T obeys $\{T,Q\}=0$. Taking into account the nilpotency of the BRST charge Q, it can be seen that observables form the BRST cohomology classes—two observables will be identified if they differ only to $\{Q,K\}$ for some K. It has been proved[7] that any observable T of a theory in BRST approach can be decomposed as T_0+"ghost terms," where T_0 is ghost-free and exactly the same with the one of the observables of the theory in the Dirac approach.[13] It is obvious the statement is also correct here.

Now we come to the critical point—how to choose a polarization. Since phase space Γ is cotangent bundle of the field configurations Φ, it seems one could have a natural polarization; i.e., either Φ or Φ_π might be chosen as "coordinate," which acts on a linear space of functional of coordinate as multiplication; the other as "momentum" acts as functional differentiation. We shall see, however, the two polarizations will probably lead two different quantum theories if spacetime manifold M is arbitrary.

According to the correspondence principle, observables become Hermitian operators with null ghost numbers, and they will commute with the BRST operator. And the BRST cohomology property of observables still remains at the quantum level. Consequently it selects a subspace of the Hilbert space as the space of "physical states" (see Henneaux in Ref. 7). On the other hand, if two operators T and $T'=T+[Q,K]$ are identified, they should have the same expectation value

$$\langle \psi_1|T|\psi_2\rangle=\langle\psi_1|T'|\psi_2\rangle \ . \tag{17}$$

It is certainly not all states in the Hilbert space that satisfy the equation, but those, we call physical states, which obey

$$Q|\psi\rangle=0 \ . \tag{18}$$

Equation (18) defines the physical subspace of the Hilbert space. Notice the nilpotency of BRST charge Q, physical states form the BRST cohomology classes: $|\psi\rangle\sim|\psi\rangle+Q|\chi\rangle$. Of course, any physical state with form $Q|\chi\rangle$ is a zero-norm state. We emphasize that all states here are only degenerate vacuum states as the Hamiltonian is zero. This is a main difference between topological field theories and conventional field theories.

To complete BRST quantization procedure, we need to examine the physical state condition (18) further. First we consider the polarization with Φ as coordinate. One always can decompose any physical state into a direct product $|\psi\rangle=|A_\mu\rangle\times|\text{ghosts}\rangle$, where $|A_\mu\rangle$ does not involve the ghosts and $|\text{ghosts}\rangle$ does not involve the gauge field either since the ghost fields are decoupled in the present case [for non-Abelian gauge theories, a similar expansion of any physical state in the power of ghosts exists (see Henneaux in Ref. 7)]. Then we can rewrite the BRST charge as

$$Q=\int_Y(-2c\,\epsilon^{\mu\nu\rho\lambda}\partial_\nu B_{\rho\lambda}-2\epsilon^{\mu\nu\rho\lambda}F_{\nu\rho}\xi_\lambda-\bar{\xi}^{\mu\nu}\partial_\nu\phi)e_\mu \ . \tag{19}$$

Because the ghost states $c|\chi\rangle$, $\xi_\lambda|\chi\rangle$, and $\bar{\xi}^{\mu\nu}\partial_\nu\phi|\chi\rangle$ are orthogonal with each other, the physical state condition (18) can be replaced by

$$\epsilon^{\mu\nu\rho\lambda}F_{\rho\lambda}|A\rangle=0 \ , \tag{20a}$$

$$\epsilon^{\mu\nu\rho\lambda}\partial_\nu B_{\rho\lambda}|A\rangle=0 \ , \tag{20b}$$

$$\bar{\xi}^{\mu\nu}\partial_\nu\phi|\text{ghost}\rangle=0 \ . \tag{20c}$$

Equation (20c) is a condition for the ghost state (another possibility is $|A\rangle=0$, which is trivial and ignored). Equation (20a) just tells us that state $|A\rangle$ has support only on flat connections, since, as we have chosen, A_μ act as multiplication. While (20b) implies that state $|A\rangle$ is gauge invariant since $\epsilon^{\mu\nu\rho\lambda}\partial_\nu B_{\rho\lambda}$ is the generator of gauge transformations of A_μ. Putting these together, we realize that the $|A_\mu\rangle$ parts of physical states are defined on the first cohomology class of M.

On the other hand, however, if we take the polarization with Φ_π as the coordinate, the arguments of physical states in (20) should be replaced by B and the conjugates of ghosts, while operator B acts as a multiplier and A as functional differentiation. By the same reason it results that $|B\rangle$ parts of physical states are defined on the second cohomology class of M.

For an arbitrary M, the dimensions of the first and the second cohomology group of M generally are unequal; therefore, the two polarization choices are not equivalent generally. However, if taking $M=\Sigma\times R^1$, the equivalence can be guaranteed.

In summary, we have investigated canonical BRST quantization of an example of a topological field theory in a manifestly covariant version. We found the restriction $M=R^1\times\Sigma$ could not be removed in this approach. This method can be generalized to other topological field models, such as the wide class proposed by Horowitz,[4] only with technical complications in general.

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