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Feynman's Relation for Particle Production near the Boundary of Phase Space*

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Feynman's relation for one-particle inclusive experiments at the edge of the fragmentation region is derived using only quasi-two-body amplitudes and single-Regge-pole exchange. A similar relation for two-particle inclusive experiments is also derived.

Recently, Feynman's result[1] about the behavior of single-particle spectra near the phase-space boundary was derived[2] using a forward three-body optical theorem, along the lines suggested by Mueller.[3] In that kind of analysis the amplitude in that region is dominated by a triple-Regge-pole diagram. I would like to point out that the result can be derived in a more conventional way, using only quasi-two-body amplitudes and a single-Regge-pole exchange.

We look at the process

$$a+b\to c+\text{anything}, \tag{1}$$

where the momenta are p_1, p_2, q, K, respectively (K is the sum of four-momenta of all the undetected outgoing particles). The invariant differential cross section is given by

$$\begin{aligned}\frac{d^3\sigma}{d^3q/q_0}&=f(s,q_\parallel,q_\perp)\\&=\frac{1}{\Delta^{1/2}(s,m_a^2,m_b^2)}\sum_n|A_n(a+b\to c)|^2\\&\quad\times\delta^4(p_1+p_2-q-K),\end{aligned} \tag{2}$$

where $\Delta(x,y,z)$ is the usual triangular function and

$$s=(p_1+p_2)^2. \tag{3}$$

$A_n(a+b\to c)$ stands for the amplitude for the production of a well-defined state together with the detected particle c. The summation extends all of these states as well as integrations over phase space. $q_\perp$ and $q_\parallel$ are, respectively, the transverse and longitudinal momentum components of particle c in the c.m. frame. It was argued by Feynman[1] that $f(s,q_\parallel,q_\perp)$ becomes a function of the variables $q_\perp$ and

$$x=2q_\parallel/\sqrt{s} \tag{4}$$

only, in the limit that s and $q_\parallel$ go to infinity with $q_\perp$ and the scaling variable x held fixed. This suggestion was later shown by Mueller[3] to follow from quite general arguments, for the whole range of the variable x $(-1\leqslant x\leqslant 1)$. There is also an amount of experimental information[4] in favor of this scaling hypothesis, and it even looks as if the limit is reached for quite low values of s. We are therefore going to take the scaling behavior of $f(s,q_\parallel,q_\perp)$ as our starting point and look at the behavior of the limiting function when the detected particle is near the boundary of the fragmentation region of, say, the projectile – that is, when $x\to 1$. We restrict ourselves to processes in which the detected particle is not the product of diffractive dissociation (that is, the projectile and the fragment cannot couple to the Pomeranchukon), for reasons which will be discussed below.

In the above specified limit it will be convenient to choose as independent variables t, the momentum transfer between the projectile and the fragment, and M^2, the missing mass squared. They are given by

$$\begin{aligned}t&=(p_1-q)^2,\\M^2&=K^2=(p_1+p_2-q)^2.\end{aligned} \tag{5}$$

M^2 is then related to the scaling variable x via

$$M^2/s\simeq 1-x, \tag{6}$$

and t is related to $q_\perp$ and x.

Let us fix t and M^2 and start to increase s. The production of a well-defined final state, when s/M^2 gets very large, will then become a quasi-two-body scattering, dominated by the highest Regge pole which can be exchanged in the t channel, as shown in Fig. 1. Therefore, all the amplitudes $A_n(a+b\to c)$ for the production of states which can couple to the leading Regge pole [and which are the dominant contributions to the sum on the right-hand side of Eq. (2)] will have the form

$$A_n(a+b\to c)\simeq s^{\alpha(t)}g_n(M^2,t), \tag{7}$$

where $\alpha(t)$ is the leading-trajectory function and $g_n(M^2,t)$ the appropriate residue function. Since the s dependence factors out from all the leading contributions, their sum will give

$$f(s,M^2,t)\simeq s^{2\alpha(t)-1}g(M^2,t). \tag{8}$$

Let us now increase M^2, keeping s always much bigger than M^2 (that is, x close to 1). Eventually we will get into the scaling region, where $f(s,M^2,t)$

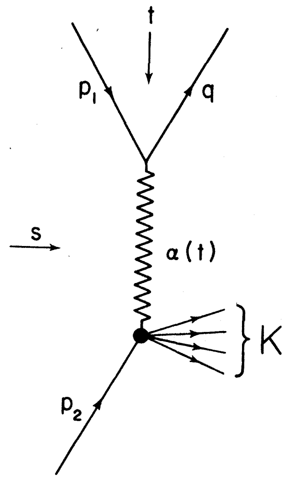

FIG. 1. Single-Regge-pole exchange in a one-particle inclusive experiment.

becomes a function of M^2/s and t only. From this and Eq. (8) we get

$$\frac{d^3\sigma}{d^3q/q_0} \underset{x\to 1;\, s\to\infty}{\overset{t \text{ fixed}}{\sim}} \left(\frac{s}{M^2}\right)^{2\alpha(t)-1}\beta(t)$$

$$=(1-x)^{1-2\alpha(t)}\beta(t), \tag{9}$$

which is Feynman's relation.[5]

The above argument is completely analogous to the one which is used in the derivation of Regge behavior of the scaling functions of electron scattering. In that case we have Regge behavior at fixed q^2 when $\nu\to\infty$, and combining this with the scaling property, the behavior in q^2 and in the scaling variable is dictated.[6]

The above derivation does not hold when the fragment c comes out as a product of diffraction dissociation[7] (Feynman's argument and the derivation of Ref. 2 also do not hold when there is Pomeranchukon exchange). The reason is that in that case,[8] the fragments of the projectile (when we look at a definite final state) always take away fixed fractions of the available momentum, that is, they have fixed x. In other words, there is no part of phase space where the dominant mechanism will be a Regge exchange, as shown in Fig. 1. Instead there will be an exchange of a Pomeranchukon which couples to the projectile and to all of its fragments, and this will produce fragments all over the fragmentation region, including $x\to 1$.

We have, therefore, a nice illustration of Wilson's idea that one should treat differently production via diffractive dissociation and multiperipheral mechanism. Feynman's relation obviously applies only to the multiperipheral component of production.

A derivation of Feynman's relation analogous to the above has already been given within the context of the dual-resonance model.[9] The aim of this note is merely to point out that the relation is quite model-independent, and it follows immediately from scaling and "old-fashioned" Regge behavior, provided we make the necessary distinction between diffractive and multiperipheral production.

The physical reasoning behind our derivation is of course closely related to that of Ref. 2. The reason why our derivation is simpler clearly stems from the fact that we assume scaling as our starting point. As shown in Ref. 3, this assumption is equivalent to performing part of the triple Regge sum and assuming Pomeranchukon dominance [if $\alpha_p(0)\neq 1$, one can divide the inclusive-experiment cross section by σ_{tot}, and the rest of the argument remains unchanged].

Our derivation can be extended to two-particle inclusive experiments which do not have a diffractive component (like, e.g., $p+p\to 2\pi+$anything; $\pi^-+p\to\pi^+\pi^-+$anything, where the produced π^- is in the proton's fragmentation region, etc.). The reaction now is

$$a+b\to c+d+\text{anything}, \tag{10}$$

and the momenta are, respectively, p_1, p_2, q_1, q_2, and K. The invariant differential cross section will now be a function of s and the momenta of the detected particles; that is,

$$\frac{d^6\sigma}{(d^3q_1/q_{1,0})(d^3q_2/q_{2,0})}=f(s,q_{1\parallel},q_{2\parallel},q_{1\perp},q_{2\perp}) \tag{11}$$

using obvious notations. The scaling hypothesis now will be that $f(s,q_{1\parallel},q_{2\parallel},q_{1\perp},q_{2\perp})$ becomes a function only of $q_{1\perp}$, $q_{2\perp}$, and

$$x_i=2q_{i\parallel}/\sqrt{s}, \quad i=1,\ 2 \tag{12}$$

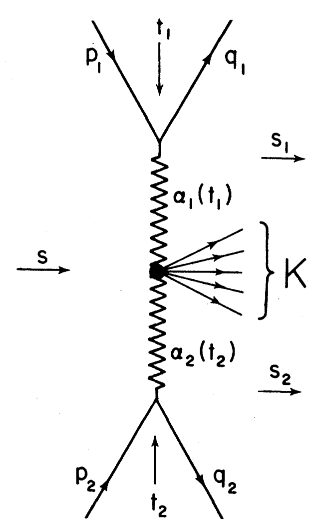

FIG. 2. Double-Regge-pole exchange in a two-particle inclusive experiment.

when s and the longitudinal momenta go to infinity, provided that the scaling variables x_i and the transverse momenta are held fixed. We now look at the scaling function near the boundary of phase space when each detected particle is a fragment of a different incoming particle – for instance, when c is a fragment of a, and d is a fragment of b. This amounts to taking the limits $x_1 \to 1$ and $x_2 \to -1$ simultaneously. It will again be more convenient to use in this region the variables M^2, s_1, s_2, t_1, and t_2, where

$$M^2 = K^2, \quad s_i = (q_i + K)^2, \quad t_i = (p_i - q_i)^2, \quad i = 1,\ 2. \tag{13}$$

The momentum transfers squared, t_i, are determined by the scaling variables x_i and the transverse momenta $q_{i\perp}$. Noticing that, at the phase-space boundary which we discuss, the missing mass is at rest, in the c.m. frame, one can immediately derive the analog of Eq. (6):

$$\frac{M^2}{s_1} \simeq 1 - x_1, \quad \frac{M^2}{s_2} \simeq 1 - |x_2|. \tag{14}$$

We proceed now exactly as before. The dominant mechanism when s_1 and s_2 go to infinity with the missing mass and the momentum transfers held fixed will be double-Regge-pole exchange, shown in Fig. 2. The analog of Eq. (7) will now read

$$A_n(a + b \to c + d) \simeq s_1^{\alpha_1(t_1)} s_2^{\alpha_2(t_2)} g_n^{(1)}(M^2, t_1) g_n^{(2)}(M^2, t_2), \tag{15}$$

where all the notation is self-explanatory. From this and the relation

$$s = s_1 s_2 / M^2, \tag{16}$$

which holds in the kinematical region at issue, we can write the analog of Eq. (8):

$$f(s_1, s_2, M^2, t_1, t_2) \simeq s_1^{2\alpha_1(t_1)-1} s_2^{2\alpha_2(t_2)-1} g^{(1)}(M^2, t_1) g^{(2)}(M^2, t_2). \tag{17}$$

Finally, increasing M^2 so that we get into the scaling region, and combining Eq. (17) with the scaling hypothesis, we get

$$\frac{d^6\sigma}{(d^3q_1/q_{1,0})(d^3q_2/q_{2,0})} \underset{x_1\to 1;\, x_2 \to -1;\, s\to\infty}{\overset{t_1, t_2 \text{ fixed}}{\sim}} \left(\frac{s_1}{M^2}\right)^{2\alpha_1(t_1)-1} \left(\frac{s_2}{M^2}\right)^{2\alpha_2(t_2)-1} \beta_1(t_1)\beta_2(t_2)$$

$$= (1 - x_1)^{1-2\alpha_1(t_1)} (1 - |x_2|)^{1-2\alpha_2(t_2)} \beta_1(t_1)\beta_2(t_2). \tag{18}$$

Integrating this equation over the variables of one of the detected particles, it reduces to the expression for single-particle production [Eq. (9)] as it should.

Note added in proof. Two-particle inclusive experiments were treated here only in the double-Regge-exchange limit and the detected particles were therefore taken as independent. In general there should be another dynamical variable which measures the correlation between them. For a recent discussion see, e.g., D. Z. Freedman, C. E. Jones, F. E. Low, and J. E. Young, Phys. Rev. Letters 26, 1197 (1971).

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[3]A. H. Mueller, Phys. Rev. D 2, 2963 (1970).

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[5]It was shown by Mueller (Ref. 3) that in the projectile fragmentation region, the cross section is independent of the target. $\beta(t)$ therefore depends only on the projectile.

[6]In both cases one commits the crime of replacing a double limit by two successive limits. This, however, is justified by the experimental observation that the scaling region is reached at finite values of the relevant variables.

[7]The exclusion of diffraction dissociation stems from purely dynamical reasoning. Equation (9) will lead to a contribution to the total cross section of the form

$$\int_{t_{\min}} dt\ \frac{\beta(t)}{1-\alpha(t)}.$$

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